Signalling Games with Multiple Senders

Asha Sadanand

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Abstract

In this paper I explore information signalling when the same information is held by more than one informed party. Generally, having more informed parties increases the times when signals can be used to convey the information to uninformed parties and implement separating equilibria. Also of importance is the context of the game between the informed players, and whether the prefer to coordinate or not in signalling the information. I show that the traditional approach to forward induction does not give appropriate results. Instead I suggest that the approach taken in Manipulated Nash Equilibrium gives results that are intuitively superior.

Department of Economics

University of Guelph

Guelph, Canada

1. Introduction and Motivating Examples

The prisoners' dilemma is a simple symmetric game with a dominant strategy equilibrium. However, sometimes when this game is presented to audiences it can be (possibly unintentionally) understood as a clever device designed to induce the prisoners to tell the truth. Actually it is far from that.

A premise of the prisoners' dilemma story is that the prisoners are known to be guilty. Then we get them to play this game that has a dominant strategy Nash equilibrium and we choose to call the actions that constitute that equilibrium as "confess", "confess". And then we say Ah ha! They confessed.

Consider, though, what happens if they are actually not guilty, but we still get them to play the game. They choose to "confess" because after all it is still the (dominant strategy) Nash equilibrium of the game that we asked them to play. Ah, ha! we say, look they are "confessing", they must be guilty. But that would be faulty logic.[1]

Payoff Matrix for Prisoners' Dilemma

Prisoner 1

		Confess	*Don't*
Prisoner 1	*Confess*	-8, -8	-1, -15
	Don't	-15, -1	-10, -10

For the game to have anything to do with whether they are guilty or not, the payoffs would have to reflect something about whether they actually committed the crime. Let us say that the crime for which they were possibly guilty was robbing a bank, so if they were guilty, they would have a huge amount of money that they could split between themselves, and denote each player's share of it by K, say, if neither confessed. Now we have two payoff matrices: the one above if they were not guilty and the one below if they were guilty.[2]

[1] Asking the suspects to play the prisoners' dilemma game, when we do not know whether they are guilty, is really a entrapment mechanism rather than a valid device to elicit the truth.

[2] Rather than being a signalling game, this version of the prisoners dilemma is a screening (or mechanism design) problem where we are considering whether the prisoners' dilemma is an effective mechanism to elicit the truth, and we will see that in some cases it is actually the opposite.

Payoff Matrix for Prisoners' Dilemma
(prisoners are guilty)

Prisoner 1

Prisoner 1		*Confess*	*Don't*
	Confess	-8, -8	-1, -15
	Don't	-15, -1	-10+K, -10+K

Notice for this game a new Nash equilibrium is introduced when K is large enough which is "Don't", "Don't", and the original "Confess", Confess" equilibrium is no longer a dominant strategy Nash equilibrium. This means that we can have (at times) the awkward and contrary situation where when confronted with the prisoners' dilemma game, guilty ones don't confess and the innocent ones do.

This brings us to the general topic of when can information that is known to some informed players be conveyed to the uninformed player? Consider another example:[3]

Suppose there are two young ladies Sally and Jane, both seeking employment at Rayco Inc. We suppose that for each, their skill levels can be either high or low, and payoffs for each one and for Rayco, depending on whether the demanding job or the undemanding job is assigned, are given in the table below.

Payoffs for worker and Rayo depending on Skill and Job Type

Rayco

Sally or Jane		*Demanding*	*Undemanding*
	High	3, 3	0, 1
	Low	2, 0	1, 3

Notice that for Rayco a good match is important and gives a high payoff of 3, a bad match has a lower payoff, and is worse if the low ability worker is involved. By contrast, the workers prefer

[3] Adapted from Farrell and Rabin [1996].

the demanding job to the undemanding job (possibly because it pays more) and this is particularly pronounced for the high ability worker.

