Subgame Perfect Equilibria in Majoritarian Bargaining

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Abstract

We study three-person bargaining games with discounting, where an alternative is accepted if it is approved by a majority of players. We characterize the set of subgame perfect equilibrium payoffs and show that for any proposal in the space of possible agreements there exists a discount factor such that given the proposal is made and accepted by one of the players in period zero. Also we construct a subgame perfect equilibrium such that arbitrary long delay of acceptance occurs with probability one.

KEYWORDS: dynamic games, bargaining, majoritarian voting, subgame perfect equilibrium, delay of acceptance.

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1 Introduction

In bargaining games two or more players are trying to reach an agreement regarding the distribution of a surplus. Rubinstein (1982) examines two-person alternating offers games and proves uniqueness of subgame perfect equilibrium in the model. In the unique subgame perfect equilibrium the first proposal is immediately accepted by the opponent. The framework provided by Rubinstein is widely used in bargaining literature and has been extended in many directions.

One extension of the Rubinstein (1982) model concerns the application to collective choice problems, see Baron and Ferejohn (1989), Harrington (1990), Baron and Kalai (1993), and Banks and Duggan (2000). These papers study multilateral bargaining games, where an alternative is accepted if it is approved by a set of players that belongs to a collection of decisive coalition. This approach makes it possible to study a variety of important institutional set-ups,including the one of majority voting. In Baron and Ferejohn (1989) n-player sequential bargaining game is considered. The proposer selection process and the order of responders are modeled by time-invariant recognition probabilities and acceptance of the simple majority is enough to implement the proposal. Baron and Ferejohn show that in the described game stationary subgame perfect equilibrium is unique and on the equilibrium path of play the proposal, that gives δ/n to a half of responding players selected at random and the rest to a player recognized as a proposer is made and accepted in period zero.

This result was used as theoretic prediction in the broad list of experimental literature, see McKelvey (1991), Frechette et al. (2005), Diermeier and Morton (2005), Miller and Vanberg (2010), and Breitmoser and Tan (2010). McKelvey (1991) studies a stochastic bargaining game with three voters and three alternatives, Frechette et al. (2005) study four experimental designs of Baron-Ferejohn model with three players, considering different weights (amount of votes of a player) and different selection probabilities, Diermeier and Morton (2005) consider a finitely-repeated version of the model consisting of five periods with a zero payoff if no agreement is reached and different recognition probabilities for players, Miller and Vanberg (2010) compare the costs of reaching agreement under majority and unanimity rules, and Breitmoser and Tan (2010) introduce an experiment of Baron-Ferejohn model with three players and discount factor equal to 0.95. Most of the papers concluded, that experimental data is hardly explained by Baron-Ferejohn prediction. In the data proposers usually offer too much to the responding players and the proposals have been accepted with too high probability. It is explained by generosity concepts, inequality averse concepts and many others. As an explanations to the experiment McKelvey (1991) mentions possibility that subjects are trying to find a non–stationary equilibrium. We show that more complicated strategies may lead to broader set of subgame perfect equilibrium payoffs that are in line with surprising results of the experiments.

In this paper we study Baron-Ferejohn model with three players. We use subgame perfect equilibrium as a solution concept. A subgame perfect equilibrium is only sensible if we know what players would do when they reach a particular decision. Thus, the subgame perfect equilibrium must express all actions at all information sets, regardless of whether that information set is actually reached in equilibrium. As it was noted by Rubinstein (1991), strategy of a player includes not only his own plan of actions, but also his opponent's beliefs, that he does not follow the plan of actions. In contrast with stationary strategies, the concept of subgame perfect equilibrium requires re-examination of the strategy every period of the game. We characterize the set of subgame equilibrium payoffs, next we construct subgame perfect equilibria, where for any given vector from the set of subgame perfect equilibria payoffs there exists discount factor, such that the given vector is proposed and accepted in period zero.

When discount factor goes to zero, the set of subgame equilibria payoffs shrinks to the unit vector, that gives one to the player, recognized as a proposer in period 0. When discount factor goes to one, the set of subgame equilibria payoffs expands to the set of feasible payoffs. It holds that for any vector in the set of feasible payoffs there exist a discount factor such that the constructed strategy profile is a subgame perfect equilibrium. Nevertheless, the usual statement of the folk theorem, stating that for sufficiently high discount factor any individually rational payoff is a subgame perfect equilibrium payoff, does not hold in our model. Baron and Ferejohn (1989) proved a folk theorem for a game with at least five players, and the provided proof does not work for the three player bargaining game. Chatterjee et. al. (1993) look at strictly superadditive games and show that for sufficiently high discount factor any individually rational outcome is possible, which does not hold in our setting. Norman (2002) shows that any interior division can be supported as a subgame perfect equilibrium outcome if players are sufficiently patient and there are sufficiently many rounds of bargaining.

There are some evidence for delay in acceptance in the experimental studies of Baron-Ferejohn model such as Miller and Vanberg (2010), Agranov and Tergiman (2012). In this paper we provide necessary and sufficient conditions for one period of delay in acceptance. We show that there is a discount factor, such that for any greater discount factor one period of delay is possible and for any smaller discount factor the subgame perfect equilibrium with delay in acceptance does not exist. Also we report on the possibility of arbitrary long delay of acceptance in subgame perfect equilibrium strategy profile when the discount factor is above a particular threshold.

So, by Rubinstein (1982) the uniqueness of the subgame perfect equilibrium for two players is proven, by Baron and Ferejohn (1989) a folk theorem for the game with five or more players is proven, and it was not investigated yet what allocations of the benefits may be supported as a subgame perfect equilibrium in a bargaining game with three players. Remarkable papers that consider majoritarian bargaining with three players are Baron and Herron (1999), who analyze a game where players have Euclidian preferences over a two dimensional space of legislative outcomes, Kalandrakis (2004), who considers a game, where in each period a new dollar is divided and fully characterize a Markov equilibrium in this model, Tsai (2009), who investigates bargaining game with incomplete information about time preferences, and Herings and Houba (2010), who look at mixed consistent subgame perfect equilibria in the bargaining game with three alternatives.

The rest of the paper is organized as follows. In Section 2 we describe the bargaining game, state the main result, and derive the bounds for subgame perfect equilibria outcomes. In Section 3 we construct subgame perfect equilibrium and complete the prove of the main result. In Section 4 we provide necessary and sufficient conditions for one period of delay in acceptance. In Section 5 we investigate arbitrary long delay of acceptance in subgame perfect equilibria strategies. Section 6 concludes.

2 The bounds for subgame perfect equilibrium payoffs

We consider a three player dynamic game of perfect information Γ. The set of players denoted by $N = \{1, 2, 3\}$ has to agree on the choice of a payoff vector in the set of feasible payoffs $V = \{x \in \mathbb{R}^3_+,\ x_1 + x_2 + x_3 \leq 1\}$. In each time period $t = 0, 1, 2, \ldots$ nature selects the proposer and the order of responders. The proposer selection process and the order of responders are modeled by time-invariant recognition probabilities. The probability that player $i \in N$ is recognized to make a proposal is equal to $1/3$ across all periods. Also the probability that player $i \in N$ is recognized to respond first is equal to $1/3$ across all periods. More precisely, in each time period $t = 0, 1, 2, \ldots$ nature selects the proposer and the order of responders by means of a permutation $\pi^t : \{1, 2, 3\} \to N$, and every permutation is chosen with equal probability. The player, who is recognized by nature as a proposer, makes a proposal $x^t \in V$. Then the other two players sequentially respond to the proposal by accepting or rejecting it. If one of the responding players accepts the proposal, the game ends and the proposal is implemented. If both players reject the proposal, the next period begins. The utility of player $i \in N$ who receives outcome x_i in period t is $\delta^t x_i$, where $\delta \in [0, 1)$ is the common discount factor.

A history h is the sequence of all actions that have occurred before a particular decision node in the game. For simplicity we suppress the elements of the sequence pertaining to the moves by the responding players: it is understood that in any non–terminal history both responders have rejected all proposals to date. With this convention any non-terminal history is of one of the following two types:

1. $h \in H_1^t$ if and only if h is of the form $h = (\pi^0, x^0, \dots, \pi^{t-1}, x^{t-1}, \pi^t)$,
2. $h \in H_2^t$ if and only if h is of the form $h = (\pi^0, x^0, \dots, \pi^{t-1}, x^{t-1}, \pi^t, x^t)$,

where π^k is a permutation of the set N and $x^k \in V$ for every k. After history $h \in H_1^t$ a proposer moves, and after history $h \in H_2^t$ responders move. Histories in H_1^t will be called *proposer histories* while those in H_2^t *responder histories.*

Strictly speaking, one has to distinguish two types of responder histories: histories after which the first responder moves and histories after which the second responder moves. We shall not make such distinction; the symbol $h \in H_2^t$ might denote any of these histories. When we talk about Player $\pi^t(3)$ casting a vote after history h, it is to be understood that the Player $\pi^t(2)$ has already rejected the proposal.

Theorem 2.1 (Baron, Ferejohn, 1989) *For all $\delta \in [0, 1)$ a configuration of pure strategies is a stationary subgame-perfect equilibrium in an infinite session, majority rule, n-member (with n odd) if and only if it has the following form: (1) a member recognized proposes to receive $1 - \delta(n-1)/2n$ and offers δ/n to $(n-1)/2$ other members selected at random; (2) each member accepts any proposal in which at least δ/n is received, and rejects otherwise. The proposal in period 0 is made and accepted.*

Let the the bounds for subgame perfect equilibrium (SPE) payoffs be denoted by

$$\begin{aligned} \overline{u}_i &= \sup\{u_i \mid u_i \text{ is a SPE payoff for player } i\}, \\ \underline{u}_i &= \inf\{u_i \mid u_i \text{ is a SPE payoff for player } i\}. \end{aligned}$$

Define

$$\underline{b} = \frac{3 - 3\delta}{9 - 6\delta - \delta^2} \tag{2.1}$$

$$\overline{b} = \frac{3 - \delta}{9 - 6\delta - \delta^2}. \tag{2.2}$$

These are plotted plotted in the Figure 1 as functions of δ. The main result of the paper is the following theorem.

