Deliberation, Leadership and Information Aggregation*

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Abstract

We analyze committees of voters who take a decision between two options as a two-stage process. In a discussion stage, voters share non verifiable information about a private signal concerning what is the best option. In a voting stage, votes are cast and one of the options is implemented. We introduce the possibility of leadership whereby a certain voter, the leader, is more influential than the rest at the discussion stage even though she is not better informed. We study information transmission and find, amongst others, a non-monotonic relation between how influential the leader is and how truthful voters are at discussion stage.

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1 Introduction

Committees are a common framework for making social choices. In a committee, its members discuss their views on the issue at stake and then choose among the different options, usually via voting. For example, a company's chief executives meet often to decide the firm's future strategy. Ministers of a cabinet meet regularly to choose the policies the government should follow. Faculty members in a university meet during staff meetings to agree on new appointments, course programs, etc.

A frequent feature of committees is that the opinions of some of its members, leaders, are taken as more relevant by its other members. This can happen even though leaders may not necessarily be better informed about the issue at hand than the other committee members. The leaders' opinions can be persuasive to other committee members for a variety of reasons. It may be that leaders are more effective at communicating their views or that some committee members believe the leaders to be better informed. Alternatively, some committee members may want to favor the leaders' views expecting something in return.

Another important feature of committees is that some of its members may be biased in their opinions and preferences for the different options available. The existence of a bias implies that a committee member may prefer a certain option even though such option need not be the best one from the committees' point of view. As an example, consider a faculty member at a university that when hiring new recruits favors the candidates that work on her same field.

The objective of this paper is to investigate the implications of leadership and preference biases on information aggregation and choices in deliberative committees. For such end, we consider a common value election setting where committee members, voters, have to decide between two options: whether to change to a certain alternative or to maintain the status quo. Voters receive a piece of private information (signal) about whether the state of nature is the alternative or the status quo. Voters then meet at a discussion stage and talk about the two options. During the discussion stage voters express their support for either of the options by reporting, truthfully or not, the value of their private signals. Afterwards, during a voting stage each voter simultaneously casts a ballot for one of the options and the alternative beats the status quo if and only if it receives at least a given number of votes.

Within the committee, there are three types of voters. First, there is the *leader*, who is characterized by two factors: her bias and her influence on other voters. The leader is biased in that she will vote against the status quo if and only if the alternative receives significantly more support at the discussion stage. The leader's influence on other voters manifests via the second type of voters: the *followers*. Followers regard the leader's views as the truth and, thus, vote for the option that is supported by the leader during the discussion stage. The final type of voters is the *objective voters*, who want the option that matches with the state of nature to be implemented.

In our results we focus first on the information revealed during the discussion stage. In particular, we are interested in understanding whether voters have incentives to truthfully reveal their signals during the discussion stage. In order to analyze voters' incentives in the deliberation stage we focus on the situations where voters' communications are pivotal in the sense that their messages in the discussion stage affect the voting decision of other voters and the voting outcome. Although each voter only observes the signal she receives, since the message a voter sends at the discussion stage is only relevant whenever she is pivotal then such voter may infer information about the signals received by the other voters precisely from the fact of being pivotal.

On top of studying the information revealed during the discussion stage we are also interested in knowing whether and how the existence of a leader affects the likelihood of implementing the option that matches with the state of nature. This is motivated by the interpretation of the leader and her followers as elements that distort the social decision process where objective voters are the only voters who have an interest in implementing the "right" option.

As a benchmark case we consider first situations where the leader is not biased. This means that she prefers whichever option receives more support during the discussion stage. In this case, there exists an equilibrium where all voters share their private signals during the discussion stage if and only if either there are only a few followers or sufficiently many followers. There being only a few followers means that followers are not able to block the selection of a certain option whenever a non-majority voting rule is in place. On the other hand, sufficiently many followers means that followers are enough as to force the implementation of any option. Once we move to the case where the leader is biased we find that an equilibrium where voters truthfully reveal their signals exist if and only if there are sufficiently many followers and the accuracy of the signal is high enough.

Our results characterize how the existence of a leader and her followers affect information revelation. Surprisingly, it is not true that more followers make truthful sharing of information more unlikely. As a matter of fact, if the leader is biased, having sufficiently many followers is a requirement for her to truthfully communicate during the discussion stage. Moreover, we also find that ceteris paribus the majority rule is the best voting rule in that it makes truthful sharing of information more likely.

Our contribution to the literature lies in the inclusion of leadership in decision committees.

To our knowledge, ours is the first attempt at modelling this phenomenon that is much present in real life situations. Before proceeding with the formal analysis we review the related literature. The Marquis de Condorcet (1785) made one of the strongest endorsement of elections (ruled by majority) as the optimal way to make social choices for common value problems. If a group of voters independently evaluate the convenience of two alternatives (candidates, policies) and each voter vote for the alternative she considers superior, then the alternative that is actually superior is more likely to receive a majority of votes (see Young (1988)). Following on the work of Austen-Smith and Banks (1996), there is a growing literature that extends this result to frameworks where voters may incur in strategic behavior and abstaining, see for instance Feddersen and Pesendorfer (1996), Feddersen and Pesendorfer (1997) and Feddersen and Pesendorfer (1998); McLennan (1998).

The papers that are more closely related to our work are those of Coughlan (2000) and Austen-Smith and Feddersen (2006). These authors extend the Condorcet Jury framework by introducing a preliminary stage where voters can share information previous to voting. Information transmitted in the deliberation stage is not verifiable and thus, the deliberation stage is regarded as a cheap-talk game.[1] Coughlan (2000) analyzes communication prior voting in a framework similar to ours and proves that sincere revelation of signals is obtained when voters are similar enough, independently of the rule used in the voting stage. We built on the model by Austen-Smith and Feddersen (2006) who assume that voters' preferences depend directly on the profile of signals received by the voters.

Finally, we should mention Gerardi and Yariv (2007) and Jackson and Tan (2011). Gerardi and Yariv (2007) show in a mechanism design environment the equivalence of the set of equilibria for different voting rule (different than unanimity) under cheap talk when voters may use dominated strategies in the voting stage. Jackson and Tan (2011) model deliberation as the transmission of verifiable information prior to voting. The strategic issues arise because whether a voter receives a signal or not is unobservable. In this case, voters may have incentives to pretend that she didn't receive info when they receive signals that do not fit their a priori preferences.

The rest of the paper is organized as follows. In Section 2 we present the model and the notation. In Section 3, we deal with the framework where there is a unique leader that is unbiased. In Section 4 we investigate a situation where the leader has a bias. In Section 5 we analyze the welfare implications of the model. In Section 6 we extend the analysis by contemplating a situation where there are two leaders. Finally, in Section 7 we conclude.

[1] There is also a relevant literature that analyzes argumentation and debates as information aggregation methods in different settings. See, for instance, Glazer and Rubinstein (2001); Spiegler (2006), and references therein.

2 Model

Consider a committee formed by $N+1$ voters where $N \geq 2$ is an even number. Voters have to decide whether to implement an alternative A or to keep the status quo Q.

There are two states of nature, $S = \{A, Q\}$, and each voter receives a private signal about the state. The private signal received by voter i is given by $\theta_i \in \{A, Q\}$ where

$$P(\theta_i = A | S = A) = P(\theta_i = Q | S = Q) = p$$

with $p \in \left[\frac{1}{2}, 1\right]$. That is, p is the accuracy of the signal and is independent of the identity of the voter. Let $\theta = (\theta_1, \ldots, \theta_{N+1}) \in \{A, Q\}^{N+1}$ denote a profile of observed signals. For each voter i let $\theta_{-i} \in \{A, Q\}^N$ be the complementary profile of signals observed by voters other than i. Define Θ_A and Θ_Q as the set of voters who receive signal A and Q respectively. Finally, let $\Theta_A \setminus i \equiv \{j \neq i \mid \theta_j = A\}$ be the set of voters excluding i who receive signal A and let $\Theta_Q \setminus i \equiv \{j \neq i \mid \theta_j = Q\}$ be the set of voters excluding i who receive signal Q.

After each player receives a signal, a discussion stage takes place. This discussion stage has the form of a non-binding straw poll where each voter reveals a message $m_i \in \{A, Q\}$ to all other voters. Let $m \in \{A, Q\}^{N+1}$ denote a profile of reported messages. For each voter i let $m_{-i} \in \{A, Q\}^N$ be the complementary profile of messages reported by voters other than i. Define M_A and M_Q as the set of voters who report signal A and Q respectively. Finally, let $M_A \setminus i \equiv \{j \neq i \mid m_j = A\}$ denote the set of voters excluding i who reveal message A and let $M_Q \setminus i \equiv \{j \neq i \mid m_j = Q\}$ be the set of voters excluding i who reveal message Q.

Once the discussion stage is over voters casts their vote during the voting stage. Let $v_i \in \{A, Q\}$ be the alternative chosen by voter i and define V_A and V_Q as the set of voters who choose option A and Q respectively. The alternative A is implemented if and only if it receives at least $q \in \left\{\frac{N}{2}+1, \ldots, N+1\right\}$ votes. Note that if $q = \frac{N}{2}+1$ then the majority rule is in place whilst if $q = N+1$ then the voting rule is the unanimity rule.