Suppose initially we simplify the game as between Rayco and to only one worker, who has private information about her type. For simplicity we suppose that each type is equally likely, and the worker can signal H or send no signal, which we denote by ϕ. The signals are costly, and we denote by ϵ the cost of signalling H truthfully, and α the cost of signaling false information, with $\epsilon < \alpha$. Then as long as the costs are not so high that all the gains are lost (that is $\epsilon < 3$ and $\alpha < 1$), there cannot be any signalling because both types prefer D and thus would want to send the signal H.[4]

Now returning to the two worker situation, suppose the nature of the information is that one of Sally and Jane is of high ability and the other is of low ability, again equally likely. The true information is known to both of the workers but not to the employer, who only knows the probabilities. We consider two cases: (i) Rayco has only one opening for each type of job, and (ii) job openings are plentiful and so Rayco can hire one or both or none in each kind of job.[5] The payoff matrix below gives Sally's Jane's and Rayco's payoffs depending on the information, either HL or LH, and Rayco's assignment of jobs. If we are in case (i) then we consider only the first two columns; otherwise all four columns are possible.

		Rayco			
		$D\ U$	$U\ D$	$U\ U$	$D\ D$
Sally, Jane	$H,\ L$	3, 1, 6	0, 2, 1	0, 1, 4	3, 2, 3
	$L,\ H$	2, 0, 1	1, 3, 6	1, 0, 4	2, 3, 3

We begin with the first case, where Rayco has only two openings, one of each type of job. If Rayco has no information it could do anything, always choose UD, always choose DU or always randomize, and get an expected payoff of 3.5. Now suppose we add to the game that before Rayco assigns any jobs, Sally and Jane are able simultaneously to signal that they are high (denote by H) or not signal (denote by ϕ). Recall, the signals are costly, and we denote by ϵ the cost of signalling the truth, and α the cost of signaling false information, $\epsilon < \alpha$. Upon receiving the signals HH,

[4] This is found in Farrell and Rabin [1996].

[5] We don't consider the case where jobs of both types are plentiful and the ability levels for Sally and Jane are independent. The reason is that in that case the resulting multi-sender game just uncouples into two separate single sender games.

$H\phi$, ϕH, or $\phi\phi$, then Rayco decides the job assignment.

If based on the observed signals Rayco concludes that Sally and Jane are HL, then the best assignment of jobs for Rayco is DU, if instead Rayco concludes LH the UD is best. If Rayco cannot figure out the information from the signals then Rayco's best response is any randomization.

Suppose Rayco interpreted a single worker signalling H to mean that that worker is of high skill, and if both signal, or none signal it cannot figure out the information. In the latter two cases Rayco's best response is to randomize; suppose Rayco randomizes with a probability of p_1 for DU if the signal is HH and a probability p_2 if the signal is $\phi\phi$. Then the first stage game facing Sally and Jane if the true information is HL (a similar one would arise for the other possible information, LH) is

Payoffs for Sally, Jane, and Rayco from signalling when the true state is HL

Jane

		H	ϕ
Sally	H	$3p_1 - \epsilon,\ 2 - p_1 - \alpha,\ 7/2$	$3 - \epsilon,\ 1,\ 6$
	ϕ	$0,\ 2 - \alpha,\ 1$	$3p_2,\ 2 - p_2,\ 7/2$

Here we see that if $\epsilon/3 < p_1$ and $p_2 < 1 - \epsilon/3$, then H is a dominant strategy for Sally, and if $1 - \alpha < p_1$ and $p_2 < \alpha$, then ϕ is a dominant strategy for Jane resulting is one dominant strategy Nash equilibrium where the truth is signalled, and Rayco assigns the workers to the correct jobs. It can be seen that when the state is LH the same conditions also result in the truth being a dominant strategy Nash equilibrium. Thus if we denote by K the $\max\{1 - \alpha, \epsilon/3\}$ and by k the $\min\{\alpha, 1 - \epsilon/3\}$ then if Rayco chooses $p_1 > K$ and $p_2 < k$ there is a dominant strategy Nash equilibrium where the workers signal the truth and Rayco makes the correct assignment. Such probabilities exist as long a s $\alpha > 0$ and $\epsilon < 3$.

