A Sequential Choice Model of Family Business Succession

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Abstract: A game theoretic model of family business succession, in which an elder sibling (E) and a younger sibling (Y) sequentially choose levels of pursuit for a managerial leadership position, is developed and analyzed. After observing chosen pursuit levels, the founder of the business selects a successor. Intuition suggests that E might enjoy a "first mover advantage." After determining a Subgame Perfect Nash Equilibrium for the specified sequential move game, the possibility of a "first mover advantage" is explored in two ways. First, direct comparisons of the equilibrium outcome are made between E and Y. It is argued that when E and Y have the same value for the position, E realizes a higher payoff than Y if and only if the founder is predisposed to choose E as the successor. Second, the potential for the sequential nature of the framework to systematically alter the payoff of a player in a particular direction is addressed by fixing the values of all relevant parameters in the model and making a comparison of a particular candidate's equilibrium expected payoff if he is E versus if he is Y. This counterfactual exercise reveals that: (i) a candidate of fixed attributes always realizes a greater probability of being chosen as successor if he is E (instead of Y) and (ii) an individual prefers being E (over being Y) if and only if the value he places on being chosen as successor and/or the predisposition of the founder to choose him as successor are relatively low.

Keywords: applied game theory; family business; management succession.

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1 Introduction

The following section develops a formal game theoretic model of competition between two interested siblings for control of a family business. We analyze a situation in which the siblings sequentially choose (i.e., elder sibling first, younger sibling second) the degree to which to pursue the position.

Our analysis focuses on the effects and impacts of sibling age/birth order on succession outcomes. Intuition would seemingly suggest that the "elder sibling" might possibly have a "first mover advantage" and that each sibling would prefer to be in the position of being the first to choose their level of pursuit. We explore this issue in part by determining conditions under which a candidate would prefer being the elder sibling versus being the younger sibling.

2 Specification of Model

Consider a situation in which a founder of a family business desires to pass control over to one of his two offspring.[1] Assume that the two offspring differ in age, letting E denote the elder sibling and Y denote the younger sibling. Each candidate $i \in \{E, Y\}$ has a particular valuation for being named the successor, denoted by V_i. The candidates will sequentially chooses levels of pursuit for the position, denoted l_E and l_Y. In practice, the choice of level of pursuit could be something as simple and informal as amount of effort, level of interest, or quality of work revealed by a candidate when working at the family business during young adulthood.

First E chooses $l_E \geq 0$, after which Y is able to observe l_E and then choose l_Y. Following these choices of l_E and l_Y, E is chosen to be the successor with probability $P_E(l_E, l_Y) = \frac{\delta_E l_E}{\delta_E l_E + \delta_Y l_Y}$ and Y is chosen to be the successor with probability $P_Y(l_Y, l_E) = \frac{\delta_Y l_Y}{\delta_E l_E + \delta_Y l_Y}$. The parameters $\delta_E > 0$ and $\delta_Y > 0$ determine how each probability depends upon the levels of pursuit chosen by the candidates. Observe the following for the function $P_E(l_E, l_Y)$ (recognizing that similar observations can be made for the function $P_Y(l_Y, l_E)$):

- $P_E(0, l_Y) = 0$ for $l_Y > 0$ (i.e., if E chooses $l_E = 0$ and Y chooses $l_Y > 0$, then E will not be chosen as successor);

- $P_E(l_E, 0) = 1$ for $l_E > 0$ (i.e., if Y chooses $l_Y = 0$ and E chooses $l_E > 0$, then E will be chosen as successor);

[1]For purposes of simplicity, we are concentrating on management succession for which the founder will ultimately choose one of his offspring and not consider an "outside option."

- $\frac{\partial P_E(l_E,l_Y)}{\partial l_E} = \frac{\delta_E \delta_Y l_Y}{(\delta_E l_E + \delta_Y l_Y)^2} > 0$ (i.e., the probability of E being named as successor increases as l_E is increased);

- $\frac{\partial^2 P_E(l_E,l_Y)}{\partial l_E^2} = \frac{-2\delta_E^2 \delta_Y l_Y}{(\delta_E l_E + \delta_Y l_Y)^3} < 0$ (i.e., the marginal impact of increasing $P_E(l_E, l_Y)$ by increasing l_E is diminishing);

- $\frac{\partial P_E(l_E,l_Y)}{\partial l_Y} = \frac{-\delta_Y \delta_E l_E}{(\delta_E l_E + \delta_Y l_Y)^2} < 0$ (i.e., the probability of E being named as successor decreases as l_Y is increased).

Note that when E and Y choose equal levels of pursuit $P_E(l, l) = \frac{\delta_E}{\delta_E + \delta_Y}$, which is:

- equal to $\frac{1}{2}$ if $\delta_E = \delta_Y$;

- greater than $\frac{1}{2}$ if $\delta_E > \delta_Y$;

- less than $\frac{1}{2}$ if $\delta_E < \delta_Y$.

That is, if $\delta_E = \delta_Y$ then both candidates are equally positioned in that each will be chosen with an equal probability of $\frac{1}{2}$ when they choose equal levels of pursuit. If instead $\delta_E > \delta_Y$, then the founder is predisposed to choose E as the successor, since E is chosen as successor with a higher probability than Y when both candidates choose the same level of pursuit. Conversely, for $\delta_E < \delta_Y$, the founder is predisposed to choose Y.

Assume that pursuing the position at the level l_i costs the candidate Cl_i, with $C > 0$ (i.e., there are constant marginal costs of $C > 0$ associated with increasing l_i). It follows that the expected payoff of E is

$$\Pi_E = P_E(l_E, l_Y) V_E - Cl_E = \frac{\delta_E l_E}{\delta_E l_E + \delta_Y l_Y} V_E - Cl_E \tag{1}$$

and the expected payoff of Y is

$$\Pi_Y = P_Y(l_Y, l_E) V_Y - Cl_Y = \frac{\delta_Y l_Y}{\delta_E l_E + \delta_Y l_Y} V_Y - Cl_Y. \tag{2}$$

The sequential move game between E and Y is analyzed via backward induction, and a Subgame Perfect Nash Equilibrium is identified as follows. First, the terminal choice by Y is analyzed to determine the optimal value of l_Y as a function of the chosen and observed level of l_E. Second, the initial choice by E is analyzed to determine the optimal value of l_E, explicitly recognizing how the initial choice of l_E influences the subsequent choice of l_Y. After determining these equilibrium levels of pursuit, denoted l_E^* and l_Y^*, the resulting equilibrium probability of being chosen as successor and expected payoff is determined for each candidate.

