Bank Competition and Lending Policy over Business Cycles*

Yan Liu†

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Abstract

Two facts of bank lending policy over business cycles, i.e., countercyclical loan spread and credit standard, are well documented in the empirical literature. Within a framework of dynamic credit market, featuring asymmetric information and imperfectly competitive banks with costly screening technology, it is showed analytically that the equilibrium (of a repeated game) captures simultaneously the two aspects of bank lending policy. Furthermore, the equilibrium countercyclicality stems exclusively from time variation of competitive intensity among banks, which is supported specifically by an often overlooked evidence contained in Fed's *Senior Loan Officer Opinion Survey*. More strikingly, when the basic framework is extended to account for variable funding cost, i.e., risk-free rate, then it can be showed that an *extended* period of historically low risk-free rate will induce *excessively* low credit standard, which provides an explanation for the unusual events in the credit market over years leading up to the Crisis.

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†Department of Economics, Stony Brook University. Email: `yan.liu@stonybrook.edu`.

I Introduction

Two facts of bank lending policy have been widely documented in the empirical literature: both loan spread (loan rate minus funding cost) and credit standard (broadly defined) tend to be countercyclical over business cycle.[1] Clearly, the countercyclicality of bank lending policy suggests itself as a potentially forceful source that amplifies business cycle fluctuations, given the prominence of bank credit—or more generally, any form of intermediated credit—in financing real economic activities. Thus, it is important to understand the underlying reasons for time variations in bank lending policy.

Intuitively, under perfect competition loan spread displays countercyclicality if associated risk is countercyclical in bank credit market. However, Aliaga-Díaz and Olivero (2011) show that countercyclicality is significant even after controlling various risk factors. Moreover, the countercyclicality of credit standard, such as standard for creditworthiness *ex ante* and collateralization requirement *ex post* loan approval decision, is even more puzzling under perfect competition (i.e., for a given loan spread), since the only criterion for granting a loan should be the time invariant positive NPV rule for any profit maximizing bank, regardless of the phase in which the business cycles is.

However, if one leaves the perfect competition paradigm and adopts an imperfect competition perspective, then it is possible to resolve the puzzling facts on the countercyclicality of loan spread and credit standard simultaneously, as showed in this paper. Actually, an explanation rests on imperfect competition is favored strongly by an often neglected piece of evidence contained in the Fed's *Senior Loan Officer Opinion Survey*, that more aggressive competition is consistently rated as the chief reason for both easing credit standard and lowering loan spread.

Now let me spell out a bit more details about the framework developed in this paper and the underlying intuitions for explaining the empirical facts about bank lending policy. The basic setup of the framework consists of a dynamic (i.e., repeated) credit market in which new borrowers apply for credit from the same set of $N < \infty$ symmetric banks to finance one-period projects and then die out. Bank lending is in simple loan contract and borrowers are protected by limited liability. There are two types of borrowers, good and bad. Good borrower have positive NPV projects whereas bad ones have only negative NPV projects, and each project can only be financed by a loan contract from one bank. Given some private benefit accruing to both types of borrowers, then only pooling equilibrium results and the

[1] For recent works on the former one, see Aliaga-Díaz and Olivero (2010, 2011) and Corbae and D'Erasmo (2012); and for the latter one, see Asea and Blomberg (1998), Lown, Morgan, and Rohatgi (2000), and Lown and Morgan (2006).

profitability of a loan on an average borrower depends only on the loan rate (assume no possibility of posting different amounts of collateral by different borrowers). Banks can (only) detect a borrower's type via a noisy screening screening technology in the form of creditworthiness test,[2] which produces (randomly) an informative yet unprecise indicator, G and B, of a borrower's type. The precision of the indicator, called screening intensity, is chosen by the testing bank; the test is costly in the screening intensity with a increasing and convex cost function. Some mild regularity conditions guarantee that whenever a test is conducted, the screening intensity is optimally chosen by the bank such that only G-borrowers receive credit and B-borrowers are denied credit. Furthermore, under the model structure, a lower screening intensity corresponds to a higher proportion of G-borrowers, thus more bad borrowers obtain credit, which in turn corresponds to a situation with easy credit standards.

The timing of the credit market in each period is specified in the following way. At the beginning of each period, banks offer loan contracts, each of which specifies a loan rate (hence a loan spread, assuming a constant funding cost that is zero), to attract borrowers. Next, each borrower chooses among contracts the most preferred one (with a uniform tie-breaking rule). The bank approached by a (unique) pool of borrowers then conducts creditworthiness test on every borrower in its pool, choosing screening intensity optimally conditional on the offered loan spread, and extends credit only to G-borrowers. At the end of the period, projects are completed and payoffs to each party are realized. Clearly, a monopoly bank will choose loan spread, and consequentially the screening intensity, so as to maximize the within-period profit, while perfectly competitive banks undercut each other to a level at which optimally chosen screening intensity can only result in zero profit on loans extended to G-borrowers.

To capture the notion of business cycles in this framework, consider a prototype binary-valued (stochastic) shock to the size of each project, hence the size of associated loan contract, which realizes at the beginning of each period, before credit market opens. The interpretation is straightforward, that a larger size corresponds to the expansionary phase and a smaller size corresponds to the recessionary phase. Banks are infinitely lived, with an objective of expected discounted profits for each. Intuitively, the (per period) monopoly profit on each loan is higher in the expansionary phase than that in the recessionary phase, given a larger loan size in the former phase. Nonetheless, the monopoly loan rate and associated screening intensity will not change, if both project payoff and screening cost are linear in the size of the loan; therefore, accordingly there is no fluctuation in bank lending policy over business cycles.

[2] This follows the literature on bank competition with screening initiated by Broecker (1990).

However, if one considers the optimal subgame perfect equilibrium among N symmetric banks competing in each period by offering loan contracts (i.e., loan spreads) subject to the shock specified above, then the equilibrium prediction changes dramatically. Essentially, the insight of Rotemberg and Saloner (1986) applies here, that banks compete more aggressively by charging lower loan rate in the expansionary phase so as to reduce the incentive for any one bank to deviate. The underlying intuition is straightforward. If equilibrium loan rate were set at too high a level in the expansionary phase, then undercutting by a single bank to acquire the entire market share would be strictly profitable under certain parametric restriction even under harshest future punishment (zero profit) by other banks. But the same is not true in the recessionary phase, as the potential profit from deviation is considerably limited due to smaller loan size. Having established that the equilibrium (under certain condition) features in a lower loan rate in the expansionary phase, it follows that the associated optimal screening intensity is lower, hence the credit standard is easier, in the expansionary phase.

II The Model: Basic Setup

Consider an environment of discrete time with infinite horizon, indexed by t. There is a *continuum* of borrowers with unit mass and $N < \infty$ symmetric banks, indexed by $i \in N \equiv \{1, \ldots, N\}$, both *risk-neutral* and *infinitely lived*. Every period, borrowers and banks interact in a *credit market*, where borrowers seeking credit constitute the demand-side while banks competing in extending credit constitute the supply-side.

II.A Borrower's Characteristics

At the beginning of time t, each one of *ex ante* identical borrowers is hit by an *investment opportunity* shock $\theta \in \Theta \equiv \{\theta^g, \theta^b\} \subset [0, 1]$ with $\Pr(\theta^g) = \mu^0$ and $\Pr(\theta^b) = 1 - \mu^0$, and is called a *type* θ borrower. For notational simplicity, Θ is identified with $\{g, b\}$, indicating "good" and "bad" respectively, with obvious meanings for g- and b-borrower. Shock θ is assumed to be iid across borrowers and over time, so that an application of a suitable law of large number (LLN) implies that the distribution of borrower's type is stationary, characterized by μ^0, yet independent over time. Consistent with the iid assumption, θ is assumed to be borrower's *private information*.

Coming with shock θ is a θ-project which makes available a constant-return-to-scale (CRS) technology to a θ-borrower: $\$(\widetilde{X}^\theta + \underline{u}^\theta)$ consumption good (perishable) will be produced at the end of time t by \$1 *capital good* input as (the only) initial investment;[3] $\widetilde{X}^\theta$

[3]Implicitly, a unit of account (call it 1 dollar or \$1) is assumed to be in place. However, I postulate that the unit of account is denominated in *real good* in order to rule out complications brought up by inflation in the subsequent dynamic analysis. Thus all interest rates discussed subsequently refer to real rates.

is random and independent across borrowers, taking value $X \in (1, \infty)$ with probability θ in success and $\lambda \in (0, 1)$ with probability $1 - \theta$ in failure; and $\underline{u}^\theta > 0$ is deterministic ($\underline{u}^g$ may differ from $\underline{u}^b$). In addition, $\widetilde{X}^\theta$ is *publicly observable* whereas $\underline{u}^\theta > 0$ is not, thus $\underline{u}^\theta$ represents all private benefits accruing to the borrower once the project is funded. CRS technology implies that a project of size z, $\forall\, z > 0$, will produce \$$z(\widetilde{X}^\theta + \underline{u}^\theta)$ output from \$$z$ input. However, a project is *indivisible* and all projects have a common size z given exogenously, thus unless a borrower can raise \$$z$, the project will not be started.

II.B Loan Contract

Borrowers receive no exogenous endowment (either consumption or capital good), neither do they possess a storage technology that transforms perishable consumption good produced before into currently usable capital good. Borrowers rely on bank credit to fund their projects. Bank lending is in one-period *loan* contract, each of which specifies a tuple $\ell = \langle R, C \rangle$ per \$1 credit: (gross) *interest rate* $R \equiv 1 + r$ to be repaid in case of no default with r denoting net interest rate, and collateral C to be transferred in case of default. Each loan is of size z and a project is not allowed to be funded by more than one loan. Borrowers are protected by *limited liability*. As a result, on the one hand, since $\underline{u}^\theta$ is not publicly observable, R is bounded from above by X; while on the other hand, hence not contractible, R is bounded from below by $1 + r^f$, since banks are assumed to have access to elastic supply of funds at risk-free rate r^f. Without loss of generality, r^f is normalized to 0. Given $R \equiv [1, X]$, a borrower defaults on the loan if and only if the project fails, thus the *default probability* of a θ-borrower is $1 - \theta$, and $1 - \lambda$ corresponds to *loss given default*. Moreover, by limited liability, the maximum value of C that can be enforced by the contract is λ. Evidently, in no circumstance will a contract ask for a $C < \lambda$. Thus, following analyses focus exclusively on loans of a form $\ell = \langle R, \lambda \rangle$, or $\ell = R$ for simplicity, where $R \in [1, X]$. For a unit (size) loan $\ell = R$ granted to a θ-project, Figure 1 depicts the payoff structure with expected payoff $u^\theta(R) = \theta(X - R) + \underline{u}^\theta > 0$ to the borrower and $\eta^\theta(R) = \theta R + (1 - \theta)\lambda - 1$ to the lending bank.

Since risk-free rate is zero, the *expected net present value* of a unit size θ-project is given by $\text{NPV}^\theta \equiv \eta^\theta(X) = \theta X + (1 - \theta)\lambda - 1$.

Assumption 1. Success probability $\theta^g > \theta^b$ is such that $\text{NPV}^g > 0 > \text{NPV}^b$. Moreover, the investment shock distribution $\mu^0 \geq \frac{1}{2}$ is such that[4]

$$\mathbb{E}^0\big[\text{NPV}^\theta\big] \equiv \mu^0 \text{NPV}^g + (1 - \mu^0)\text{NPV}^b > 0.$$

[4]Assuming $\mu^0 \geq \frac{1}{2}$ is only for expositional convenience; for analysis below, all we need is $\mu^0\theta^g - (1 - \mu^0)\theta^b > 0$, which is implied by $\mu^0 \geq \frac{1}{2}$.

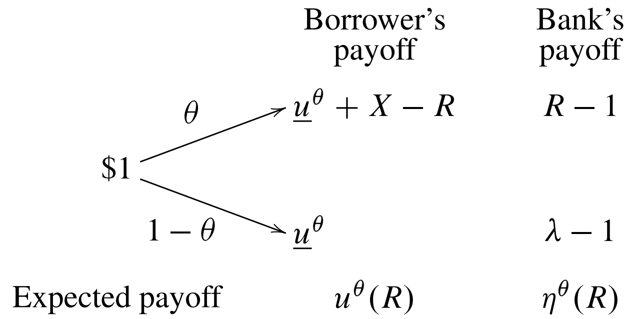

FIGURE 1: Payoff structure of a unit loan $\ell = R$ to a θ-borrower, $\theta \in \Theta$.

