Dynamic Congestion Games: The Price of Seasonality

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We consider a dynamic congestion game, based on the model of Ford and Fulkerson of flows on networks. We are given a simple directed network where a source is linked to a destination by finitely many parallel edges. Each edge has a transit time and a capacity. Time is discrete and at each stage, finitely many players arrive at the source and choose a path to the destination. The number of arrivals per stage is throughout assumed to be periodic.

The framework that we will choose is an atomic version of the nonatomic setting of Koch and Skutella (2011).The games will be played on a finite graph with one source and one destination where each edge of the graph is characterized by a free flow time (this can be seen as the cost of traveling that edge when it is empty) and a capacity. The players will arrive at a constant rate and will choose which road to take to go from source to destination (with the possibility to update their decision once they reach intermediate nodes on the graph). Their aim will be to reach destination in the shortest possible time, given the choice of the other players. Players who choose an edge will exit it after the time it takes to travel it unless the number of players exceeds the capacity, in which case the players will wait at the end of the edge. The novel aspect of our approach is that, rather than just looking at the steady state flow, we will focus on the transient phase that leads to the steady state. This will be extremely important to determine the equilibrium costs. We will look first at the case where the constant flow exhausts the capacity of the network and we show that, similarly to the static case, in the worst equilibrium, once the steady state is reached, the cost along any possible chosen path will be the same. The initial transient phase will achieve this equalization. This will allow us to show that in steady state the worst equilibrium flow will coincide with the optimal flow. The two will differ only in the initial transient phase. Although the steady state flows in equilibrium and at the social optimum are the same, the costs that they engender differ. Based on this observation we will be able to compute the price of anarchy for this class of games. The case of inflow below the total capacity will be solved by suitably adapting the case at a capacity.

In real life inputs of routing networks are usually not steady, but there are often seasonalities, for instance on some routes traffic is higher during rush hours, on others it is higher during the weekend, etc. Therefore, once we have solved the case of a constant inflow, we look at the case of periodic inflow, where at single times the capacity constraint could be violated, but it is respected over the period, so that the system does not explode. The violation of the capacity at some periodic times will produce an additional delay and we will examine what this delay is at the equilibrium and at the optimum. Again we will compute the price of anarchy in this case. To the best of our knowledge, network routing games with periodic input have not been studied in the literature.