Voters' preferences over the option implemented are not homogeneous. In particular, voters can be of three types: voter i is either a *leader*, $i \in L$, she is a *follower*, $i \in F$, or she is an *objective* voter $i \in O$. Initially, we assume that the set L is a singleton so there is a unique leader l, i.e. $\{l\} = L$. The preferences of each voter can be represented by a utility function $u : \{L, F, O\} \times \{A, Q\} \times \{A, Q\}^{N+1} \times \{A, Q\}^{N+1} \to \{A, Q\}$ where the first argument is the type of the voter, the second argument is the option implemented, the third argument is the profile of signals and the fourth argument is the profile of messages. We assume voters are expected utility maximizers.

The leader is characterized by the fact that his message at the discussion stage has a great influence in some voters. Moreover, the leader may have a biased against either option in that she prefers the alternative A if and only if there is, in her opinion, sufficient evidence in favour of A. We model this as the leader requiring at least $b \in \{0, \ldots, N+1\}$ signals in favour of A in order to prefer the alternative A during the voting stage. Given that $p \geq \frac{1}{2}$, we say that the leader is *unbiased* if $b = \frac{N}{2} + 1$ as in this case he prefers the option that is more likely to match with the state of nature given the signals received by all players. We say that the leaser is *biased* if $b \neq \frac{N}{2} + 1$.[2] We represent the preferences of the leader by the following utility function:

$$u(L, A, \theta, m) = 1 - u(L, Q, \theta, m) \quad = \quad \begin{cases} 1 & \text{if } \#\Theta_A \geq b, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

The preferences of the followers depend completely on the message revealed by the leader. Followers simply want the option revealed by the leader to win the election. Their preferences can be represented by the following utility function:

$$u(F, A, \theta, m) = 1 - u(F, Q, \theta, m) \quad = \quad \begin{cases} 1 & m_l = A, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The final type of voters are the objective voters, who want to chose the alternative that matches with the state of nature. Thus, given that the accuracy of the signals is greater or equal to $\frac{1}{2}$ objective voters prefer the alternative A to the status quo Q if and only if there are at least $\frac{N}{2} + 1$ voters who received signal A. Objective voters' preferences can be represented by the following utility function:

$$u(O, A, \theta, m) = 1 - u(O, Q, \theta) \quad = \quad \begin{cases} 1 & \text{if } \#\Theta_A \geq \frac{N}{2} + 1, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

The messages sent during the discussion stage are given by the message strategy which is mapping from the voter's type and the signal she receives to the set of options, $\sigma^m : \{L, F, O\} \times \{A, Q\} \to \{A, Q\}$. The realization of σ^m for a certain player i is given by m_i. How voters choose between the two options during the voting stage is determined by the voting strategy. This is a map from the voters' type, the signal she receives and the set of message profiles of all voters to the set of available options, $\sigma^v : \{L, F, O\} \times \{A, Q\} \times \{A, Q\}^{N+1} \to \{A, Q\}$. The realization of σ^v for a certain player i is given by v_i.

[2] Although we are assuming ad-hoc that the leader could be biased, there are different arguments that could justify the existence of such bias from the optimality, and Bayesian, point of view. For instance, Coughlan (2000) considers a framework where the voters want to make the "right" choice but the cost of not implementing the right choice is different depending on the state of nature. Alternatively, in Jackson and Tan (2011) voters' preferences have a private component whereby a voter may require more (or less) than half of the signals to prefer the alternative to the status quo.

It is assumed that all that is relevant to the game presented is common knowledge except for the realized value of the private signals: every voter only observes its own signal. Once voters receive their signals and before they reveal their messages they update their beliefs on the signals received by other voters using Bayesian updating.

We are interested in studying the circumstances under which there is full information transmission. We say that voter i in group $G \in \{L, F, O\}$ truthfully reveals her signal if $\sigma^m(G, \theta_i) = \theta_i$.

Given the utility function of each voter, if all voters truthfully reveal their signals then the unique weakly dominant voting strategies for any voter i are given by:

$$\sigma^v(L, \theta_i, m) = \begin{cases} A & \text{if } \#M_A \geq b, \\ Q & \text{otherwise.} \end{cases} \tag{4}$$

$$\sigma^v(F, \theta_i, m) = \begin{cases} A & \text{if } m_l = A, \\ Q & \text{otherwise.} \end{cases} \tag{5}$$

$$\sigma^v(O, \theta_i, m) = \begin{cases} A & \text{if } \#M_A \geq \frac{N}{2} + 1, \\ Q & \text{otherwise.} \end{cases} \tag{6}$$

Define $\mathbf{v} : \{A, Q\}^{N+1} \to \{A, Q\}$ as the option that is implemented given a profile of messages in $\{Q, A\}^{N+1}$ when the voting strategies are given in (4), (5) and (6). In an abuse of notation, we omit the number of voters of each type in the description of $\mathbf{v}$.

Definition 1. *The message strategy σ_i^m is a truth-telling best response for voter i of type G if $\sigma^m(G, \theta_i) = \theta_i$ and for any message strategy $\bar{\sigma}^m$*

$$\mathbb{E}\left[u(G, \mathbf{v}(\theta), \theta, \theta) \mid \theta_i\right] \geq \mathbb{E}\left[u(G, \mathbf{v}(\theta_{-i}, \bar{\sigma}^m(G, \theta_i)), \theta, (\theta_{-i}, \bar{\sigma}^m(G, \theta_i))) \mid \theta_i\right],$$

where $\mathbb{E}$ denotes the expected value operator and $m_i = \sigma^m(G, \theta_i) = \theta_i$.

Now we are in condition to introduce the equilibrium concept we are interested in:

Definition 2. *A profile of message strategies $(\sigma_1^m, \ldots, \sigma_{N+1}^m)$ is a fully revealing (Bayesian Nash) equilibrium (FRE) if and only if for each voter i the message strategy σ_i^m is a truth-telling best response.*

A FRE is a sequential (and perfect) Bayesian Nash equilibrium of the dynamic two stage incomplete information game where voters report their true signals and vote accordingly in the voting stage. Therefore, in a FRE each voter truthfully reveals her signal and no voter has incentives to change neither his message during the discussion stage nor his vote during the voting stage.

Given that when all voters truthfully report their signals there is a unique optimal voting strategy, it is sufficient to look at individual incentives to report truthfully during the discussion stage. To this end, when checking whether reporting truthfully is a best response strategy for a voter we only need to consider the situations where her report may influence the final outcome if all other voters are truthful. We refer to such situations by saying that the voter is *pivotal*. Formally, given the profile of messages θ voter i is pivotal if there is a $m_i \neq \theta_i$ such that $\mathbf{v}\left(\theta\right) \neq \mathbf{v}\left(\left(\theta_{-i}, m_i\right)\right)$.

There are convenient implications of the assumption of the use of undominated strategies in the voting stage. When voters use undominated voting strategies, each voter's vote does not depend on the message she reports. Moreover, since by reporting a certain option a pivotal voter can only increase the support for such option if a voter is pivotal then the voting outcome must coincide with the message this voter reveals.

3 Unbiased Leader

We analyze first the voters' strategic incentives in the communication stage when the leader is unbiased, $b = \frac{N}{2} + 1$. We have the following result:

Proposition 1. *Assume that the leader is unbiased.*

- *If either $\#F \geq q$ or $\#F < (N+1) - (q-1)$ then there is a fully revealing equilibrium.*
- *If $q > \#F \geq (N+1) - (q-1)$ then there is no fully revealing equilibrium.*

Proof. Assume first that $\#F \geq q$ or $\#F < (N+1) - (q-1)$. We consider the strategic incentives in the message stage for the different types of voters.

Leader Consider the leader l and assume that the remaining voters truthfully report their signal. That is, for each $i \neq l$, $m_i = \theta_i$. First, we have to characterize the circumstances under which l is pivotal.

Assume that $\#F \geq q$. Since for each $f \in F$ and each $m'_l \in \{A, Q\}$, $v_f(m'_l, m_{-l}) = m'_l$, then $\mathbf{v}(m'_l, m_{-l}) = m'_l$ and the leader is pivotal.

Assume that $\#F < (N+1) - (q-1)$. If $\#\Theta_A \setminus l \neq \#\Theta_Q \setminus l$ then all objective voters and the leader vote for a certain option that is independent on the leader's report. Since $\#(O+1) \geq q$, for each $m_l, m'_l \in \{A, Q\}$, $\mathbf{v}(m_l, m_{-l}) = \mathbf{v}(m'_l, m_{-l})$ and the leader is not pivotal. If $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$ and $q \neq N+1$ then all objective voters and followers vote according to the leader's report and since $q \neq N+1$ the leader is pivotal. Finally, if

$\#\Theta_A \setminus l = \#\Theta_Q \setminus l$ and $q = N+1$, then the alternative A is the voting outcome when the leader receives $\theta_l = A$ and reports $m_l = A$, and Q is the voting outcome otherwise. Thus, the leader is pivotal only when $\theta_l = A$.

Summing up, provided that either $\#F \geq q$ or $\#F < (N+1) - (q-1)$, the leader is pivotal if and only if either:

- $\#F \geq q$ or,
- $(N+1) - (q-1) > \#F$ $(\#O \geq q-1)$ and either
  - $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$ and $q \neq N+1$ or,
  - $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$, $q = N+1$ and $\theta_l = A$,

Next, we explore whether there exists a truth-telling best response for the leader whenever she is pivotal and all other voters are truthful.

If $\#F \geq q$ then the leader learns nothing about the signals of other players from being pivotal. Thus, since since $p \geq \frac{1}{2}$ implies $P\left(\#\Theta_A > \#\Theta_Q \middle| \theta_l = A\right) = P\left(\#\Theta_A < \#\Theta_Q \middle| \theta_l = Q\right) \geq \frac{1}{2}$, $m_l = \theta_l$ is a best response for l.