In words, g-project has a higher success probability, and parameters $(\mu^0, \theta^g, \theta^b, X, \lambda)$ are such that: (i) g-project has a positive (expected) NPV whereas b-project has a negative (expected) NPV (this also explains why g-borrower is "good" while b-borrower is "bad"); and (ii) *ex ante*, each project has a positive (expected) NPV. Note that, by LLN, $\mathbb{E}^0[\text{NPV}^{\theta}]$ also equals to the average NPV across all projects per unit size.

It is easy to show that $\eta^{\theta}(R)$ is strictly increasing in R, $\eta^g(R) > \eta^b(R)$, and $\eta^b(R) < 0 \;\forall\, R \in [1, X]$. Since $\eta^g(X) = \text{NPV}^g > 0$ and $\eta^g(1) < 0$, there is a unique $\underline{R} \in [1, X]$ such that $\eta^g(\underline{R}) = 0$ and $\eta^g(R) > 0$ for all $R > \underline{R}$. Evidently, no bank will ever offer a loan with $R < \underline{R}$, as the expected payoff is negative even for a g-borrower. Accordingly, let

$$\mathscr{L}^i = \left\{R(\ell) \middle| \ell \in \mathscr{L}^i\right\} \subset \mathscr{R} \equiv [\underline{R}, X]$$

denote both the set of loans and the associated index set, offered by bank $i \in N$; and let $\mathscr{L} = \cup_{i \in N} \mathscr{L}^i$ denote the set of all loans in the market (symbol $\cup$ denotes *disjoint union*).

II.C Information Asymmetry

Since $\text{NPV}^b < 0$, there exists no loan contract $\langle R, C \rangle$ such that lending bank's expected payoff from a b-borrower is greater than 1. As a result, if information is symmetric in the credit market, i.e., banks know the type of each individual borrower, then lending decision is trivial: b-borrowers will be denied credit unambiguously and a loan of size z will be offered to each g-borrowers. To make the bank's lending decision nontrivial, some form of asymmetric information is necessary. In particular, I assume that banks do not know directly the type of any individual borrower, nonetheless they do know the prior type distribution μ^0. Note that this is consistent with the assumption that borrower's type is private information. This pure adverse selection problem is the only information friction present in this framework. No effort is made for investigating implications of moral hazard problem on the part of either borrowers or banks.

II.D Screening Technology

Despite that on average lending to the entire population of borrowers can be profitable, as $\mathbb{E}^0[\mathrm{NPV}^\theta] > 0$, a profit maximizing bank always has incentive to identify a loan application from a b-borrower since $\mathrm{NPV}^b < 0$. To this end, I assume that each bank i has available a screening technology T^i, taking a form of creditworthiness test as in Broecker (1990), albeit *costly* and *noisy*. The test works as follows.

Test upon fresh borrower For a *fresh* borrower, i.e., a borrower without a *publicly visible* test record, a test with *screening intensity* q generates an *indicator* $\phi \in \Phi \equiv \{G, B\}$ which reveals some (but not all) information about the unobservable type of the borrower. In particular, following Thakor (1996), ϕ is random and correlated with θ in a specific way elucidated in the following assumption:

Assumption 2. Indicator ϕ generated in a test with screening intensity q upon a fresh borrower satisfies

$$\Pr(\phi = G|\theta = g) = \Pr(\phi = B|\theta = b) = q \in \mathbb{Q} \equiv \left[\tfrac{1}{2}, 1\right]. \tag{1}$$

In words, conditioning on the true type being g (b resp.), the probability of observing an indicator G (B resp.) is q.[5] Evidently, with q close to 1, ϕ becomes a fairly accurate indicator for θ. Indeed, the accuracy of the test is fully determined by the screening intensity q, as demonstrated by the posterior probabilities

$$\begin{aligned} \nu^G(q,\mu) &\equiv \Pr(g|G;q,\mu) = \frac{q\mu}{\Pr(G|q,\mu)} = \frac{q\mu}{q\mu + (1-q)(1-\mu)} \\ \nu^B(q,\mu) &\equiv \Pr(g|B;q,\mu) = \frac{(1-q)\mu}{\Pr(B|q,\mu)} = \frac{(1-q)\mu}{(1-q)\mu + q(1-\mu)} \end{aligned} \tag{2}$$

for an *arbitrary* prior $\Pr(g) = \mu \in \mathcal{U} \equiv [0,1]$.[6] For a brief illustration, suppose $\mu \in \mathrm{Int}\,\mathcal{U}$ (interior of $\mathcal{U}$) and suspending (q,μ). When $q = 1$, $\Pr(g|G) = \Pr(b|B) = 1$, so that ϕ is a perfect indicator of θ regardless of μ. In general, when $q \in \mathrm{Int}\,\mathbb{Q}$, there is $\Pr(g|G) > \mu > \Pr(g|B)$, thus ϕ is *informative* yet noisy for θ. Lastly, when $q = \frac{1}{2}$, $\Pr(g|G) = \Pr(g|B) = \mu$

[5] The specification that the test is symmetric on both types, $\Pr(G|g) = \Pr(B|b) = q$, is inconsequential. A straightforward generalization, as in the original formulation of Broecker (1990), has the form $\Pr(G|g) = q_g$ and $\Pr(B|b) = q_b$, so that screening intensity is allowed to be chosen separately for identifying different borrower's type. See Gehrig (1998) for a model incorporates this alternative specification.

[6] Here μ may be different from μ^0, the distribution of shock θ. Note $\nu^\phi(q,\mu)$ is well defined — indeed, smooth — over $\mathbb{Q} \times \mathcal{U}$, *except* for two points, i.e., $(1,0)$ for $\nu^G(\cdot)$ and $(1,1)$ for $\nu^B(\cdot)$. However, subsequent analysis does not depend on specific values assigned for $\nu^G(1,0)$ and $\nu^B(0,1)$. See Claim B.1 in Appendix B.1 for various properties of $\nu^\phi(q,\mu)$.

and $\Pr(b|G) = \Pr(b|B) = 1 - \mu$ and ϕ ceases to be informative. To sum up, by assigning a (fresh) borrower to some *class* G or B according to ϕ, a test provides with the bank some information on the type distribution of a borrower.

Central to this framework is that screening intensity q will be chosen optimally by a bank at a cost, as captured by a per test cost function $c(q)$ for a unit loan.[7] Screening cost displays constant return to scale in loan size, thus it costs $zc(q)$ for a test on a loan of size z.[8] As standard, $c(\cdot)$ is assumed to be increasing and convex, with $c\left(\frac{1}{2}\right) = 0$. Evidently, test result is no longer informative when $q = \frac{1}{2}$, therefore the bank should be indifferent about whether to conduct the test or not.

TEST UPON REVISITED BORROWER For a *revisited* ϕ-borrower, i.e., a borrower with a known test record ϕ, a new test will not reveal extra information regarding to borrower's type beyond that already contained in ϕ.[9] In particular, following assumption holds:

Assumption 3. Indicator ϕ_+ generated by a test upon a revisited ϕ-borrower satisfies

$$\Pr(\phi_+|\theta = g, \phi) = \Pr(\phi_+|\theta = b, \phi), \quad \phi_+,\ \phi \in \Phi. \tag{3}$$

Note that ϕ needs not to be the result of a test upon a fresh borrower. The relevant point is on the availability of such a previous test result to the bank considering a new test on the same borrower. On the one hand, a revisited borrower may be tested before yet test result was not revealed, so that the borrower will still be tested as a fresh one at a cost $c_+ = c(q)$ with intensity q. On the other hand, however ϕ is revealed, the next bank screening this borrower will take into account this information; indeed, the bank is not allowed to simply ignore a known ϕ. A test upon a revisited ϕ-borrower costs $c_+ = \bar{c}_+ \geq 0$. Nonetheless, as showed later, the precise form of the cost structure for such a test is irrelevant for optimal behavior of any party.

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[7]In the literature of bank lending with screening, the screening intensity is typically assumed to be constant, either costless as in Broecker (1990) and Riordan (1993) or of a fixed cost as in Thakor (1996), Cao and Shi (2001), and Gehrig and Stenbacka (2004, 2011). Several exceptions to my notice include Chan, Greenbaum, and Thakor (1986), Gehrig (1998), and Ruckes (2004), yet all of which have setups differ from this paper.

[8]This is a fairly strong assumption. However, as showed later, abstracting away scale economy in screening technology greatly simplifies subsequent algebra. More importantly, throughout this paper loan size is exogenous, thus allowing for scale economy in screening technology introduces only mechanical effect which sheds little extra light on the optimal screening problem facing each bank.

[9]This is for sure an extreme case. A general specification takes the form $\Pr(G|g, \phi) = \mathcal{F}(\phi) + \epsilon$, $\Pr(G|b, \phi) = \mathcal{F}(\phi) - \epsilon$, $\mathcal{F}(\phi) \in (0, 1)$, $\forall\, \phi \in \{G, B\}$. As long as $\epsilon > 0$ is small enough, i.e., a new test does not generate *too much* extra information for a revisited borrower, identical conclusions will be derived subsequently based on this slightly more general specification.

This assumption is new to the literature of bank screening (via creditworthiness test), and its principal implication will become transparent after specifying the timing within a period. Nonetheless, several remarks are in place to motivate this fairly strong assumption. Basic economics argument predicts that viable production technology used by any firm should have stable performance as a basic requirement, otherwise the technology will be modified in one way or another. In the same spirit, a bank's screening technology should be expected to generate close to identical *posterior type distribution* for the same borrower, no matter how many times the test has been conducted. As a result, for two identical indicators ϕ and ϕ_+ generated by consecutive tests of a bank, it is reasonable to expect that $\Pr(\theta|\phi_+, \phi)$ be close to $\Pr(\theta|\phi)$. Imposing equality then turns out to imply eq.(3). Indeed, Claim B.2 of Appendix B.1 demonstrates that they are precisely equivalent. Along the same line of argument, if the screening technology is such that the very initial test result persists in subsequent repetition of test by the same bank, i.e., $\Pr(\phi_+|\phi) = 1$ for $\phi_+ = \phi$, then not only eq.(3) holds but both sides equal to 1 (Claim B.3, Appendix B.1).

To carry the argument one step further, from single-bank repetitive-test to multi-bank sequential-test, a particular interpretation of the assumption of symmetric banks comes into stage. Typically, the assumption of symmetric firms means common production technology, i.e., given identical inputs, symmetric firms end up with the same quantity and/or quality of output. In the context of symmetric screening technology, taking as input a revisited borrower ϕ initially tested by bank i, a subsequent test by bank j should give rise to the same posterior type distribution as that of bank i in a repeated test. Put in another way, this reflects the idea that screening technology works in more or less identical way across banks. Further comments on alternative specifications of screening technology are relegated to Subsection V.A.

II.E Timing of the Lending Process

Bank lending process in each period consists of five stages:

1. All Banks offer sets of loans simultaneously and publicly, $\mathscr{L}^i$ for each bank $i \in N$, and $\mathscr{L} = \cup_{i\in N}\mathscr{L}^i$.

2. Each borrower chooses a loan $\ell \in \mathscr{L}$ to apply for as a fresh borrower. Borrowers choosing a loan ℓ offered by $N(\ell) > 1$ banks independently apply for one bank out of $N(\ell)$ with equal probability.

3. A bank offering ℓ decides whether to screen or not borrowers at ℓ. Upon screening, the bank chooses screening intensity q and conduct a test on every borrower. A lending decision, i.e., approving (A) or denying (D) an borrower, is made after observing test

result ϕ, which is not revealed in any verifiable form by the testing bank. Otherwise, the bank denies borrowers at ℓ altogether incurring zero cost.

4. A borrower approved by bank i in stage 3 at ℓ has two options: stay with i and get a loan ℓ of size z, or leave i and apply for a loan ℓ_+ from another bank $j \neq i$. A borrower denied by bank i in stage 3 can also apply for ℓ_+ from $j \neq i$.

5. Banks make lending decision as in stage 3 on revisited borrowers, conditional on all information revealed in previous stages. In particular, banks know for which contract a revisited borrower applied in stage 2.

II.F A Game-Theoretic Formulation

One interpretation of the lending process advocates a game-theoretic perspective. Coupled with payoff structure for each loan and cost structure for each test, the timing of the lending process determines a lending game among two types of (continuum) borrowers and N banks within each period. This is an extensive form game with incomplete information. Common knowledge about the entire game is presumed for all players. The corresponding game tree is illustrated in Figure 2, where a shaded box contains the relevant player, a rounded box indicates result of certain action, and a row vector in parenthesis at an terminal node represents payoffs to the relevant borrower (the first one) and bank (the second one). Figure 2 also makes clear the lending game has a particular structure: First, conditional on banks' choices $\mathscr{L} = \cup_{i \in N} \mathscr{L}^i$, the subsequent game (stage 2–5) consists of a (proper) subgame; second, stage 2–3 and 4–5 consist of two rounds of lending interaction; and third, when $\mathscr{L}$ contains more than two loans, each round of lending interaction resembles a *signaling* game. The signaling game structure stems from the fact that on the one hand each bank makes its screening choice (D or q) conditional on a belief $\mu(\ell) = \Pr(\theta = g|\ell) \in \mathscr{U}$ held on the prior type distribution at ℓ, while on the other hand borrowers will take into account the screening choice of each bank when choosing a loan $\ell \in \mathscr{L}$ to apply, as made explicit in the next Section. Here ℓ serves the role of a (potential) signal for borrower's type. Intuitively, these specific structures will have certain implications on the (static) equilibrium notion of the lending game to be made precise in the next Subsection.