Theorem 2.2 (Main result) *It holds that $\overline{u}_1 = \overline{u}_2 = \overline{u}_3 = \overline{b}$ and $\underline{u}_1 = \underline{u}_2 = \underline{u}_3 = \underline{b}$.*

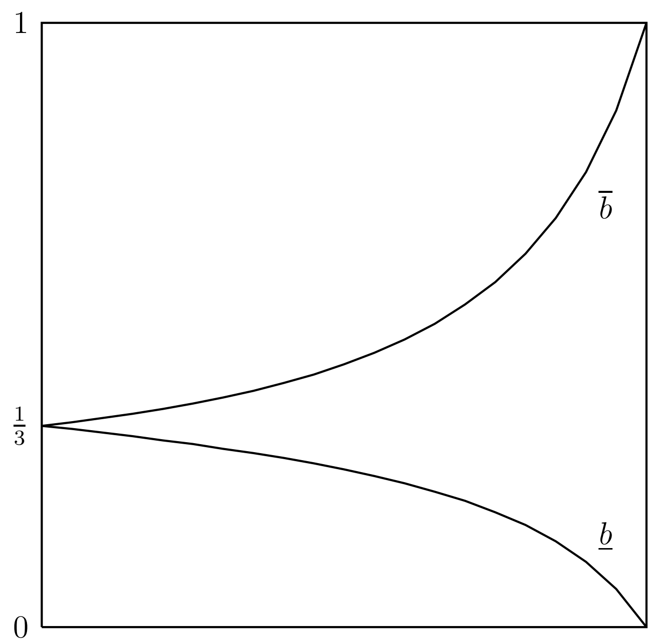


Figure 1: $\underline{b}$ and $\overline{b}$ as functions of δ.

The proof of the Theorem 2.2 consists of two parts. The first part, carried out in the remainder of this section, consist in showing that in any subgame perfect equilbrium the payoff to any player is bounded below by $\underline{b}$ and above $\overline{b}$, that is $\underline{b} \leq \underline{u}_i$ and $\overline{u}_i \leq \overline{b}$. In the next section we show that these bounds are tight: we explicitly construct a subgame perfect equilibrium where Player 2 receives payoff $\overline{b}$ and Player 3 the payoff $\underline{b}$.

Due to the symmetry of the game it is clear that $\overline{u}_1 = \overline{u}_2 = \overline{u}_3$ and $\underline{u}_1 = \underline{u}_2 = \underline{u}_3$. We henceforth write simply $\overline{u}$ instead of $\overline{u}_i$ and $\underline{u}$ instead of $\underline{u}_i$. In the remainder of this section we prove that $\underline{b} \leq \underline{u}$ and $\overline{u} \leq \overline{b}$. The proof proceeds in a succession of claims.

Claim 2.3 *Consider a proposer history h ending with a permutation π. In any subgame perfect equilibrium σ:*

[1] *A proposal $x \in V$ by player $\pi(1)$ such that $x_{\pi(2)} > \delta\overline{u}$ or $x_{\pi(3)} > \delta\overline{u}$ is accepted by player $\pi(2)$ or by player $\pi(3)$.*

[2] *A proposal $x \in V$ by player $\pi(1)$ such that $x_{\pi(2)} < \delta\underline{u}$ and $x_{\pi(3)} < \delta\underline{u}$ is rejected by both players $\pi(2)$ and $\pi(3)$.*

Proof: [1] Let $x \in V$ be such that $x_i > \delta\overline{u}$ for some $i \in \{\pi(2), \pi(3)\}$. Suppose that according to the strategy σ the proposal x is accepted by neither player $\pi(2)$ nor $\pi(3)$. Consider the responder history (h, x). The payoff to player i on the strategy σ at (h, x) is at most $\delta\overline{u}$. On the other hand, accepting the proposal x yields player i the payoff x_i. Thus player i has a one–shot profitable deviation at (h, x), leading to a contradiction.

[2] Let $x \in V$ be such that $x_i < \delta \underline{u}$ for both $i \in \{\pi(2), \pi(3)\}$. Consider player $\pi(3)$'s action at the responder history (h, x). Since accepting x yields the payoff x_i and rejecting x gives at least $\delta \underline{u}$, player $\pi(3)$ must reject. Consider now player $\pi(2)$'s action at (h, x). Rejection of x by player $\pi(2)$ is followed by the rejection of x by player $\pi(3)$ and so yields player $\pi(2)$ the payoff of at least $\delta \underline{u}$. We conclude that player $\pi(2)$ must reject x. $\square$

Claim 2.4 *It holds that* $\underline{u} + \overline{u} \leq 1$.

Proof: Let $u = (u_1, u_2, u_3)$ be a subgame perfect equilibrium payoff vector. Then $u_1 + u_2 \leq u_1 + u_2 + u_3 \leq 1$. Therefore $u_1 \leq 1 - u_2$. Hence

$$\overline{u} = \sup\{u_1\} \leq \sup\{1 - u_2\} = 1 - \inf\{u_2\} = 1 - \underline{u},$$

where the supremum and the infimum are taken over all subgame perfect equilibrium payoff vectors u. $\square$

Claim 2.5 *Consider period* 0 *proposer history* $h = (\pi)$. *Let* $v = (v_1, v_2, v_3)$ *be the payoffs on a subgame perfect equilibrium* σ *conditional on history* h *being reached. It holds that*

[1] $1 - \delta \overline{u} \leq v_{\pi(1)}$,

[2] $v_{\pi(1)} \leq 1 - \delta \underline{u}$,

[3] $v_{\pi(2)} \leq \delta \overline{u}$ *and* $v_{\pi(3)} \leq \delta \overline{u}$.

Proof: [1] By Claim 2.3 the proposal x where $x_{\pi(1)} = 1 - \epsilon - \delta \overline{u}$, $x_{\pi(2)} = \epsilon + \delta \overline{u}$ and $x_{\pi(3)} = 0$ is accepted for each $\epsilon > 0$. Since σ is a subgame perfect equilibrium we have $1 - \epsilon - \delta \overline{u} \leq v_{\pi(1)}$ for each $\epsilon > 0$. Therefore $1 - \delta \overline{u} \leq v_{\pi(1)}$.

[2] Let x be player $\pi(1)$'s proposal at h under the strategy σ. Suppose first that x is rejected under σ. Then clearly $v_{\pi(1)} \leq \delta \overline{u}$. It follows from the preceding claim that $\delta \overline{u} \leq 1 - \delta \underline{u}$ and therefore that $v_{\pi(1)} \leq 1 - \delta \underline{u}$. Suppose now x is accepted. Then by Claim 2.3 it is the case that $x_{\pi(2)} \geq \delta \underline{u}$ or $x_{\pi(3)} \geq \delta \underline{u}$. Therefore $v_{\pi(1)} = x_{\pi(1)} \leq 1 - x_{\pi(2)} - x_{\pi(3)} \leq 1 - \delta \underline{u}$.

[3] We have $v_{\pi(2)} \leq 1 - v_{\pi(1)} - v_{\pi(3)} \leq 1 - v_{\pi(1)} \leq \delta \overline{u}$ where the last inequality follows from [1]. The argument for $v_{\pi(3)}$ is similar. $\square$

Claim 2.6 *It holds that*

$$\frac{1 - \delta \overline{u}}{3} \leq \underline{u} \leq \overline{u} \leq \frac{1 - \delta \underline{u}}{3 - 2\delta}.$$

Proof: Let σ be a subgame perfect equilibrium with payoff vector $u = (u_1, u_2, u_3)$. For each permutation $\pi \in \Pi$ let $v^\pi = (v_1^\pi, v_2^\pi, v_3^\pi)$ denote the payoff on σ conditional on nature choosing permutation π. Since each permutation is equally likely we have

$$u = \tfrac{1}{6} \sum_{\pi \in \Pi} v^\pi.$$

There are exactly two permutation where player 1 is the proposer, namely $(1, 2, 3)$ and $(1, 3, 2)$. For each of these two permutations π it holds that $1 - \delta\overline{u} \leq v_1^\pi \leq 1 - \delta\underline{u}$, by parts [1–2] of the preceding claim. In each of the remaining permutations π player 1 is a responding player and so $0 \leq v_1^\pi \leq \delta\overline{u}$ by part [3] of the preceding claim. We conclude that

$$\tfrac{2}{6}(1 - \delta\overline{u}) + \tfrac{4}{6}0 \leq u_1 \leq \tfrac{2}{6}(1 - \delta\underline{u}) + \tfrac{4}{6}\delta\overline{u}.$$

Since the inequalities holds for each subgame perfect equilibrium payoff u, we have

$$\tfrac{1}{3}(1 - \delta\overline{u}) \leq \underline{u} \leq \overline{u} \leq \tfrac{1}{3}(1 - \delta\underline{u}) + \tfrac{2}{3}\delta\overline{u}.$$

Rearranging the last inequality yields the result. □

We are now in a position to prove the first half of Theorem 2.2, namely: the payoff to any player in any subgame perfect equilibrium of the game is bounded below by $\underline{b}$ and above by $\overline{b}$.

Theorem 2.7 *It holds that $\underline{b} \leq \underline{u}$ and $\overline{u} \leq \overline{b}$.*

Proof: Define the functions $f, g : [0, 1] \to [0, 1]$ by

$$f(x) = \frac{1 - \delta x}{3} \text{ and } g(x) = \frac{1 - \delta x}{3 - 2\delta}$$

Also let $h = g \circ f$. Then h is given by

$$h(x) = \frac{3 - \delta + \delta^2 x}{9 - 6\delta}.$$

It is easy to see that h is a contraction and that $\overline{b}$ is a fixed point of h.

The preceding claim implies that

$$f(\overline{u}) \leq \underline{u} \text{ and } \overline{u} \leq g(\underline{u}).$$

Using the fact that g is a decreasing function we obtain $\overline{u} \leq g(\underline{u}) \leq g(f(\overline{u}))$. Hence $\overline{u} \leq h(\overline{u})$. Since h is an increasing function, we can iterate the last inequality to obtain $\overline{u} \leq h^n(\overline{u})$. By the Banach theorem $h^n(\overline{u})$ converges to $\overline{b}$. Hence we obtain $\overline{u} \leq \overline{b}$. Therefore $f(\overline{b}) \leq f(\overline{u}) \leq \underline{u}$. An easy computation shows that $\underline{b} = f(\overline{b})$. This completes the proof. □

3 The subgame perfect equilibrium strategies

In the previous section we have shown that the payoff to any player in any subgame perfect equilibrium is bounded below by $\underline{b}$ and above by $\overline{b}$. In this section we show that these bounds are tight. We explicitly construct a subgame perfect equilibrium strategy yielding player 2 the payoff $\overline{b}$ and player 3 the payoff $\underline{b}$.