From here the possibility of a "first mover advantage" is explored in two different ways. First, direct comparisons of the equilibrium outcome are made between E and Y. Within this discussion, it is argued that when E and Y have the same value for being chosen as successor (i.e., when $V_E = V_Y$), the elder sibling realizes a higher payoff than the younger sibling if and only if the founder is predisposed to choose E over Y (i.e., when $\delta_E > \delta_Y$). In a purely symmetric environment (i.e., $V_E = V_Y$ and $\delta_E = \delta_Y$) the two candidates have identical equilibrium payoffs, suggesting that there is not an advantage for being the candidate who chooses his level of pursuit first. It is also explained how observations on succession outcomes may reveal a built in bias on the part of the founder.

Second, the potential for the sequential nature of the framework to systematically alter the payoff of a player in a particular direction is addressed by fixing the values of all relevant parameters in the model and making a comparison of a candidate's equilibrium expected payoff if he is the elder sibling versus if he is the younger sibling. As a result of this counterfactual exercise, conditions (in terms of the values of the parameters of the model) are determined under which a candidate would prefer being the elder sibling versus being the younger sibling. It is argued that a candidate who has values of δ_i and V_i that are collectively "relatively low" would prefer being the elder sibling over being the younger sibling. Thus, in some sense, in a framework in which levels of pursuit are chosen sequentially, there is an advantage to being the elder sibling for precisely such candidates.

3 Identification of Equilibrium

To solve the game via backward induction start by considering the choice of l_Y by Y, having observed the value of l_E chosen by E. Recall, the expected payoff of Y is given by (2). Partial differentiation of this function with respect to l_Y yields the following first order condition for maximization[2]:

$$\frac{\partial \Pi_Y}{\partial l_Y} = \frac{\delta_Y \delta_E l_E}{(\delta_E l_E + \delta_Y l_Y)^2} V_Y - C = 0. \tag{3}$$

Condition (3) is satisfied if and only if:

$$\delta_E l_E + \delta_Y l_Y = \sqrt{\frac{V_Y}{C} \delta_Y \delta_E l_E} \tag{4}$$

$$\Leftrightarrow l_Y = \sqrt{\frac{\delta_E}{\delta_Y} l_E} \left(\sqrt{\frac{V_Y}{C}} - \sqrt{\frac{\delta_E}{\delta_Y} l_E} \right).$$

[2]Note, the second order condition for maximization is satisfied, since $\frac{\partial^2 P_Y(l_Y, l_E)}{\partial l_Y^2} < 0$.

Recognize that this is the optimal choice of l_Y only so long as E has chosen a sufficiently small level of l_E. More precisely, if $l_E \geq \frac{\delta_Y V_Y}{\delta_E C}$, then $\left.\frac{\partial \Pi_Y}{\partial l_Y}\right|_{l_Y=0} \leq 0$, in which case the optimal choice for Y is $l_Y = 0$. That is, E is able to completely dissuade Y from pursing the position by choosing a sufficiently large level of l_E. Thus, the optimal choice of l_Y by Y as a function of the value of l_E initially chosen by E is:

$$l_Y^*(l_E) = \begin{cases} \sqrt{\frac{\delta_E}{\delta_Y} l_E}\left(\sqrt{\frac{V_Y}{C}} - \sqrt{\frac{\delta_E}{\delta_Y} l_E}\right), & if \quad l_E < \frac{\delta_Y V_Y}{\delta_E C} \\ 0, & if \quad l_E \geq \frac{\delta_Y V_Y}{\delta_E C} \end{cases}. \tag{5}$$

Now consider the initial choice of l_E by E, which is made under the recognition that Y will subsequently choose $l_Y = l_Y^*(l_E)$ as specified in (5). Recall, the expected payoff of E is given by (1). Equation (4) specifies a condition that the optimal value of l_Y must satisfy (at an interior solution). Substituting the expression on the right hand side of (4) into (1) and simplifying reveals that (for $l_E < \frac{\delta_Y V_Y}{\delta_E C}$) the payoff of E as a function of l_E can be expressed as:

$$\Pi_E(l_E) = V_E \sqrt{\frac{\delta_E C}{\delta_Y V_Y}} \sqrt{l_E} - C l_E. \tag{6}$$

Differentiating (6) leads to the following first order condition for maximization[3]:

$$\Pi_E'(l_E) = \frac{1}{2} V_E \sqrt{\frac{\delta_E C}{\delta_Y V_Y}} \sqrt{\frac{1}{l_E}} - C = 0. \tag{7}$$

Condition (7) is satisfied if and only if:

$$l_E = \frac{\delta_E V_E^2}{4C\delta_Y V_Y}.$$

But recall that E can dissuade Y from pursuing the position by choosing $l_E \geq \frac{\delta_Y V_Y}{\delta_E C}$, in which case E is guaranteed to be named the successor and realize a payoff of $\Pi_E(l_E) = V_E - C l_E$. Clearly the optimal choice in this range is $l_E = \frac{\delta_Y V_Y}{\delta_E C} \equiv \bar{l}_E$ (since a larger value of l_E is more costly but provides no additional benefit). Choosing $l_E = \bar{l}_E$ in order to dissuade Y from pursuing the position is best for E if and only if

$$\begin{gathered} \Pi_E'\left(\bar{l}_E\right) \geq 0 \\ \Leftrightarrow \frac{1}{2} V_E \sqrt{\frac{\delta_E C}{\delta_Y V_Y}} \sqrt{\frac{\delta_E C}{\delta_Y V_Y}} - C \geq 0 \\ \Leftrightarrow \delta_E V_E \geq 2\delta_Y V_Y. \end{gathered}$$

[3]From this condition it is clear that the second order condition for maximization is satisfied.

Thus, the choice of l_E by E at the unique Subgame Perfect Nash Equilibrium is:

$$l_E^* = \begin{cases} \frac{\delta_E V_E^2}{4C\delta_Y V_Y}, & if \quad \delta_E V_E < 2\delta_Y V_Y \\ \frac{\delta_Y V_Y}{\delta_E C}, & if \quad \delta_E V_E \geq 2\delta_Y V_Y \end{cases}. \tag{8}$$

Subsequently, Y will choose $l_Y = l_Y^*(l_E^*)$, which evaluating (5) at (8) and simplifying is:

$$l_Y^* = \begin{cases} \frac{\delta_E V_E (2\delta_Y V_Y - \delta_E V_E)}{4\delta_Y^2 C V_Y}, & if \quad \delta_E V_E < 2\delta_Y V_Y \\ 0, & if \quad \delta_E V_E \geq 2\delta_Y V_Y \end{cases}. \tag{9}$$