If $\#F < (N+1) - (q-1)$ and $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$, then if $\theta_l = A$ we have that $\#\Theta_A > \#\Theta_Q$ and if $\theta_l = Q$ then $\#\Theta_A < \#\Theta_Q$. Thus, $u(L, A, \theta, m) > u(L, Q, \theta, m)$ if and only if $\theta_l = A$ and $m_l = \theta_l$ is a best response for l.

Followers Consider an arbitrary follower f. If $\#F \geq q$ then f is not pivotal as in this case the outcome of the voting stage depends entirely on the message revealed by the leader. If $\#\Theta_A \setminus f \neq \#\Theta_Q \setminus f$ then f is not pivotal as her message does not influence any vote. Finally, if $\#F < (N+1) - (q-1)$ $(\#O + 1 \geq q)$ and $\#\Theta_A \setminus f = \#\Theta_Q \setminus f$, then m_f determines the vote of the leader and all the objective voters. Since $\#O + 1 \geq q$, f is pivotal. Moreover, since $P\left(\theta_l = A | \#\Theta_A \setminus f = \#\Theta_Q \setminus f\right) = P\left(\theta_l = Q | \#\Theta_A \setminus f = \#\Theta_Q \setminus f\right) = \frac{1}{2}$, $m_f = \theta_f$ is is a best response for f.

Objective Voters Consider an arbitrary objective voter o. A necessary condition for o to be pivotal is that $\#\Theta_A \setminus o = \#\Theta_Q \setminus o$ as otherwise her message does not influence any vote. If $\theta_o = A$ then $\#\Theta_A \setminus o = \#\Theta_Q \setminus o$ implies $\#\Theta_A > \#\Theta_Q$ and, similarly, $\theta_o = Q$ implies $\#\Theta_A < \#\Theta_Q$. Thus, with the same arguments we use for the leader, $m_o = \theta_o$ is the best response for o.

In conclusion, whenever $\#F \geq q$ or $\#F < (N+1) - (q-1)$ there exists a truth-telling best response for every voter. Hence, if either $\#F \geq q$ or $\#F < (N+1) - (q-1)$ there is a FRE.

To conclude the proof assume now that $q > \#F \geq (N+1) - (q-1)$ and consider an arbitrary follower f. If $\#\Theta_A \setminus f \neq \#\Theta_Q \setminus f$, then f is not pivotal as her message does not influence any vote. Moreover, if $q > \#F \geq (N+1) - (q-1)$ and $\theta_l = Q$ then all followers vote for Q and the number of votes f can influence are at most $q-1$. Thus, f is not pivotal. However, if $q > \#F \geq (N+1) - (q-1)$, $\#\Theta_A \setminus f = \#\Theta_Q \setminus f$ and $\theta_l = A$, m_f determines the vote of the leader and all objective voters. If $m_f = A$ then all voters vote for A and A is the voting outcome. If $m_f = Q$ then only the followers vote for A and since $\#O + 1 \geq q$ then Q is the voting outcome and f is pivotal. Note that since $m_l = \theta_l = A$, $u(F, A, \theta, m) > u(F, Q, \theta, m)$, and $m_f = A$ is the best-response for f independently of the signal she receives. Hence, if $q > \#F \geq (N+1) - (q-1)$ then whenever a follower is pivotal she has incentives to miss-report. Thus, there is no FRE with $q > \#F \geq (N+1) - (q-1)$. □

Given the result in Proposition 1, we have that a FRE is possible if and only if either there are either sufficiently few or sufficiently many followers. When there are sufficiently many followers ($\#F \geq q$) then followers are enough as to force the implementation of the alternative A. In this case the leader is pivotal always and, moreover, she is the only pivotal voter. Hence, her vote always completely determines the option to be implemented. Therefore, the leader learns nothing from being pivotal and, thus, the only information about the signals of other voters she has is her own signal. In consequence, since the leader is unbiased and the accuracy of the signal is at least $\frac{1}{2}$, if she receives a certain signals she believes this signal is the one that most other voters receive. Thus, she has incentives to truthfully reveal her signal.

If there are sufficiently few followers ($\#F < N+1-(q-1)$) then the number of followers is not enough to block the implementation of A in case all other voters vote for the alternative. Thus, if the leader is pivotal then she can influence the vote of objective voters which can only happen if the rest of voters received as many A signals as Q signals. In this case, an unbiased leader wants the alternative that matches with her own signal to be implemented and, thus, reports truthfully.

Finally, when the number of followers is not enough as too completely determine the outcome of the voting stage ($\#F < q$) but their number is enough to block the implementation of A in case all non-followers vote for the alternative ($\#F \geq N+1-(q-1)$) a FRE is not possible. In this situation, whenever a follower is pivotal it must be that the leader reports A. Thus, any follower has incentives to reveal A regardless of her own signal.

A consequence of the result in proposition 1 is the following:

Corollary 1. *Assume that the leader is unbiased:*

i) Under the majority rule $\left(q = \frac{N}{2} + 1\right)$ *there always exists a FRE.*

ii) Under the unanimity rule ($q = N + 1$) if there is at least one follower then there does not exist a FRE.

From the information revelation point of view, the majority rule outperforms any other voting rule as it makes truthful sharing of information possible for any number of followers in case the leader is unbiased. For all other super-majority rules ($q \neq \frac{N}{2} + 1$) there exists a number of followers such that truthful sharing of information is not possible.

Proposition 1 shows that there are some circumstance where a FRE is not possible. We now turn our attention to study situations where although no equilibrium where all voters tell the truth exists equilibria where most voters tell the truth are possible. In particular, whenever a FRE does not exists because a type of voter has incentives to miss-report their signal we construct a sequential Bayesian equilibrium where a set of voters truthfully report their signals and each voter's decision in the voting stage is based on the information reported by these truthful voters and her own signal. So far we have analyzed the existence of perfect Bayesian equilibria under the prior probability beliefs that for each pair of voters i, j, $i \neq j$, $P(\theta_i = m_i \mid m_i, \theta_j) = 1$. Now we analyze the existence of perfect Bayesian equilibria under the assumption that there is a group of voters whose report is not informative, while the remaining voters report truthfully the signal they have received.

Definition 3. *We say that voter i is uninformative if $\sigma_i^m(\theta_i)$ is independent on θ_i. Let K be the set of uninformative voters and let K_A be the set of uninformative voters who receive signal A.*

We analyze first the optimal voting strategies for each type of voter under the existence of a a group of voters whose messages are not informative.

Lemma 1. *Assume the leader is unbiased, $b = \frac{N}{2} + 1$. Assume further that $K \neq \emptyset$ and all $j \notin K$ report truthfully. For each voter i the unique weakly dominant voting strategies is given by:*

$$\sigma^v(L, \theta_j, m) = \begin{cases} A & \text{if } i \notin K \text{ and } \#(M_A \setminus K) \geq \frac{N+1-\#K}{2}, \\ A & \text{if } i \in K \text{ and } \#(M_A \setminus K) + \mathbb{1}_{\theta_i = A} \geq \frac{N+1-(\#K-1)}{2}, \\ Q & \text{otherwise.} \end{cases} \tag{7}$$

$$\sigma^v(F, \theta_i, m) = m_l. \tag{8}$$

$$\sigma^v(O, \theta_i, m) = \begin{cases} A & \text{if } i \notin K \text{ and } \#(M_A \setminus K) \geq \frac{N+1-\#K}{2}, \\ A & \text{if } i \in K \text{ and } \#(M_A \setminus K) + \mathbb{1}_{\theta_i = A} \geq \frac{N+1-(\#K-1)}{2}, \\ Q & \text{otherwise.} \end{cases} \tag{9}$$

where $\mathbb{1}_{\theta_j = A}$ is the indicator function that takes the value 1 if $\theta_j = A$ and 0 otherwise.

Proof. We proceed by consider each type of voter individually:

Leader Assume first that $l \notin K$. Since the leader is unbiased she derives one unit of utility from voting the alternative A if and only if the number of signals in the population is greater or equal than $\frac{N}{2}+1$. Given that the report of the voters in K is not informative, the leader only considers $N+1-\#K$ truthful signals. Let $x \equiv \{j \notin K \mid m_j = A\} \in \{0,1,\ldots,N+1-\#K\}$ Thus, the leader votes for A if and only if $P\left(\#\Theta_A \geq \frac{N}{2}+1|\#(\Theta_A \setminus K)=x\right) \geq P\left(\#\Theta_A < \frac{N}{2}+1|\#(\Theta_A \setminus K)=x\right)$. This can be rewritten as

$$P\left(\#K_A \geq \frac{N}{2}+1-x|\#(\Theta_A \setminus K)=x\right) \geq P\left(\#K_A < \frac{N}{2}+1-x|\#(\Theta_A \setminus K)=x\right)$$

which holds if and only if

$$\begin{aligned}
\sum_{i=\frac{N}{2}+1-x}^{\#K} P\left(\#K_A = i|\#(\Theta_A \setminus K)=x\right) &\geq \sum_{i=0}^{\frac{N}{2}-x} P\left(\#K_A = i|\#(\Theta_A \setminus K)=x\right), \\
\sum_{i=\frac{N}{2}+1-x}^{\#K} P\left(\#K_A = i \cap \#(\Theta_A \setminus K)=x\right) &\geq \sum_{i=0}^{\frac{N}{2}-x} P\left(\#K_A = i \cap \#(\Theta_A \setminus K)=x\right).
\end{aligned}$$

The inequality above can be rewritten as

$$\begin{aligned}
&\sum_{i=\frac{N}{2}+1-x}^{\#K} \binom{\#K}{i} \left(p^{i+x}(1-p)^{N+1-i-x} + (1-p)^{i+x}p^{N+1-i-x}\right) \geq \\
&\qquad \sum_{i=0}^{\frac{N}{2}-x} \binom{\#K}{i} \left(p^{i+x}(1-p)^{N+1-i-x} + (1-p)^{i+x}p^{N+1-i-x}\right).
\end{aligned}$$

Comparing term by term the components of both sums at either side of the inequality above leads to the conclusion that the inequality holds if and only if $\#K + x \geq N+1-x$. Thus, the leader votes for A if and only if $x \geq \frac{N+1-\#K}{2}$. The proof for the case where $i \in K$ follows easily from the arguments above and, hence, its proof is omitted.