We define

$$\begin{aligned} V_1 &= \{(v_1^1, v_2^1, v_3^1) | \quad v_1^1 \geq 1 - \delta\overline{b} \quad \text{and} \quad [v_2^1 \geq \delta\underline{b} \quad \text{or} \quad v_3^1 \geq \delta\underline{b}]\} \\ V_2 &= \{(v_1^2, v_2^2, v_3^2) | \quad v_2^2 \geq 1 - \delta\overline{b} \quad \text{and} \quad [v_3^2 \geq \delta\underline{b} \quad \text{or} \quad v_1^2 \geq \delta\underline{b}]\} \\ V_3 &= \{(v_1^3, v_2^3, v_3^3) | \quad v_3^3 \geq 1 - \delta\overline{b} \quad \text{and} \quad [v_1^3 \geq \delta\underline{b} \quad \text{or} \quad v_2^3 \geq \delta\underline{b}]\}. \end{aligned}$$

Theorem 3.1 *Take $a_1 \in V_1$, $a_2 \in V_2$, and $a_3 \in V_3$. Then there exists subgame perfect equilibrium σ such that for each permutation π^0 the proposal $a_{\pi^0(1)}$ is made and accepted in period* 0*.*

Assuming Theorem 3.1 we complete the proof of Theorem 2.2. Take $a_1 = (1-\delta\overline{b}, \delta\overline{b}, 0)$, $a_2 = (\delta\underline{b}, 1 - \delta\underline{b}, 0)$, and $a_3 = (0, \delta\overline{b}, 1 - \delta\overline{b})$. Clearly $a_i \in V_i$. Now let σ be the strategy as in Theorem 3.1. Then the expected payoff for Player 2 on σ is $\overline{b}$, and the expected payoff for Player 3 is $\underline{b}$, where we have used the equations

$$\underline{b} = \tfrac{1}{3}(1 - \delta\overline{b}) \text{ and } \overline{b} = \tfrac{1}{3}(1 - \delta\underline{b}) + \tfrac{2}{3}\delta\overline{b}. \tag{3.1}$$

The rest of this section is dedicated to the proof of Theorem 3.1. The theorem is proven separately for the case $\delta \leq 3/5$ and the case $\delta > 3/5$, but in both cases the strategy σ is constructed along the same lines.

We first introduce some additional notation. Consider a history $h = (\pi^0, x^0, \ldots, \pi^t, x^t)$. We denote the proposer in period t by p^t, in other words $\pi^t(1) = p^t$. We denote by j^t the responding player in period t, who has been proposed a higher share of the cake than the other responding player, i.e.

$$j^t = \begin{cases} \pi^t(2), & \text{if} \quad [x^t_{\pi^t(2)} \geq x^t_{\pi^t(3)}], \\ \pi^t(3), & \text{if} \quad [x^t_{\pi^t(3)} > x^t_{\pi^t(2)}]. \end{cases}$$

The other responding player in period t is denoted by k^t. We write x_i^t rather than $x_{i^t}^t$. A proposer history $h = (\pi^0, x^0, \ldots, \pi^t, x^t, \pi^{t+1})$ uniquely defines the sequence

$$h^* = (p^0, x^0, j^0, k^0, \ldots, p^t, x^t, j^t, k^t, p^{t+1}).$$

Sequences of this form will be used to define the proposer strategy, denoted by $\rho^0, \rho^1, \ldots$. These will be defined recursively as follows: Set $\rho^0(p^0) = a_{\pi^0(1)}$ and for each $t \geq 0$ let

$$\rho^{t+1}(p^0, \ldots, p^t, x^t, j^t, k^t, p^{t+1}) = f(p^t, x^t, j^t, k^t, p^{t+1}, \rho^t(p^0, \ldots, p^t)),$$

where the function f will be specified below. With the minor abuse of notation we write $\rho^t(h)$ rather than $\rho^t(h^*)$. Note that for $\delta > 3/5$ the following holds: $\underline{b} < \overline{b}/2$, which plays important role in the construction.

3.1 The proof of Theorem 3.1 for $\delta \leq 3/5$

Suppose $\delta \leq 3/5$. The function f is defined below in Table 1 and Table 2. The first three columns specify the players p^t, j^t, k^t, and the remaining three columns specify the values $f(p^t, x^t, j^t, k^t, p^{t+1}, y^t)$ depending on the value of p^{t+1}. Table one applies if $x^t = y^t$ and Table 2 applies if $x^t \neq y^t$. Thus in particular, if after the proposer history h in period t the proposer deviates from the strategy σ and makes a proposal $x^t \neq \rho^t(h)$, then the proposals in period $t+1$ are determined by Table 2. If in period t the proposer complies with the strategy σ and makes a proposal $x^t = \rho^t(h)$, then the proposals in period $t+1$ are determined by Table 1.

Table 1: $f(p^t, x^t, j^t, k^t, p^{t+1}, x^t)$.

p^t	j^t	k^t	$p^{t+1} = 1$	$p^{t+1} = 2$	$p^{t+1} = 3$
1	2	3	$(1-\delta\underline{b}, 0, \delta\underline{b})$	$(\delta\overline{b}, 1-\delta\overline{b}, 0)$	$(\delta\overline{b}, 0, 1-\delta\overline{b})$
1	3	2	$(1-\delta\underline{b}, \delta\underline{b}, 0)$	$(\delta\overline{b}, 1-\delta\overline{b}, 0)$	$(\delta\overline{b}, 0, 1-\delta\overline{b})$
2	1	3	$(1-\delta\overline{b}, \delta\overline{b}, 0)$	$(0, 1-\delta\underline{b}, \delta\underline{b})$	$(0, \delta\overline{b}, 1-\delta\overline{b})$
2	3	1	$(1-\delta\overline{b}, \delta\overline{b}, 0)$	$(\delta\underline{b}, 1-\delta\underline{b}, 0)$	$(0, \delta\overline{b}, 1-\delta\overline{b})$
3	1	2	$(1-\delta\overline{b}, 0, \delta\overline{b})$	$(0, 1-\delta\overline{b}, \delta\overline{b})$	$(0, \delta\underline{b}, 1-\delta\underline{b})$
3	2	1	$(1-\delta\overline{b}, 0, \delta\overline{b})$	$(0, 1-\delta\overline{b}, \delta\overline{b})$	$(\delta\underline{b}, 0, 1-\delta\underline{b})$

Table 2: $f(p^t, x^t, j^t, k^t, p^{t+1}, y^t)$ where $x^t \neq y^t$.

p^t	j^t	k^t	$p^{t+1} = 1$	$p^{t+1} = 2$	$p^{t+1} = 3$
1	2	3	$(1-\delta\overline{b}, \delta\overline{b}, 0)$	$(\delta\underline{b}, 1-\delta\underline{b}, 0)$	$(0, \delta\overline{b}, 1-\delta\overline{b})$
1	3	2	$(1-\delta\overline{b}, 0, \delta\overline{b})$	$(0, 1-\delta\overline{b}, \delta\overline{b})$	$(\delta\underline{b}, 0, 1-\delta\underline{b})$
2	1	3	$(1-\delta\underline{b}, \delta\underline{b}, 0)$	$(\delta\overline{b}, 1-\delta\overline{b}, 0)$	$(\delta\overline{b}, 0, 1-\delta\overline{b})$
2	3	1	$(1-\delta\overline{b}, 0, \delta\overline{b})$	$(0, 1-\delta\overline{b}, \delta\overline{b})$	$(0, \delta\underline{b}, 1-\delta\underline{b})$
3	1	2	$(1-\delta\underline{b}, 0, \delta\underline{b})$	$(\delta\overline{b}, 1-\delta\overline{b}, 0)$	$(\delta\overline{b}, 0, 1-\delta\overline{b})$
3	2	1	$(1-\delta\overline{b}, \delta\overline{b}, 0)$	$(0, 1-\delta\underline{b}, \delta\underline{b})$	$(0, \delta\overline{b}, 1-\delta\overline{b})$

We define the strategy σ as follows:

1. For each $t \geq 0$, for each history $h = (\pi^0, x^0, \ldots, \pi^{t-1}, x^{t-1}, \pi^t)$, Player p^t proposes $\rho^t(h)$

2. After history $h = (\pi^0, x^0, \ldots, \pi^{t-1}, x^{t-1}, \pi^t, x^t)$ Player j^t accepts x^t if

   - $x^t = \rho^t(h)$ or
   - $x^t_j \geq \delta\overline{b}$,

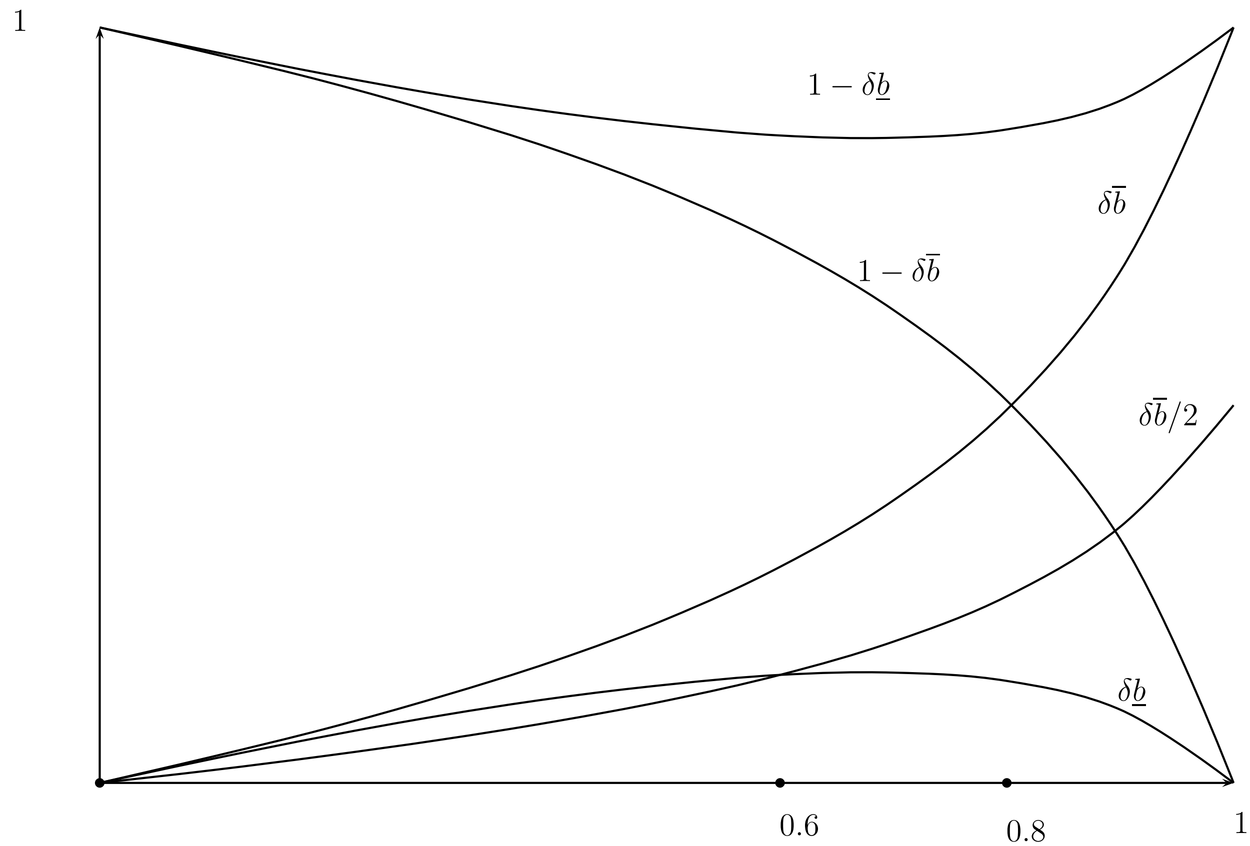
   and rejects otherwise.