If instead Rayco believed that all signals were meaningless, and so always randomized with probability weights p for UD and $1 - p$ for DU, then we would have pooling equilibria where none of the workers would find it worthwhile to signal. Starting with the pooling outcome, suppose Rayco observed one of the workers, say Sally, choosing H. If upon signalling Rayco assigned the signalling player U for sure, the signalling player would be worse off since they incur the cost of the signal, in addition to having the worse job for sure. So, it must be that a player is signalling because they hope to convince Rayco to give them D. We use the Cho-Kreps intuitive criterion. If Sally uses the signal H it certainly can be that the information is HL since Sally's payoff increases from $3p$ to $3-\epsilon$ (for p small enough). But instead if the true state is LH, but Rayco still assigns DU because of Sally's signal, this results in Sally's payoff decreasing from $2-p$ to $2-\alpha$ (for p small enough). Thus the pooling equilibria break when $p < \min\{\alpha, 1-\epsilon/3\}$ using the intuitive criterion; otherwise either the wrong player type wants to signal, or the right player type doesn't want to signal, or both and so the pooling equilibrium survives the intuitive criterion.

In other words if the cost of signalling the truth is very low ($\epsilon \approx 0$) and if the cost of misrepresenting the truth is sufficiently large ($\alpha \approx 1$ or $\alpha > 1$) pooling equilibria break down under the intuitive criterion. But pooling equilibria outside that range survive the intuitive criterion.

Now we consider the case where Rayco has plenty of jobs of either kind, then with no information Rayco would want to assign both to the undemanding job.[6] Suppose again Rayco interpreted a single worker signalling H to mean that that worker is of high skill, and if both signal, or none signal it cannot figure out the information. Then the first stage game facing Sally and Jane is

Payoffs for Sally, Jane, and Rayco from signalling when the true state is HL

		Jane: H	Jane: ϕ
Sally	H	$-\epsilon,\ 1-\alpha,\ 4$	$3-\epsilon,\ 1,\ 6$
	ϕ	$0,\ 2-\alpha,\ 1$	$0,\ 1,\ 4$

Now we notice that whenever $\epsilon < 3$ and $\alpha < 1$ there are two Nash equilibria: one where the true state is signalled, and one where the wrong state is signalled. If the workers were choosing the equilibrium with the wrong state being signalled, if this was part of any equilibrium Rayco would expect it. In that case since it cannot distinguish between right and wrong signals Rayco

[6] We note that this game does not decouple into two separate games, one between Sally and Rayco and one between Jane and Rayco because the informed players in each case have the same information.

would never offer DU or UD, and would instead choose the pooling offers of UU. Thus, in any separating equilibrium the workers only choose to signal the true state. However, we do not have the dominant strategy solution in the signaling portion of the game.

With the same parameter values, in the pooling equilibrium, Rayco believes that all signals are meaningless and so always offers only UU. Correspondingly the workers never signal anything since it is costly. But suppose that in this pooling environment Rayco observes a worker, say Sally, signalling H. As before we conclude that the signaller must be hoping that Rayco will assign D.

If Rayco did this, then Sally's payoff goes up to $3 - \epsilon$ if the true information is HL and Sally gains $2 - \epsilon$. If instead the state is LH Sally gains $1 - \alpha$ from the new assignment. Then for $\alpha < 1$ Cho-Kreps fails to break the pooling solution. Thus, here we find that both pooling and separating equilibria exist in this case.

In the rest of this paper I will discuss some more examples that provide further insights into how increasing the number of informed players affects information transmission, and how it is influenced by how their interests are aligned. Based on the examples I will provide some general results on signalling with several informed parties.

2. Some Related Literature

We are interested in the simplest form of a multi-player signalling game where at the outset Nature determines the state of the world. In the first stage there are number of informed agents (know the state of the world) who each send a signal. These signals are received by a single uninformed player who then takes a move and the game ends.

There are number of commonly used refinements for signalling games such as the Cho-Kreps Intuitive Criterion, or the Banks and Sobel Divine equilibrium, D1 and D2, or the Grossman and Perry Perfect Sequential Equilibrium, and so on. The main practical issue with extending existing refinements is that with multiple informed players there are multiple signals being received by the uninformed party and these refinement concepts are not designed to deal with multiple signals about the same information. A few instances of multiple informed agents have been studied in the applied literature. There when multiple sender situations are encountered, generally some additional assumptions are made such as out of equilibrium messages should be interpreted to be a single deviation by one informed player rather than multiple deviations by various informed players, or putting some restrictions on beliefs.[7]