These levels of pursuit by the candidates lead to equilibrium probabilities of being named successor of

$$P_E^* = P_E(l_E^*, l_Y^*) = \begin{cases} \frac{\delta_E V_E}{2\delta_Y V_Y}, & if \quad \delta_E V_E < 2\delta_Y V_Y \\ 1, & if \quad \delta_E V_E \geq 2\delta_Y V_Y \end{cases} \tag{10}$$

for E and of

$$P_Y^* = P_Y(l_Y^*, l_E^*) = \begin{cases} \frac{2\delta_Y V_Y - \delta_E V_E}{2\delta_Y V_Y}, & if \quad \delta_E V_E < 2\delta_Y V_Y \\ 0, & if \quad \delta_E V_E \geq 2\delta_Y V_Y \end{cases} \tag{11}$$

for Y. Finally, the equilibrium payoffs of E and Y, obtained by evaluating (1) and (2) at (8) and (9), are

$$\Pi_E^* = \begin{cases} \frac{\delta_E V_E^2}{4\delta_Y V_Y}, & if \quad \delta_E V_E < 2\delta_Y V_Y \\ V_E - \frac{\delta_Y}{\delta_E} V_Y, & if \quad \delta_E V_E \geq 2\delta_Y V_Y \end{cases} \tag{12}$$

and

$$\Pi_Y^* = \begin{cases} \frac{(2\delta_Y V_Y - \delta_E V_E)^2}{4\delta_Y^2 V_Y}, & if \quad \delta_E V_E < 2\delta_Y V_Y \\ 0, & if \quad \delta_E V_E \geq 2\delta_Y V_Y \end{cases} \tag{13}$$

respectively.

Table 1 provides a summary of how the equilibrium levels of l_E^*, l_Y^*, P_E^*, P_Y^*, Π_E^*, and Π_Y^* each depend upon the exogenous parameters δ_E, δ_Y, V_E, V_Y, and C. Each cell within this table contains an ordered pair, such as $(+, -)$. The first term in each ordered pair indicates the directional change for the relevant expression at the "interior solution" (which arises when $\delta_E V_E < 2\delta_Y V_Y$), and the second term in each ordered pair indicates the directional change for the relevant expression at the "corner solution" (which arises when $\delta_E V_E \geq 2\delta_Y V_Y$). An entry of: $+$ indicates that the equilibrium value is increasing in the corresponding exogenous parameter; $-$ indicates that the equilibrium value is decreasing in the corresponding exogenous parameter; 0 indicates that the equilibrium value does not depend upon the corresponding exogenous parameter; and *nm* indicates that the equilibrium value is non-monotonic (over the relevant range) in the corresponding exogenous parameter.

For example, the pair $(+, -)$ in the upper-most, left-most cell indicates that (all other factors fixed) an increase in δ_E (which corresponds to the founder becoming more predisposed to choose E as the successor) would result in an increase in the value of l_E^* when the interior solution arises and a decrease in the value of l_E^* when the corner solution arises. These observations follow directly from an inspection of Equation (8), which reveals:

$$\frac{\partial l_E^*}{\partial \delta_E} = \begin{cases} \frac{V_E^2}{4C\delta_Y V_Y} > 0, & if \quad \delta_E V_E < 2\delta_Y V_Y \\ -\frac{\delta_Y V_Y}{\delta_E^2 C} < 0, & if \quad \delta_E V_E \geq 2\delta_Y V_Y \end{cases}$$

Since an interior solution is realized if and only if $\delta_E V_E < 2\delta_Y V_Y$, these observations actually reveal that over the full range of possible values of δ_E the equilibrium level of l_E^* is non-monotonic in δ_E.

This non-monotonic relation is fairly intuitive. First consider parameter values for which an interior solution is realized. Equation (7) reveals that the marginal benefit to E from increasing l_E is $\frac{1}{2}V_E\sqrt{\frac{\delta_E C}{\delta_Y V_Y}}\sqrt{\frac{1}{l_E}}$, which is greater when δ_E is larger in value. From this observation (along with a recognition, also revealed by Equation (7), that the marginal cost from increasing l_E does not depend upon δ_E), we can readily see why a larger value of l_E^* is preferred (at the interior solution) when δ_E is larger. Further, recognize that $\bar{l}_E = \frac{\delta_Y V_Y}{\delta_E C}$ (the minimum level of l_E for which Y will be dissuaded from pursuing the position) is decreasing in δ_E. Recall, when opting to dissuade Y from pursuing the position it is best for E to do so by choosing the smallest value of l_E which induces such a response from Y. Thus, once δ_E is increased to the point at which $\delta_E V_E \geq 2\delta_Y V_Y$ (so that a corner solution is realized), further increases in δ_E lead to a decrease in the equilibrium value of l_E^*, precisely because Y can be dissuaded from pursuing the position by a lower choice of l_E.

From Table 1, we see that the only equilibrium expression which is non-monotonic over either the range of interior solutions or the range of corner solutions is the expression for l_Y^* with respect to a change in δ_Y. From (9) it follows that $\frac{\partial l_Y^*}{\partial \delta_Y} = \frac{\delta_E V_E(\delta_E V_E - \delta_Y V_Y)}{2\delta_Y^3 C V_Y}$ for $\delta_E V_E < 2\delta_Y V_Y$. Thus, $\frac{\partial l_Y^*}{\partial \delta_Y} > 0$ for $\delta_E V_E > \delta_Y V_Y$, whereas $\frac{\partial l_Y^*}{\partial \delta_Y} < 0$ for $\delta_E V_E < \delta_Y V_Y$ (each of which can occur for $\delta_E V_E < 2\delta_Y V_Y$).

This impact on the behavior of l_Y^* with respect to a change in δ_Y can be explained as follows. Start by focusing on relatively small values of δ_Y and consider how the optimal choice of l_Y^* changes as δ_Y increases (i.e., as the founder becomes more predisposed to choosing Y as the successor). For relatively small values of δ_Y we have $2\delta_Y V_Y \leq \delta_E V_E$, in which case the optimal choice of Y is $l_Y^* = 0$ (i.e., to not pursue the position). This is best because the founder is so relatively predisposed to *not* choose Y as the successor. Once δ_Y is increased to the point where $2\delta_Y V_Y = \delta_E V_E$ we still have $l_Y^* = 0$, whereas once δ_Y is large enough so that $2\delta_Y V_Y > \delta_E V_E$ we have $l_Y^* > 0$. That is, once the founder becomes sufficiently open to the choice of naming Y as the successor, Y chooses to actively pursue the position. But, as δ_Y becomes very large, the founder becomes so predisposed to naming Y as the successor that E in essence gives up and Y is able to virtually guarantee getting the position by exerting minimal effort. That is, as $\delta_Y \to \infty$: $l_E^* \to 0$; $P_Y^* \to 1$; and $l_Y^* \to 0$.