3. After history $h = (\pi^0, x^0, \ldots, \pi^{t-1}, x^{t-1}, \pi^t, x^t)$ Player k^t accepts x^t if

   - $x^t = \rho^t(h)$ and $x^t_k \geq \delta^2\underline{b}/3 + \delta\underline{b}$ or
   - $x^t \neq \rho^t(h)$ and $x^t_k \geq \delta\underline{b}$,

   and rejects otherwise.

The following graph helps to understand the values of equilibrium proposals depending on δ, which is measured on X axis.



The intuition of the strategy profile is the following. Table 1 determines $\rho^t(h)$ after the history $h = (\pi^0, x^0, \dots, x^{t-1}, \pi^t)$ such that $x^{t-1} = \rho^{t-1}(\pi^0, x^0, \dots, \pi^{t-1})$. In other words it prescribes the proposal to be made after equilibrium proposal was made and rejected in previous period. According to the Table 1 Player p^{t-1} obtains highest possible expected payoff of $\overline{b}$, as an encouragement for making a proposal prescribed by the strategy profile in the previous period; Player j^{t-1} obtains lowest possible expected payoff of $\underline{b}$, as a punishment for rejecting the proposal prescribed by the strategy profile; Player k^{t-1} receives the rest, which is $\underline{b} + \delta\underline{b}/3$.

Table 2 determines $\rho^t(h)$ after the history $h = (\pi^0, x^0, \dots, x^{t-1}, \pi^t)$ such that $x^{t-1} \neq \rho^{t-1}(\pi^0, x^0, \dots, \pi^{t-1})$. In other words it prescribes the proposal to be made after the deviation of the proposer in previous period. According to the Table 2 Player p^{t-1} obtains expected payoff of $\underline{b} + \delta\underline{b}/3$, as a punishment for deviating in the previous period. Player j^{t-1} obtains highest possible expected payoff of $\overline{b}$, as an encouragement for rejecting the proposal that was not prescribed by the strategy profile; Player k^{t-1} receives the rest, which is $\underline{b}$.

The proof of the fact that the strategy profile $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ described above is a subgame perfect equilibrium is divided into five steps: Claim 3.2 states that σ has "no delay property", Claim 3.3 shows that after any responder history h the responder, who receives the lower share has no profitable one-shot deviations from σ at h. Claim 3.4 show that after any responder history h the responder, who receives the higher share has no profitable one-shot deviations from σ at h. Claim 3.5 show that proposals $x^t \neq \rho^t(h)$ which gives to the proposer more than $1 - \delta\overline{b}$ are rejected by both responders. And finally, Claim 3.6 show that after any proposer history h the proposer has no profitable one-shot deviations from σ at h. We show this by verifying that the one-shot deviation principle is satisfied.

Claim 3.2 *The strategy profile σ has the following no delay property: if after history $h = (\pi^0, x^0, \dots, \pi^{t-1}, x^{t-1})$ players play according to the strategy σ, $\rho^t(h)$ is proposed and accepted by one of the responders and the game ends.*

Proof: The Claim is immediate from the above definition: indeed, Player j^t according to the strategy profile accepts offers x^t, such that $x^t = \rho^t$. $\square$

Claim 3.3 *The responder who is offered the smaller share after history $h = (\pi^0, x^0, \dots, \pi^t, x^t)$, i.e. player k^t, has no profitable one-shot deviations from σ at h.*

Proof: According to the strategy profile, Player k^t accepts x^t if $[x^t \neq \rho^t(h)$ and $x_k^t \geq \delta\underline{b}]$ or $[x^t = \rho^t(h)$ and $x_k^t \geq \delta^2\underline{b}/3 + \delta\underline{b}]$, and rejects otherwise. We consider four cases:

1. $x^t = \rho^t(h)$ and $x_k^t < \delta^2\underline{b}/3 + \delta\underline{b}$,
2. $x^t = \rho^t(h)$ and $x_k^t \geq \delta^2\underline{b}/3 + \delta\underline{b}$,
3. $x^t \neq \rho^t(h)$ and $x_k^t < \delta\underline{b}$,
4. $x^t \neq \rho^t(h)$ and $x_k^t \geq \delta\underline{b}$.

1) Case $x^t = \rho^t(h)$ and $x_k^t < \delta^2\underline{b}/3 + \delta\underline{b}$.

According to the strategy profile, Player k^t rejects the proposal, which leads to an expected payoff of $\delta^{t+1}(\delta\underline{b}/3 + (1 - \delta\overline{b})/3) = \delta^{t+1}(\delta\underline{b}/3 + \underline{b}) = \delta^t(\delta^2\underline{b}/3 + \delta\underline{b})$, in the case when Player j^t rejects also. If Player k^t deviates and accepts, it gives him a payoff less than $\delta^t(\delta^2\underline{b}/3 + \delta\underline{b})$. So acceptance is not a profitable deviation.

2) Case $x^t = \rho^t(h)$ and $x_k^t \geq \delta^2\underline{b}/3 + \delta\underline{b}$.

According to the strategy profile, Player k^t accepts the proposal, which leads to a payoff of at least $\delta^t(\delta^2\underline{b}/3+\delta\underline{b})$. If Player k^t deviates and rejects, in the case when Player j^t rejects also, an expected payoff is $\delta^t(\delta^2\underline{b}/3 + \delta\underline{b})$. So rejection is not a profitable deviation.

3) Case $x^t \neq \rho^t(h)$ and $x_k^t < \delta\underline{b}$.

According to the strategy profile, Player k^t rejects the proposal, leading to an expected payoff of $\delta^{t+1}(1-\delta\overline{b})/3 = \delta^{t+1}\underline{b}$, in the case when Player j^t rejects also. If Player k^t deviates and accepts, it leads to a payoff less than $\delta^{t+1}\underline{b}$. So acceptance is not a profitable deviation.

4) Case $x^t \neq \rho^t(h)$ and $x_k^t \geq \delta\underline{b}$.

According to the strategy profile, Player k^t accepts the proposal, which leads to a payoff of at least $\delta^{t+1}\underline{b}$. If Player k^t deviates and rejects, in the case when Player j^t rejects also, an expected payoff is $\delta^{t+1}\underline{b}$. So rejection is not a profitable deviation. □

Claim 3.4 *The responder who is offered larger share at history $h = (\pi^0, x^0, \ldots, \pi^t, x^t)$, i.e. player j^t, has no profitable one–shot deviations from σ at h.*

Proof: According to the strategy profile, Player j^t accepts x^t if $[x^t = \rho^t(h)]$ or $[x_j^t \geq \delta\overline{b}]$, and rejects otherwise. We consider three cases:

1. $x^t = \rho^t(h)$,
2. $x^t \neq \rho^t(h)$ and $x_j^t \geq \delta\overline{b}$,

3. $x^t \neq \rho^t(h)$ and $x^t_j < \delta\overline{b}$,

1) Case $x^t = \rho^t(h)$.

Note, that if $x^t = \rho^t(h)$, then $x^t_j \geq \delta\underline{b}$. Any proposal prescribed by the strategy profile by Table 1 provides $x^t_j = \delta\underline{b}$ or $x^t_j = \delta\overline{b}$, any proposal prescribed by the strategy profile by Table 2 provides $x^t_j = \delta\underline{b}$, and if $x^t = a_i$, $i = 1, 2, 3$, then $x^t_j \geq \delta\underline{b}$. According to the strategy profile, Player j^t accepts, which leads to payoff $\delta^t x^t_j \geq \delta^{t+1}\underline{b}$. If Player j^t rejects x^t, it leads to an expected payoff of $\delta^{t+1}(1 - \delta\overline{b})/3 = \delta^{t+1}\underline{b}$, in the case when Player k^t rejects also. So rejection is not a profitable deviation.

2) Case $x^t \neq \rho^t(h)$ and $x^t_j \geq \delta\overline{b}$.

According to the strategy profile, Player j^t accepts, which leads to a payoff of at least $\delta^{t+1}\overline{b}$. If Player j deviates and rejects x^t, it leads to an expected payoff of $\delta^{t+1}((1-\delta\underline{b})/3 + 2\delta\overline{b}/3) = \delta^{t+1}\overline{b}$, in the case when Player k^t rejects also. So rejection is not a profitable deviation.

3) Case $x^t \neq \rho^t(h)$ and $x^t_j < \delta\overline{b}$.

According to the strategy profile, Player j^t rejects, which leads to an expected payoff of $\delta^{t+1}((1 - \delta\underline{b})/3 + 2\delta\overline{b}/3) = \delta^{t+1}\overline{b}$, in the case when Player k^t rejects also. If Player j^t deviates and accepts he receives a payoff less than or equal to $\delta^{t+1}\overline{b}$. So acceptance is not a profitable deviation. □

We have checked that responders have no profitable one-shot deviations. We presently turn our attention to the proposers. The following claim states that whenever a proposer deviates from σ and demands himself a share larger than $1 - \delta\overline{b}$, the proposal will be rejected.

Claim 3.5 *After history $h = (\pi^0, x^0, \ldots, \pi^{t-1}, x^{t-1}, \pi^t)$ proposals $x^t \neq \rho^t(h)$ such that $x^t_{p^t} > 1 - \delta\overline{b}$ are rejected by both responders.*

Proof: Note that $x^t_k + x^t_j < \delta\overline{b}$. Since by definition player k^t is the responder who is offered a smaller share, we have $x^t_k < \delta\overline{b}/2$. Moreover, $\delta\overline{b}/2 \leq \delta\underline{b}$ as follows from the assumption that $\delta \leq 3/5$. Thus $x^t_k < \delta\underline{b}$. It follows by the definition of σ that Player k^t rejects the proposal x^t. Player j^t rejects the proposal x^t, since $x^t_j < \delta\overline{b}$. □

Claim 3.6 *The proposer, i.e. Player p^t, after history $h = (\pi^0, x^0, \ldots, \pi^{t-1}, x^{t-1}, \pi^t)$ has no profitable one-shot deviations from σ at h.*

Proof: Following σ leads to a payoff greater than or equal to $\delta^t(1 - \delta\overline{b})$ for Player p^t. Indeed, this follows since both Tables 1 and 2 provide that $x_{p^t} = 1 - \delta\overline{b}$ or $x_{p^t} = 1 - \delta\underline{b}$, and the proposal $\rho(h)$ is accepted.