However, if we were to develop a general theory along the lines of existing refinements, we would first need to establish some ideas and terminology that cast the multiplayer signalling game closer to the single player signalling game. The starting point for each refinements is always in reference to a particular equilibrium. We will call the entire vector of signals sent by the informed players as the message, while the individual choices for the informed players will be referred to as signals. If some informed player(s) deviate(s) from their equilibrium signal(s), it becomes a different message. And then we can ask whether, if the uninformed party takes its best response (based on its beliefs about which states this could possibly happen – and this is where there is some variation across the different refinements), can the deviating player(s) be better off as compared to the original equilibrium. However, since the deviation (often away from a pooling outcome) is an attempt to signal a particular state (or subset of states) that is known to all the informed players, the other informed players may also want to change their signals.

So we might then ask does there exist a new message that agrees with the observed deviation(s), but also allows the other informed players to change their signals as consistent with the new state(s) being signalled. Moreover, these new signals and the corresponding best response of the informed

[7] See Hertzendorf and Overgaard [2001] for an example.

player should not make the players with the new signals any worse off, and the whole thing should make the initial deviator(s) strictly better off. Finally, the off equilibrium message (the new signals together with the deviations(s)) should have the potential to be part of a Nash equilibrium. If all this is true then the uninformed party should put some non-zero weight on this state given the off-equilibrium deviation signal(s) it has just received (as prescribed by the specific refinement being extended).

This is a very complex process for which we have managed to give an intuitive outline but would require even more complex structure to operationalize precisely. Moreover we notice that all of these refinements require a number of different sets of beliefs for each information set. First of all there are the equilibrium beliefs for the player that is on the move at that information set. In addition if a sender were to deviate and actually reach an off-equilibrium information set, the main effort of the refinements literature has gone into redefining an additional set of sensible beliefs for that same information set which should hold if we actually saw this deviation, relative to the initial equilibrium.

In many games even after refinement there is a multiplicity of equilibrium beliefs, some with better payoff outcomes and some worse for the player whose beliefs are concerned. Conventional game theory is agnostic about how to select among these numerous beliefs, but the theory is firm about dispelling the notion that beliefs are a matter of choice (or even strategic choice) for the player concerned.[8]

Thus we see that an unreached (or off-equilibrium) information set has the complicated situation of having several beliefs associated with it; which beliefs are adopted can be pivotal to the outcome; and players are not thought to be strategic about their beliefs. Occam's razor leads us to think that a different theory with some parsimony in the proliferation of beliefs may be better.

In the alternative approach that we will develop here, we take the opposite stand by not utilizing *any* beliefs at all for the off-equilibrium information sets, neither equilibrium beliefs nor counterfactual beliefs associated with hypothetical deviations. While this flies in the face of all the literature developed since sequential equilibrium (Kreps and Wilson [1982]) it is still worth considering because of its simplicity. If we were to query real people about all their various beliefs as specified in the literature, they would be hard pressed to even come up with their equilibrium beliefs on off equilibrium information sets, let alone ones for counterfactual deviations of other

[8] Conflicts with the adage that if you believe strongly enough in something, it will happen.

players.

The idea is based on a thought process equilibrium concept called Manipulated Nash Equilibrium (Amershi, Sadanand and Sadanand [1982,1992]), its development was roughly contemporaneous with Kreps and wilson sequential equilibrium, and which was later termed Virtual Observability (Weber, Camerer, Knez [2004]). With this approach equilibrium is arrived upon by a sequence of thought processes on the part of the players, that when it converges, is the Manipulated Nash Equilibrium.

In Amershi, Sadanand, and Sadanand, we develop an algorithm that allows us to find the eventual equilibrium that is reached from the thought process dynamics. The algorithm quite simply involves progressively "erasing" information sets. A first order manipulation by a player is where an immediate follower can observe what has occurred in the information set where they are moving. (They still need to make only one choice at the information set, but they can know which nodes(s) they are at.) A second order manipulation is where two player ahead can have full information, and so on. A Manipulated Nash Equilibrium is an outcome that is immune to all orders of manipulation by any player. The algorithm checks for the putative equilibrium to be a Nash equilibrium in all the games where varying numbers of information sets have been "erased" (from zero to the maximum) for any and all strategic players.[9] Beliefs never come into play directly, because in each instance we only seek Nash equilibrium, however, as information sets are "erased" players definitely keep in mind that they need to anticipate earlier players' thought processes. So as such then, the Manipulated Nash equilibrium only specifies actions (and in certain cases beliefs) along the equilibrium path, but nothing is specified for off equilibrium regions of the game. In other words the focus is only on the the equilibrium outcome, not on off equilibrium choices and beliefs, or on any notions of what players say they would do if some non-equilibrium actions occurred, and how they revised their beliefs after an unexpected signal is observed.