The bulk of the results reported in Table 1 are intuitive and expected. For example, we see that Π_E^* (the equilibrium payoff of E) is: increasing in δ_E; decreasing in δ_Y; increasing in V_E; decreasing in V_Y; and not dependent upon C. That is, the payoff of E is larger when the founder is more inclined to choose E as the successor or when E values the position more highly. Similarly, the payoff of E is smaller when the founder is more inclined to choose Y as the successor or when Y values the position more highly.[4] In some sense, the intuitive nature of these results serves as a verification that the model reasonably captures the important ways in which the two siblings interact and compete with each other in their quest to be chosen as successor. Within the next section, an examination of the equilibrium outcome is undertaken, in order to determine if this process potentially treats the candidates in an unequal, biased manner.

4 Comparison of Outcome Across Candidates

This section contains a detailed inspection of the equilibrium, following two different approaches. First, direct comparisons between E and Y at the equilibrium outcome are made. Second, fixing the fundamental characteristics of each candidate (i.e., for "Sibling i" fix δ_i and V_i, and for "Sibling j" fix δ_j and V_j) a counterfactual analysis is conducted to determine if a candidate would prefer to be the elder sibling or the younger sibling.

4.1 Direct Comparisons of Equilibrium Outcomes

In this subsection, direct comparisons of the equilibrium levels of pursuit, probabilities of being chosen successor, and payoffs between E and Y are made. Throughout this discussion, keep in mind that while it may be difficult in practice to observe values of payoffs and levels of pursuit, the outcome of the succession process (in the present context, reflected by the probability with which each sibling is chosen as the successor) is more easily observable (particularly on an aggregate level).

[4]The fact that the payoff of E does not depend upon the magnitude of C is a consequence of the simplifying assumption that the costs of pursuing the position are common across the two siblings, so that a larger value of C results in not only higher costs of pursuit for E but also for Y as well.

For $\delta_E V_E \geq 2\delta_Y V_Y$ we have the corner solution in which Y chooses to not pursue the position. Recognize that (all other factors fixed) this arises when either: δ_E is sufficiently large (i.e., the founder is relatively inclined to choose E); V_E is sufficiently large (i.e., E places a sufficiently large value on the position); δ_Y is sufficiently small (i.e., the founder is relatively *not* inclined to choose Y); or V_Y is sufficiently small (i.e., Y places a sufficiently small value on the position). In such cases we trivially have that: E pursues the position to a greater degree than Y; E is named the successor with greater probability than Y; and E realizes a greater payoff than Y.

For situations with $\delta_E V_E < 2\delta_Y V_Y$, we have the interior solution under which both candidates pursue the position. In such instances we see, from (8) and (9), that

$$l_E^* \geq l_Y^* \Leftrightarrow \frac{\delta_E V_E^2}{4C\delta_Y V_Y} \geq \frac{\delta_E V_E \left(2\delta_Y V_Y - \delta_E V_E\right)}{4\delta_Y^2 C V_Y}$$

$$\Leftrightarrow \delta_Y V_E + \delta_E V_E - 2\delta_Y V_Y \geq 0. \tag{14}$$

For parameter values satisfying $\delta_E V_E < 2\delta_Y V_Y$ this inequality may be either satisfied or violated. That is, at an interior solution, E may choose either a higher level or lower level of pursuit for the position (depending upon the parameters of the model).

From (10) and (11) we see that

$$P_E^* \geq P_Y^* \Leftrightarrow \frac{\delta_E V_E}{2\delta_Y V_Y} \geq \frac{2\delta_Y V_Y - \delta_E V_E}{2\delta_Y V_Y}$$

$$\Leftrightarrow \delta_E V_E - \delta_Y V_Y \geq 0. \tag{15}$$

Again, this inequality may be either satisfied or violated, implying that in equilibrium the probability of E being named successor may be either greater than or less than the probability of Y being named successor (depending upon the parameters of the model).

Finally, from (12) and (13) we see that

$$\Pi_E^* \geq \Pi_Y^* \Leftrightarrow \frac{\delta_E V_E^2}{4\delta_Y V_Y} \geq \frac{\left(2\delta_Y V_Y - \delta_E V_E\right)^2}{4\delta_Y^2 V_Y}$$

$$\Leftrightarrow \delta_E V_E \left(\delta_Y V_E - \delta_E V_E\right) - 4\delta_Y V_Y \left(\delta_Y V_Y - \delta_E V_E\right) \geq 0. \tag{16}$$

Once again, this condition may either be satisfied or violated, revealing that the equilibrium payoff of E may be either greater than or less than the equilibrium payoff of Y (depending upon the parameters of the model).

To easily see that each of these three comparisons could go in either direction, consider $V_E = V_Y$ (in which case the condition for having an interior solution becomes $\delta_E < 2\delta_Y$). When the two siblings have the same value for being chosen as the successor, we readily see that $l_E^* \geq l_Y^*$ if and only if $\delta_E \geq \delta_Y$ (from (14)) and $P_E^* \geq P_Y^*$ if and only if $\delta_E \geq \delta_Y$ from (15). Further, from (16) it follows that $\Pi_E^* \geq \Pi_Y^*$ if and only if:

$$-\delta_E^2 + 5\delta_E\delta_Y - 4\delta_Y^2 \geq 0 \Leftrightarrow (\delta_E - \delta_Y)(4\delta_Y - \delta_E) \geq 0.$$

Since we are focusing on a situation in which the interior solution is realized, $4\delta_Y - \delta_E > 0$, implying that $\Pi_E^* \geq \Pi_Y^*$ if and only if $\delta_E \geq \delta_Y$.[5]

That is, when the two siblings have identical values for becoming the successor, the comparison of the equilibrium outcome depends solely and directly upon the degree to which the founder is predisposed to choose either the elder or younger candidate. If $\delta_E > \delta_Y$ (i.e., if the founder is predisposed to choose E over Y), then E will pursue the position more aggressively, be chosen as successor with a greater probability, and ultimately realize a greater expected payoff than Y. In contrast, if $\delta_E < \delta_Y$ (i.e., if the founder is predisposed to choose Y over E), then E will pursue the position less aggressively, be chosen as successor with a lower probability, and ultimately realize a lower expected payoff than Y.

Further, when $V_E = V_Y$ and $\delta_E = \delta_Y$ (i.e., in a purely symmetric environment) the two candidates will pursue the position to the same degree, be chosen successor with equal probability, and realize identical payoffs. The fact that the two candidates have identical equilibrium payoffs when $V_E = V_Y$ and $\delta_E = \delta_Y$ suggests that in such a purely symmetric setting there is not a bias toward the candidate who chooses his level of pursuit first. This should be viewed as a somewhat comforting result, since it suggests that if the "order of choice" is determined in an exogenous and arbitrary fashion (e.g., by birth order), the process is not rigged against or in favor of either candidate. As a consequence, it would seem as if the founder need not take any deliberate actions to alter the sequence of choice out of concerns for achieving a more equitable outcome.