If Player p^t proposes $x^t \neq \rho^t(h)$ such that $x^t_{p^t} > 1 - \delta\overline{b}$, then the proposal is rejected as argued Claim 3.5 and an expected payoff for Player p^t is equal to $\delta^{t+1}((1-\delta\overline{b})/3 + \delta\underline{b}/3) \leq \delta^t(1-\delta\overline{b})$.

If Player p^t proposes $x^t \neq \rho^t(h)$ such that $x^t_{p^t} \leq 1 - \delta\overline{b}$, then the proposal is either accepted and leads to payoff $\delta^t x^t_{p^t} \leq \delta^t(1-\delta\overline{b})$, or rejected and leads to an expected payoff of $\delta^{t+1}((1-\delta\overline{b})/3 + \delta\underline{b}/3) \leq \delta^t(1-\delta\overline{b})$. In both cases, the payoff of Player p^t is less than or equal to $\delta^t(1-\delta\overline{b})$. □

3.2 The proof of Theorem 3.1 for $\delta > 3/5$

Suppose $\delta > 3/5$. The function f is defined by Table 1 above and Table 3 below. As before Table 1 gives the values $f(p^t, x^t, j^t, k^t, p^{t+1}, y^t)$ when $x^t = y^t$ and Table 3 when $x^t \neq y^t$. The function θ is defined by the equation

$$\theta(x^t_k) = \begin{cases} x^t_k/\delta - \underline{b}, & \text{if } \delta\underline{b} < x^t_k < \delta\overline{b}/2, \\ 0, & \text{otherwise.} \end{cases} \tag{3.2}$$

Table 3: $f(p^t, x^t, j^t, k^t, p^{t+1}, y^t)$, where $x^t \neq y^t$ and $\theta = \theta(x^t_k)$.

p^t	j^t	k^t	$p^{t+1} = 1$	$p^{t+1} = 2$	$p^{t+1} = 3$
1	2	3	$(1-\delta\overline{b}, \delta\overline{b}-\theta, \theta)$	$(\delta\underline{b}, 1-\delta\underline{b}-\theta, \theta)$	$(0, \delta\overline{b}-\theta, 1-\delta\overline{b}+\theta)$
1	3	2	$(1-\delta\overline{b}, \theta, \delta\overline{b}-\theta)$	$(0, 1-\delta\overline{b}+\theta, \delta\overline{b}-\theta)$	$(\delta\underline{b}, \theta, 1-\delta\underline{b}-\theta)$
2	1	3	$(1-\delta\underline{b}-\theta, \delta\underline{b}, \theta)$	$(\delta\overline{b}-\theta, 1-\delta\overline{b}, \theta)$	$(\delta\overline{b}-\theta, 0, 1-\delta\overline{b}+\theta)$
2	3	1	$(1-\delta\overline{b}+\theta, 0, \delta\overline{b}-\theta)$	$(\theta, 1-\delta\overline{b}, \delta\overline{b}-\theta)$	$(\theta, \delta\underline{b}, 1-\delta\underline{b}-\theta)$
3	1	2	$(1-\delta\underline{b}-\theta, \theta, \delta\underline{b})$	$(\delta\overline{b}-\theta, 1-\delta\overline{b}+\theta, 0)$	$(\delta\overline{b}-\theta, \theta, 1-\delta\overline{b})$
3	2	1	$(1-\delta\overline{b}+\theta, \delta\overline{b}-\theta, 0)$	$(\theta, 1-\delta\underline{b}-\theta, \delta\underline{b})$	$(\theta, \delta\overline{b}-\theta, 1-\delta\overline{b})$

Define the strategy σ as follows:

1. For each $t \geq 0$, for each history $h = (\pi^0, x^0, \dots, \pi^{t-1}, x^{t-1}, \pi^t)$, Player p^t proposes $\rho^t(h)$
2. After history $h = (\pi^0, x^0, \dots, \pi^{t-1}, x^{t-1}, \pi^t, x^t)$ Player j^t accepts x^t if
   - $x^t = \rho^t(h)$ or
   - $x^t_k \leq \delta\underline{b}$ and $x^t_j \geq \delta\overline{b}$ or

- $x_k^t \geq \delta\overline{b}/2$ and $x_j^t \geq \delta\overline{b}$ or
- $\delta\underline{b} < x_k^t < \delta\overline{b}/2$ and $x_j^t \geq \delta\overline{b} + \delta\underline{b} - x_k^t$,

and rejects otherwise.

3. After history $h = (\pi^0, x^0, \ldots, \pi^{t-1}, x^{t-1}, \pi^t, x^t)$ Player k^t accepts x^t if

- $x^t = \rho^t(h)$ and $x_k^t \geq \delta^2\underline{b}/3 + \delta\underline{b}$ or
- $x^t \neq \rho^t(h)$ and $x_k^t \geq \delta\overline{b}/2$,

and rejects otherwise.

Note, that when $\theta(x_k^{t-1}) = 0$, Table 3 is identical to Table 2. Therefore, when $\theta(x_k^{t-1}) = 0$ the intuition of the strategy profile is the same for any δ. In the case $\theta(x_k^{t-1}) \neq 0$ Player j^{t-1} obtains an expected payoff of $\overline{b} - \theta(x_k^{t-1})$ - Player j^{t-1} is still rewarded for rejecting the proposal that was not prescribed by the strategy profile, but he shares a part of the reward with Player k^{t-1}; Player k^{t-1} receives the rest, which is x_k^{t-1}/δ. It makes Player k^{t-1} indifferent between accepting and rejecting

The proof of the fact that the strategy profile $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ described above is a subgame perfect equilibrium is divided into five steps, that are the same as in the proof for $\delta \leq 3/5$: Claim 3.7 states that σ has "no delay property", Claim 3.8 shows that after any responder history h the responder, who receives the lower share has no profitable one-shot deviations from σ at h. Claim 3.10 show that after any responder history h the responder, who receives the higher share has no profitable one-shot deviations from σ at h. Claim 3.11 show that proposals $x^t \neq \rho^t(h)$ which gives to the proposer more than $1 - \delta\overline{b}$ are rejected by both responders. And finally, Claim 3.12 show that after any proposer history h the proposer has no profitable one-shot deviations from σ at h.

Claim 3.7 *The strategy profile σ has the following no delay property: if after history $h = (\pi^0, x^0, \ldots, \pi^{t-1}, x^{t-1})$ players play according to the strategy σ, $\rho^t(h)$ is proposed and accepted by one of the responders and the game ends.*

Proof: The Claim is immediate from the above definition: indeed, Player j^t according to the strategy profile accepts offers x^t, such that $x^t = \rho^t$. □

Claim 3.8 *The responder who is offered smaller share, i.e. player k^t, after history $h = (\pi^0, x^0, \ldots, \pi^t, x^t)$ has no profitable one-shot deviations from σ at h.*

Proof: According to the strategy Player k^t accepts x^t if $[x^t \neq \rho^t(h)$ and $x_k^t \geq \delta\overline{b}/2]$ or $[x^t = \rho^t(h)$ and $x_k^t \geq \delta^2\underline{b}/3 + \delta\underline{b}]$, and rejects otherwise. We consider five cases:

1. $x^t = \rho^t(h)$ and $x_k^t < \delta^2\underline{b}/3 + \delta\underline{b}$ - same proof as in Claim 3.3,

2. $x^t = \rho^t(h)$ and $x_k^t \geq \delta^2\underline{b}/3 + \delta\underline{b}$ - same proof as in Claim 3.3,

3. $x^t \neq \rho^t(h)$ and $x_k^t \leq \delta\underline{b}$- same proof as in Claim 3.3,

4. $x^t \neq \rho^t(h)$ and $\delta\underline{b} < x_k^t < \delta\overline{b}/2$,

5. $x^t \neq \rho^t(h)$ and $x_k^t \geq \delta\overline{b}/2$.

4) Case $x^t \neq \rho^t(h)$ and $\delta\underline{b} < x_k^t < \delta\overline{b}/2$.

According to the strategy profile, Player k^t rejects the proposal. Letting $\theta = \theta(x_k^t) = \delta^{-1}x_k^t - \underline{b}$, and using Equation (3.1) we compute the rejection to give the expected payoff of $\delta^{t+1}\left(\frac{1}{3}(1-\delta\overline{b}+\theta) + \frac{1}{3}\theta + \frac{1}{3}\theta\right) = \delta^{t+1}\left(\frac{1}{3}(1-\delta\overline{b}) + \theta\right) = \delta^{t+1}(\underline{b}+\theta) = \delta^t x_k^t$, in the case when Player j^t rejects also. If Player k^t deviates and accepts, he obtain a payoff of $\delta^t x_k^t$. So acceptance is not a profitable deviation.

5) Case $x^t \neq \rho^t(h)$ and $x_k^t \geq \delta\overline{b}/2$.