It is worth to stress that in this framework, neither any restriction is imposed on the number of loans permitted to be offered by banks, nor a borrower is allowed only one chance in applying for a loan. The only seemingly restrictive assumption is that a borrower can apply for one loan in each round of lending interaction, yet supplemented with extra opportunities of applying for other loans in a second round. The sequential nature of this assumption is in no way too unrealistic. Indeed, a serious screening process by a bank on an borrower is a

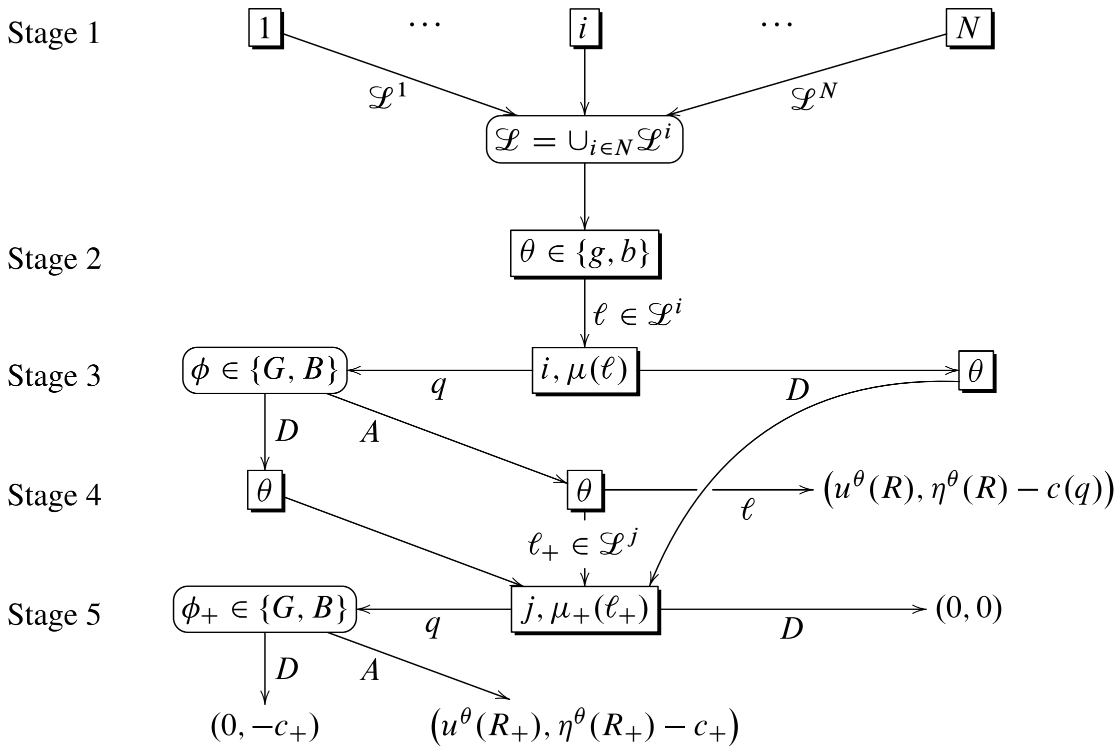

FIGURE 2: The lending game within a period.

Notes: Shaded box indicates the player making decisions in a stage, with $\mu(\ell) = \Pr(g|\ell)$ denoting the belief held by i for ℓ (likewise for $\mu_+(\ell_+)$). Labels along arrows indicate actions chosen by players. Rounded box shows the outcome of a player's action upon which subsequent action can be conditioned. Payoffs for a unit loan are specified in parentheses with borrower ordered first, and R (R_+) corresponds to ℓ (ℓ_+). Cost $c(q)$ is not shown when i denies θ after screening in stage 4.

substantial one in that some intimate interaction between the bank and borrower is required before any solid lending decision can be made, thus it seems reasonable to assume that a borrower can interact with only one bank at a time given various constraints (time, file work, etc.) facing the borrower. Moreover, the postulate of merely one extra round of lending interaction is innocuous; straightforward extensions can be readily made for more than one round, yet this adds no more insight but only complicates the model.

As a final remark on the lending game, strategic interaction *among* banks occurs only in stage 1, i.e., banks compete *directly* with each other for market share. In the subgame, in turn, strategic interaction occurs between two types of borrowers, each type as a whole, and all banks as a whole. Bank's decision on whether to screen or not, and (if the former) the screening intensity q, have no *direct* impact on other bank's screening decision. As shown below, under some suitable conditions, optimal screening choice depends only on the common prior $\mu(\ell)$ and $\mu_+(\ell)$ at each loan ℓ.

There can be another interpretation of the lending process, in particular for stage 2–5 (i.e., the subgame $\mathscr{L}$), which favors a typical competitive rational expectations perspective with almost nil strategic concern except for stage 1. This interpretation will be discussed at the end of Subsection III.D, in together with other comments on the timing specification and the game theoretic formulation advocated here.

III Analysis of the Static Game

III.A Solution Concept

For expositional purpose, call all banks player α and borrowers player β. Individual bank or borrower corresponds to distinct incarnation of α or β respectively. the *action space* $\mathcal{A}^i \equiv \mathcal{A}^\alpha$ for each bank i consists of $\{\mathscr{L} | \text{any subset of } \mathscr{R}\}$ for stage 1, and $\{D, \mathbb{Q} \times \{D, A\}\}$ for both stage 3 and 5 where the first D in the braces effectively means "no screen." A (pure behavior) strategy σ^i of bank i specifies for each *information set* of i an action from $\mathcal{A}^\alpha$. It is worth to stress that each $\ell \in \mathscr{L}^i$ is an information set of bank i in the subgame starting from $\mathscr{L}$, so is each realized ϕ from a test. The description of the bank side completes with a (common) belief system $\mu(\ell)$, $\mu_+(\ell)$ $\forall\, \ell \in \mathscr{L}$, with the possibility that $\mu_+(\cdot)$ be different from $\mu(\cdot)$. In addition, posterior belief $\nu^\phi(\cdot)$ at ϕ is determined by $\mu(\cdot)$ and $\mu_+(\cdot)$ for stage 3 and 5 respectively, in together with screening intensity q. Turning to the borrower side. Conditional $\mathscr{L}$, the action space $\mathcal{A}^\theta$ for a θ-borrower consists of $\mathscr{L}$ in stage 2 and $\mathscr{L}_+ \equiv \cup_{j \neq i} \mathscr{L}^j$ in stage 4 if a loan from i is chosen in stage 2 (hence $\mathscr{L}_+$ is borrower-bank specific). A (behavior) strategy σ^θ of borrower θ is defined accordingly, except that in contrast to σ^i, a priori there is no restriction on borrower's strategy to be pure; σ^θ is allowed to be mixed, so that arbitrary prior type distribution over $\mathscr{L}$ is possible whenever $\mathscr{L}$ contains more than one loan.

The solution concept used for the static lending game is the standard sequential equilibrium of Kreps and Wilson (1982). Precisely, an *equilibrium* is a profile of

$$\langle (\sigma^i)_{i \in N}, \mu(\cdot), \mu_+(\cdot); (\sigma^\theta)_{\theta \in \Theta} \rangle$$

such that belief systems $\mu(\cdot)$ and $\mu_+(\cdot)$ are *consistent* with the strategy profiles $(\sigma^i)_{i \in N}$ and $(\sigma^\theta)_{\theta \in \Theta}$, and *sequential rationality* is satisfied at each information set of each player. For notational simplicity, denote $\boldsymbol{\sigma} = \langle \boldsymbol{\sigma}^\alpha, \boldsymbol{\sigma}^\beta \rangle \equiv \langle (\sigma^i)_{i \in N}, (\sigma^\theta)_{\theta \in \Theta} \rangle$ the entire strategy profile. Moreover, since banks are symmetric in this setup, it is natural to restrain the analysis to the case of *symmetric equilibrium* in which $\sigma^i = \sigma^\alpha$ for all $i \in N$,[10] thus $\boldsymbol{\sigma}^\alpha = (\sigma^\alpha)_{1\times N}$. As mentioned before, conditional on $\mathscr{L}$, subsequent lending interactions become a proper subgame, denoted by $\mathscr{L}$, of the original lending game, therefore, *any* sequential equilibrium profile is subgame perfect conditional on $\mathscr{L}$. Consequently, backward induction can be used to solve the lending game: Determine the sequential equilibrium of each subgame $\mathscr{L}$ first, and then determine the equilibrium action $\mathscr{L}^i$ for each bank i in stage 1.

As is well-known, in general there are too many sequential equilibria in a signaling-like game. This is also true for the subgame $\mathscr{L}$ of the lending game, for each round of lending interaction resembles a signaling game. To be sure, when $\mathscr{L}$ contains only one loan contract, the lending game terminates at stage 3, since there is no other loan for borrowers to apply one more time. Accordingly, the number of sequential equilibria can be reduced considerably; indeed, the lending interaction between borrowers and banks is no longer a genuine game, since borrowers have only one loan to apply and banks only need to optimize the screening intensity for a $1/N$ share of all borrowers. Nonetheless, restricting $\mathscr{L}$ to be a singleton set is an overly stringent assumption, so is any explicit restriction on the action space $\mathcal{A}^\alpha$ or $\mathcal{A}^\theta$. Clearly, it would be desirable to have certain mild restrictions on the *strategic behavior* of borrowers and banks such that some particularly simple equilibrium outcome emerges.

In the rest of this Section, bank's screening choice and borrower's application choice will be explored in turn, accompanied by discussions for certain strategic restrictions, such that some particularly simple outcome holds necessary for any resulting sequential equilibrium. The guiding principle is on strategic complexity: players opt for strategies of low complexity whenever *doing so entails no direct impact upon immediate payoff*. Essentially, a program for refining the set of sequential equilibria is followed, though no effort is devoted to formalize this strategic complexity consideration.[11]

III.B Bank's Decision

Suppose a bank is in stage 3. Given a fresh borrower for a loan $\ell = R \in \mathscr{R}$, the bank needs to choose whether or not to screen the borrower, and if screen, at what screening intensity. The borrower can be of two types, $\theta = g$ or b, with expected payoff $\eta^\theta(\ell)$ for a unit loan. Bank's choice depends on the expected payoff of a loan ℓ to this borrower, which in turns

[10] Symmetric strategy on the part of borrowers of identical type has been incorporated into the definition of σ^θ, where a borrower's strategy only depends on type θ hence is identical across a continuum of borrowers of the same type.

[11] Strategic complexity considered here is indeed, in certain sense made clear below, a self-evident one. The treatment differs from the formal conception of strategic complexity based on automata (Turning machine) in the repeated game literature (see Rubinstein 1986, Abreu and Rubinstein 1988, and references therein).

depends on the (common) prior type distribution. Let μ denote this prior, i.e., $\mu = \Pr(g|\ell)$. As noted before, a test T with intensity $q \in \mathbb{Q}$ can generate some information about the unknown type of the borrower. In particular, bank's expected payoff from a unit loan to a class ϕ borrower, $\phi \in \Phi$, is

$$\eta^{\phi}(q, R, \mu) = \nu^{\phi}(q, \mu)\eta^{g}(R) + \big(1 - \nu^{\phi}(q, \mu)\big)\eta^{b}(R), \tag{4}$$

where $\nu^{\phi}(q, \mu) = \Pr(g|\phi; q, \mu)$ is given by eq.(2). It is straightforward to verify that $\nu^{\phi}(\cdot)$ is continuously differentiable over $\mathbb{Q} \times \mathcal{R} \times \mathrm{Int}\mathcal{U}$. Following Lemma summarizes several properties of $\eta^{\phi}(\cdot)$, where ∂_x denotes partial derivative with respect to x.