But it should be recognized that a built in bias on the part of the founder can tip the scales in favor of one candidate or the other. Further, observed differences in succession outcomes over an entire population can be a revelation of such a built in bias. For example, suppose that over the entire population, we systematically observe that elder siblings are chosen as successors more frequently than younger siblings. What can be made of this observed outcome?

Suppose that in each instance of a succession decision the values of V_E and V_Y are determined as independent random draws from a common probability distribution. This implicitly assumes that there is no *a priori* difference between the two siblings. After values

[5]Similarly, for situations with $\delta_E = \delta_Y = \delta$ we could easily see that $l_E^* \geq l_Y^*$, $P_E^* \geq P_Y^*$, and $\Pi_E^* \geq \Pi_Y^*$ each occur if and only if $V_E \geq V_Y$.

of V_E and V_Y are realized, the siblings interact and a successor is chosen under the framework described and analyzed thus far. Over the entire population, we will systematically observe elder siblings being chosen as successors more frequently than younger siblings when $P_E^* > P_Y^*$. For this discussion, let $P_E^* = \frac{\delta_E V_E}{2\delta_Y V_Y} = p$ and realize that we are considering $p > \frac{1}{2}$. The equation $\frac{\delta_E V_E}{2\delta_Y V_Y} = p$ can be rearranged as $2\delta_Y V_Y p = \delta_E V_E$. Since the values of V_E and V_Y are determined as independent random draws from a common probability distribution, for this condition to hold in expectation would require $2\delta_Y p = \delta_E$ or, equivalently, $p\delta_Y = \frac{1}{2}\delta_E$. Since we are focusing on circumstances with $p > \frac{1}{2}$, this necessitates $\delta_E > \delta_Y$. That is, under the assumption that there are no *a priori* differences between the siblings in terms of their valuations for the position, an observation that elder siblings are chosen as successors more frequently than younger siblings would reveal a built in predisposition or bias on the part of the founder to choose the elder child as the successor.

4.2 Counterfactual Comparisons of Equilibrium Outcomes

The previous subsection made direct comparisons of different aspects of the equilibrium outcome between the two siblings. In this subsection a slightly different approach is taken in order to determine if there is a elder advantage or disadvantage to the sequential succession process as modeled. Consider a situation in which there are two siblings of fixed characteristics and suppose the preference on the part of the founder for one individual over the other is based upon the actual attributes of the individuals and not simply birth order. That it, "Sibling i" has a value of V_i for being named the successor, "Sibling j" has a value of V_j for being named the successor, and the inclination of the founder's succession choice is reflected by the parameters δ_i and δ_j.

By way of a formal analysis, we determine if there is a "first mover advantage" by answering the the question: Would a "Sibling i" characterized by V_i and δ_i facing a rival "Sibling j" characterized by V_j and δ_j prefer to be the elder sibling or the younger sibling? That is, if a candidate knew the values placed on being chosen as successor and the parameters in the probability functions for both himself and his sibling, would he prefer to be the elder sibling or the younger sibling. For a candidate of such fixed attributes, a preference for being the elder sibling reveals an elder sibling advantage, whereas a preference for being the younger sibling reveals an elder sibling disadvantage.

Suppose $V_i = \alpha V$ and $V_j = V$. Thus, $\alpha = \frac{V_i}{V_j}$, from which it is apparent that $\alpha > 1$ corresponds to a situation in which $V_i > V_j$ and $\alpha < 1$ corresponds to a situation in which $V_i < V_j$. Similarly, suppose $\delta_i = \tau\delta$ and $\delta_j = \delta$. Consequently, $\tau = \frac{\delta_i}{\delta_j}$, from which it is apparent that $\tau > 1$ corresponds to a case in which $\delta_i > \delta_j$ and $\tau < 1$ corresponds to a case in which $\delta_i < \delta_j$. If i is the elder sibling, then an interior solution is realized if and only if:

$$\delta_E V_E < 2\delta_Y V_Y \Leftrightarrow \alpha\tau < 2.$$

If i is instead the younger sibling, then an interior solution is realized if and only if:

$$\delta_E V_E < 2\delta_Y V_Y \Leftrightarrow 1 < 2\alpha\tau \Leftrightarrow \frac{1}{2} < \alpha\tau.$$

Thus, this process of sequential choice results in an interior solution irrespective of whether i is the elder or younger sibling so long as $\alpha\tau \in \left[\frac{1}{2}, 2\right]$. If $\alpha\tau < \frac{1}{2}$, then an interior solution is realized when i is the elder sibling whereas i will be induced to not pursue the position when i is the younger sibling. Similarly, for $\alpha\tau > 2$, an interior solution is realized when i is the younger sibling whereas i will induce j to not pursue the position when i is the elder sibling.

Let $\tilde{\Pi}_E$ denote the payoff of "Sibling i" if he were the elder sibling, and let $\tilde{\Pi}_Y$ denote the payoff of "Sibling i" if he were the younger sibling. Evaluating (12) at the relevant parameter values (and correctly recognizing when an interior versus corner solution arises) yields:

$$\tilde{\Pi}_E = \begin{cases} \frac{\alpha^2\tau V}{4}, & if \quad \alpha\tau \leq 2 \\ \left(\alpha - \frac{1}{\tau}\right)V, & if \quad \alpha\tau > 2 \end{cases}.$$

A similar evaluation of (13) yields:

$$\tilde{\Pi}_Y = \begin{cases} 0, & if \quad \alpha\tau < \frac{1}{2} \\ \frac{(2\alpha\tau-1)^2 V}{4\alpha\tau^2}, & if \quad \alpha\tau > \frac{1}{2} \end{cases}.$$

The aim is to make a full comparison of $\tilde{\Pi}_E$ to $\tilde{\Pi}_Y$ at all possible parameter values. Letting $z \equiv \alpha\tau$, recognize that $\tilde{\Pi}_E > \tilde{\Pi}_Y$ if and only if

$$\theta_\Pi(z) \equiv \frac{\tilde{\Pi}_Y}{\tilde{\Pi}_E} = \begin{cases} 0, & if \quad z < \frac{1}{2} \\ \frac{(2z-1)^2}{z^3}, & if \quad z \in \left[\frac{1}{2}, 2\right] \\ 1 + \frac{1}{4z(z-1)}, & if \quad z > 2 \end{cases}$$

is less than 1.

Clearly $\theta_\Pi(z) < 1$ for $z < \frac{1}{2}$ and $\theta_\Pi(z) > 1$ for $z > 2$. Focusing on $z \in \left[\frac{1}{2}, 2\right]$, observe that $\theta_\Pi\left(\frac{1}{2}\right) = 0$, $\theta_\Pi(1) = 1$, and $\theta_\Pi(2) = \frac{9}{8}$. Further, for z in this range, $\theta'_\Pi(z) = (3-2z)\frac{2z-1}{z^4}$. It is straightforward to see that $\theta'_\Pi(z) \geq 0$ for $z \in \left[\frac{1}{2}, \frac{3}{2}\right]$ whereas $\theta'_\Pi(z) < 0$ for $z \in \left(\frac{3}{2}, 2\right]$. Thus, $\theta_\Pi(z) < 1$ for $z \leq 1$ and $\theta_\Pi(z) > 1$ for $z > 1$. Figure 1 illustrates $\theta_\Pi(z)$.