According to the strategy profile, Player k^t accepts the proposal, which leads to a payoff of at least $\delta^t\overline{b}/2$. If Player k^t deviates and rejects, in the case when Player j^t rejects also, he obtain an expected payoff of $\delta^{t+1}((1-\delta\overline{b})/3) = \delta^{t+1}\underline{b} < \delta^t\delta\overline{b}/2$ for $\delta > 3/5$. So rejection is not a profitable deviation. □

Claim 3.9 *It holds that $\delta\overline{b} - \theta(z) \geq \delta\underline{b}$ for each $z \in [0,1]$.*

Proof: It follows from the definition of θ that $\theta(z) \leq \frac{1}{2}\overline{b} - \underline{b}$. Hence

$$\delta\overline{b} - \theta(z) \geq \delta\overline{b} - \tfrac{1}{2}\overline{b} + \underline{b} = (\delta - \tfrac{1}{2})\overline{b} + \underline{b} \geq (\delta - \tfrac{1}{2})2\underline{b} + \underline{b} = 2\delta\underline{b} \geq \delta\underline{b}.$$

where we have used the fact that $\delta \geq 3/5$ and $\overline{b} \geq 2\underline{b}$. □

Claim 3.10 *The responder who is offered larger share, i.e. player j^t, after history $h = (\pi^0, x^0, \ldots, \pi^t, x^t)$ has no profitable one–shot deviations from σ at h.*

Proof: According to the strategy profile, Player j^t accepts x^t if

- $x^t = \rho^t(h)$ or
- $x_k^t \leq \delta\underline{b}$ and $x_j^t \geq \delta\overline{b}$ or
- $x_k^t \geq \delta\overline{b}/2$ and $x_j^t \geq \delta\overline{b}$ or

- $\delta\underline{b} < x_k^t < \delta\overline{b}/2$ and $x_j^t \geq \delta\overline{b} + \delta\underline{b} - x_k^t$,

and rejects otherwise. We consider five cases:

1. $x^t = \rho^t(h)$,
2. $x^t \neq \rho^t(h)$ and $[x_k^t \leq \delta\underline{b}$ or $x_k^t \geq \delta\overline{b}/2]$ and $x_j^t \geq \delta\overline{b}$ - same proof as in Claim 3.4,
3. $x^t \neq \rho^t(h)$ and $[x_k^t \leq \delta\underline{b}$ or $x_k^t \geq \delta\overline{b}/2]$ and $x_j^t < \delta\overline{b}$,
4. $x^t \neq \rho^t(h)$ and $\delta\underline{b} < x_k^t < \delta\overline{b}/2$ and $x_j^t \geq \delta\overline{b} + \delta\underline{b} - x_k^t$,
5. $x^t \neq \rho^t(h)$ and $\delta\underline{b} < x_k^t < \delta\overline{b}/2$ and $x_j^t < \delta\overline{b} + \delta\underline{b} - x_k^t$.

1) Case $x^t = \rho^t(h)$.

First we argue that if $x^t = \rho^t(h)$, then $x_j^t \geq \delta\underline{b}$. Any proposal in Table 1 provides $x_j^t = \delta\underline{b}$ or $x_j^t = \delta\overline{b}$. Any proposal in Table 3 offers the responders the shares $\delta\underline{b}$ and θ, or the shares $\delta\overline{b} - \theta$ and 0. In view of the preceding claim, in either case one of the two responders is offered at least $\delta\underline{b}$, so $x_j^t \geq \delta\underline{b}$. Finally if $x^t = a$ then $x_j^t \geq \delta\underline{b}$.

According to the strategy profile, Player j^t accepts, leading to a payoff $\delta^t x_j^t \geq \delta^{t+1}\underline{b}$. If Player j^t deviates and rejects x^t, in the case when Player k^t rejects also, it leads to an expected payoff of $\delta^{t+1}(1-\delta\overline{b})/3 = \delta^{t+1}\underline{b}$. So rejection is not a profitable deviation.

3) Case $x^t \neq \rho^t(h)$ and $[x_k^t \leq \delta\underline{b}$ or $x_k^t \geq \delta\overline{b}/2]$ and $x_j^t < \delta\overline{b}$.

According to the strategy profile, Player j^t rejects, leading to an expected payoff of $\delta^{t+1}((1-\delta\underline{b})/3 + 2\delta\overline{b}/3) = \delta^{t+1}\overline{b}$, in the case when Player k^t rejects also. If Player j^t deviates and accepts he receives a payoff less than $\delta^{t+1}\overline{b}$. So acceptance is not a profitable deviation.

4) Case $x^t \neq \rho^t(h)$ and $\delta\underline{b} < x_k^t < \delta\overline{b}/2$ and $x_j^t \geq \delta\overline{b} + \delta\underline{b} - x_k^t$.

According to the strategy profile, Player j^t accepts, leading to a payoff of at least $\delta^{t+1}\overline{b} + \delta^{t+1}\underline{b} - \delta^t x_k^t$. If Player j^t deviates and rejects, in the case when Player k^t rejects also, it leads to an expected payoff of $\delta^{t+1}(\frac{1}{3}(1-\delta\underline{b}-\theta) + \frac{2}{3}(\delta\overline{b}-\theta)) = \delta^{t+1}(\overline{b}-\theta) = \delta^{t+1}\overline{b} + \delta^{t+1}\underline{b} - \delta^t x_k^t$, where we used (3.1) and where $\theta = \theta(x_k^t) = \delta^{-1}x_k^t - \underline{b}$. So rejection is not a profitable deviation.

5) Case $x^t \neq \rho^t(h)$ and $\delta\underline{b} < x_k^t < \delta\overline{b}/2$ and $x_j^t < \delta\overline{b} + \delta\underline{b} - x_k^t$.

According to the strategy profile, Player j^t rejects, leading to an expected payoff of $\delta^{t+1}\overline{b} + \delta^{t+1}\underline{b} - \delta^t x_k^t$, in the case when Player k^t rejects also. If Player j^t deviates and accepts, it gives him a payoff of at most $\delta^{t+1}\overline{b} + \delta^{t+1}\underline{b} - \delta^t x_k^t$. So acceptance is not a profitable deviation. □

Claim 3.11 *After history $h = (\pi^0, x^0, \ldots, \pi^{t-1}, x^{t-1}, \pi^t)$ proposals $x^t \neq \rho^t(h)$ such that $x^t_{p^t} > 1 - \delta\overline{b}$ are rejected by both responders.*

Proof: Note, that $x^t_j + x^t_k < \delta\overline{b}$. Since k^t is the responder who is offered smaller share, we have $x^t_k \leq \delta\overline{b}/2$. We consider 2 cases:

1. $x^t_k \leq \delta\underline{b}$,
2. $\delta\underline{b} < x^t_k < \delta\overline{b}/2$.

Case 1. $x^t_k \leq \delta\underline{b}$.

According to the strategy profile, Player k^t rejects the proposal, as $x^t_k < \delta\overline{b}/2$.

According to the strategy profile, Player j^t rejects the proposal, as $x^t_j < \delta\overline{b}$.

Case 2. $\delta\underline{b} < x^t_k < \delta\overline{b}/2$.

According to the strategy profile, Player k^t rejects the proposal, as $x^t_k < \delta\overline{b}/2$.

According to the strategy profile, Player j^t rejects the proposal, as $x^t_j < 1 - x^t_p - x^t_k \leq 1 - (1 - \delta\overline{b}) - x^t_k = \delta\overline{b} - x^t_k < \delta\overline{b} - x^t_k + \delta\underline{b}$. □

Claim 3.12 *The proposer, i.e. Player p^t, after history $h = (\pi^0, x^0, \ldots, \pi^{t-1}, x^{t-1})$ has no profitable one-shot deviations from σ at h.*

Proof: First we argue that following σ leads to a payoff at least $\delta^t(1 - \delta\overline{b})$ for Player p^t. According to σ the proposer demands one of the four shares: $1 - \delta\overline{b}$, $1 - \delta\underline{b}$, $1 - \delta\overline{b} + \theta$, or the share $1 - \delta\underline{b} - \theta$. Each of these is at least $1 - \delta\overline{b}$. In particular, the fact that $1 - \delta\underline{b} - \theta$ is at least $1 - \delta\overline{b}$ follows from Claim 3.9.

If Player p^t proposes $x^t \neq \rho^t(h)$ such that $x^t_{p^t} > 1 - \delta\overline{b}$, then the proposal is rejected according to Claim 3.11 and the expected payoff for Player p^t is equal to $\delta^{t+1}((1 - \delta\overline{b})/3 + \delta\underline{b}/3) \leq \delta^t(1 - \delta\overline{b})$, which holds for every $\delta \in [0, 1)$.

If Player p^t proposes $x^t \neq \rho^t(h)$ such that $x^t_{p^t} \leq 1 - \delta\overline{b}$, then the proposal is either accepted and leads to payoff $\delta^t x^t_{p^t} \leq \delta^t(1 - \delta\overline{b})$, or rejected and leads to an expected payoff of $\delta^{t+1}((1 - \delta\overline{b})/3 + \delta\underline{b}/3) \leq \delta^t(1 - \delta\overline{b})$. In both cases the payoff of Player p^t is less than or equal to $\delta^t(1 - \delta\overline{b})$. □

Now we can compare our result with experimental literature, where for the game with three players, a stationary subgame perfect equilibrium predicts a payoff of $1 - \delta/3$ to the proposer in period 0 and a payoff of $\delta/3$ to one of the responders. We have shown that broader set of payoffs is consistent with subgame perfect equilibrium strategies.

In one of the first experimental investigations of the Baron-Ferejohn model, McKelvey (1991) studies a stochastic bargaining game with three voters and three alternatives. He concludes that the data is not precisely predicted by the solution of the game. In the data proposers usually offer too much to the responding players and the proposals have been accepted with too high probability.

Diermeier and Morton (2005) consider a finitely-repeated version of the Baron-Ferejohn model to obtain more accurate predictions that in the model of McKelvey (1991). Experiment consists of five periods with a zero payoff if no agreement is reached. Diermeier and Morton use different recognition probabilities for players, in contrast with Baron-Ferejohn model. Similar to McKelvey, Diermeier and Morton find little support for the predictions of the Baron-Ferejohn model. First, in 1/3 of the cases positive amount was proposed to all players, not just tothe members of the minimal winning coalition. Which is allowed in our paper. Second, proposers do not select the coalition partner with the lowest continuation value. Third, proposers do not use their power as it was predicted by the Baron-Ferejohn model for stationary strategies. They consistently offer too much to other coalition members. Fourth, a significant percentage of first period proposals above the continuation value were rejected.

In Breitmoser and Tan (2010) the following treatment is considered:

Game 1 (PB95). In each round, one player is recognized as proposer by a uniform draw from N. This player chooses x, $x \in \mathbb{R}^3_+$, $x_1 + x_2 + x_3 \leq 24$, and the other players vote on x. If one of them accepts, then the payoffs are x. Otherwise, the payoffs are 0 with probability .05 and a new round begins with probability .95. In other terms $\delta = 0.95$.

The prediction of Baron-Ferejohn in stationary strategies gives 16.4 to the proposer and 7.6 to one of the responders. The sample estimates are 10.69 (with standard deviation 3.44) to the proposer, 8.61 (with standard deviation 2.39) to Player j, and 4.12 (with standard deviation 3.14) to Player k. The gap between proposer and second voter payoffs was smaller than the stationary subgame perfect equilibrium prediction. Note, that our result predicts payoff to the proposer in the range from 4.5 ($1-\delta\overline{b}$) to 22.49 ($1-\delta\underline{b}$); payoff to the player, who receives higher share is predicted to be in the range from 1.5 ($\delta\underline{b}$) to 16.11 ($\delta\overline{b}$). Moreover, the responder received on average the continuation payoff, while the other voter got more than nothing - contrary to stationary equilibrium predictions, but also in line with our result. These observations of Breitmoser and Tan (2010) are similar with those of previous studies, which also noted the under-realization of proposer power and the generosity shown to voters outside the minimal winning coalition as in Frechette et al. (2005). Frechette et al. (2005) study four experimental designs of Baron-Ferejohn model with three players, considering different weights (amount of votes of a player) and different selection probabilities. Frechette etal. suggestthat players rely on a"fair"reference point of $1/n$ shareof the benefits when making decisions. Offers below that share are consistently rejected in all treatments, while shares above $1/n$ are usually accepted.