Lemma 1. *Under Assumption 1 and 2.* $\forall\, (q, R, \mu) \in \mathrm{Int}\mathbb{Q} \times \mathcal{R} \times \mathrm{Int}\mathcal{U}$, $\partial_R \eta^{\phi}(q, R, \mu)$, $\partial_\mu \eta^{\phi}(q, R, \mu) > 0$, $\forall\, \phi \in \Phi$, $\partial_q \eta^{G}(q, R, \mu) > 0$, *and* $\partial_q \eta^{B}(q, R, \mu) < 0$. *For a given* $R \in (\underline{R}, X]$, *there exists a unique* $a = \underline{a}(R) \in \mathrm{Int}\mathcal{U}$ *such that* $a\eta^{g}(R) + (1-a)\eta^{b}(R) = 0$. *Moreover,*

- *(a)* $\forall\, \mu \geq \underline{a}(R)$, $\eta^{G}(q, R, \mu) \geq 0$, $\forall\, q \in \mathbb{Q}$ *(equality holds only for* $\mu = \underline{a}(R)$ *and* $q = 1/2$*); and there exists* $q^{B}(R, \mu) \in \mathbb{Q}$ *such that* $\eta^{B}(q, R, \mu) < 0$ *iff* $q > q^{B}(R, \mu)$ *where* $q^{B}(R, \mu)$ *is strictly increasing in* R *and* μ*;*

- *(b)* $\forall\, \mu < \underline{a}(R)$, $\eta^{B}(q, R, \mu) < 0$, $\forall\, q \in \mathbb{Q}$*; and there exists* $q^{G}(R, \mu) \in \mathbb{Q}$ *such that* $\eta^{G}(q, R, \mu) > 0$ *iff* $q > q^{G}(R, \mu)$ *where* $q^{G}(R, \mu)$ *is strictly decreasing in* R *and* μ.

The proof is in Appendix C (p.38). Intuitively, expected payoff is increasing in loan rate and proportion of good borrowers. Moreover, since screening technology is informative, and *ex post* borrower composition assigned to class G improves with the accuracy of test, $\eta^{G}(\cdot)$ is increasing in q, whereas the converse is true for $\eta^{B}(\cdot)$. In the same spirit, given a high enough *ex ante* proportion μ of good borrower, *ex post* composition of class G can not be worse than μ, so $\eta^{G}(\cdot) > 0$, while with a sufficiently high screening intensity it is always possible reduce the proportion of good borrowers in class B such that the expected payoff from such a class is negative. Lastly, when *ex ante* composition is bad enough, one has to screen borrowers with hight enough intensity such that expected payoff from class G is positive.

Suppose a test has been conducted and an indicator ϕ is available. Now, the bank needs to decide whether or not to approve the loan application in light of information revealed by ϕ. Any strategy specifying an action at this point is necessarily a *decision rule*, since it will be applied systematically across all borrowers, perhaps at different loan contracts, tested by the bank.[12] A particularly *simple* decision rule suggests itself in this case: approval when $\eta^\phi(\cdot) \geq 0$ and denial when $\eta^\phi(\cdot) < 0$. Ignoring the strategic environment, this standard NPV rule is optimal to the bank apparently. However, the strategic nature of the multi-round lending interaction does render this simple NPV rule questionable as the *unique* equilibrium outcome. In particular, if the bank is able, or even willing, to get involved in sophisticated strategic reasonings, then it is no longer *obvious* that such a simple NPV rule should actually prevail in any equilibrium.[13]

The underlying twist is twofold. First of all, if the bank is uncertain about whether an approved borrower will stay with it, then the optimality of this simple decision rule is called into question. More importantly, any particular decision rule to be applied upon tested borrowers by one bank, such as the simple NPV rule, affects *indirectly* type distributions of revisited borrowers over all loan contracts in stage 4, hence *indirectly* the payoffs of all players (including the bank itself) in stage 5.

However, the validity of adopting the simple NPV rule can be ensured by resorting to two extra conditions on bank's behavior. The first one is a specific *expectation* held by the bank in question, henceforth labeled by E-α, which states that

> *whenever a loan application is approved by a bank, the applying borrower will indeed choose to stay with it and not to apply another loan.*

Under this hypothetic expectation, and *suppose* for a moment that the bank ignores any indirect impact upon payoffs in stage 5, then sequential rationality implies that the simple NPV rule is the unique best response of the bank, given payoff zero from denying an borrower (gross of screening cost).

The second condition concerns with the supposed ignorance of the bank on indirect impacts of a particular decision rule. This is where the strategic complexity consideration comes into play. Assessing precisely and fully all strategic consequences of an alternative decision rule is a vastly complicated task for any bank in this setup. Once taking into account

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[12] Recall that there is a continuum of borrowers. Except for the extreme case in which the bank in question either is approached by no borrower or decides to reject all borrowers, the bank should always be dealing with many borrowers.

[13] As a first glance may suggest, provided that the particular lending interaction between a bank and a borrower is a *proper* subgame where lending decision D or A leads to directly a terminal node, *backward induction* or *dominance criterion* should immediately solicit the simple NPV rule as the unique best response, regardless strategic considerations in other respects. However, this reasoning is not valid since the bank is playing identical subgames with many (perhaps continuum) borrowers simultaneously, and any decision rule necessarily applies to all those borrowers at the same time. It is precisely this last feature that makes a complete assessment of any decision rule quite complicated.

other banks' lending decision rules (stage 3) and borrowers' application choices (stage 4), the optimality of the simple NPV rule is no longer guaranteed, since indirect benefit (calculated in stage 3) from obtaining (in stage 5) a ϕ_+-borrower with $\eta^{\phi+}(\cdot) > 0$ denied by some other bank (in stage 3, perhaps that bank knows this borrower is of positive expected payoff) may outweigh direct cost of denying a ϕ-borrower with $\eta^{\phi}(\cdot) > 0$ (in stage 3). Computing precisely relevant payoffs for given strategies is intricate *per se*, not even to mention the large variety of other players' strategies to be considered comprehensively by the bank in question. In contrast, by focusing exclusively on the *immediate* payoff of a lending decision from a ϕ borrower, that is 0 for D and $\eta^{\phi}(\cdot)$ for A, not only a lending decision can be readily made in a systematic manner, i.e., the simple NPV rule, but the entire obscure computation associated with the complicated strategic consideration can be circumvented.

Above discussion is summarized in the following Lemma, which pins down bank's optimal lending decision rule in stage 3.

Lemma 2. *Suppose a bank obtains indicator ϕ after testing a borrower at ℓ in stage 3. Then under E-α and strategic complexity consideration, denying the loan application iff $\eta^{\phi}(\ell) < 0$ is the unique best response of the bank.*

It is worth mentioning that the simple NPV rule will always be used in stage 5 whenever a test is conducted on a revisited borrower, as the lending process terminates after bank's lending decision. Moreover, it will be proved later on that both the simple NPV rule and borrower's behavior prescribed by E-α are equilibrium outcomes in a particular equilibrium. As a result, strategic complexity consideration can be viewed as a sharp refinement that elicits a unique equilibrium of simple outcomes.

In light of Lemma 2, *ex ante* a test, bank's expected payoff $\eta(q, R, \mu)$, from screening with intensity q a borrower applying for a unit loan is

$$\eta(q, R, \mu) = \Pr(G|q, \mu) \max\{\eta^{G}(q, R, \mu), 0\} + \Pr(B|q, \mu) \max\{\eta^{B}(q, R, \mu), 0\}, \quad (5)$$

where the probability of observing an indicator ϕ, $\Pr(\phi|q, \mu)\ \forall\, \phi \in \Phi$ and $(q, \mu) \in \mathbb{Q} \times \mathrm{Int}\mathcal{U}$, is given in eq.(2). Note that $\eta(q, R, \mu)$ is well-defined and continuous over $\mathbb{Q} \times \mathcal{R} \times \mathcal{U}$, with $\eta(q, R, 1) = \eta^{g}(R)$ and $\eta(q, R, 0) = 0$. Bank's lending decision, following the simple NPV rule, is reflected in the $\max\{\cdot\}$ operator, since a ϕ-borrower with $\eta^{\phi}(\cdot) < 0$ will be denied for sure after screening. Evidently, $\eta(q, R, \mu)$ is bounded from below by 0. In addition, it turns out to be useful to denote

$$\eta^{ND}(R, \mu) = \mu\eta^{g}(R) + (1 - \mu)\eta^{b}(R)$$

bank's expected payoff from always approving a unit loan to a borrower at $\ell = R$ with μ (*ND* for "no denial"). Since $\eta(q, R, \mu)$ represents (per unit) expected payoff from screening for a bank considering a test on a borrower, it is important to know the behavior of this function over the parameter space in order to analyze bank's optimal screening choice later. This is shown in the following Lemma.

Lemma 3. *Under Assumption 1 and 2. Fix an arbitrary $R \in (\underline{R}, X]$, and let $\underline{a}(R)$ and $q^{\phi}(R, \mu)\ \forall\, \phi \in \Phi$ and $\mu \in \operatorname{Int}\mathcal{U}$ be those given in Lemma 1. Define*

$$\Delta_\eta(R, \mu) = \mu\eta^g(R) - (1-\mu)\eta^b(R). \tag{6}$$

Then $\Delta_\eta(R, \mu) > 0\ \forall\, \mu \in \mathcal{U}$. And $\eta(q, R, \mu)$ can be expressed as follows:

(*a*) *When $\mu \geq \underline{a}(R)$,*

$$\eta(q, R, \mu) = \begin{cases} \eta^{ND}(R, \mu), & \text{if } \frac{1}{2} \leq q \leq q^B(R, \mu); \\ q\Delta_\eta(R, \mu) + (1-\mu)\eta^b(R), & \text{if } q^B(R, \mu) < q \leq 1. \end{cases}$$

Moreover, $\eta^{ND}(R, \mu) \geq 0$, with equality only if $\mu = \underline{a}(R)$ and $q = \frac{1}{2}$, in which case $\eta^{\phi}\big(\frac{1}{2}, R, \underline{a}(R)\big) = 0\ \forall\, \phi \in \Phi$.

(*b*) *When $\mu < \underline{a}(R)$,*

$$\eta(q, R, \mu) = \begin{cases} 0, & \text{if } \frac{1}{2} \leq q \leq q^G(R, \mu); \\ q\Delta_\eta(R, \mu) + (1-\mu)\eta^b(R), & \text{if } q^G(R, \mu) < q \leq 1. \end{cases}$$

The proof is in Appendix C (p.39). Putting in a simple way, this Lemma shows that for given R and μ, expected payoff from screening $\eta(q, R, \mu)$ is piece-wise linear in screening intensity q, and is strictly increasing for all q above certain threshold. When $\eta(q, R, \mu)$ is linearly increasing, the slope is given by $\partial_q \eta(q, R, \mu) = \Delta_\eta(R, \mu)$. Therefore, $\Delta_\eta(R, \mu)$ also represents the *marginal benefit of screening*. Note that $\Delta_\eta(R, \mu) > 0\ \forall\, R > \underline{R}$, regardless of μ, and equals to 0 only if $R = \underline{R}$ and $\mu = 1$. So one concludes that the marginal benefit of screening is always positive, except for one point, over the parameter space $\mathcal{R} \times \mathcal{U}$.

Next, consider the problem of whether a bank should ever screen a borrower in stage 3, taking as given $\ell = R$ and μ. Again, the very first question to be dealt with is the complication caused by potential strategic consequences of a specific decision rule of screening. Recall that a bank has the option of not screening, hance denying altogether, any borrower applying for a loan. Thus any strategy, or equivalently any rule, governing the screening decision necessarily entails certain implications for the payoffs of all players in the rest of the lending game. As in the case of lending decision after a test, here once again, the strategic complexity consideration suggests the bank in question focusing on exclusively on the immediate payoff associated with the screening decision: zero if the bank decides not to screen any borrower versus the net (expected) payoff $\eta(q, R, \mu) - c(q)$ if the bank decides to screen a borrower with intensity q. The bank has the option of choosing any $q \in \mathbb{Q}$. As a result, the bank is willing to actually conduct a test only if the (unit) *profit*

$$\pi(R, \mu) = \max_{q \in \mathbb{Q}} \eta(q, R, \mu) - c(q)$$

from a (unit) loan to a borrower screened *optimally* by the bank is not negative; otherwise, the bank will deny all borrowers at $\ell = R$ with prior distribution μ. Also, let

$$\mathcal{BQ}(\pi(R, \mu)) = \operatorname*{argmax}_{q \in \mathbb{Q}} \eta(q, R, \mu) - c(q)$$

denote the set of profit maximizing q for given R, μ.