In terms of $\tilde{\Pi}_E$, $\tilde{\Pi}_Y$, α, and τ, these observations are that: $\tilde{\Pi}_E > \tilde{\Pi}_Y$ for $\alpha\tau < 1$; $\tilde{\Pi}_E = \tilde{\Pi}_Y$ for $\alpha\tau = 1$; and $\tilde{\Pi}_E < \tilde{\Pi}_Y$ for $\alpha\tau > 1$. So, we see that a "Sibling i" with $V_i = \alpha V$

and $\delta_i = \tau\delta$ (facing a sibling with $V_j = V$ and $\delta_j = \delta$) would prefer to be the elder sibling when α and τ collectively are "relatively small" (i.e., when their product is less than 1). At a very basic level, recognize that this clearly holds when both $\alpha \leq 1$ and $\tau \leq 1$. As a result, "Sibling i" prefers to be the elder sibling if he values the position less than his sibling and the founder is predisposed to choose his sibling.

More generally, for each possible arbitrary value of $\alpha > 0$, it follows that "Sibling i": prefers to be the elder sibling if $\tau < \frac{1}{\alpha}$; is indifferent between being the elder versus younger sibling if $\tau = \frac{1}{\alpha}$; and prefers to be the younger sibling if $\tau > \frac{1}{\alpha}$. Note, similar observations can be made regarding a cut-off value of α for each possible arbitrary value of $\tau > 0$. Such observations reinforce the notion that "Sibling i" prefers to be the elder sibling so long as α and τ are "sufficiently small," which correspond to δ_i and V_i being "relatively low" in value. In such situations, refer to "Sibling i" as "relatively weak."

Insight into why "Sibling i" has a preference for being the elder sibling when and only when he is "relatively weak" can be obtained by making comparisons of the equilibrium choice of level of pursuit and equilibrium probability of being chosen as successor for such a candidate if he were E versus if he were Y. Let $\tilde{l}_E$ denote the level of pursuit by "Sibling i" if he were the elder sibling, and let $\tilde{l}_Y$ denote the level of pursuit by "Sibling i" if he were the younger sibling. Evaluating (8) at the relevant parameter values (and correctly recognizing when an interior versus corner solution will be realized) yields:

$$\tilde{l}_E = \begin{cases} \frac{\alpha^2\tau V}{4C}, & if \quad \alpha\tau \leq 2 \\ \frac{V}{\tau C}, & if \quad \alpha\tau > 2 \end{cases}.$$

A similar evaluation of (9) yields:

$$\tilde{l}_Y = \begin{cases} 0, & if \quad \alpha\tau < \frac{1}{2} \\ \frac{(2\alpha\tau-1)V}{4\alpha\tau^2 C}, & if \quad \alpha\tau > \frac{1}{2} \end{cases}.$$

As was done for $\tilde{\Pi}_E$ and $\tilde{\Pi}_Y$, the aim is to make a full comparison of $\tilde{l}_E$ to $\tilde{l}_Y$ at all possible parameter values. Again letting $z \equiv \alpha\tau$, recognize that $\tilde{l}_E > \tilde{l}_Y$ if and only if

$$\theta_l(z) \equiv \frac{\tilde{l}_Y}{\tilde{l}_E} = \begin{cases} 0, & if \quad z < \frac{1}{2} \\ \frac{2z-1}{z^3}, & if \quad z \in \left[\frac{1}{2}, 2\right] \\ \frac{2z-1}{4z}, & if \quad z > 2 \end{cases}$$

is less than 1. This is clearly the case for $z < \frac{1}{2}$ and $z > 2$.

Focusing on $z \in \left[\frac{1}{2}, 2\right]$, first recognize that $\theta_l\left(\frac{1}{2}\right) = 0$, $\theta_l(1) = 1$, and $\theta_l(2) = \frac{3}{8}$. Further, $\theta_l'(z) = \frac{3-4z}{z^4}$, from which it immediately follows that $\theta_l'(z) > 0$ for $z \in \left[\frac{1}{2}, \frac{3}{4}\right)$ and $\theta_l'(z) < 0$ for $z \in \left(\frac{3}{4}, 2\right]$ (implying that $\theta_l(z)$ achieves its maximum value at $z = \frac{3}{4}$). Thus, for $z \in \left[\frac{1}{2}, 2\right]$ we see that there exists a unique $\bar{z} \in \left(\frac{1}{2}, \frac{3}{4}\right)$ such that $\theta_l(z) > 1$ if and only if $z \in (\bar{z}, 1)$.[6] Figure 2 provides an illustration of $\theta_l(z)$.

This examination of $\theta_l(z)$ reveals that "Sibling i" would choose a higher level of pursuit if he is the younger sibling (compared to his chosen level of pursuit if he were the older sibling) if and only if he is just "slightly weaker" (i.e., $z \in (\bar{z}, 1)$). Otherwise – that is, when he is "relatively strong" (i.e., $z > 1$) or "drastically weaker" (i.e., $z < \bar{z}$) – he would choose a higher level of pursuit if he were the elder sibling.

Shifting attention to the equilibrium probability of being chosen as successor for "Sibling i," let $\tilde{P}_E$ denote this probability if he were the elder sibling and let $\tilde{P}_Y$ denote this probability if he were the younger sibling. Evaluating (10) at the relevant parameter values (and correctly recognizing when an interior versus corner solution will be realized) yields:

$$\tilde{P}_E = \begin{cases} \frac{\alpha\tau}{2}, & if \quad \alpha\tau \leq 2 \\ 1, & if \quad \alpha\tau > 2 \end{cases}.$$

A similar evaluation of (11) yields:

$$\tilde{P}_Y = \begin{cases} 0, & if \quad \alpha\tau < \frac{1}{2} \\ \frac{2\alpha\tau - 1}{2\alpha\tau}, & if \quad \alpha\tau > \frac{1}{2} \end{cases}.$$

Again, the aim is to make a full comparison of $\tilde{P}_E$ to $\tilde{P}_Y$ at all possible parameter values. With $z \equiv \alpha\tau$, recognize that $\tilde{P}_E > \tilde{P}_Y$ if and only if

$$\theta_P(z) \equiv \frac{\tilde{P}_Y}{\tilde{P}_E} = \begin{cases} 0, & if \quad z < \frac{1}{2} \\ \frac{2z-1}{z^2}, & if \quad z \in \left[\frac{1}{2}, 2\right] \\ \frac{2z-1}{2z}, & if \quad z > 2 \end{cases}$$

is less than 1. This is once again clearly the case for $z < \frac{1}{2}$ and $z > 2$. For $z \in \left[\frac{1}{2}, 2\right]$, first observe that $\theta_P\left(\frac{1}{2}\right) = 0$, $\theta_P(1) = 1$, and $\theta_P(2) = \frac{3}{4}$. Further, $\theta_P'(z) = \frac{2(1-z)}{z^3}$, which immediately reveals that $\theta_P(z)$ is maximized at $z = 1$. Thus, $\theta_P(z) \leq 1$ for all possible values of z. This function is illustrated in Figure 3.