Miller and Vanberg (2010) compares the costs of reaching agreement under majority and unanimity rule in the context of an experimental bargaining game. In the game every period the pie shrinks by 10%, so $\delta = 0.9$. Miller and Vanberg use the prediction of Baron-Ferejohn model as a benchmark, but they also consider delay as a hypothesis, which is not predicted by Baron-Ferejohn, but in line with our result. Under both rules, they find patterns very similar to those reported on in previous literature. Proposers demand a higher share than they allocate to non-proposers, but the difference is still far from the equilibrium prediction. Interestingly, approximately half of the proposals in the first period are three-way equal splits and only one out of five allocates 0 to one of the non-proposers. In our solution three-way equal splits are SPE payoffs for $\delta \geq 0.84$, which is the case for the experiment. In the last 10 periods, more than 75% of proposals include a zero- offer and the proportion of three-way equal splits is consistently below 15%. Thus, it looks as though many subjects were initially inclined to propose equal splits and learned over time to form minimum winning coalitions.

In many experiments we see behavior that is not in line with stationary equilibria, but consistent with SPE. We show that more complicated strategies may lead to broader set of SPE payoffs that are in line with surprising results of the experiments.

4 Equilibria with one period delay

There are several results about delay in acceptance in subgame perfect equilibrium in the theoretical and the experimental literature. Miller and Vanberg (2010) show that delay occurs more often under unanimity rule than under majority rule in a experimental framework. Agranov and Tergiman (2012) make an experiment based on the Baron-Ferejohn model with the possibility of unrestricted cheap-talk communication before a proposal is submitted. In both treatments - with and without communication- about 15% of the games results in delay in acceptance. Some evidence for delay in acceptance in the experiments based on the Baron-Ferejohn model was found, which is not predicted by Baron-Ferejohn theoretical benchmark, where the equilibrium proposal is made and accepted in period 0.

In this section we find necessary and sufficient conditions for one period delay.

A strategy σ is said to have a *one–period delay* if, irrespectively of the moves of nature, the proposal in period 0 is rejected by both responders, and the proposal in period 1 is accepted by at least one responder. Even more precisely, σ has one–period delay if for all permutations π^0 and π^1 the proposal $\sigma(\pi^0)$ is rejected by both players $\pi^0(2)$ and $\pi^0(3)$ while the proposal $\sigma(\pi^0, \sigma(\pi^0), \pi^1)$ is accepted by $\pi^1(2)$ or $\pi^1(3)$.

Theorem 4.1 *There exists an subgame perfect equilibrium with one period delay if and only if* $\delta \geq 0.804$.

A simple computation reveals that the condition $\delta \geq 0.804$ is equivalent to the inequality $1 - \delta\overline{b} \leq \delta\overline{b}$. The latter plays a crucial role in the proofs below. We first prove the only if part of the theorem.

Claim 4.2 *Suppose there is a subgame perfect equilibrium with one period delay. Then* $\delta \geq 0.804$.

Proof: Let σ be a subgame perfect equilibrium with one period delay. For concreteness consider the permutation $\pi^0 = (1, 2, 3)$. Following σ after the history (π^0) leads to the payoff of at most $\delta\overline{u}$ to player 1. On the other hand for each $\epsilon > 0$ the proposal $(1 - \delta\overline{u} - \epsilon, \delta\overline{u} + \epsilon, 0)$ is accepted by Claim 2.3. Since σ is subgame perfect we must have $1 - \delta\overline{u} - \epsilon \leq \delta\overline{u}$ for each $\epsilon > 0$. Hence $1 - \delta\overline{u} \leq \delta\overline{u}$. Now by our main result (Theorem 2.2) $\overline{u} = \overline{b}$. Thus $1 - \delta\overline{b} \leq \delta\overline{b}$, which leads to $\delta \geq 0.804$ $\square$

To prove the if part of the theorem we construct a subgame perfect equilibrium strategy profile $\widehat{\sigma}$ such that on the equilibrium path of play the proposer in period 0 (Player p^0) demands the entire surplus, the proposal is rejected, the proposal in period 1 is accepted leading to the payoff $\delta\overline{b}$ to player p^0. Any deviation by Player p^0 results in a payoff of at most $1 - \delta\overline{b}$. Under the assumption that $1 - \delta\overline{b} \leq \delta\overline{b}$ the deviation is not profitable.

We define the strategy profile τ_1 to be the strategy profile provided by Theorem 3.1 with $a_1 = (1 - \delta\underline{b}, \delta\underline{b}, 0)$, $a_2 = (\delta\overline{b}, 1 - \delta\overline{b}, 0)$, and $a_3 = (\delta\overline{b}, 0, 1 - \delta\overline{b})$. Following the strategy profile τ_1 leads to an expected payoff of $\overline{b}$ for Player 1.

We define the strategy profile τ_2 to be the strategy profile provided by Theorem 3.1 with $a_1 = (1 - \delta\overline{b}, \delta\overline{b}, 0)$, $a_2 = (0, 1 - \delta\underline{b}, \delta\underline{b})$, and $a_3 = (0, \delta\overline{b}, 1 - \delta\overline{b})$. Following the strategy profile τ_2 leads to an expected payoff of $\overline{b}$ for Player 2.

We define the strategy profile τ_3 to be the strategy profile provided by Theorem 3.1 with $a_1 = (1 - \delta\overline{b}, 0, \delta\overline{b})$, $a_2 = (0, 1 - \delta\overline{b}, \delta\overline{b})$, and $a_3 = (0, \delta\underline{b}, 1 - \delta\underline{b})$. Following the strategy profile τ_3 leads to an expected payoff of $\overline{b}$ for Player 3.

Finally we define the strategy profile γ as the strategy profile provided by Theorem 3.1 with $a_1 = (1 - \delta\overline{b}, \delta\overline{b}, 0)$, $a_2 = (0, 1 - \delta\overline{b}, \delta\overline{b})$, and $a_3 = (\delta\overline{b}, 0, 1 - \delta\overline{b})$.

For period $t = 0$ we define

$$\begin{aligned}
\widehat{\sigma}(\pi^0) &= e_{p^0} \\
\widehat{\sigma}(\pi^0, e_{p^0}) &= r \\
\widehat{\sigma}(\pi^0, x^0) &= \gamma(\pi^0, x^0), \text{ if } x^0 \neq e_{p^0}.
\end{aligned}$$

The latter equality is in fact a pair of equations, one for each of the two responding players. The same remark applies to all formulae below where $\widehat{\sigma}$ is defined at a responder history. For histories of period $t \geq 1$ following the proposal e_{p^0} in period $t = 0$ we define

$$\begin{aligned}
\widehat{\sigma}(\pi^0, e_{p^0}, \pi^1, x^1, \ldots, \pi^t) &= \tau_{p^0}(\pi^1, x^1, \ldots, \pi^t) \\
\widehat{\sigma}(\pi^0, e_{p^0}, \pi^1, x^1, \ldots, \pi^t, x^t) &= \tau_{p^0}(\pi^1, x^1 \ldots, \pi^t, x^t).
\end{aligned}$$

For histories of period $t \geq 1$ following the proposal $x^0 \neq e_{p^0}$ in period $t = 0$ we define

$$\begin{aligned}
\widehat{\sigma}(\pi^0, x^0, \pi^1, x^1, \ldots, \pi^t) &= \gamma(\pi^0, x^0, \pi^1, x^1, \ldots, \pi^t) \\
\widehat{\sigma}(\pi^0, x^0, \pi^1, x^1, \ldots, \pi^t, x^t) &= \gamma(\pi^0, x^0, \pi^1, x^1, \ldots, \pi^t, x^t).
\end{aligned}$$

Let $0 \leq j \leq t$. For a history h in period t we define "the tail" of h starting from period j, h^{-j}, as follows: for $h = (\pi^0, x^0, \ldots, \pi^t)$ we define $h^{-j} = (\pi^j, x^j, \ldots, \pi^t)$, and for $h = (\pi^0, x^0, \ldots, \pi^t, x^t)$ we define $h^{-j} = (\pi^j, x^j, \ldots, \pi^t, x^t)$. In particular $h^{-0} = h$. We write $h \geq h_0$ if $h = h_0$ or history h extends history h_0.

The claim below makes an almost trivial observation: Suppose that the strategy σ is a subgame perfect equilibrium of Γ. Suppose furthermore that the strategy $\widehat{\sigma}$ requires that, as soon as some given history h_0 in period j has been reached, all events preceding period j be deleted from memory, and strategy σ be executed. Then $\widehat{\sigma}$ is subgame perfect in the subgame starting at h_0. Notice that h_0 can be a proposer or a responder history.

Claim 4.3 *Let σ be a subgame perfect equilibrium of Γ and h_0 a history in period j. Suppose that $\widehat{\sigma}(h) = \sigma(h^{-j})$ for each history $h \geq h_0$. Then there are no profitable one–shot deviations from $\widehat{\sigma}$ at $h \geq h_0$.*

Proof: The subgame starting at history h is strategically equivalent to that starting at h^{-j}. Hence, if there were a profitable one–shot deviation from $\widehat{\sigma}$ at h, there would be one from σ at h^{-j}. □

Claim 4.4 *There are no profitable one–shot deviations from $\widehat{\sigma}$.*

Proof: Consider the proposer history (π^0). We compare the strategies γ and $\widehat{\sigma}$. At history (π^0) the strategy γ requires that the proposal $\gamma(\pi^0)$ is accepted, and player p^0 receives the payoff $1-\delta\overline{b}$. The strategy $\widehat{\sigma}$ prescribes that player p^0 propose e_{p^0}, the proposal be rejected, and as of period 1 strategy τ_{p^0} be played, leading to the payoff $\delta\overline{b}$ to player p^0. Notice that $1-\delta\overline{b} \leq \delta\overline{b}$ since $\delta \leq 0.804$.