One mild clarification on terminology is required here. As noted above, $\eta(q, R, \mu) \geq 0 = c\left(\frac{1}{2}\right)$, therefore by choosing $q = \frac{1}{2}$ the net payoff is bounded from 0 and the profit $\pi(R, \mu) \geq 0$ by definition. However, by Lemma 3, for any $R \in \mathcal{R}$, $\eta(q, R, \mu) = 0$ if and only if (q, μ) belongs to either $\left\{(q, \mu) \middle| \mu \in \left[0, \underline{a}(R)\right), q \in \left[\frac{1}{2}, q^G(R, \mu)\right)\right\}$ or $\left\{\left(\frac{1}{2}, \underline{a}(R)\right)\right\} \cup \left\{(q, \mu) \middle| \mu \in \left[0, \underline{a}(R)\right), q = q^G(R, \mu)\right\}$. It can be easily checked that the latter set a segment of the boundary of the former region, over which $\eta^{\phi}(q, R, \mu) < 0 \; \forall \, \phi \in \Phi$. It follows that $\forall \, R \in \mathcal{R}$ and $\mu \in [0, \underline{a}(R)]$, as long as $\eta(q, R, \mu) = 0$, both G- and B-borrower will be denied even after being screened with $q \in \left[\frac{1}{2}, q^G(R, \mu)\right)$, except for one point $q = q^G(R, \mu)$ where $\eta^G(q, R, \mu) = 0$. Alternatively, the statement that bank chooses not to screen (i.e., deny all borrowers) at (R, μ) if $\eta(q, R, \mu) = 0$ for all $q \in \mathcal{BQ}(0)$ is *generically* true.

Subject to this qualification, bank's screening decision rule can be stated formally as follows.

Lemma 4. *Under E-α and strategic complexity consideration, the unique best response for a bank in stage 3 is that denying all borrowers at $\ell = R$ with prior distribution μ without screening iff $\pi(R, \mu) = 0$ and $\eta(q, R, \mu) = 0 \; \forall \, q \in \mathcal{BQ}(0)$.*

As a consequence of this Lemma, screening decision depends on crucially on when $\pi(R, \mu) = 0$ for a given loan $\ell = R$ with prior μ. To further characterize the unit profit function $\pi(R, \mu)$ with optimal screening, following regularity assumption on the screening cost function $c(q)$ is imposed.

Assumption 4. Screening cost $c(q)$ is continuously differentiable with second order derivative well defined over $\mathbb{Q}$. Moreover,

(a) there is a $\mathfrak{q} \in \left[\frac{1}{2}, 1\right)$ such that $c(q) = 0$ over $\mathbb{Q}^0 \equiv \left[\frac{1}{2}, \mathfrak{q}\right]$, $c'(q)$ and $c''(q) > 0$ over $\mathbb{Q}^1 \equiv (\mathfrak{q}, 1]$ with $c'(\mathfrak{q}) = 0$;

(b) $c'(1) > \Delta_\eta(X, \mu^0)$, and $c(1) < -(1-\mu^0)\eta^b(X)$; in addition

(c) $c''(q) \geq \mathfrak{C} \equiv \max\left\{\mathfrak{C}^1, \mathfrak{C}^2\right\}$, $\forall\, q \in \mathbb{Q}^1$, where

$$\mathfrak{C}^1 = 2\big(\mu^0 - (1-\mu^0)\theta^b/\theta^g\big)\big(\eta^g(X) + \underline{u}^g\big),$$
$$\mathfrak{C}^2 = \frac{c'(\overline{q}) - (2\mu^0)^2\Delta_\eta(\underline{R}, \mu^0)}{1-\overline{q}},$$

with $\mathfrak{C}^1 > 0$ and $\overline{q} \equiv \mu^0\eta^g(X)/\Delta_\eta(X, \mu^0) \in \left(\frac{1}{2}, 1\right)$ according to Assumption 1.[14]

By setting $\mathfrak{q} = \frac{1}{2}$, part ($a$) becomes a standard strictly increasing and convex cost assumption. So that by allowing $\mathfrak{q} > \frac{1}{2}$, part ($a$) is slightly weaker than the standard one.[15] Part (b) and (c) imposes two regularity conditions on $c(q)$, of which the full implications will become clear below.[16]

Before stating the main result of this paper, it is helpful to first define a function. Specifically, $\forall\, (R, \mu) \in \mathcal{R} \times \mathcal{U}$, let

$$q^*(R, \mu) = (c')^{-1}\big(\Delta_\eta(R, \mu)\big), \tag{7}$$

[14]Some simple algebra shows that $\underline{R} = \lambda + (1-\lambda)/\theta^g$, hence $-\eta^b(\underline{R}) = (1-\theta^b/\theta^g)(1-\lambda) > 0$, and $\Delta_\eta(\underline{R}, \mu^0) = \mu^0\eta^g(\underline{R}) - (1-\mu^0)\eta^b(\underline{R}) = (1-\mu^0)(1-\theta^b/\theta^g)(1-\lambda) > 0$.

[15]Assuming $c(q) = 0$ over $\mathbb{Q}^0$ captures the idea that sloppy screening practice does not require exerting significant effort by a bank's loan officers/underwriters; even though some information about the borrower can be generated, as $q > \frac{1}{2}$, it is based on some superficial and easily exploitable attributes of the application profile, and the information is not sufficient to differentiate good ones from bad ones since $\eta^G(q, \cdot)$ and $\eta^B(q, \cdot)$ are of the same sign for q sufficiently low.

[16]As a prelude, part (b) ensures that banks deny B-borrowers in stage 3 when screening optimally even in the case of charging X, i.e., highest possible interest rate (recall $\mathbb{E}^0\big[\mathrm{NPV}^\theta\big] > 0$, thus banks make no loss in lending to *all* borrowers when charging $R = X$); whereas part (c) ensures that both g- and b-borrowers find it optimal to apply first in stage 2 the loan with the lowest interest rates, even after accounting for possible denials in stage 3. A legitimate concern is whether Assumption 1 and 4 together render the parameter space (one component of which is the space of screening cost functions) empty, as Assumption 4 shows an upper bound on $c(1)$ as well as lower bounds on $c'(1)$ and $c''(q)$, of which most relevant terms are in turn constrained by Assumption 1. It turns out that under some additional, yet both *mild* and *intuitive*, parametric restrictions, the parameter space satisfying all Assumptions is non-empty. In particular, Assumption 4 imposes no severe restrictions on $c(\cdot)$. Detailed discussion is relegated to Appendix B.2.

where function $(c')^{-1} : \big[0, \max\{\eta^g(X), -\eta^b(\underline{R}), c'(1)\}\big] \to [\mathfrak{q}, 1]$ is defined as follows

$$(c')^{-1}(x) = \begin{cases} q \in [\mathfrak{q}, 1], & \text{if } 0 \le x \le c'(1) \text{ and } c'(q) = x; \\ 1, & \text{if } c'(1) < x \le \max\{\eta^g(X), -\eta^b(\underline{R})\}. \end{cases}$$

Broadly speaking, $(c')^{-1}(\cdot)$ denotes the inverse of $c'(\cdot)$ over $\mathbb{Q}^1$, with the domain extended from $[0, c'(1)]$ to the whole *range* of $\Delta_\eta(R, \mu)$ $\forall\, (R, \mu) \in \mathscr{R} \times \mathscr{U}$.[17] Since $c'(\cdot)$ is strictly increasing over $[\mathfrak{q}, 1]$, $(c')^{-1}(\cdot)$ is well-defined over $[0, c'(1)]$. In addition, $(c')^{-1}(x) = \mathfrak{q}$ only at $x = 0$, and it is continuously differentiable over $[0, c'(1))$ with $\mathrm{d}(c')^{-1}(x)/\mathrm{d}x = 1/c''\big((c')^{-1}(x)\big) > 0$.[18]

Equipped with Assumption 4, following proposition fully characterizes optimal screening decision of a bank in stage 3.

Proposition 1. *Under Assumption 1–2 and 4 (a). For any $R \in (\underline{R}, X]$, there exists a unique $\underline{\mu}(R) \in (0, \underline{a}(R))$ and a unique $\overline{\mu}(R) \in (\underline{a}(R), 1)$ such that*

(*a*) $\forall\, \mu \in \big(\overline{\mu}(R), 1\big]$, $\pi(R, \mu) > 0$, $\mathcal{BQ}(\pi(R, \mu)) = \mathbb{Q}^0$ *where* $\eta^\phi(q, R, \mu) > 0\ \forall\, \phi \in \Phi$;

(*b*) $\forall\, \mu \in \big[0, \underline{\mu}(R)\big)$, $\pi(R, \mu) = 0$, $\mathcal{BQ}(\pi(R, \mu)) = \mathbb{Q}^0$ *where* $\eta^\phi(q, R, \mu) < 0\ \forall\, \phi \in \Phi$;

(*c*) $\forall\, \mu \in \big(\underline{\mu}(R), \overline{\mu}(R)\big)$, $\pi(R, \mu) > 0$, $\mathcal{BQ}(\pi(R, \mu)) = q^*(R, \mu)$, *and* $\eta^G(q, R, \mu) > 0 > \eta^B(q, R, \mu)$ *at* $q = q^*(R, \mu)$;

(*d*) *for* $\mu = \overline{\mu}(R)$, $\pi(R, \mu) > 0$, $\mathcal{BQ}(\pi(R, \mu)) = \mathbb{Q}^0 \cup \{q^*(R, \mu)\}$ *where* $\eta^\phi(q, R, \mu) > 0\ \forall\, \phi \in \Phi$ *except for* $\eta^B(q, R, \mu) < 0$ *at* $q = q^*(R, \mu)$; *and*

(*e*) *for* $\mu = \underline{\mu}(R)$, $\pi(R, \mu) = 0$, $\mathcal{BQ}(\pi(R, \mu)) = \mathbb{Q}^0 \cup \{q^*(R, \mu)\}$ *where* $\eta^\phi(q, R, \mu) < 0\ \forall\, \phi \in \Phi$ *except for* $\eta^G(q, R, \mu) > 0$ *at* $q = q^*(R, \mu)$.

Remark. For $R \in (\underline{R}, X]$, $\Delta_\eta(R, \mu) > 0\ \forall\, \mu \in \mathscr{U}$, thus $q^*(R, \mu) > \mathfrak{q}$. As a result, for $\mu = \overline{\mu}(R)$ and $\underline{\mu}(R)$, $\mathcal{BQ}(\pi(R, \mu))$ has two disconnect components $\mathbb{Q}^0$ and $\{q^*(R, \mu)\}$.

The proof is contained in Appendix C (p.39). The principal conclusion of this Proposition is part (*c*), which asserts that for interim prior type distribution, the optimal screening

[17]Since $\Delta_\eta(R, \mu) = \mu\eta^g(R) - (1-\mu)\eta^b(R) \ge 0$, the range of $\Delta_\eta(\cdot)$ is $\big[0, \max\{\eta^g(X), -\eta^b(\underline{R})\}\big]$. Part (*b*) of Assumption 4 only requires that $c'(1) > \Delta_\eta(X, \mu^0)$, it remains possible that $\max\{\eta^g(X), -\eta^b(\underline{R})\} > c'(1) > \Delta_\eta(X, \mu^0)$. Consequently the domain of function $(c')^{-1}$ is $\big[0, \max\{\eta^g(X), -\eta^b(\underline{R}), c'(1)\}\big]$.

[18]Moreover, this last observation also implies that there is a kink of $(c')^{-1}(x)$ at $x = c'(1)$ provided that $c'(1) < \max\{\eta^g(X), -\eta^b(\underline{R})\}$, since $\mathrm{d}(c')^{-1}(x)/\mathrm{d}x|_{x=c'(1)-} = 1/c''\big((c')^{-1}(1)\big) > 0$, while $\mathrm{d}(c')^{-1}(x)/\mathrm{d}x|_{x=c'(1)+} = 0$ as $(c')^{-1}(x) = 1\ \forall\, x > c'(1)$.

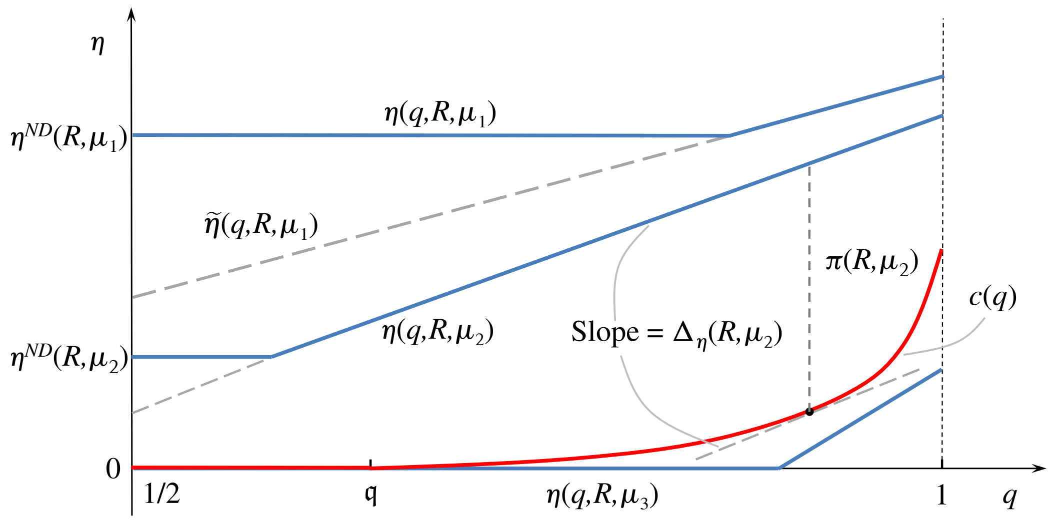

FIGURE 3: Payoffs and costs from screening a (unit) loan application.