Applying the Manipulated Nash approach to our signalling games is straightforward. In addition to Nature's move here are only two stages to this game: First all the informed players move together by each choosing a signal, then the uninformed party moves. Nature is obviously not a strategic player, and so is excluded from any thought process dynamics. "Erasing" information sets means that the responder can know what the true information is, but can take only one action at each information set.

[9] Nature is not thought to be a strategic player.

For completeness we mention another strand of literature that deals with multi dimensional signalling spaces in conjunction with multiple senders in the context of cheap talk. The seminal paper here is Battaglini [2002] where he cleverly shows that as long as the receiver's ideal point does not lie on the contract curve between the two sender's ideal points, direct mechanisms where senders are just asked to report will be fully revealing. Essentially the receiver is able to find two directions to form a new basis for the information space such that each element is orthogonal to one of the sender's level sets, and that sender's information is used on along that direction. Thus although each sender reports not eh entire multidimensional information, his report is only used in determining one component of the newly formed basis of the information space. This is an important contribution because it appears that the one dimensional information spaces are the exception in that it may be impossible to have fully revealing reports it hat setting. But whenever there are multiple dimensions, and as long as there are enough senders as compared to the dimensions, and as long a stye receiver's bliss point is not on the contract curve, fully revealing is completely possible. Another paper addressing similar questions are Wolinski [2002].

Finally there is another literature on experimental results of players working in teams in signalling games (an example is Cooper and Kagel [2005]) where they find that working in teams allows better learning and thinking about forward induction.

3. Analysis of Some Multi-Sender Games

We can apply this to our game with Sally, Jane and Rayco. Recall the version where Rayco had one job of each kind. Starting from a polling outcome, if there is a first order Manipulation by the informed player of high ability, to signal their ability this will be known to Rayco when the information set is "erased" and the demanding job will be offered; however if a low ability player attempts such a manipulation it will not be followed with the demanding job. Thus only the separating equilibrium survives and is the Manipulated Nash Equilibrium; all pooling equilibria are eliminated (all values of p).

Now recall that the version where Rayco had plenty of jobs of each kind, we were left with both the pooling equilibrium where Rayco assigned both young ladies the undemanding job, and the separating one where the high ability type worker would signal their ability and be consequently assigned the demanding job, while the other received the undemanding job. A very similar argument leaves us with just the separating outcome. If we start with the pooling outcome, we can see that the high ability worker can make a first order manipulation where they deviate to the signal H, the uninformed party would know it and assign the demanding job. The low ability worker could never manage this because when the information set is "erased" the responder will see that the ability is low. Thus the pooling outcome is not a Manipulated Nash equilibrium; it is easily established that the unique Manipulated Nash equilibrium is the separating outcome.

We introduce a few more abstract examples (they do not have a story) which emphasize that in the multiple informed player setting it is not enough to ask (as the existing refinements do) whether a single informed player would benefit from deviation if the true state was recognized by the deviation. The reason is that although a deviating player may benefit, to think that it would just end with the single deviation may be unreasonable given the incentives between the informed players.

Consider the following example where two informed players can each be either in state A or state B, and they both observe each others' state. There are fours states: AA, AB, BA, and BB. The signals they send each are similarly, a or b, resulting in four possible messages for the uninformed player: aa, ab, ba, and bb. The figure below give that payoffs for all the players based on the state, the signals and responders action.