Followers Since the followers always prefer the alternative that matches the report of the leader, they always vote according to the leader's report..

Objective Voters The result follows directly from the arguments in the analysis of the leader's optimal voting strategy. □

Define $\mathbf{v}' : \{A,Q\}^{N+1} \rightarrow \{A,Q\}$ as the option that is implemented given a profile of messages in $\{A,Q\}^{N+1}$ when the voting strategies are given in (7), (8) and (9). Notice that in an abuse of notation we are omitting the number of voters of each type in the description of $\mathbf{v}'$.

The next two definitions introduce the equilibrium concept under the prior belief that there is a group of uninformative voters and the remaining voters truthfully report the signal they receive.

Definition 4. *Given the set of voters who miss-report, K, the message strategy σ^m is a truth-telling best response for voter $i \notin K$ of type $G \in \{L, F, O\}$ if and only if $m_i = \sigma^m(G, \theta_i) = \theta_i$ and for any message strategy $\bar{\sigma}^m$*

$$E\left[u(G, \mathbf{v}'(m), \theta, m) \mid \theta_i\right] \geq E\left[u(G, \mathbf{v}'(m_{-i}, \bar{\sigma}^m(G, \theta_i)), \theta, (m_{-i}, \bar{\sigma}^m(G, \theta_i))) \mid \theta_i\right]$$

where m is the message profile such that $m_j = \theta_j$ if $j \notin K$ and m_j is independent on θ_j if $j \in K$.

Definition 5. *A profile of message strategies $(\sigma_1^m, \ldots, \sigma_{N+1}^m)$ is a partially revealing (Bayesian Nash) equilibrium (PRE) if and only if there exists a set of voters $K \in \{0, \ldots, N+1\}$ such that for all voter $i \notin K$ the message strategy σ_i^m is a truth-telling best response given H and for all voter $j \in K$ the message strategy σ_j^m does not depend on θ_j.*

A PRE is a perfect (sequential) Bayesian equilibrium under the (fulfilled) prior belief that the messages reported by uninformative voters do not provide relevant information, and the remaining voters truthfully report their signal. Notice that the voting strategies in (4), (5) and (6) are equivalent to those in (7), (8) and (9) if $b = \frac{N}{2} + 1$ and $K = \emptyset$. Similarly, a FRE is a PRE where $K = \emptyset$.

In the next result we investigate the existence of a PRE in situations where a FRE is not possible. Thus, we restrict our attention to situations where $q > \#F > N + 1 - (q - 1)$. Moreover, if a PRE exists then we focus on equilibria where the set of uninformative voters is minimal.

Since followers' only concern is the message reported by the leader and, given that $q > \#F > N+1-(q-1)$, they can always force the voting outcome to be Q, the followers are the natural candidates to be uninformative voters. We show that, in general, we can construct a PRE with the followers and the leader as uninformative voters. Only under very special configurations of the accuracy of the signal p and the number of follower $\#F$ a PRE with $K = F$ can be devised.

Proposition 2. *Assume that the leader is unbiased and $q > F \geq N + 1 - (q - 1)$. Denote $F' = F \cup \{l\}$,*

- *If $q \neq N + 1$ and*

$$P\left(\#(\Theta_A \setminus F') = \#(\Theta_Q \setminus F') | \#(\Theta_A \setminus F') \geq \#(\Theta_Q \setminus F')\right) \geq \frac{1}{2},$$

  then there exists a PRE with the followers as the set of uninformative voters.

- *If either* $q = N + 1$ *or*

$$P\left(\#(\Theta_A \setminus F') = \#(\Theta_Q \setminus F') | \#(\Theta_A \setminus F') \geq \#(\Theta_Q \setminus F')\right) \geq \frac{1}{2},$$

there is no PRE with with the followers as the set of uninformative voters, but there exists a PRE with with the followers and the leader as the set of uninformative voters.

Proof. Assume that the leader is unbiased and $q > F \geq N + 1 - (q - 1)$. Assume there is a group of uninformative voters K. Note that if voter j belongs to K the she is not pivotal as in a PRE uninformative voter's messages are ignored. We proceed by studying the individual incentives of at the discussion stage of each of the different types of voters to check whether their messages can be informative.

Leader Let $K' = K \setminus \{l\}$. Given that $q > \#F \geq (N+1) - (q-1)$, there are three possible scenarios where the leader is pivotal:

- $\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K')$. In this case both the leader and objective voters vote for A independently on the leader's report. However, since $\#F \geq (N + 1) - (q - 1)$, the voting outcome is A only if followers also vote for A. Since the followers' vote is determined by the message revealed by the leader we have that leader is pivotal.

- $\#(\Theta_A \setminus K') = \#(\Theta_Q \setminus K')$. In this situation followers and objective voters vote according to the leader's message, while the leader herself votes according to θ_l. If $q \neq N + 1$, then the leader's report determines the voting outcome and l is pivotal. If $q = N + 1$, then the voting outcome is A when the leader receives $\theta_l = A$ and Q otherwise. Therefore, the leader is pivotal if and only if either $q \neq N+1$ or $q = N+1$ and $\theta_l = A$.

- $\#(\Theta_A \setminus K') < \#(\Theta_Q \setminus K')$. In this scenario the leader and objective voters vote for Q and, thus, the voting outcome is Q. Therefore, the leader is not pivotal.

Assume that $q \neq N+1$. The leader is pivotal if and only if $\#(\Theta_A \setminus K') \geq \#(\Theta_Q \setminus K')$. If $\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K')$ then the leader prefers alternative A to Q and her best response is to report $m_l = A$ independently of the signal she receives. If $\#(\Theta_A \setminus K') = \#(\Theta_Q \setminus K')$, then the leader prefers the voting outcome to coincide with the signal she has received, and the her best response is to report $m_l = \theta_l$. Therefore, if

$$P\left(\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K') | \#(\Theta_A \setminus K') \geq \#(\Theta_Q \setminus K')\right) \geq \frac{1}{2}$$

the leader reports A independently of the signal she has received. On the contrary, if

$$P\left(\#(\Theta_A \setminus K') = \#(\Theta_Q \setminus K') | \#(\Theta_A \setminus K') \geq \#(\Theta_Q \setminus K')\right) \geq \frac{1}{2}$$

then the leader's best response is to report $m_l = \theta_l$.

Consider now the case where $q = N+1$. If $\theta_l = Q$ then the leader is pivotal if and only if $\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K')$. Thus, since $\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K')$ implies $\#(\Theta_A \setminus K) \geq \#(\Theta_Q \setminus K)$ the leader has incentives to report A regardless of her signal and, thus, she has incentives to miss-report.

Finally, if $q = N+1$ and $\theta_l = A$ then the leader is pivotal if and only if $\#(\Theta_A \setminus K') \geq \#(\Theta_Q \setminus K')$. Thus, given that $\#(\Theta_A \setminus K') \geq \#(\Theta_Q \setminus K')$ and $\theta_l = A$ imply $\#(\Theta_A \setminus K) > \#(\Theta_Q \setminus K)$ the leader has incentives to report truthfully.

Followers Consider an arbitrary follower f. Given the voting strategies in (7), (8) and (9) if $q > \#F \geq (N+1)-(q-1)$ then the alternative A is implemented if and only if all followers (including f) vote for A, which can happen only if $m_l = A$. Thus, a necessary condition for a follower to be pivotal is that $m_l = A$. Hence, such follower has incentives to report A regardless on her signal.

Objective Voters Consider an arbitrary objective voter o and let $K'' = K \setminus \{o\}$. A necessary condition for objective voter o to be pivotal is that $\#(\Theta_A \setminus K'') < \frac{N+1-\#K''}{2}$ and $\#(\Theta_A \setminus K'') + 1 \geq \frac{N+1-\#K''}{2}$ as otherwise her message does not influence any vote. Thus, o prefers the voting outcome to coincide with the signal she has received and she sends the message $m_o = \theta_o$.