Notes: This figure illustrates situations for a generic $R \in (\underline{R}, X]$ and various μ, with $\mu_1 > \overline{\mu}(R) > \mu_2 > \underline{\mu}(R) > \mu_3$. Moreover, $\widetilde{\eta}(q, R, \mu) \equiv q\Delta_\eta(R, \mu) + (1-\mu)\eta^b(R)\ \forall\, q \in \mathbb{Q}$, and $\widetilde{\eta}(\frac{1}{2}, R, \mu) = \frac{1}{2}\eta^{ND}(R, \mu)$. The kinks of $\eta(\cdot, \mu_1)$ and $\eta(\cdot, \mu_2)$ are at $q^G(R, \mu_1)$ and $q^G(R, \mu_2)$ respectively, and the kink of $\eta(\cdot, \mu_3)$ is at $q^B(R, \mu_3)$. The black dot denotes the optimal screening intensity $q^*(R, \mu_2)$. In this illustration, $\Delta_\eta(R, \mu)$ is decreasing in μ.

intensity is uniquely pinned down by eq.(7). The reason is straightforward: The marginal benefit from screening is $\Delta_\eta(R, \mu)$, and the marginal cost is $c'(q)$, bank's optimization of screening intensity subject to $q \leq 1$ results in $c'(q^*) \leq \Delta_\eta(R, \mu)$ where inequality holds only for $q^* = 1$ and $c'(1) < \Delta_\eta(R, \mu)$. As a result, optimal screening decision $q^*(R, \mu)$ can be expressed as in eq.(7). The main intuition of this proposition is illustrated in Figure 3. For μ large enough, the payoff from always approving a borrower, $\eta^{ND}(R, \mu)$, is even higher then the net payoff from screening optimally at $q^*(R, \mu)$, thus a bank will choose q sufficiently low such that $c(q) = 0$. In contrast, for μ small enough, the net payoff from screening optimally at $q^*(R, \mu)$ is still negative, i.e., positive payoff from all G-borrowers is more than offset by the screening cost, thus a bank will simply choose to deny all borrower and receive zero profit.

For each $R \in (\underline{R}, X]$, there is a multiplicity of $\mathcal{BQ}(\pi(R, \mu))$ for $\mu \in \mathcal{U} \setminus \big(\underline{\mu}(R), \overline{\mu}(R)\big)$. However, except for two boundary points $\underline{\mu}(R)$ and $\overline{\mu}(R)$, bank's lending decision is identical regardless what q is chosen from $\mathcal{BQ}(\pi(R, \mu))$: Lemma 4 and the preceding Proposition implies that a bank approves all borrowers at $\ell = R$ if $\mu > \overline{\mu}(R)$ and deny all if $\mu < \underline{\mu}(R)$ by choosing arbitrary $q \in \mathbb{Q}^0$. When $\mu = \overline{\mu}(R)$ or $\underline{\mu}(R)$, a bank is indifferent between $q \in \mathbb{Q}^0$ and $q = q^*(R, \mu)$. For convenience of subsequent discussion, let a bank choose

$q^*(R,\mu)$ in these two cases. Under this convention, next Proposition fully characterizes bank's lending decision rule.

Proposition 2. *Under* E-α *and strategic complexity consideration. Fix a loan* $\ell = R \in (\underline{R}, X]$ *in stage 3. A bank approves all borrowers at* R *if* $\mu > \overline{\mu}(R)$ *while denies all if* $\mu < \underline{\mu}(R)$*. When* $\mu \in [\underline{\mu}(R), \overline{\mu}(R)]$*, a bank chooses optimally an intensity* $q^*(R,\mu) > \mathfrak{q}$ *to screen each borrower, and depending on test result* $\phi = G$ *or* B*, approves all* G*-borrowers while denies all* B*-borrowers.*

To be sure, this Proposition depicts bank's optimal behavior in stage 3 only. For potentially different screening cost structure c_+ in stage 5, the optimal screening decision by then will be different accordingly. However, as a preclude, it will be showed later that only previously denied borrowers, hence with a negative expected payoffs as of stage 3, will apply again for a loan in stage 5. So that in conjunction with Assumption 3, no bank will approve these borrower for the prospect of identifying a good borrower is sufficiently low. Yet to reach this conclusion, first will be addressed borrower's problem in stage 2 and 4. As a final remark, the last Proposition justifies our restriction on bank's strategy to be pure, at least in the subgame $\mathscr{L}$. There is no point of randomizing lending decision rule for a bank, which essentially reflects the fact that one bank's action has no direct impact upon other bank's payoff in the subgame.

III.C Borrower's Decision

In this Subsection, consider the loan application problem facing a θ-borrower, $\theta \in \Theta = \{g, b\}$, in stage 2. Since a borrower is presumed to apply for one loan, and in light of Proposition 2, denial rate (probability) may differ at distinct loan, a borrower has incentives to take into account denial rate at each loan for calculating expected payoff.

Continue to denote $\mathscr{L}$ the set of loan available in stage 2, and let $p^\theta(\ell)$ denote the approval rate (i.e., one minus denial rate) at $\ell \in \mathscr{L}$ for a θ-borrower. Given that there is a continuum of both g- and b-borrowers, for each *individual* borrower, prior type distribution $\mu \in \mathscr{U}$ at a loan $\ell = R$ is determined by *all other* borrowers' mixed strategy profile $\boldsymbol{\sigma}^\beta$.[19] Therefore, once $\boldsymbol{\sigma}^\beta = (\sigma^\theta)_{\theta\in\Theta}$ is fixed, μ becomes common knowledge, among not only borrowers but also banks, and will not be altered by an individual borrower's unilateral deviation. Moreover, once (R,μ) becomes common knowledge, lending decision rule, including

[19] If one is concerned with the possibility that a continuum set $\mathscr{L}$ of loans is offered and precisely each individual borrower chooses a distinct loan thus prior type distribution at each loan is degenerate, then feel free to make an additional assumption restricting $\mathscr{L}$ to be finite. Indeed, since $\mathscr{L}$ is offered by banks in stage 1, in no equilibrium will a bank find it profitable to offer a continuum of loans.

optimal screening intensity, of a bank which has offered $\ell = R$ is determined according to Proposition 2. Recall that this lending decision rule is the best response of a bank facing (R, μ), thus in any equilibrium the individual borrower in question will rightly anticipate it and hence calculate denial rate accordingly.

Preceding argument suggests that $p^\theta(\ell)$ has the following functional form:

$$p^g(\ell) \text{ and } p^b(\ell) = \begin{cases} 1, & \text{if } \mu \in \big(\overline{\mu}(R), 1\big] \\ q \text{ and } 1-q \text{ resp.}, & \text{if } \mu \in \big[\underline{\mu}(R), \overline{\mu}(R)\big] \\ 0, & \text{if } \mu \in \big[0, \underline{\mu}(R)\big) \end{cases} \tag{8}$$

where ℓ effectively refers to both R and μ, and $q = q^*(R, \mu)$. Given q, a good borrower will be assigned to class G, hence approved, with probability $q > \frac{1}{2}$, whereas a bad borrower will be assigned to class G and denied with probability $1 - q < \frac{1}{2}$.

Note that expected payoff for a θ-borrower in stage 2, *ex ante* the screening process, is actually associated with borrower's (pure) strategy in both stage 2 and stage 4, i.e., (ℓ, ℓ_+). In particular, ℓ_+ is of a form $\big(\ell_+^A, \ell_+^D\big)$, where ℓ_+^A, ℓ_+^D denote loans chosen in stage 4 conditional on being approved (A) or denied (D) respectively in stage 3. Moreover, conditional on A, borrower's strategy is a binary choice such that either $\ell_+^A = \ell \in \mathscr{L}^i \subset \mathscr{L}$ or $\ell_+^A \in \mathscr{L}_+ = \cup_{j \neq i} \mathscr{L}^j$, i.e., either choose to stay with bank i at ℓ or apply for a loan from other banks. Lastly, let $p_+^{\theta,\phi}(\ell_+) \in [0, 1]$ denote the approval rate at a loan R_+ with μ_+ in stage 4 for a θ-borrower assigned to class ϕ in stage 3.[20] With these notations, the *ex ante* expected payoff $U^\theta(\ell, \ell_+)$ for a θ-borrower employing strategy (ℓ, ℓ_+) can be written as

$$U^\theta(\ell, \ell_+) = U^{\theta,A}(\ell, \ell_+) p^\theta(\ell) + U^{\theta,D}(\ell_+)\big(1 - p^\theta(\ell)\big), \tag{9}$$

in which

$$U^{\theta,A}(\ell, \ell_+) = \begin{cases} u^\theta(\ell), & \text{if } \ell_+^A = \ell \\ u^\theta\big(\ell_+^A\big) p_+^{\theta,\phi}\big(\ell_+^A\big), & \text{if } \ell_+^A \in \mathscr{L}_+ \end{cases}$$
$$U^{\theta,D}(\ell_+) = u^\theta\big(\ell_+^D\big) p_+^{\theta,\phi}\big(\ell_+^D\big)$$

denote conditional payoffs from being approved and denied in stage 3 respectively, where $u^\theta(\ell) \equiv u^\theta(R) = \theta(X - R) + \underline{u}^\theta$ for $\ell = R$. Taking strategy profile $\boldsymbol{\sigma} = \langle \boldsymbol{\sigma}^\alpha, \boldsymbol{\sigma}^\beta \rangle$

[20]Of course, the bank approached by this (θ, ϕ)-borrower may or may not know the previous test result ϕ. Even if the bank does not know ϕ, approval rate $p_+^{\theta,\phi}(\cdot)$ determined by bank's screening decision may differ from $p^\theta(\cdot)$, because, as for $p^\theta(\cdot)$, in principle $p_+^{\theta,\phi}(\cdot)$ may depend on the cost structure c_+ (particularly $\bar{c}_+$) in stage 5. However, as to be showed below, the precise form of this approval rate is irrelevant for our analysis.

of all other player as given, an individual θ-borrower's problem is to maximize $U^\theta(\ell, \ell_+)$ over $\left\{(\ell, \ell_+) \middle| \ell \in \mathscr{L}, \ell_+ = \left(\ell_+^A, \ell_+^D\right) \in (\{\ell\} \cup \mathscr{L}_+) \times \mathscr{L}_+\right\}$, where disjoint union $\{\ell\} \cup \mathscr{L}_+$ represents the binary choice to be made upon approval.

With the aid of the explicit expression of $U(\ell, \ell_+)$ given in eq.(9), a couple of results regarding to borrower's optimal application behavior can be established formally. First, we claim that, conditional on choosing not to stay with ℓ, i.e., to leave the bank applied for in stage 2, it is never optimal for a borrower to choose $\ell_+ = \left(\ell_+^A, \ell_+^D\right) \in \mathscr{L}_+ \times \mathscr{L}_+$ such that $u^\theta\left(\ell_+^A\right) p_+^{\theta,\phi}\left(\ell_+^A\right) \neq u^\theta\left(\ell_+^D\right) p_+^{\theta,\phi}\left(\ell_+^D\right)$. The reason is straightforward to explain. For instance, suppose $\ell_+^A = \ell_1$, $\ell_+^D = \ell_2$, and $u^\theta(\ell_1) p_+^{\theta,\phi}(\ell_1) > u^\theta(\ell_2) p_+^{\theta,\phi}(\ell_2)$ without lose of generality. Then, by modifying strategy $\ell_+ = (\ell_1, \ell_2)$ to $\ell_+' = (\ell_1, \ell_1)$, we have

$$
\begin{aligned}
U^\theta(\ell, \ell_+) &= u^\theta(\ell_1) p_+^{\theta,\phi}(\ell_1) p^\theta(\ell) + u^\theta(\ell_2) p_+^{\theta,\phi}(\ell_2)\big(1 - p^\theta(\ell)\big) \\
&\qquad < u^\theta(\ell_1) p_+^{\theta,\phi}(\ell_1) p^\theta(\ell) + u^\theta(\ell_1) p_+^{\theta,\phi}(\ell_1)\big(1 - p^\theta(\ell)\big) = U^\theta(\ell, \ell_+'),
\end{aligned}
$$

i.e., (ℓ, ℓ_+) can be improved upon by replacing ℓ_2 with ℓ_1. Putting in another way, choosing (ℓ, ℓ_1, ℓ_2) with ℓ_1, $\ell_2 \neq \ell$ and $u^\theta(\ell_1) p_+^{\theta,\phi}(\ell_1) \neq u^\theta(\ell_2) p_+^{\theta,\phi}(\ell_2)$ is a dominated strategy. Notably, preceding reasoning is valid for *any* specification of $p^\theta(\cdot)$ and $p_+^{\theta,\phi}(\cdot)$, meaning that a borrower can effectively reach this conclusion without knowing precisely absolute magnitudes of approval rates but only relative likelihood of approval.