Example 1

State		X	Y	X	Y	X	Y
AA	0, 0, 15	-5, -5, 10	-1, -1, 0	-5, -5, 10	-1, -1, 0	-5, -5, 10	-1, -1, 0
AB	0, 0, 15	-5, -5, 0	8, 10, 6	-5, -5, 10	7, 7, 4	-5, -5,10	7, 7, 4
BA	0, 0, 15	-5, -5, 10	7, 7, 4	-5, -5, 0	10, 8, 6	-5, -5, 10	7, 7, 4
BB	0, 0, 15	-5, -5, 10	7, 7, 3	-5, -5, 10	7, 7, 3	-5, -5, 0	10, 10, 6
	aa	ab		ba		bb	

Here we see that there is a pooling equilibrium where both informed players always signal a and the responder plans to choose action X if any other signal is received. There is also a separating equilibrium where every one signals the truth and the responder always chooses action Y. Starting with the pooling equilibrium, if we consider manipulations by the senders we see that they can immediately move to the separating outcome and it improves their payoffs. So it would be a successful manipulation from the pooling equilibrium. However, if we instead consider the usual approach of deviations using beliefs, a single deviator cannot achieve this deviation since it would be met with the responder choosing X, as it is the best response to a single deviation. To see this, suppose it was player 2 that deviates to truthful signals. Then the responder would receive the message aa when the true states are either AA or BA, and would receive ab when the true states are either AB or BB. Only the latter would be an out of equilibrium message, and the responder then could determine that if he chose X his expected payoff will be $\frac{0+10}{2} = 5$ but if he chose Y his expected payoff would be $\frac{6+3}{2} = 4.5$ which is lower. So we would need both senders to deviate at once to upset the pooling equilibrium.

Consider the following modification of the previous game.

Example 2

State		X	Y	X	Y	X	Y
AA	0, 0, 15	-5, -5, 10	0, 0, 0	-5, -5, 10	0, 0, 0	-5, -5, 10	0, 0, 0
AB	0, 0, 15	-5, -5, 0	8, 10, 6	-5, -5, 10	9, 10, 4	-5, -5, 10	9, 7, 4
BA	0, 0, 15	-5, -5, 10	10, 9, 4	-5, -5, 0	10, 8, 6	-5, -5, 10	7, 9, 4
BB	0, 0, 15	-5, -5, 10	10, 0, 3	-5, -5, 10	0, 10, 3	-5, -5, 0	11, 11, 6
	aa	ab		ba		bb	

Here we have a pooling equilibrium where all senders give the message a and also a semi-separating equilibrium where all send the signal a, except when the true state is BB and then they all send the signal b. Consider deviations from the latter equilibrium from a to b by player 2, say, when a is his choice in the semi-separating equilibrium. Then the true state could either be AB or BA. If we are using a regular refinement such as the Intuitive Criterion, we ask which of the types of player 2 would make this deviation if the responder knew it was them. That would be both types and then the best response would be Y. So then this equilibrium would fail the Intuitive Criterion.

However, notice that this deviation is not really stable because if the true state were AB player 1 would want to switch his signal to b also. So the only reasonable state where such a deviation by Player 2 could work is BA. But in that state the responder prefers X to Y, so then player 2 would not deviate in the first place. Thus the semi separating equilibrium cannot be broken and is a Manipulated Nash Equilibrium.

The pooling outcome can easily been seem to not be stable against manipulations by the informed players towards the semi-separating outcome. And again using conventional refinements a deviation by a single player cannot upset the pooling outcome. Thus the only Manipulated Nash equilibrium is the semi-separating outcome.

Now consider a third version of this game. It too has the pooling equilibrium at where all senders always send a (and the responder always X). But there is another equilibrium where type AA signal aa and the rest send bb. Again we start at the usual pooling outcome and see that selecting this latter outcome is outside the commonly used refinements because it requires all three types (AA, AB, and BB) to deviate at once.

Example 3

State		X	Y	X	Y	X	Y
AA	0, 0, 15	-5, -5, 10	0, 0, 0	-5, -5, 10	0, 0, 0	-5, -5, 10	0, 0, 0
AB	0, 0, 15	-5, -5, 0	-5, 10, 8	-5, -5, 10	7, 7, 12	-5, -5, 10	7, 7, 7
BA	0, 0, 15	-5, -5, 10	7, 7, 12	-5, -5, 0	10, -5, 8	-5, -5, 10	7, 7, 7
BB	0, 0, 15	-5, -5, 10	7, 7, 1	-5, -5, 10	7, 7, 1	-5, -5, 0	10, 10, 7
	aa	ab		ba		bb	