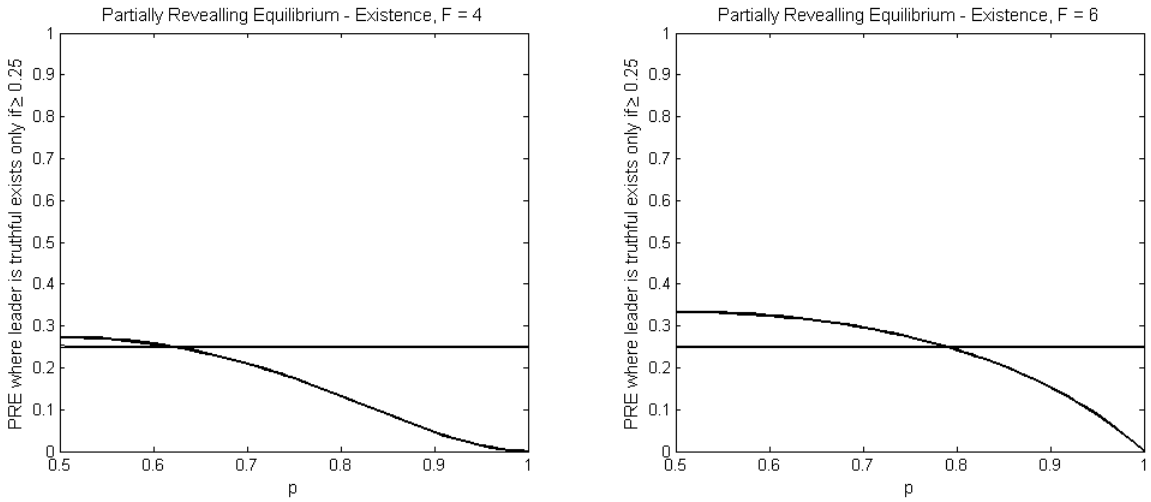
To conclude the proof, note that the message of the followers is never informative: $F \subseteq K$. Moreover, as we have just shown for objective voters it is always a best response to truthfully report their signal whilst the leader is truthful only if $q \neq N+1$ and the signal are accurate enough. □

Proposition 2 implies that if $\#F$ is odd then there is no PRE with $K = F$. This is because if $\#F$ odd then since $N+1$ is odd we have that $\#(\Theta_A \setminus F') + \#(\Theta_Q \setminus F')$ is odd and, thus, $\#(\Theta_A \setminus F') \neq \#(\Theta_Q \setminus F')$. If $\#F$ is even the condition for the existence of a PRE with $K = F$ can be rewritten as

$$\frac{\binom{N-\#F}{\frac{N-\#F}{2}}(p(1-p))^{\frac{N-\#F}{2}}}{\sum_{i=\frac{N-\#F}{2}}^{N-\#F}\binom{N-\#F}{i}\left[p^i(1-p)^{N-\#F-i}+(1-p)^i p^{N-\#F-i}\right]} \geq \frac{1}{4}.$$

In Figure 1 we plot the inequality above to illustrate the values of the accuracy of the signal p for which a PRE where the leader is truthful is possible.

Figure 1: $N + 1 = 9$, $q = 7$.



4 Biased Leader

Consider now the situation where the leader is biased, i.e. $b \neq \frac{N}{2} + 1$. The existence of a biased leaders affects the strategic incentives of all voters. Effectively, this fact precludes the possibility of full information aggregation unless the leader has enough followers and the accuracy of the signals is high enough.

Proposition 3. *Assume that the leader has a bias $b \neq \frac{N}{2} + 1$. A FRE exists if and only if $\#F \geq q$ and p is high enough relative to b.*

Proof. Assume first that $\#F \geq q$. From the arguments in Proposition 1, if $\#F \geq q$ then the leader is always pivotal and she is the only pivotal voter. Thus, to check the existence of a FRE it suffices to explore the incentives of the leader to truthfully reveal her signal. If $\#F \geq q$ the leader learns nothing about the signals of other players from the fact that she is pivotal. Thus, if $b > \frac{N}{2} + 1$ and $\theta_l = Q$ then $P\left(\#\Theta_A < b|\theta_l = Q\right) > \frac{1}{2}$ and the leader has incentives to truthfully report his signal. If $b > \frac{N}{2} + 1$ and $\theta_l = A$ then whether the leader wants to report truthfully or not depends on whether or not $\#\Theta_A \geq b$ given $\theta_l = A$, which depends on the accuracy of the signal p and the value of b. If $b < \frac{N}{2} + 1$ and $\theta_l = Q$ then again whether the leader wants to report truthfully or not depends on de accuracy of the signal p and the value of b as in the previous case. Finally, if $b < \frac{N}{2} + 1$ and $\theta_l = A$ then $P\left(\#\Theta_A \geq b|\theta_l = A\right) > \frac{1}{2}$ and the leader wants to truthfully report his signal.

We can implicitly compute the value of p relative to b for the leader to truthfully reveal his signal if either $b > \frac{N}{2} + 1$ and $\theta_l = A$ or $b < \frac{N}{2} + 1$ and $\theta_l = Q$. If $b > \frac{N}{2} + 1$ and $\theta_l = A$

then the leader wants to truthfully reveal his signal if

$$\begin{aligned}P\left(\#\Theta_A \geq b|\theta_l = A\right) &= \sum_{i=b-1}^{N} \binom{N}{i} \left[p^{i+1}(1-p)^{N-i} + p^{N-i}(1-p)^{i+1}\right] \\ &\geq \frac{1}{2}.\end{aligned}$$

On the other hand, if $b < \frac{N}{2}+1$ and $\theta_l = Q$ then the leader wants to truthfully reveal his signal if

$$\begin{aligned}P\left(\#\Theta_A < b|\theta_l = Q\right) &= P\left(\#\Theta_Q \geq N+1-b|\theta_l = Q\right) \\ &= \sum_{i=N-b}^{N} \binom{N}{i} \left[p^{i+1}(1-p)^{N-i} + p^{N-i}(1-p)^{i+1}\right] \\ &\geq \frac{1}{2}.\end{aligned}$$

In conclusion, if $\#F \geq q$ then the leader follows a truth-telling best response if and only if p is high enough relative to b. Moreover, since followers and objective voters are never pivotal, truth-telling is always a best response for them. Thus, if $\#F \geq q$ then there exists a FRE if and only if p is high enough relative to b.

Assume now that $\#F < q$. We show that in this situation for at least one type of voter it is not a best-response strategy to report their true signal.

Firstly, consider the case where $\#F > N+1-q$ ($\#O < q-1$). Let f be an arbitrary follower. Note that whenever $\theta_l = Q$, then every follower votes for Q and Q is the voting outcome independently of the signal reported by f. However, if $\theta_l = A$, then all the followers vote for A but they do not suffice to determine the voting outcome. Therefore, whenever f is pivotal it must be that $\theta_l = A$. Since in this situation f wants the voting outcome to be the alternative A independently of the signal she receives, her best response is always to report $m_f = A$, which precludes the existence of a FRE.

Next consider consider the case where $\#F < N+1-q$ ($\#O \geq q$). In this situation objective voters completely determine the outcome of the voting stage. With the arguments in the proof of Proposition 1, the leader is pivotal if and only if $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$. Assume first that $b > \frac{N}{2}+1$. If l is pivotal, then $\#\Theta_A < b$ and l prefers Q over A independently of θ_l. Hence, the leader's best response is always to report $m_l = Q$. Assume now that $b < \frac{N}{2}+1$, then if the leader is pivotal $\#\Theta_A \geq b$ and l prefers A to be implemented independently of θ_l. Hence, the leader's best response is always to to report $m_l = A$. Thus, the leader's best response if $\#F < N+1-q$ is to report the signal that matches her bias, which precludes the existence of a FRE.

Finally, consider the case where $\#F = N + 1 - q$ ($\#O = q - 1$). We have two possible situations: $q \neq N + 1$ and $q = N + 1$. We analyze first the case where $q \neq N + 1$. Assume that $b > \frac{N}{2} + 1$ and consider the leader's incentives in the following three possible scenarios:

- $\#\Theta_A \setminus l > \#\Theta_Q \setminus l$. In this case every objective voter votes for A regardless of the leader's report. Note that the vote of the leader, v_l, does not depend on m_l. If $\#\Theta_A \geq b$, then $v_l = A$ and the voting outcome is A independently of m_l. Thus, l is not pivotal. If $\#\Theta_A < b$ then $v_l = Q$. If $m_l = A$, then every follower and every objective voter voter for A and the voting outcome is A. If $m_l = Q$ then the leader and the followers vote for Q and the voting outcome is Q. Hence l is pivotal only if $\#\Theta_A < b$.

- $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$. In this situation followers and objective voters always vote according to the leader message, and are enough to determine the voting outcome. That is, l is pivotal. Notice that $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$ and $b > \frac{N}{2} + 1$ implies $\#\Theta_A < b$.

- $\#\Theta_A \setminus l < \#\Theta_Q \setminus l$. In this case every objective voter votes for Q regardless of the leader's report. Since $\#F < q$ the voting outcome is Q regardless of the message sent by the leader and, thus, l is not pivotal.

