A conditional contribution mechanism for the provision of public goods

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Extended abstract

By definition public goods are non-excludable and have non-rivalry in consumption. This creates incentives to freeride on the contributions of others. Multiple mechanisms have been proposed to solve this issue. However, all these mechanisms have some drawbacks. Some rely on the existence of a central authority, need the enforceability of transfers between agents, or have high requirements of knowledge for agents to reach the proposed equilibrium of the mechanism. The conditional contribution mechanism (CCM) proposed in this paper needs none of those requirements. Furthermore, all agents have the possibility to contribute nothing, i.e. participation does not need to be enforceable.

In public good experiments about half the agents behave as conditional cooperators with a tendency towards selfishness. The proposed mechanism makes use of this fact and enables agents to conditionally contribute. When agents condition their contribution on the contribution of other agents, those agents have the incentive to fulfill this condition by contributing themselves. Thus, the contribution of one agent can in fact increase the amount contributed to the public good by much more than just his own contribution. This is the driving force behind the mechanism that solves the incentive problem. And this incentive might even be stronger than the tendency towards selfishness of the agents.

First, within the considered binary public good environment $n \in \mathbb{N}$ agents, labeled i, each have an endowment of one monetary unit, which they can invest into the public good or keep to themselves. An outcome is defined as $z := \{z_1, \ldots, z_n\} \in Z := \{0,1\}^n$, where $z_i = 1$ means that agent $i \in I$ contributes to the public good and $z_i = 0$ means that agent i does not contribute to the public good. For notational convenience I define $z_0 := \{0, \ldots, 0\}$ as the outcome without any contributions. All agents have a certain valuation $\theta_i \in [0, 1)$ for

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the public good, which is private information. The utility of agent i of a certain outcome is defined by

$$u_i(z) = 1 - z_i + \theta_i \sum_{j=1}^{n} z_j$$

In this environment I introduce the Binary Conditional Contribution Mechanism (BCCM). The mechanism is denoted by $G := \{M, g\}$. The message space of the mechanism is given by $M = \prod_{i=1}^{n} M_i$ with $M_i := \{0, \ldots, n\}$. A message of agent i is called m_i and $m_i = k$ translates into: “I contribute, if k others contribute as well.“ Note that the message space includes the options to unconditionally contribute ($m_i = 0$) and never contribute ($m_i = n$). The message profile of all agents but agent i will be denoted by m_{-i}. $M_{-i} := \prod_{j \neq i} M_j$ is defined in the same way. The outcome function g of the mechanism maps the message profile $m \in M$ to the outcome $z \in Z$ with the highest level of contribution which is consistent with the message profile:

$$\bar{k}(m) := \max\{k \in \{0, \ldots, n\} | \sum_{i=1}^{n} \mathbf{1}(m_i < k) \geq k\}$$
$$g(m) = z, \text{ with } z_i = 1 \Leftrightarrow m_i < \bar{k}(m)$$

Here $\mathbf{1}(m_i < k)$ represents the indicator function which is 1 if $m_i < k$ and zero otherwise. The BCCM has multiple Nash equilibria with outcomes from zero to full (or almost full, if $\theta_i < \frac{1}{n}$ for some agents) contribution[1]. An additional solution concept is needed to select from those equilibria. Therefore unexploitable better response dynamics (UBRD) are introduced. In UBRD agents switch from message m_i to m_i' if m_i' is a better response to m and if m_i' is unexploitable. A message m_i' is unexploitable in outcome z if for any $m_{-i} \in M_{-i}$ either $u_i(g(m_i', m_{-i})) \geq u_i(z)$ or $g(m_i', m_{-i})_i = 0$ holds. Here $g(m_i', m_{-i})_i$ is the ith component of the outcome $g(m_i', m_{-i})$.

A dynamic solution concept is chosen because the most common static solution concepts fail to explain how the equilibrium is achieved. There are many motivations for dynamic concepts: Firstly, the public good game can be repeated. Secondly, agents encounter similar one shot public good scenarios multiple times. Thirdly, the public good can be supplied step by step and supply is increased whenever the new round of the mechanism yields a higher outcome. Fourthly, if none of these points apply the mechanism could be adjusted to be not played once but a lager number of times. The outcome implemented will then be the highest level agreed upon in those rounds.

The outcomes of dynamic solution concepts are defined as the outcomes of their recurrent classes. A recurrent class of a dynamic concept is a set of message profiles, which if ever reached by the dynamics is never left and which contains no smaller set with the same property. It is shown that, if there exist outcomes, which are strict Pareto improvements

[1] Any mechanism which grants agents the possibility to freeride (or to not participate, which is equivalent) will have a Nash equilibrium with zero contributions.

over z_0 for all contributing agents, those outcomes are exactly the outcomes of recurrent classes of the BCCM under UBRD (Proposition 1).

The second environment is an extension of the first to non-binary contributions. Every agent has some endowment w_i, which can be invested into the public good from zero to w_i in small discrete steps. This resembles the real world smallest indivisible monetary unit (e.g. Cents in the USA, or the Euro-zone). The natural extension of the BCCM lets agents announce some amount of their money α_i as well as a threshold of total participation β_i. Total contribution must be at least β_i such that agent i is willing to contribute α_i. This mechanism does not give agents enough flexibility to explore possible Pareto improvements once they achieved some positive level of contribution. This leads in most cases to equilibria under the dynamics, which are not Pareto optimal (Proposition 2).

This issue can be solved by the conditional contribution mechanism (CCM). In this mechanism agents announce two (or more) tuples of the form (α_i, β_i). The outcome is defined by the condition that the highest amount, which is consistent with the messages chosen, is contributed to the public good . This leads to a well defined outcome function (Proposition 3). Agents have now the opportunity to keep one tuple fix. This will in general be responsible for the current outcome. Agents can change the other tuple(s) to explore possibilities for further Pareto improvements.

The central result of the paper is that $z \in Z$ is an outcome of a recurrent class of the CCM under UBRD if and only if z is Pareto optimal and a strict Pareto improvement over z_0, if at least one such outcome exists (Proposition 4).

If the environment is further generalized to cover monotone increasing instead of linear valuation functions not all Pareto optimal allocations are in the core anymore. Thus the outcome changes slightly. In this environment $z \in Z$ is an outcome of a recurrent class of the CCM under UBRD if and only if z is in the core and any deviation of a coalition from z makes at least one agent in that coalition strictly worse of. As before this holds if at least one such outcome exists (Proposition 5).

The paper presents a new mechanism for the provision of public goods. This mechanism needs no central authority, no enforceable transfers and works for heterogeneous valuations of the public good, which are private information. With the proposed solution concept of UBRD the CCM leads to Pareto optimal outcomes, which are furthermore Pareto improvements over the status quo (z_0).