Now consider an example where we see that some pooling and some separating equilibria survive. Here in example 4 suppose the the two possible information states are equally likely, and this prior distribution is of course known by the sender. Notice that there are two pooling equilibria where all senders pool at aa and the receiver always chooses X, and where all senders pool at bb and the recover always chooses X. There are also the separating equilibria where senders choose aa if the information is A and if it is B they choose bb, or the reverse where senders choose aa if the information is B and if it is A they choose bb,and in each case the receiver still chooses X always. If we wanted to see if any of these are immune to forward induction, we might start with the intuitive criterion. If we look at the pooling equilibrium at bb, and suppose the recover notices a deviation to ab, the it must have been player 1 that deviated. Checking for equilibrium dominance 1A's equilibrium payoff is 1, by deviating he can get 0 or -5, both are lower so it cannot be 1A that is deviating. Now looking at 1B, the equilibrium payoff is 0, by deviating she gets 1 or -1, so in this case there is no equilibrium dominance. So, 1B might deviate. If the receiver thought it was 1B, then they know that the information state is B and then the best response would be Y, which is indeed player 1B's preferred response whereby she gets more than in the equilibrium under consideration, so by the usual interpretation of the intuitive criterion this deviation would be a success.

But when there are multiple senders, it may not be reasonable to simply stop here. We also need to consider what the reaction of the other sender(s) would be before we ought to declare the deviation as a success. Indeed here Player 2B upon seeing this deviation (or in truth upon anticipating this deviation) would deviate to a, so that the observed message would be aa. Here Player 2 is an unwilling but helpless participant to the deviation. Player 2 far preferred the pooling equilibrium at bb, but that equilibrium simply breaks down with forward induction. Notice that the polling at aa does not break down with a similar move by player 2. Separating equilibria, of course, never breakdown with forward induction.

Example 4

State	X	Y	X	Y	X	Y	X	Y
A	3, 0, 2	-5, -5, 0	-5, -5, 6	0, 0, 1	-1, -1, 2	0, 1, 6	1, 2, 2	-5, -5, 3
B	2, 1, 2	-5, -5, 3	-1, -1, 2	1, 0, 6	-5, -5, 6	0, 0, 1	0, 3, 2	-5, -5, 0
	aa		ab		ba		bb	

And finally in example 5 we see a different story. Players are able to deviate successfully according to the regular intuitive criterion, but when we think about what the other senders will do afterwards, it becomes clear that the deviator would not deviate in the first place. Here again we have all the previous equilibria. Starting with pooling at aa notice that if there was a deviation to ba The receiver may logically conclude that it must have been player 1B and so be prepared to respond with Y, but of course Player 2B can figure all this out and seeing that then he would be getting uniformly -5, would further deviate to bb with a response of the receiver of X. But this is really counter productive for Player 1B who is now left with a payoff of 0 when previously he had 2, and so it is reasonable to think that player 1B would not even start this sequence of events, although the regular intuitive criterion would predict that the deviation would occur.

Example 5

State	X	Y	X	Y	X	Y	X	Y
A	3, 0, 2	-5, -5, 0	-1, -1, 2	1, 0, 6	-5, -5, 6	2, -5, 1	1, 2, 2	-5, -5, 0
B	2, 1, 2	-5, -5, 3	-5, -5, 6	2, 0, 1	-5, -5, 2	5, -5, 6	0, 3, 2	-5, -5, 0
	aa		ab		ba		bb	

4. Conclusion

These examples serve to show that the conventional refinements (such as Cho-Kreps Intuitive Criterion, which was focus in this paper because we were mostly considering only two information states, but also Banks and Sobel Devine equilibrium, and Grossman and Perry Perfect Sequential Equilibrium) that allow single player deviations, fall short in capturing all of the necessary aspects required when considering signalling games with multiple informed agents. We instead suggest that the concept of Manipulated Nash equilibrium, which allows deviations by multiple players at a time, and allows the players to go though the thought process of the other players in analyzing the game, better captures all the incentives in a game with several informed players, and it avoids the ad hoc process of assigning beliefs at off-equilibrium information sets, and further the need to redefine beliefs for counterfactual situations where players actually reach off-equilibrium information sets.

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