Evolution of Reciprocity in Asymmetric Social Dilemmas[1]

Introduction

Reciprocity is a key mechanism to evolving cooperation in 2x2, repeated, symmetric Prisoner's Dilemmas. We extend this basic set-up along two dimensions. First, in a two-population - low and high types - evolutionary framework we let the payoffs accruing to the two population types differ either in the benefit and the cost of cooperation. Second, we investigate n-player PD games, arguably a more realistic scenario for situations typically referred to as tragedies of the commons. With these ingredients we introduce a generalized Tit-for-Tat behavioral rule for the repeated, n-player asymmetric, social dilemma game that commands "cooperate" provided that certain thresholds in the number of cooperators is reached in the low and high type population, respectively. Thus, Tit-for-Tatters in each population may evolve different degrees of "toughness", i.e. low or high cooperation *thresholds*.

Asymmetric trigger strategies

For each repeated game $n/2$ (n even) players drawn randomly from each population and play a n-player asymmetric PD. Let i, j denote the number of players of type I, J, respectively that played cooperatively. A repeated trigger strategy for conditional cooperators in the two populations could be defined as:

$$TFT^i_{\alpha_1,\beta_1}, TFT^j_{\alpha_2,\beta_2}, \{\alpha_1, \beta_1, \alpha_2, \beta_2\} \in [0, \frac{n}{2} - 1] \tag{1}$$

$TFT^i_{\alpha_1,\beta_1}$: *"start with C_i and play C_i if at least α_1 type I and β_1 type J players cooperated in previous round, otherwise play D_i"*

$TFT^j_{\alpha_2,\beta_2}$: *"start with C_j and play C_j if at least α_2 type I and β_2 type J players cooperated in previous round, otherwise play D_j"*

In the repeated game, expected payoffs for each rule $\{TFT^i_{\alpha_1,\beta_1}, AllD^i, TFT^j_{\alpha_2,\beta_2}, AllD^j\}$

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in a population of evolving rules denoted by $\Pi_{TFT^i_{\alpha_1,\beta_1}}, \Pi_{AllD^i}, \Pi_{TFT^j_{\alpha_2,\beta_2}}, \Pi_{AllD^j}$

Fractions of each behavioral rule in populations I, J are updated according to replicator dynamics.

$$\dot{\rho}_1 = \rho_1(1-\rho_1)(\Pi_{TFT^i_{\alpha_1,\beta_1}} - \Pi_{AllD^i}) = f_1(\rho_1, \rho_2, \Delta\Pi^i) \quad (2)$$

$$\dot{\rho}_2 = \rho_2(1-\rho_2)(\Pi_{TFT^j_{\alpha_2,\beta_2}} - \Pi_{AllD^j}) = f_2(\rho_1, \rho_2, \Delta\Pi^j) \quad (3)$$

Computation of $\Delta\Pi^i$ and $\Delta\Pi^j$ for $\{\alpha_1 \le \alpha_2, \beta_1 \le \beta_2\}$

Fig. 1 displays the possible play paths of TFT players for the given ordering of thresholds enabling us to compute, for all $k, l \in [0, \frac{n}{2}-1]$ sample draws, the corresponding repeated game payoffs. Next, expectations (i.e.summation) of all samples (k,l)-induced repeated game payoffs over the binomial distributions B_1, B_2 would result in the repeated game payoff for a type $I(J)$ player using $TFT^i_{\alpha_1,\beta_1}$ $(TFT^j_{\alpha_2,\beta_2})$ rule.

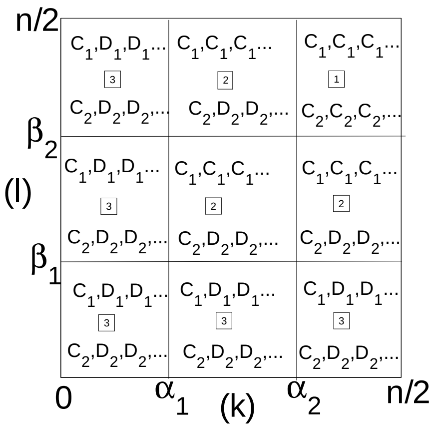
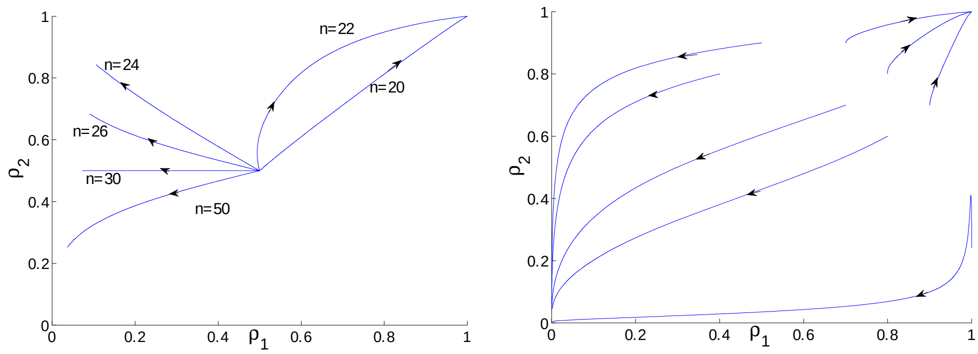