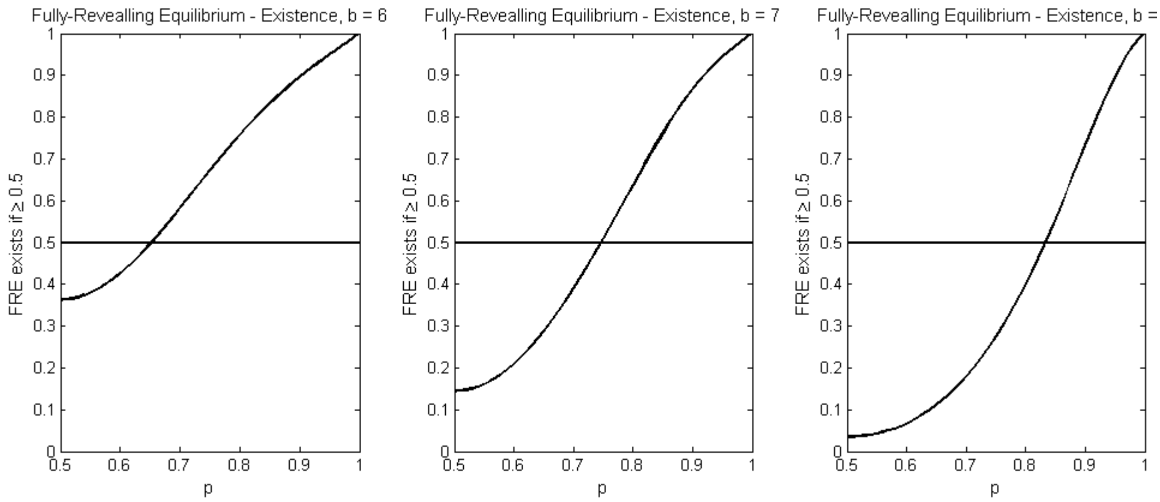
Summing up, if $b > \frac{N}{2} + 1$ and l is pivotal then $\#\Theta_A < b$ and the leader's best response is always to report $m_l = Q$, following her bias. A parallel argument applies to prove that if $b < \frac{N}{2} + 1$ and l is pivotal then $\Theta_A \geq b$ and l's best response is to report $m_l = A$ regardless of her signal.

To conclude, consider the case $\#F = N + 1 - q$ and $q = N + 1$. Clearly, this implies that $\#F = \emptyset$. Assume that $b > \frac{N}{2} + 1$. Since there is no follower and the leader's vote does not depend on her signal, the only possibility for the leader to be pivotal is that her message affects the vote of the objective voters. This fact implies that whenever l is pivotal $\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l)$ and, thus, $\#\Theta_A < b$. Therefore, the leader's best response is always to report $m_l = Q$ independently of the signal she receives. If $b < \frac{N}{2} + 1$ similar arguments apply and we have that whenever l is pivotal then $\#\Theta_A > b$ and the leader's best response is to report $m_l = A$ regardless of her signal. Hence, a biased leader's best response if all other voters report truthfully is to report the signal that favors her bias. That is, a FRE is not possible. $\square$

Proposition 3 states that with a biased leader truthful information transmission occurs only if there are enough followers ($\#F \geq q$). Thus, in comparison with the situation where the leader is unbiased, the case where there are few followers ($\#F < N + 1 - (q - 1)$) is not compatible with a FRE any more. The reason for this is that when $\#F < N + 1 - (q - 1)$ the leader is pivotal if and only if her message can influence objective voters. This is possible only if the number of signals in favor of both alternatives, excluding the leader's own signal, is the same. Since the leader is biased, this means that independently on the signal she receives she has incentives to report according to her bias: reveal A if $b < \frac{N}{2} + 1$ and reveal Q if $b > \frac{N}{2} + 1$.

In Figure 2 we explore a situation where $N + 1 = 9$ and $F \geq q$ and show the values of p for which a FRE is possible given different bias levels. The higher the bias of the leader the more accurate is the precision of the signal needed for a FRE to exist.

Figure 2: $N + 1 = 9$, $F \geq q$.



Before moving to the welfare analysis, two remarks are in order. Proposition 3 deals with the case where the leader is the only voter who is biased. Although a FRE does not exits in the situations when objective voters suffice to determine the voting outcome, if objective voters had the same bias as the leader then a FRE will be possible. In this case, whenever the leader (or any objective voter) is pivotal her report determines the votes of all the voters that share her same preferences. Thus, whether the leader prefers A or Q depends on the signal she receives and consequently she has incentives to reveal truthfully. Conversely, if the leader is unbiased and objective voters are biased (all with the same bias) then the converse arguments shows that a FRE is not possible. Finally, it is clear that if the leader is biased and $\#F < N + 1 - q$ ($\#O \geq q$) then a PRE exist where $F \cup \{l\}$ is the set of uninformative voters.

5 Welfare Analysis

In this section we turn our attention to the welfare analysis. We focus on how likely the option that matches with the state of nature is to be implemented. Throughout this section we assume that the leader is unbiased ($b = \frac{N}{2} + 1$). To simplify the exposition we reduce significantly the number of cases to be considered by focusing on situations where either a FRE exists or if not then a PRE exists where the leader reports truthfully.

The objective of the committee is then to choose the "right" option in the sense that the voting outcome matches the actual state of nature. Given the arguments in Coughlan (2000) if all voters are identical (unbiased leader, no followers) the two stage deliberation process would most likely select the right choice for any voting rule q. However the existence of a leader and a group of followers introduces a distortion: even though all voters report true information at the discussion stage some members may not use all relevant information when voting.

Proposition 4. *Assume that if either $F \geq q$ or $F < N + -(q-1)$ then voters strategies constitute a FRE. Moreover if $q > \#F \geq N+1-(q-1)$ then assume voters strategies constitute a PRE where the set of uninformative voters is $K = F$. The probability of implementing the option that matches with the state of nature is given by:*

$$P\left(\mathbf{v}(\theta) = S\right) = \begin{cases} p & \textit{if } \#F \geq q, \\ \sum_{i=\frac{N-\#F-1}{2}}^{N} \binom{N}{i} p^{i+1}(1-p)^{N-i} & \textit{if } q > \#F \geq N+1-(q-1), \\ \sum_{i=\frac{N}{2}+1}^{N+1} \binom{N+1}{i} p^{i}(1-p)^{N+1-i} & \textit{otherwise.} \end{cases}$$

Proof. If $\#F \geq q$ then in a FRE the option implemented coincides with the message revealed by the leader. Thus, the probability that the option implemented matches with the state of nature given that the leader is truthful equals the probability that the signal of the leader coincides with the state of nature, i.e. p.

Assume that $q > \#F \geq N+1-(q-1)$ and the conditions in proposition 2 for the leader not to be uninformative are satisfied. Since $q > \#F \geq N+1-(q-1)$ implies $\#O < q-1$ then the option implemented is A only if the leader's message is A and at least one non-follower votes for A. Given that all non-followers, i.e. objective voters and the leader, vote for the same option then the alternative A is implemented if and only if all voters vote for it. Thus, in a PRE where the leader truthfully reveals her signal A is implemented if and only if $\theta_l = A$ and $\#(\Theta_A \setminus F) \geq \frac{N+1-\#F}{2}$ (see equation (9)). Thus, as both states are equally likely the probability of implementing the option that matches with the state of nature is given by

$$\frac{1}{2}P\left(\theta_l = A \cap \#(\Theta_A \setminus F) \geq \frac{N+1-\#F}{2}|S=A\right) + \frac{1}{2}P\left(\theta_l = Q \cup \#(\Theta_A \setminus F) < \frac{N+1-\#F}{2}|S=Q\right),$$

which can be rewritten as

$$\sum_{i=\frac{N-\#F-1}{2}}^{N} \binom{N}{i} p^{i+1}(1-p)^{N-i}$$

If $\#F < (N+1)-(q-1)$ then $\#O \geq q-1$ and since the leader is unbiased in a FRE the option implemented coincides with the option objective voters and the leader vote for. The option objective voters and the leader vote for is A if and only if $\#\Theta_A \geq \frac{N}{2}+1$. Thus, the probability of implementing the option that matches with the state of nature is given by

$$\begin{aligned} P\left(\#\Theta_A \geq \frac{N}{2}+1 \middle| S=A\right) &= P\left(\#\Theta_Q \geq \frac{N}{2}+1 \middle| S=Q\right) \\ &= \sum_{i=\frac{N}{2}+1}^{N+1} \binom{N+1}{i} p^i(1-p)^{N+1-i}. \end{aligned}$$

□

If all voters where objective voters then the probability of implementing the option that coincides with the state of nature equals $\sum_{i=\frac{N}{2}+1}^{N+1}\binom{N+1}{i}p^i(1-p)^{N+1-i}$. Thus, by proposition 4 we can infer that the existence of a leader and her followers decreases the likelihood of implementing the option that matches with the state of nature except when all voters are truthful and the number of followers is below $\#F < N+1-(q-1)$. Thus, under the majority rule ($q = \frac{N}{2}+1$) given that a FRE always exists if the number of followers is less than half the number of voters ($\#F < \frac{N}{2}+1$)) then the probability of implementing the option that matches with the state of nature is the same as if all voters were objective voters.

6 Multiple Leaders

In this section we extend our results to situations where the set of leaders contains two voters $L = \{l, l'\}$. While the preferences and voting behavior of leaders and objective voters are not affected by the existence of more than one leader, followers' behavior is affected. We assume that followers act as a type of objective voters that only care about the signals both leaders send: followers strictly prefer a certain option if and only if both leaders send the same signal and are indifferent between the two options if and only if the leaders send different signals.

Formally, the utility function that represents the preferences of the followers in the scenario with two leaders l and l' is given by

$$u(F, A, \theta, m) = 1 - u(F, Q, \theta, m) \quad = \quad \begin{cases} 1 & \text{if } m_l = m_{l'} = A, \\ \frac{1}{2} & \text{if } m_l \neq m_{l'}, \\ 0 & \text{otherwise.} \end{cases}$$

A weakly dominant voting strategy for the followers when all other voters are truthful can be expressed by

$$\sigma^v(F, \theta_i, m) \quad = \quad \begin{cases} A & \text{if } m_l = m_{l'} = A, \\ Q & \text{otherwise.} \end{cases} \tag{10}$$

Note that in order to reduce the number of cases that need to be considered we are assuming that in case followers are indifferent between the alternative A and the status quo Q then they vote for the status quo.