Next, let's consider the choice to be made upon approval (A) at $\ell \in \mathscr{L}^i$, i.e., whether to stay with bank i or to leave and apply for a loan from a bank other than i. For clarity, let's label this loan by ℓ_1. Sequential rationality requires that a borrower chooses ℓ_1 and stays with bank i if and only if $u^\theta(\ell_1) \geq \max_{\ell' \in \mathscr{L}_+} u^\theta(\ell') p_+^{\theta,\phi}(\ell')$. Suppose there is a $\ell_2 \in \mathscr{L}_+$ such that $u^\theta(\ell_1) < u^\theta(\ell_2) p_+^{\theta,\phi}(\ell_2)$, so that given the chance of applying for another loan upon A, the borrower would be willing to discard ℓ_1 and leave bank i. We claim that the strategy of a form $\left(\ell, \ell_+^A, \ell_+^D\right) = (\ell_1, \ell_2, \ell_3)$, for some $\ell_3 \in \mathscr{L}_+$, is a weakly dominated strategy, if ℓ_2 *is offered by more than two banks*. In light of the previous claim, without lose of generality, we can replace ℓ_3 with ℓ_2 and consider the strategy in the form of (ℓ_1, ℓ_2, ℓ_2) only. The expected payoff associated with this strategy is

$$
U^\theta(\ell_1, \ell_2, \ell_2) = u^\theta(\ell_2) p_+^{\theta,\phi}(\ell_2) p^\theta(\ell_1) + u^\theta(\ell_2) p_+^{\theta,\phi}(\ell_2)\big(1 - p^\theta(\ell_1)\big) = u^\theta(\ell_2) p_+^{\theta,\phi}(\ell_2).
$$

Since $p_+^{\theta,\phi}(\ell_2) \leq 1$ in any event, expected payoff from a modified strategy (ℓ_2, ℓ_2, ℓ_2) is such that

$$
\begin{aligned}
U^\theta(\ell_2, \ell_2, \ell_2) &= u^\theta(\ell_2) p^\theta(\ell_2) + u^\theta(\ell_2) p_+^{\theta,\phi}(\ell_2)\big(1 - p^\theta(\ell_2)\big) \\
&\geq u^\theta(\ell_2) p_+^{\theta,\phi}(\ell_2) p^\theta(\ell_2) + u^\theta(\ell_2) p_+^{\theta,\phi}(\ell_2)\big(1 - p^\theta(\ell_2)\big) = U^\theta(\ell_1, \ell_2, \ell_2).
\end{aligned}
$$

Note that the result that (ℓ_1, ℓ_2, ℓ_2) is weakly dominated by (ℓ_2, ℓ_2, ℓ_2) depends crucially on the premise that ℓ_2 is offered by at least two banks; insofar this is the case, upon being denied by a bank at ℓ_2, the borrower can apply for ℓ_2 again from some bank other than the first one, which makes the dominating strategy (ℓ_2, ℓ_2, ℓ_2) feasible.

This last claim is of some significance. Paraphrasing in a slightly different way, it reads: to leave bank i upon approval for a loan ℓ' from other bank is weakly dominated provided that ℓ' is offered by more than one bank. Or still in another way, given symmetric strategy profile among banks, $\boldsymbol{\sigma}^\alpha = (\sigma^\alpha)_{1\times N}$, staying with the bank from which a borrower initially applies for a loan upon approval weakly dominates leaving the bank and applying for a loan from any other banks. This is precisely what is needed to justify the particular expectation E-α elucidated in the previous Subsection.

Yet by far, E-α has not been fully justified, as demonstrated by the qualification that ℓ_2 be offered by more than one bank. The remaining case to be considered is where ℓ_2 is offered by only one bank. In specific, consider a strategy (ℓ_1, ℓ_2, ℓ_2) where $\ell_1 \in \mathscr{L}^i$ and $\ell_2 \in \mathscr{L}^j$ for a unique $j \neq i$, i.e., ℓ_2 is offered uniquely by bank j. Now, a modified strategy, (ℓ_2, ℓ_2, ℓ_2), as the one used for demonstrating the previous claim, is no longer feasible to the borrower in question, because once denied by bank j, nowhere in the market can be found ℓ_2 to apply for again, i.e., the borrower has to end up with $\ell_+^D \neq \ell_2$. Nonetheless, we claim that strategy (ℓ_1, ℓ_2, ℓ_2) is still weakly dominated by (ℓ_2, ℓ_2, ℓ_1). The argument runs as follows. First, observe that since

$$U^\theta(\ell_2, \ell_2, \ell_1) = u^\theta(\ell_2)p^\theta(\ell_2) + u^\theta(\ell_1)p_+^{\theta,\phi}(\ell_1)\big(1 - p^\theta(\ell_2)\big),$$

for $U^\theta(\ell_2, \ell_2, \ell_1) < U^\theta(\ell_1, \ell_2, \ell_2) = u^\theta(\ell_2)p_+^{\theta,\phi}(\ell_2)$, it has to be the case that $p^\theta(\ell_2) < p_+^{\theta,\phi}(\ell_2)$. Suppose the preceding two inequalities hold, i.e., approval rate at ℓ_2 is higher in the second round lending interaction than the first and (ℓ_1, ℓ_2, ℓ_2) is the preferred to (ℓ_2, ℓ_2, ℓ_1). Identify the set of borrowers applying for $\ell_2 \in \mathscr{L}^j$ in the second round by the type distribution μ. Sine both approved and denied borrowers in the first round screening process opt for ℓ_2 in the second round, the first round test does not generate any useful information for bank j to condition on. As a result, j is left with the only option of treating μ as fresh borrowers. But, in doing so, approval rate, $p_+^{\theta,\phi}(\ell_2)$, determined by j's optimal screening decision, will be identical to those given by eq.(8) for a loan ℓ_2 with μ.[21] In other words, had any borrower of μ applied for ℓ_2 in the very first round, this borrower would have been tested optimally by j with the approval rate being exactly the same as that in the second round, i.e., $p^\theta(\ell_2) = p_+^{\theta,\phi}(\ell_2)$. This contradiction proves what we claimed before.

[21]Note there is no more lending interaction after stage 5, so that bank j must choose to screen borrowers in μ *optimally*, which then leads to those approval rates given in eq.(8)

Combining the preceding two claims yields the following Proposition, which justifies the expectation E-α stated above.

Proposition 3. *For any borrower The strategy of leaving a bank initially approached upon approval and applying for a loan from some other bank is weakly dominated by the strategy of staying with the bank once being approved.*

Remark. Virtually identical reasoning shows that any strategy of applying for a loan ℓ with $p^{\theta}(\ell) = 0$ ($\forall\, \theta \in \Theta$ according to eq.8) in stage 2 is a dominated one, provided that there is some other loan ℓ' with $p^{\theta}(\ell') > 0$. In other words, it is never optimal for any borrower to simply dismiss the first round loan opportunity.

In a nutshell, this proposition is merely a formal exposition of the intuitive idea that borrowers, when provided with opportunities of choosing among a set of loans to apply for, always choose to settle down upon approval and secure credit from those banks, rather then search for loans with uncertain approval prospect due to screening activities. We consider this idea to be a realistic one, especially given our intention of modeling a serious screening process, both to banks and borrowers: If credit is not readily available upon application, why dismiss security now (upon approval) and seek for uncertainty elsewhere.

III.D Simplifying the Static Game

Based on previous analysis of bank's screening decision and borrower's application decision, it is then possible to simplify the original lending game. In particular, we will show that the equilibrium outcome of the second round lending interaction, i.e., stage 4 and 5, is determined mechanically by equilibrium outcome in the first round lending interaction, i.e., stage 2 and 3, under Assumption 3. Thus the second round interaction plays no essential role and consequently can be suspended in equilibrium analysis. This leads to a reduction of the original 5-stage lending game to a 3-stage game, which can then be analyzed in a more straightforward manner.

To begin with our reasoning, first note that by Proposition 3 no borrower approved in stage 3 will choose to apply for another loan in stage 4, therefore, only borrowers who are denied in their first round loan application will choose to participate the second round application in stage 4. Next, by Proposition 2, a borrower will be denied credit at a given loan $\ell = R$ in stage 3 only if the expected payoff from a loan to such a borrower is negative to the lending bank. There are two cases to consider: all borrowers are denied at ℓ or only B-borrowers are denied. The former case corresponds to the situation where $p^{\theta}(\ell) = 0$, $\forall\, \theta \in \Theta$, according to eq.(8), which will not occur in any equilibrium by the Remark following 3.

Therefore, only the latter case is possible, i.e., banks conclude that all borrowers applying for loans in stage 4 are of class B.

In this way, banks are informed *indirectly* previous test results of all revisited borrowers. On top of this information, banks also know where those revisited borrowers came from, i.e., the loan for which each of those B-borrowers applied in stage 2. To fix notation, suppose a B-borrower was denied at loan $\ell = R$, with prior type distribution $\mu \in [\underline{\mu}(R), \overline{\mu}(R)] \subset (0,1)$, in stage 3, and is applying for a loan $\ell_+ = R_+$ at bank i in stage 4. By Proposition 2, this borrower was screened optimally by the previous testing bank at intensity $q^*(R,\mu) \in (\mathrm{q}, 1)$, thus the posterior probability of this borrower being type g is given by $\nu^B(q^*(R,\mu),\mu) \in (0,\mu)$ according to eq.(2). As bank i knows that this borrower is of class B, in together with (R,μ), it knows as well $\nu^B(q^*(R,\mu),\mu)$, which becomes i's prior now. According to Assumption 3, were i to test this revisited B-borrower again, it would find that the new test be completely uninformative about the borrower's type, i.e., posterior be identical to prior $\nu^B(q^*(R,\mu),\mu)$, while be required to pay a cost $\bar{c}_+ > 0$. As a result, bank i will optimally choose not to test this B-borrower again, but calculate expected payoff of a unit loan to such a borrower using prior $\nu^B(\cdot)$ directly as

$$\begin{aligned}&\eta_+\big(R_+, \nu^B(q^*(R,\mu),\mu)\big)\\ &\equiv \nu^B(q^*(R,\mu),\mu)\eta^g(R_+) + \big(1-\nu^B(q^*(R,\mu),\mu)\big)\eta^b(R_+),\end{aligned}$$

and subsequently approve the loan if $\eta_+\big(R_+, \nu^B(q^*(R,\mu),\mu)\big) \geq 0$ while deny it otherwise.

Turn to the problem of the revisited B-borrower. Although test result was not revealed by the testing bank in stage 3, the borrower B could still rightly infer the test result, i.e., $\phi = B$, from the denial decision of the bank. As a result, B knows that, regardless of his/her true type θ, any bank pondering about whether to approve or not a loan to B is using $\nu^B(q^*(R,\mu),\mu)$ as its prior. In order to maximize expected payoff from a loan, subject to being approved, B will apply for a minimal R_+ such that $\eta_+\big(R_+, \nu^B(q^*(R,\mu),\mu)\big) \geq 0$. The candidate interest rate is trivially determined as the unique $R_+ \in (\underline{R}, \infty)$ that solves

$$\eta_+\big(R_+, \nu^B(q^*(R,\mu),\mu)\big) = 0, \tag{10}$$

for $\eta_+\big(R_+, \nu^B(q^*(R,\mu),\mu)\big)$ is strictly increasing in R_+ and $\eta_+\big(R, \nu^B(q^*(R,\mu),\mu)\big) < 0$ (this is why B is denied). Denote the unique solution of eq.(10) by $\overline{R}_+(R,\mu)$, for any (R,μ) such that $R \in (\underline{R}, X]$ and $\mu \in [\underline{\mu}(R), \overline{\mu}(R)]$. Evidently $\overline{R}_+(R,\mu) > R$, with the possibility of exceeding X provided that $\nu^B(q^*(R,\mu),\mu)$ is small enough. If $\overline{R}_+(R,\mu) \in (\underline{R}, X]$,[22]

[22] We are not concerned with the issue of whether $\overline{R}_+(R,\mu) \in \mathscr{L}_+$ for B or not, as it will be showed that equilibrium strategy of bank's takes the form $\mathscr{L}^i = [R, X]$ where $R \geq \underline{R}$. Intuitively, had a bank offered some R, it would be happy to offer any interest rate above R.