Figure 1: Paths of Tit-for-Tat play over all possible sampling configurations for the $\alpha_1 < \alpha_2$, $\beta_1 < \beta_2$ theresholds ordering

Preliminary results

Given the complicated, non-linear structure of the dynamical system (2)-(3) we report, as an illustration, numerical results for varying number of players n. Ceteris paribus, increasing the number of players worsens the social dilemma for symmetric thresholds($\alpha_1 =$

$\alpha_2 = 10; \beta_1 = \beta_2 = 10$) as depicted in Fig. 2a below. Whereas for relatively small number of participants reciprocity could still evolve in the two populations if a certain threshold is reached then unconditional defectors take over. The remaining benefit and costs parameters are set to $B = 40, b = 30; c_h = 2, c_l = 1; w = 0.9$. For a fixed number of players $n = 20$ and symmetric thresholds across populations (i.e. ; $\alpha_1 = \beta_1 = 0$ and $\alpha_2 = \beta_2 = 10$), Panel (b) displays a situation of co-existing equilibria if the two populations' thresholds are sufficiently asymmetric *within* each population.(e.g. $\alpha_1 \ll \alpha_2$). The rest of the parameters is set to $B = 20, b = 10; c_h = 2, c_l = 1; w = 0.9$.



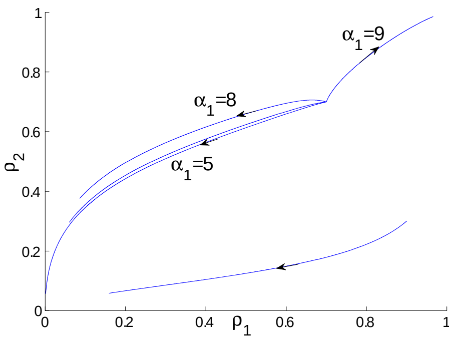
(a) Long-run attractors for varying number of players

(b) Phase plot $\alpha_1 = 0, \alpha_2 = 10; \beta_1 = 0, \beta_2 = 10$

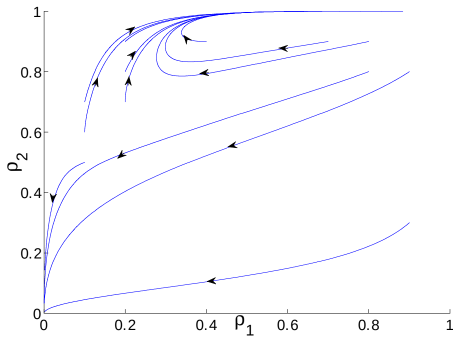
Figure 2: n-player asymmetric Prisoner's Dilemma with evolving ecologies of behavioral rules. Long-run equilibria for increasing number of players (panel (a)). phase portrait for symmetric thresholds (panel (b))

Role of cooperation thresholds

Fig. 3a shows trajectories originating at a given initial fractions of TFT players for increasing $\alpha_1 \in \{5, 7, 8, 9\}$ and the other parameters set to $n = 20; \alpha_2 = 10; \beta_1 = 9, \beta_2 = 10; B = 20, b = 10; c_h = 2, c_l = 1; w = 0.9$. Unless type I conditions on *all* self-typed players playing cooperate (i.e.$\alpha_1 = 9$) the dynamics with asymmetric players cannot sustain cooperation. Panel (b) displays the phase plot for a situation with a complacent type I player($\alpha_1 = 0, \alpha_2 = 0$) and an extremely stringent type II ($\beta_1 = 10, \beta_2 = 10$) leading to bi-stability, albeit with asymmetric basins of attraction: the all tit-for-tat equilibrium attracts only a limited set of initial conditions). Remaining parameters: $n = 20, B = 20, b = 10; c_h = 2, c_l = 1; w = 0.9$.



(a) Stable equilibria for varying α



(b) Co-existing equilibria

Figure 3: n-player asymmetric Prisoner's Dilemma with evolving ecologies of behavioral rules. Long-run equilibria for increasing thresholds for type I player(Panel (a). phase portrait for asymmetric thresholds (panel (b))

The evolutionary success of the generalized TFT strategy conditioning on own and other type reaching a critical mass of cooperators is evaluated within an ecology of repeated rules appended with unconditional defectors.Preliminary results suggest that there exists regions in the relevant parameter space - i.e. discount factor, the two types' thresholds, the asymmetric benefits and costs, etc. - such that (partial) cooperation may emerge as long-run attractor of a monotone selection evolutionary dynamic.