This discussion of $\theta_P(z)$ reveals that "Sibling i" always achieves a higher probability of being chosen as successor if he is the elder sibling (as opposed to if he is the younger sibling). This observation, in and of itself, suggests a type of elder sibling advantage. Further, it may at first appear as if this observation is at odds with the previous observation summarized by Equation (15). However, there is no inconsistency. Within the previous subsection, Equation (15) stated a condition which allowed for a comparison of the equilibrium probability of E being chosen as the successor *vis-à-vis* the probability of his rival

[6] Note, $\bar{z} \approx .61803$.

Y being chosen as the successor. Within that previous discussion, it was noted that the younger sibling may in fact be named successor with a greater probability than the elder sibling. The current discussion makes a different comparison. Presently, the fundamental attributes of "Sibling i" are fixed, and it is observed that any candidate of such fixed attributes personally realizes a greater probability of being named successor if he is the elder sibling as opposed to the younger sibling.

Collectively the discussions of $\theta_l(z)$ and $\theta_P(z)$ provide insight into why "Sibling i" has a preference for being the elder sibling when and only when he is "relatively weak." As is evident from the initial specification of payoffs in Equations (1) and (2), the payoff of "Sibling i" depends fundamentally upon his chosen level of pursuit and the resulting probability with which he will be named successor. His payoff is greater if his chosen level of pursuit is lower or if the probability with which he will be named successor is higher.

Focusing first on $z \in (\bar{z}, 1)$, "Sibling i" would choose a higher level of pursuit but be named successor with a lower probability if he were Y instead of E. Thus, he has a clear preference for being the elder sibling when he is only "slightly weaker."

For both very small values of z (i.e., $z < \bar{z}$, in which case "Sibling i" is "drastically weaker") and for large values of z (i.e., $z > 1$, in which case "Sibling i" is "relatively stronger") "Sibling i" will choose a higher level of pursuit and will be named successor with a greater probability if he is E as opposed to Y. The former effect makes being Y more desirable, while the latter effect makes being E more desirable.

For $z < \bar{z}$, the values of $\tilde{l}_E$, $\tilde{l}_Y$, $\tilde{P}_E$, and $\tilde{P}_Y$ are such that the latter effect dominates, thereby giving i a strict preference for being E. This is partly intuitive since a "Sibling i" that is "drastically weaker" (i.e., one with an extremely low value of z) would be completely dissuaded from pursuing the position if he were the younger sibling.

Conversely, for $z > 1$, the values of $\tilde{l}_E$, $\tilde{l}_Y$, $\tilde{P}_E$, and $\tilde{P}_Y$ are such that the former effect dominates, thereby giving i a strict preference for being Y. When $z > 1$ (i.e., when "Sibling i" is "relatively strong"), he is able to realize a substantial decrease in his level of pursuit if he is Y (relative to what he would choose if he were E) with only a small sacrifice in his probability of being chosen successor. These differences are revealed from an inspection of the functions $\theta_l(z)$ and $\theta_P(z)$ in Figures 2 and 3. At $z = 1$, $\theta_l(z) = \theta_P(z) = 1$. As z is increased up to $z = 2$ (the level at which each function achieves its minimum), the value of $\theta_P(z)$ decreases to only $\theta_P(2) = \frac{3}{4}$ while the value of $\theta_l(z)$ decreases all the way down to $\theta_l(2) = \frac{3}{8}$. That is, at $z = 1$ "Sibling i" would choose the exact same level of pursuit and be chosen as the successor with the same probability whether he is E or Y. In contrast, when $z = 2$ his chosen level of pursuit if he is Y is only $\frac{3}{8}$ of the level he would choose if he were E, but his probability of being named successor if he is Y (while lower than what it would be if he were E) is still $\frac{3}{4}$ of the level he would realize if he were E. As z is increased beyond $z = 2$, both $\theta_l(z)$ and $\theta_P(z)$ increase. But, in the limit as $z \to \infty$: $\theta_P(z) \to 1$ while $\theta_l(z) \to \frac{1}{2}$. That is, in the limit (when "Sibling i" is "drastically stronger"), he prefers being Y because (relative to what he would realize if he were E) he is able to exert half the level of pursuit while enjoying essentially the same probability of being chosen as successor.

In practice, a sibling of fixed characteristic will be either the elder or younger sibling. Thus, fixing V_i, δ_i, V_j, and δ_j, and subsequently defining $\tau = \frac{\delta_i}{\delta_j}$ and $\alpha = \frac{V_i}{V_j}$, "Sibling i" has a strict preference for being the elder sibling when and only when $\alpha\tau < 1$ as discussed above. However, recognize that from the perspective of "Sibling j," we could define $\alpha_j = \frac{V_j}{V_i} = \frac{1}{\alpha}$ and $\tau_j = \frac{\delta_j}{\delta_i} = \frac{1}{\tau}$ and apply all of the results of the discussion above to compare the outcome for "Sibling j" dependent upon whether he is the elder or younger sibling. Recognize that $\alpha_j\tau_j = \frac{1}{\alpha\tau}$, so that $\alpha_j\tau_j > 1$ if and only if $\alpha\tau < 1$.[7]

Therefore, focusing on the payoffs of the candidates, one of two distinct situations will be realized for the actual values of V_i, δ_i, V_j, and δ_j. Without loss of generality, suppose that "Sibling i" is the elder sibling and "Sibling j" is the younger sibling. First consider $\alpha\tau < 1$. Recognize that in such situations "Sibling i" prefers being the elder sibling over being the younger sibling, and furthermore, "Sibling j" prefers being the younger sibling over being the elder sibling. Thus, in terms of the arbitrarily determined order of choice in the sequential succession process, we have a situation of mutual harmony in which each candidate prefers to be in his actual position (as opposed to hypothetically being in the position of his sibling).