We argue that there are no profitable one–shot deviations from $\widehat{\sigma}$ at (π^0). Take some $x^0 \neq e_{p^0}$. Crucially, after the history (π^0, x^0) the strategy $\widehat{\sigma}$ coincides with γ since according to our definition $\widehat{\sigma}(h) = \gamma(h)$ for every history $h \geq (\pi^0, x^0)$. Hence, playing x^0 at history (π^0) and following $\widehat{\sigma}$ thereafter results in exactly the same payoff as playing x^0 at π^0 and following γ thereafter. Let this common payoff be denoted by v.

Since γ is subgame perfect, no player has a profitable one–shot deviation from γ. In particular, we must have $v \leq 1-\delta\overline{b}$. Hence also $v \leq \delta\overline{b}$, implying that x_0 is not a profitable one–shot deviation from $\widehat{\sigma}$.

Consider the responder history (π^0, e_{p^0}). According to $\widehat{\sigma}$ responders rejects the proposal. Accepting the proposal gives both responders the payoff 0 and is clearly not a profitable deviation from $\widehat{\sigma}$.

For all other histories the one–shot deviation follows from Claim 4.3. Indeed, take an $x^0 \neq e_{p^0}$. For histories $h \geq (\pi^0, x^0)$ Claim 4.3 applies with $h_0 = (\pi^0, x^0)$, $j = 0$, and $\sigma = \gamma$. Take any permutation π^1. For all histories $h \geq (\pi^0, e_{p^0}, \pi^1)$ Claim 4.3 applies with $h_0 = (\pi^0, e_{p^0}, \pi^1)$, $j = 1$, and $\sigma = \tau_{p^0}$. □

5 Arbitrarily long delay

In this section we show that any finite delay is compatible with subgame perfection, provided that the players are patient enough. By analogy with the case of 1–period delay, we say that the strategy σ has k–period delay if, irrespectively of the moves of nature, the proposals in period $0, \ldots, k-1$ are rejected, and the proposal in period k is accepted.

Theorem 5.1 *Given a $k \geq 1$ there exists a strategy $\widetilde{\sigma}$ having k–period delay, and a $\widetilde{\delta} < 1$ such that $\widetilde{\sigma}$ is subgame perfect in the game with a discount factor $\delta > \widetilde{\delta}$.*

We let $\widetilde{\delta}$ be the unique value of δ solving the equation

$$\frac{\delta^k}{3} = 1 - \delta \frac{3-\delta}{9-6\delta-\delta^2}.$$

Notice that the right hand side of the equation is exactly $1-\delta\overline{b}$. This is a decreasing function of δ with value 1 at $\delta = 0$ and value 0 at point $\delta = 1$. Hence the solution exists and is unique. Moreover for each $\delta > \widetilde{\delta}$ it holds that $\delta^k/3 > 1-\delta\overline{b}$.

We can estimate $\widetilde{\delta}$ from above as follows:

$$\begin{aligned} 0 &= \widetilde{\delta}^{k+2} + 6\widetilde{\delta}^{k+1} - 9\widetilde{\delta}^{k} - 27\widetilde{\delta} + 27 \\ &\leq \widetilde{\delta}^{k} + 6\widetilde{\delta}^{k} - 9\widetilde{\delta}^{k} - 27\widetilde{\delta}^{k} + 27 \\ &= -29\widetilde{\delta}^{k} + 27, \end{aligned}$$

from which it follows that $\widetilde{\delta} \leq (27/29)^{1/k}$.

For $\delta > \widetilde{\delta}$ the vector $(\delta^k/3, \delta^k/3, \delta^k/3)$ belongs to the sets V_1, V_2, and V_3 since $\delta^k/3 > 1 - \delta\overline{b}$ and $1 - \delta\overline{b} \geq \delta\underline{b}$. We let γ be the strategy profile provided by Theorem 3.1 with $a_1 = a_2 = a_3 = (\delta^k/3, \delta^k/3, \delta^k/3)$. The vector $(1/3, 1/3, 1/3)$ also belongs to the sets V_1, V_2, and V_3. Let τ be the strategy profile provided by Theorem 3.1 with $a_1 = a_2 = a_3 = (1/3, 1/3, 1/3)$.

We construct a strategy profile $\widetilde{\sigma}$ such that on the equilibrium path of play in periods $0, \ldots, k-1$ the proposers demand the entire surplus, and their proposals are rejected. If period k is reached without deviations, the memory of preceding periods is deleted and as of period k the strategy τ is followed. In particular, on the equilibrium path of play equal split is proposed and accepted in period k. However, as soon as a proposer deviates in some period $\ell < k$, the memory of preceding periods is deleted, and the strategy γ is followed.

We use the notation introduced in the previous section. Partition the set H of histories into pairwise disjoint sets $D_{-1}, D_0, \ldots, D_k$ as follows:

$$\begin{aligned} D_{-1} &= \{h \in H \mid (\pi^0, e_{\pi^0(0)}, \ldots, \pi^k, e_{\pi^k(0)}) \geq h\} \\ D_0 &= \{h \in H \mid h \geq (\pi^0, x^0) \text{ where } x^0 \neq e_{\pi^0(0)}\} \\ D_\ell &= \left\{h \in H \mid h \geq (\pi^0, e_{\pi^0(0)}, \ldots, \pi^{\ell-1}, e_{\pi^{\ell-1}(0)}, \pi^\ell, x^\ell) \text{ where } x^\ell \neq e_{\pi^\ell(0)}\right\} \end{aligned}$$

for $0 < \ell < k$ and

$$D_k = \left\{h \in H \mid h \geq \left(\pi^0, e_{\pi^0(0)}, \ldots, \pi^{k-1}, e_{\pi^{k-1}(0)}, \pi^k\right)\right\}.$$

Define the strategy $\widetilde{\sigma}$ as follows:

$$\widetilde{\sigma}(\pi^0) = e_{\pi^0(0)} \tag{5.1}$$

$$\widetilde{\sigma}(\pi^0, e_{\pi^0(0)}, \ldots, \pi^{\ell-1}, e_{\pi^{\ell-1}(0)}, \pi^\ell) = e_{\pi^\ell(0)} \quad \text{for } 1 \leq \ell < k, \tag{5.2}$$

$$\widetilde{\sigma}(\pi^0, e_{\pi^0(0)}, \ldots, \pi^\ell, e_{\pi^\ell(0)}) = r \quad \text{for } 0 \leq \ell < k. \tag{5.3}$$

$$\widetilde{\sigma}(h) = \tau(h^{-k}) \quad \text{for } h \in D_k. \tag{5.4}$$

$$\widetilde{\sigma}(h) = \gamma(h^{-\ell}) \quad \text{for } h \in D_\ell,\ 0 \leq \ell < k. \tag{5.5}$$

Equations (5.1)–(5.3) specify $\widetilde{\sigma}$ on the set D_{-1} of histories, Equations (5.5) on $D_0, \ldots, D_{k-1}$, and Equations (5.4) on D_k.

From (5.4) we get

$$\widetilde{\sigma}(\pi^0, e_{\pi^0(0)}, \ldots, \pi^{k-1}, e_{\pi^{k-1}(0)}, \pi^k) = \tau(\pi^k) = \left(\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\right) \quad (5.6)$$

$$\widetilde{\sigma}(\pi^0, e_{\pi^0(0)}, \ldots, \pi^{k-1}, e_{\pi^{k-1}(0)}, \pi^k, \left(\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\right)) = \tau\left(\pi^k, \left(\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\right)\right) = a. \quad (5.7)$$

Equations (5.1), (5.2), (5.3), (5.6), and (5.7) specify the equilibrium path of play under $\widetilde{\sigma}$: The proposals $e_{\pi^0(0)}, \ldots, e_{\pi^{k-1}(0)}$ are to be made and rejected, followed by the proposal $(1/3, 1/3, 1/3)$, which is to be accepted. Thus the strategy $\widetilde{\sigma}$ has a delay of k periods. All players receive the payoff $\delta^k/3$. Equation (5.4) specifies the play of the game as of period k, provided that no deviations occurred in the preceding periods. Equation (5.5) specifies the continuation play following a deviation from the equilibrium play in period $\ell < k$.

In the case $k = 1$ the above construction resembles that in the previous section. The difference is that here the play following the histories of the form $(\pi^0, e_{\pi^0(0)}, \pi^1)$ as defined by Equation (5.4) is independent of the permutation π^0 whereas in the preceding section it does depend on $\pi^0(1)$.

Claim 5.2 *There are no one–shot profitable deviations from $\widetilde{\sigma}$.*

Proof: Take the proposer history $h = (\pi^0, e_{\pi^0(0)}, \ldots, \pi^{\ell-1}, e_{\pi^{\ell-1}(0)}, \pi^\ell)$ for $0 \leq \ell < k$. Fix any proposal $x^\ell \neq e_{\pi^\ell(0)}$.

We compare the strategies γ and $\widetilde{\sigma}$. The strategy γ dictates that at history (π^ℓ) Player $\pi^\ell(0)$ make the proposal $(\delta^k/3, \delta^k/3, \delta^k/3)$ and the proposal be accepted. Let v' be the payoff to Player $\pi^\ell(0)$ that results from making the proposal x^ℓ at history (π^ℓ), provided that the continuation play is according to γ. Since γ is a subgame perfect, it holds that $v' \leq \delta^k/3$.

Following the strategy $\widetilde{\sigma}$ at h yields the payoff $\delta^k/3$. Let v be the payoff to Player $\pi^\ell(0)$ that results from making the proposal x^ℓ at history h, provided that the continuation play is according to $\widetilde{\sigma}$. It follows from Equation (5.5) that the payoffs v and v' are related by $v = \delta^\ell v'$. So $v \leq \delta^\ell \delta^k/3 \leq \delta^k/3$. We conclude that proposing x^ℓ is not a profitable one–shot deviation from $\widetilde{\sigma}$ at h.

Consider the responder history $h = (\pi^0, e_{\pi^0(0)}, \ldots, \pi^\ell, e_{\pi^\ell(0)})$ for $0 \leq \ell < k$. The strategy $\widetilde{\sigma}$ requires rejection. Deviation to acceptance yields zero to both responders, and is hence not a profitable.

For histories in $D_0, \ldots, D_k$ the one–shot deviation property follows by an application of Claim 4.3. □

6 Conclusion

In this paper we consider a three-person majority voting bargaining game with complete information and discounting. In our model players moves are sequential and the order of responding players is irrelevant, despite the fact that one acceptance is enough to implement the proposal. We construct subgame perfect equilibria in which any given vector from the set of feasible payoffs is equilibrium outcome for some discount factor. Our result is in line with many experimental papers, where we observe behavior that was not consistent with stationary equilibria. We show that inefficiencies are possible in the form of delay: we provide necessary and sufficient conditions for one period of delay in acceptance and construct subgame perfect equilibrium where arbitrary long delay occurs with probability one.

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