Proposition 5. *Let $L = \{l, l'\}$ and assume that both leaders are unbiased.*

- *If either $\#F \geq q$ or $\#F < (N+1) - (q-1)$ then there is a FRE.*
- *If $q > \#F \geq (N+1) - (q-1)$ then there is no FRE.*

Proof. Assume first that $\#F \geq q$ or $\#F < (N+1) - (q-1)$. We consider the individual incentives to deviate from the truth-telling best response for each of the different types of voters.

Leader Consider the leader l and assume that for each $i \neq l$, $m_i = \theta_i$. We consider different cases.

Consider first the case $\#F \geq q$. If $\theta_{l'} = A$ then for each $f \in F$ we have that $v_f = m_l$ and since $\#F \geq q$ it is true that l is pivotal. If $\theta_{l'} = Q$ then for each $v_f = Q$ for all $f \in F$ and, therefore, l is not pivotal.

If $\#F \geq q$ and l is pivotal then l learns that $\theta_{l'} = A$. If $\theta_l = A$, then

$$P\left(\Theta_A \geq \frac{N}{2} + 1 | \theta_l = \theta_{l'} = A\right) > \frac{1}{2}.$$

Therefore, l's best response is to report truthfully. Moreover, if $\theta_l = Q$, then since

$$P\left(\Theta_A \geq \frac{N}{2} + 1 | \theta_l = Q, \theta_{l'} = A\right) \quad = \quad P\left(\Theta_A < \frac{N}{2} + 1 | \theta_l = Q, \theta_{l'} = A\right)$$

l is indifferent between reporting A and reporting Q. In particular, reporting $m_l = Q$ is a best response. Therefore, if $\#F \geq q$ then truth-telling is a best response for l.

Consider next the case where $\#F < N + 1 - q$, which in turn implies $\#O \geq q - 1$. In this situation the other leader, l', and objective voters' vote determine the outcome of the election. Since l' is unbiased and for each $o \in O$ and each $m_l \in \{A, Q\}$ we have that objective voters and l' vote for the same option. Thus, if $\#(\Theta_A \setminus l) \neq \#(\Theta_Q \setminus l)$ then l is not pivotal as her message can influence $\#F$ votes only. On the other hand, if $\#(\Theta_A \setminus l) = \#(\Theta_A \setminus l')$ then l's message determines the vote of all voters (except her own vote) and, thus, she is pivotal. Therefore, if $\#F < N+1-(q-1)$ then whenever l is pivotal $\#(\Theta_A \setminus l) \neq \#(\Theta_Q \setminus l)$ and the option she prefers coincides with θ_l. In conclusion, $m_l = \theta_l$ and l has incentives to truthfully report her signal.

Consider now the situation where $\#F = N + 1 - q$, which in turn implies $\#O = q - 2$. In this case followers together with l are enough to block the implementation of A at the voting stage if they all vote for Q. If $\#(\Theta_A \setminus l) \neq \#(\Theta_Q \setminus l)$ then for each $m_l, m_{l'} \in \{A, Q\}$, and each $o \in O$, $v_l(m_l, \theta_{-l}) = v_l(m_l', \theta_{-l}) = v_{l'}(m_l, \theta_{-l}) = v_{l'}(m_l', \theta_{-l}) = v_o(m_l, \theta_{-l}) = v_o(m_l', \theta_{-l})$ and l is not pivotal. Assume thus that $\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l)$. There are two cases to be considered:

- If $\theta_{l'} = A$, then for each $i \neq l$ and each $m_l \in \{A, Q\}$, $v_i(m_l, \theta_{-l}) = m_l$ and $v_l(m_l, \theta_{-l}) = \theta_l$. Thus, $\mathbf{v}(m_l, \theta_{-l}) = m_l$ and l is pivotal.
- If $\theta_{l'} = Q$ then for each $f \in F$ and each $m_l \in \{A, Q\}$, $v_f(m_l, \theta_{-l}) = Q$. Moreover, for each $o \in O$, $v_o(m_l, \theta_{-l}) = v_{l'}(m_l, \theta_{-l}) = m_l$ and $v_l(m_l, \theta_{-l}) = \theta_l$. Assume that $\theta_l = A$. If $m_l = A$ then $\mathbf{v}(m_l, \theta_{-l}) = A$, while if $m_l = Q$ then only l votes for A and $\mathbf{v}(m_l', \theta_{-l}) = Q$. Thus, l is pivotal. Finally assume that $\theta_l = Q$, then for each $f \in F$ and each $m_l, m_l' \in \{A, Q\}$ $v_l(m_l, \theta_{-l}) = v_l(m_l', \theta_{-l}) = v_f(m_l, \theta_{-l}) = v_f(m_l', \theta_{-l}) = Q$ and $\mathbf{v}(m_l, \theta_{-l}) = \mathbf{v}(m_l', \theta_{-l}) = Q$. Therefore, l is not pivotal.

Thus, if $\#F < N + 1 - (q - 1)$ whenever l is pivotal $\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l)$ and l's best response is to send the message that coincides with her signal. Thus, $m_l = \theta_l$ and l reports truthfully.

Followers The analysis of the incentives of the followers is parallel to the case where there is a unique leader. If $\#F \geq q$ then the followers are never pivotal. If $\#F < N + 1 - (q - 1)$ (i.e. $\#O \geq q - 2$) then a follower f is pivotal if and only if $\#(\Theta_A \setminus f) = \#(\Theta_Q \setminus f)$. Since

$$P\left(\theta_l = \theta_{l'} | \#(\Theta_A \setminus f) = \#(\Theta_Q \setminus f)\right) = \frac{1}{2}$$

then followers are indifferent between sending the message $m_f = A$ or $m_f = Q$. Thus, it is a weakly dominant strategy to send the message that coincides with their signal. Hence, $m_f = \theta_f$ and followers report truthfully.

Objective Voters The analysis of objective voters incentives is parallel to the case with a unique leader. If $\#F \geq q$ then the objective voters are never pivotal. If $\#F < N+1-(q-1)$ (i.e. $\#O \geq q-2$) then a objective voter o is pivotal if and only if $\#(\Theta_A \setminus o) = \#(\Theta_Q \setminus o)$. Thus, with the same arguments as the ones used in the proof of Proposition 1 $m_o = \theta_o$ is the best response for o.

To conclude the proof, assume that $q > \#F \geq (N+1)-(q-1)$, which in turn implies $\#O \leq q-3$. In this case followers suffice to block the implementation of alternative A. For an arbitrary follower f to be pivotal it is necessary that $\theta_l = \theta_{l'} = A$ and $\#(\Theta_A \setminus f) = \#(\Theta_Q \setminus f)$. In this case, f prefers A to Q independently of the signal she receives and, hence, she has incentives to miss-report. □

Comparing the results in propositions 1 and 5 we can see that the existence of a second leader does not change the incentives to reveal information truthfully during the discussion stage. Next we extend this finding to the case where leaders are biased:

Proposition 6. *Let $L = \{l, l'\}$ with respective biases b, b' and assume that at least one of the leaders is biased, i.e. either $b \neq \frac{N}{2}+1$ or $b' \neq \frac{N}{2}+1$. There exists a FRE if and only if $\#F \geq q$, $b, b' \geq \frac{N}{2}+1$ and p is high enough relative to b and b'.*

Proof. Assume first that $\#F \geq q$. In this case followers and objective voters are never pivotal, thus, for each $i \in \{F, O\}$ we have that $m_i = \theta_i$ is a best response and both followers and objective voters report truthfully.

Consider leader l and assume that $b \geq \frac{N}{2}+1$. In this case leader l is pivotal only $\theta_{l'} = A$. Thus, if $\theta_l = A$ then since l has no information about the signals of neither followers nor objective voters she prefers A to Q if and only if

$$P\left(\#\Theta_A \geq b | \theta_l = \theta_{l'} = A\right) \quad \geq \quad \frac{1}{2}.$$

Note that the inequality above holds is true for $p = 1$ and since $P\left(\#\Theta_A \geq b | \theta_l = \theta_{l'} = A\right)$ is an increasing polynomial in p, for each b there is an interior value of p such that the inequality is satisfied: l prefers A to Q and $m_l = A$ is a best response for l. On the other hand, if $\theta_l = Q$ then $P\left(\#\Theta_A \leq \frac{N}{2} | \theta_l = A, \theta_{l'} = Q\right) \geq \frac{1}{2}$. Therefore, $P\left(\#\Theta_A \geq b | \theta_l = A, \theta_{l'} = Q\right) \leq \frac{1}{2}$ and l reports $m_l = Q$. Hence, if p is high enough relative to b we have that $m_l = \theta_l$ is the best-response for l. The same logic applies to l' and, thus, a FRE exists if $\#F \geq q$, $b, b' \geq \frac{N}{2} + 1$ and p is high enough relative to b and b'.

Consider now that $b < \frac{N}{2} + 1$. By similar arguments as the ones used above, whenever l is pivotal it must be true that $\theta_{l'} = A$. Thus, if $\theta_l = A$, then

$$P\left(\#\Theta_A \geq b|\theta_l = \theta_{l'} = A\right) \geq P\left(\#\Theta_A \geq \frac{N}{2} + 1|\theta_l = \theta_{l'} = A\right)$$

implies

$$P\left(\#\Theta_A \geq b|\theta_l = \theta_{l'} = A\right) \geq \frac{1}{2},$$

Therefore, l prefers A to Q and truth-telling is a best response. On the other hand, if $\theta_l = Q$ then

$$\begin{aligned} P\left(\#\Theta_A \geq \frac{N}{2} + 1|\theta_l = Q, \theta_{l'} = A\right) &= P\left(\#\Theta_Q \geq \frac{N}{2} + 1|\theta_l = Q, \theta_{l'} = A\right) \\ &= \frac{1}{2}. \end{aligned}$$

Since $P\left(\#\Theta_A \geq b|\theta_l = Q, \theta_{l'} = A\right) \geq P\left(\#\Theta_A \geq \frac{N}{2} + 1|\theta_l = Q, \theta_{l'} = A\right)$ then l prefers A to Q and $m_l = A$ is the best response for l. Thus, l has incentives to miss-report and a FRE does not exist.