Now instead consider $\alpha\tau > 1$. Recognize that in such situations "Sibling i" prefers being the younger sibling over being the elder sibling, while "Sibling j" prefers being the elder sibling over being the younger sibling. Thus, in terms of the arbitrarily determined order of choice in the sequential succession process, we have a situation of mutual resentment in which each candidate would prefer to be in the position of his sibling (instead of the position in which he actually finds himself). We might expect that, in such scenarios, the siblings themselves might try to do anything that they could within their power to alter the order in which they choose their levels of pursuit for the position. For example, during young adulthood the elder sibling might temporarily pursue career options outside of the family business, not because he has no interest in acquiring control of the business, but rather because he prefers for his younger sibling to choose his level of pursuit for the position first.

[7] This observation is simply that "Sibling j" is "relatively strong" if and only if "Sibling i" is "relatively weak."

It is important to note that this situation arises when the elder sibling is “relatively strong” (e.g., if $\delta_i = \delta_j$ this occurs precisely when $V_i > V_j$). Thus, such an attempt by “Sibling i” to “delay his choice” or “have his sibling choose first” does not reveal any disinterest on the part of the sibling, but, on the contrary, would reveal a greater value for ultimately being named the successor.

In summary, this counterfactual analysis (in which the fundamental characteristics of each candidate are fixed and comparisons are made after changing the order in which the candidates choose their level of pursuit) reveals that the succession process is, in certain respects, biased. First, a candidate of fixed attributes always realizes a greater probability of being chosen as successor if he is the elder candidate instead of the younger candidate. Again, this suggests one clear advantage to being the elder sibling. Second, a candidate of fixed attributes realizes a greater equilibrium payoff if he is the elder candidate if and only if he is “relatively weak” (i.e., if and only if $\alpha\tau < 1$). For such candidates there is a clear advantage to being the elder sibling. In contrast, there is a clear advantage to being the younger sibling if and only if $\alpha\tau > 1$. Finally, it was noted that in practice the realized setting will be one of either mutual harmony (in which E prefers being in the position of the elder sibling and Y prefers being in the position of the younger sibling) or mutual resentment (in which E prefers being in the position of the younger sibling and Y prefers being in the position of the elder sibling).

Table 1: Comparative Statics of Equilibrium

Directional impact on...	of a change in: δ_E	δ_Y	V_E	V_Y	C
l_E^*	$(+,-)$	$(-,+)$	$(+,0)$	$(-,+)$	$(-,-)$
l_Y^*	$(+,0)$	$(nm,0)$	$(+,0)$	$(+,0)$	$(-,0)$
P_E^*	$(+,0)$	$(-,0)$	$(+,0)$	$(-,0)$	$(0,0)$
P_Y^*	$(-,0)$	$(+,0)$	$(-,0)$	$(+,0)$	$(0,0)$
Π_E^*	$(+,+)$	$(-,-)$	$(+,+)$	$(-,-)$	$(0,0)$
Π_Y^*	$(-,0)$	$(+,0)$	$(-,0)$	$(+,0)$	$(0,0)$

Figure 1 – Illustration of $\theta_{\Pi}(z)$

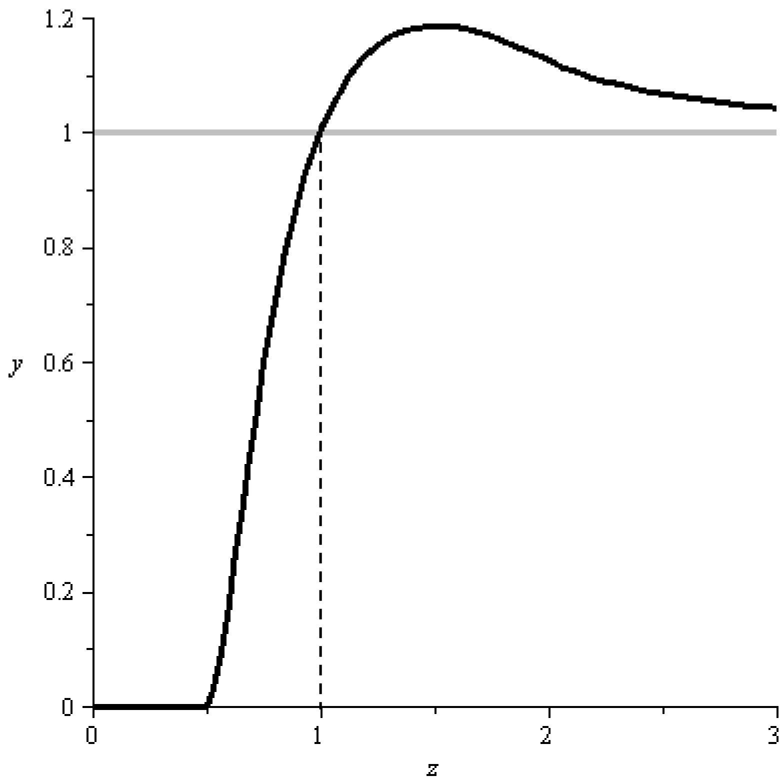

Figure 2 – Illustration of $\theta_l(z)$

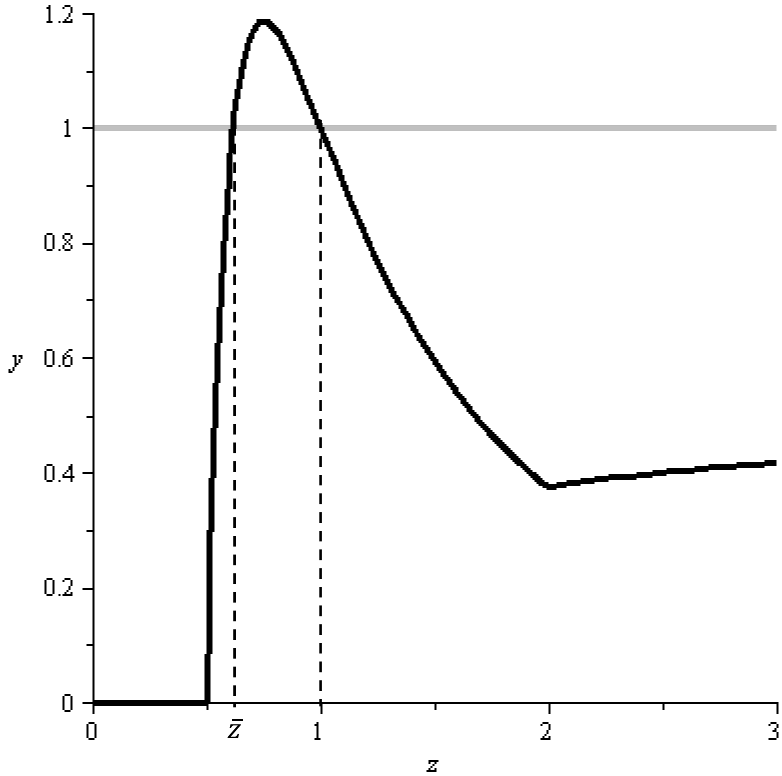

Figure 3 – Illustration of $\theta_P(z)$