Assume now that $\#F < q$, we need to show that there is always a voter whose best response whenever all other voters truthfully report their signals is to miss-report. We proceed by considering three possible scenarios: $q > \#F \geq (N+1) - (q-1)$, $\#F \leq N - q - 1$ and $\#F \in \{N - q, N - (q-1)\}$.

Consider first the case in which followers are enough to veto the alternative A: $q > \#F \geq (N+1) - (q-1)$. Consider a follower f, whenever f is pivotal it must be that $\theta_l = \theta_{l'} = A$. Thus, provided f is pivotal she prefers A to Q and $m_f = A$ is f's best response strategy independently on her signal. That is, she miss-reports.

Consider now the case where objective voters are enough to determine the voting outcome: $\#O \geq q$ ($\#F \leq N - q - 1$). A biased leader l is pivotal if and only if $\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l)$. If $b > \frac{N}{2} + 1$ then $\#\Theta_A < b$ and l prefers Q to A. Thus, $m_l = Q$ is l's best response independently of θ_l. On the other hand, if $b < \frac{N}{2} + 1$ then $\#\Theta_A \geq b$ and l prefers A to Q. Thus, $m_l = A$ is l's best response independently of θ_l. Hence, the best response for a biased leader is to report according to her bias and a FRE is not possible.

It only remains to study the case where $\#O \in \{q-2, q-1\}$ ($\#F \in \{N-q, N-(q-1)\}$). Consider first a situation where $b > \frac{N}{2} + 1$ and $b > b'$. If $\theta_{l'} = Q$ then all followers vote for Q. Moreover, since l's voting behavior does not depend on her message, for l to be pivotal her report has to change the voting behavior of either objective voters: $\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l) = \frac{N}{2}$, or the other leader l': $(\#\Theta_A \setminus l) = b' - 1$. Thus, since $b > \frac{N}{2} + 1$ and $b > b'$ if l is pivotal and $\theta_{l'} = Q$ then l prefers Q to A independently of θ_l and thus miss-reports. If, on the other hand, $\theta_{l'} = A$ then l's report affects the vote of all followers. If the message sent by l affects the vote of either objective voters or the other leader l' it must be that either $\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l) = \frac{N}{2}$ or $(\#\Theta_A \setminus l) = b' - 1$ and again l has incentives to missreport. Suppose then that m_l does not affect the vote of neither objective voters nor the other leader l'.

If both the objective voters and the other leader l' vote for Q independently of m_l then $\#\Theta_A < b$ and l prefers Q to A. Since in this situations both leaders and objective voters vote for Q and $\#O \in \{q-2, q-1\}$ the outcome is Q regardless of l's report and l is not pivotal. Assume now that objective voters and the other leader l' vote for A independently of m_l. If in this situation l prefers A to Q at the voting stage once all signals are revealed (she knows her own signal and all other voters are truthful at the discussion stage) then the voting outcome is A and again l is not pivotal. If instead l prefers Q to A at the voting stage once all signals are revealed then as objective voters and the other leader l' vote for A independently of m_l if $m_l = A$ we have that l is the only voter that votes for Q (thus A is implemented) whilst if $m_l = Q$ then followers and l vote for Q and the objective voters and l' vote for A. In this case if $\#O = q - 1$ then the outcome is always A and l is not pivotal. On the other hand if $\#O = q - 2$ then the followers and l are enough to veto A and the voting outcome is Q. Hence, l is pivotal if $\#O = q - 2$, $\theta_{l'} = A$ and l prefers Q to A at the voting stage once all signals are revealed. Finally, if objective voters and the remaining leader vote for different alternatives it is true that $\#\Theta_A < b$ and l prefers Q to A. Hence, whenever l is pivotal and she prefers Q to A at the voting stage once all signals are revealed (independently on whether she received signal A or Q). Hence, if l is pivotal she has incentives to miss-report by sending the message Q at the discussion stage regardless on the signal she receives. A symmetric argument applies to the case where $b < \frac{N}{2} + 1$ and $b < b'$ to prove that l's best response is to report A regardless on her signal.

Consider now the case where $b > \frac{N}{2} + 1$ but $b = b'$. Note that l is pivotal only if l's report changes the voting decision of either followers, objective voters or the decision of the other leader l'. If l's message determines the voting decision of the followers but leaves unchanged the decision of objective voters and the remaining leader then as in the previous paragraph l prefers Q to A regardless on her signal. On the other hand, since $b' \neq \frac{N}{2} + 1$ if l is pivotal then her message either determines the voting decision of objective voters $(\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l))$ or the voting decision of l' $(\#(\Theta_A \setminus l) = b - 1)$. If l is pivotal because her message determines the vote of objective voters then as $\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l) = \frac{N}{2}$ implies $\#\Theta_A < b$ we have that l prefers Q to A regardless on her signal. Conversely, if l is pivotal because her message determines the vote of the other leader l' then $\#\Theta_A = b-1$ then whether she prefers A or Q is determined by her own signal. If $P\left(\#(\Theta_A \setminus l) = \frac{N}{2}|\theta_l\right) \geq P\left(\#(\Theta_A \setminus l) = b-1|\theta_l\right)$ then l is at least as likely to be pivotal because $\#(\Theta_A \setminus l) = \frac{N}{2}$ as she is because $(\Theta_A \setminus l) = b-1$. Hence, l has incentive to report Q regardless on her signal and, thus, she miss-reports. A symmetric argument applies to the case where $b < \frac{N}{2}+1$ and $b = b'$ to prove that l's best response is to report A regardless on her signal.

In order to conclude the proof assume that $b > \frac{N}{2}+1$, $b = b'$ and $P\left(\#(\Theta_A \setminus l) = \frac{N}{2}|\theta_l\right) < P\left(\#(\Theta_A \setminus l) = b-1|\theta_l\right)$. For any arbitrary objective voter $o \in O$ this last inequality implies $P\left(\#(\Theta_A \setminus l) = \frac{N}{2}|\theta_o\right) < P\left(\#(\Theta_A \setminus l) = b-1|\theta_o\right)$. Since followers' voting decision is only influenced by both leaders' reports, o is pivotal only if either $\#(\Theta_A \setminus o) = \#(\Theta_Q \setminus o)$ (her message determines the voting decision of the remaining objective voters) or $\#(\Theta_A \setminus o) = b-1$ (her message determines the voting decision of the leaders). If o is pivotal because she changes the decision of the leaders then $\#\Theta_A \geq \frac{N}{2}+1$ and l prefers A to Q. Conversely, if l is pivotal because she changes the objective voters then whether se prefers A or Q depends on her own signal. Since $P\left(\#(\Theta_A \setminus l) = \frac{N}{2}|\theta_o\right) < P\left(\#(\Theta_A \setminus l) = b-1|\theta_o\right)$ it is more likely that o is pivotal because her message determines the voting behavior of the leaders $(\#(\Theta_A \setminus l) = b-1)$ than because $\#(\Theta_A \setminus l) = \frac{N}{2}$. Hence, o's best response is to report A regardless on the signal she receives. A symmetric argument applies to the case where $b < \frac{N}{2}+1$ and $b = b'$ to prove that o's best response is to report Q regardless on her signal. $\square$

7 Conclusions

In this paper we have analyzed information aggregation in deliberative committees under the presence of leadership. Deliberation is modeled as a cheap talk game where voters share non verifiable information about the right choice to be made. We have shown that the presence of leaders and voters who follow the reports of the leader may introduce distortions in the decision process. Specifically, the followers may have incentives to misreport the information they receive to obtain additional support for the option that matches the leader report when needed. Conversely, the leader may have incentives to compensate the followers' effect if when the signal she receives does not coincide with the right choice. The issues are more prevalent when the leader may have an "a priori" bias for one of the alternatives. Surprisingly, the problem is alleviated if there are many followers, since the leader knows that her report determines the final outcome and if the signal is accurate enough, it is more likely that the signal she receives coincides with the right choice (even if her preferences are biased). Similar results and intuitions apply to the more realistic case in which the leadership in the committee is shared between two voters.

We want to conclude highlighting some possible extensions and directions of further research. The key concept of this paper is that the discussion stage defines the preferences of the members of the committee about the final outcome. We have chosen the most simple framework and modeled deliberation as cheap talk revelation games. There is interesting literature on debate, strategic argumentation, and persuasion that could inspire new lines of research that incorporate additional realism in the definition of the preferences of the members of the committee (see for instance Glazer and Rubinstein (2001) Spiegler (2006) and references therein). In this paper, we have also made abstraction of reputation issues and dynamic component of the leadership. Since we have dealt with two option elections and static direct revelation games, our focus on pure strategies and FRE and PRE becomes natural. In a dynamic setting, reputational effects could compensate the direct incentives of biased leaders to support the options that confirm her bias, but the analysis would need to consider more elaborated (mixed) strategies and beliefs that support such strategies in equilibrium.

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