B's application will be approved for sure, and $\overline{R}_+(R,\mu)$ will generate maximum payoff to B among all loans that B will not be denied. Otherwise, as $\overline{R}_+(R,\mu) > X$, B will be denied for sure from any loan by any bank.

By combining above discussion and Proposition 3, type θ borrower's *ex ante* expected payoff, eq.(9), can be refined as:

$$U^\theta(R,\mu) = u^\theta(R)p^\theta(R,\mu) + u^\theta\big(\overline{R}_+(R,\mu)\big)\big(1-p^\theta(R,\mu)\big)\mathbb{1}_{\{\overline{R}_+(R,\mu)\le X\}}, \tag{11}$$

together with the requirement that $p^\theta(\cdot) > 0$, where $\mathbb{1}_{\{\cdot\}}$ denotes indicator function, and μ is the (common) prior held by all banks on borrower composition at loan R.[23] In principle, bank's prior μ should depend on R, i.e., $\mu = \mu(R)$. So far as $\mu(R)$ is specified in one way or another, $U^\theta(\cdot)$ becomes a genuine function of solely R. It is conceivable that properties of $U^\theta(\cdot)$, hence optimal choice of θ-borrower, depend on crucially the *entire* function $\mu(R)\ \forall\ R \in (\underline{R}, X]$. Yet equilibrium outcome is likely to be concentrated on a few values of R. Therefore, the renowned problem of specifying *properly* out-of-equilibrium beliefs emerges in our setup, and the choice of $\mu(R)$ becomes a vital issue for determining equilibria of the static lending game.

To solicit a proper $\mu(R)$ as an equilibrium candidate (or, indeed, what we believe to be *the* proper one), we break our reasoning into two steps. First, we pin down a unique $\mu(R)$ under the temporary hypothesis that all borrowers confine their calculation of expected payoffs to the first round lending interaction only, i.e., the first term on the RHS of eq.(11).[24] This behavior pattern will be consistent with borrower's payoff maximization of $U^\theta(R,\mu)$, if all borrowers hold a specific expectation, henceforth labeled by E-β, which reads as

> *banks' prior belief (function)* $\mu(R)$ *is such that* $\max_{R\in\mathscr{L}} U^\theta(R,\mu(R))$ *leads to the same solution as* $\max_{R\in\mathscr{L}} u^\theta(R)p^\theta(R,\mu(R))$.

As a second step, we will verify that the particular $\mu(R)$ determined in the first step does satisfy expectation E-β.

STEP 1: SOLICITING $\mu(R)$ Using standard terminology, we call an equilibrium outcome in a single round lending interaction a *pooling equilibrium*, if both types of borrowers choose

---

[23]Under the notation adopted in eq.(9), we have $p_+^{\theta,B}(R_+) = \mathbb{1}_{\{R_+\le X\}}$ not depending on θ; and $p_+^{\theta,G}(\cdot)$ does not enter into eq.(11), hence is irrelevant.

[24]We argue that this hypothesis is a realistic one. Intuitively, when choosing the loan to apply for in a first time, a borrower may well be concerned more about expected payoff from this application alone, rather than taking into account for expected payoff in any continuation process at a completely equal footing. Although this may induce certain inconsistency with borrower's objective function given in eq.(9) (and in eq.11), this potential difficulty is resolved under E-β.

the same loan $\ell = R$; otherwise, we call it a *separating equilibrium*. For the latter, it can be divided into two subclasses: complete separating and quasi separating equilibrium. The following Lemma shows that only pooling equilibrium is possible under E-β.

Lemma 5. *Under E-β the only possible equilibrium outcome is pooling in the first round lending interaction.*

The proof is contained in Appendix C (p.42). It is worth to point out that the underlying reason for the unique equilibrium outcome being pooling goes entirely the payoff function specification (restricted to the first round lending) of borrowers, given optimal lending decision responses of banks. As shown in the proof, any form of separating equilibrium outcome does not satisfy incentive compatibility constraint that prevents one type of borrowers from mimicking the loan choice of another type of borrowers. From another perspective, the pooling result does not rely on the use of a particular equilibrium refinement scheme, such as the anticipatory equilibrium of Wilson (1977). [25]

Putting in an equivalent way, this pooling result also shows that there is no room for mixed strategy by any borrower. Moreover, since any loan $\ell = R \in \mathscr{R}$, once chosen by borrowers in any equilibrium, must be associated with a pooling equilibrium outcome, the type distribution at R is uniquely determined by μ^0, i.e., the primitive distribution of the investment shock. As a consequence, the unique common prior of all banks is trivially $\mu(R) = \mu^0 \; \forall \; R$. Formally, following is a direct corollary of the preceding Lemma.

Corollary 6. *Under E-β in any equilibrium all borrowers use pure strategy and the unique common prior of all banks on borrower's type distribution at any loan $\ell = R \in \mathscr{L}$ is a constant μ^0 regardless of R.*

STEP 2: VERIFYING E-β Once $\mu(R) = \mu^0$ is determined, there is no difficulty in verifying E-β under the stated assumption. Note that according to our formulation, even if banks approve some applications in the second round lending process, the (expected) profit is 0. Thus only profit from the first round lending matters for bank's loan pricing decision. As a preliminary step, we first characterize the bank's profit function $\pi(R, \mu)$

[25] To illustrate the nature of this result a bit, let's consider the case in which approval rates are q and $1-q$ respectively for g- and b-borrowers at $\ell = R$. The corresponding payoff functions are $u^g(R)q$ and $u^b(R)(1-q)$. Evidently, only the former is increasing in q whereas the latter is decreasing in q. Thus the payoff function specification used here violates the typical condition that informed parties' payoffs be *uniformly* increasing or *uniformly* decreasing in uninformed parties' actions. In tandem with a single crossing property (which is trivially satisfied here) and a uniform monotonicity of payoffs in informed parties' own actions, the aforementioned conditions guarantee incentive compatibility be satisfied for at least some separating equilibrium outcome.

from a unit loan under the prior specification $\mu(R) = \mu^0$. For notational simplicity, let $\pi(R) \equiv \pi(R, \mu^0) = \max_{q \in \mathbb{Q}} \eta(q, R, \mu^0) - c(q)$, $q^*(R) \equiv q^*(R, \mu^0) = (c')^{-1}\big(\Delta_\eta(R)\big)$, and $\Delta_\eta(R) \equiv \Delta_\eta(R, \mu^0)$, then we have following Lemma:

Lemma 7. *Under Assumption 1–2 and 4 (a)–(b), there exists a unique $R^0 \in (\underline{R}, X)$ such that (i) $\pi(R) > 0$ iff $R \in (R^0, X]$, (ii) $\pi(R) = 0$ iff $R \in [\underline{R}, R^0]$, (iii) all B-borrowers be denied credit, and (iv) over $[R^0, X]$, both $\pi(R)$ and $q^*(R) \in (\mathfrak{q}, 1)$ are continuous and strictly increasing.*

Remark. Recall that $q^*(R)$ is defined $\forall\, R \in \mathcal{R}$, and it can be easily checked that $q^*(R)$ is strictly increasing over $\mathcal{R}$.

The proof is contained in Appendix C (p.43). Note that when borrower's composition is fixed at μ^0, optimal screening intensity $q^*(R)$ is always interior. This result stems from part (b) of Assumption 4: (i) $c(1) < -(1-\mu^0)\eta^b(X)$, or equivalently, $\eta^{ND}(X, \mu^0) < \mu^0 \eta^g(X, \mu) - c(1)$, dictates that it is always optimal for banks to choose $q^* > \mathfrak{q}$ even when charging the highest possible interest rate X; and (ii), $c'(1) > \Delta_\eta(X) = \Delta_\eta(X, \mu^0)$ leads to an optimal screening intensity q^* being always strictly less than 1.

As a direct implication of the preceding Lemma, borrower's expected payoff $U^\theta(R) \equiv U^\theta(R, \mu^0)$ given in eq.(11) can be written as

$$
\begin{aligned}
U^g(R) &= u^g(R) \cdot q^*(R) + u^g\big(\overline{R}_+(R)\big) \cdot (1 - q^*(R)) \cdot \mathbb{1}_{\{\overline{R}_+(R) \le X\}}, \\
U^b(R) &= u^b(R) \cdot (1 - q^*(R)) + u^b\big(\overline{R}_+(R)\big) \cdot q^*(R) \cdot \mathbb{1}_{\{\overline{R}_+(R) \le X\}},
\end{aligned}
$$

where $\overline{R}_+(R) \equiv \overline{R}_+(R, \mu^0)$. Following Lemma verifies E-β, that $U^\theta(R)$ is decreasing in R, under stated Assumptions.

Lemma 8. *Under Assumption 4 (c), $U^\theta(R)$ is decreasing in R over $\mathcal{R} = [\underline{R}, X]$ $\forall\, \theta \in \Theta$.*

The proof is contained in Appendix C (p.43).

III.E Two cases of Static Equilibrium

E.1 Monopoly

Having established key properties of optimal decisions of banks and borrowers, we can proceed to analyze equilibria of the static lending game. In so doing, we will focus on two cases in sequel: equilibrium with a monopoly bank ($N = 1$) and zero profit equilibrium of perfect competition ($N > 1$ with free entry).

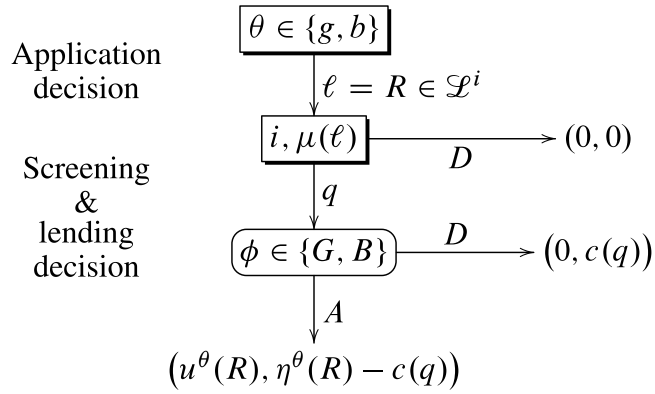

FIGURE 4: The simplified subgame $\mathscr{L} = \cup_{i \in N} \mathscr{L}^i$ within a period.

Notes: Shaded box indicates the player making decisions in a stage, with $\mu(\ell) = \Pr(g|\ell)$ denoting the belief held by i for ℓ. Labels along arrows indicate actions chosen by players. Rounded box shows the outcome of a player's action upon which subsequent action can be conditioned. Payoffs for a unit loan are specified in parentheses with borrower ordered first.

E.2 PERFECT COMPETITION

IV REPEATED GAME AMONG BANKS

IV.A SETUP AND SOLUTION CONCEPT

IV.B ANALYSIS OF THE DYNAMIC EQUILIBRIUM

IV.C SHOCKS IN OTHER FORMS

V ROBUSTNESS OF THE BASIC FRAMEWORK

V.A SCREENING PROCESS

I tend to argue that these seemingly stringent properties of the screening technology are not entirely unreasonable. What's being called a creditworthiness test is all about exploiting to the maximum extent under cost consideration a borrower's profile including both application filing and background investigation if any. Test result is based on the judgement made against the processing of the entire profile. A pure repeat of a test amounts to going through once more the borrower's profile, and it is not at all surprising that the same test result should be drawn out as no new information about the borrowers is gathered. Even if somewhat more effort is devoted to uncovering previously neglected facets of the borrower, one may reasonably doubt the effectiveness of this sort of marginal improvement in overturning the established conclusion backed by the existent profile. As an analogy, it is fairly evident that overturning a verdict of a court without definitive, convincing, and newly-assembled evidence is indeed too unlikely to happen.

An even more forceful argument is based on a less well known observation on the now prevailing practice of credit analyses within in the banking industry. Benefiting from advancement of information technology, large-scale, statistic-based analysis of credit data (including credit history data via credit bureaus and registers, procedural filing data for credit applications, in plus any "hard information" from other sources) is now commonly employed in banking industry. One concrete example is the use of binary choice model for predicting credit performance either at the loan level or the borrower level. Fed with the *same* data, almost identical statistical model used at different banks can only be hoped to generate very similar, if not at all identical, *quantitative*, let alon *qualitative*, result. Taking literally the ideal case considered in this paper, identical statistical test model used by all bank necessarily lead to identical test result, conditional on a given borrower, which actually justifies the argument for expecting $\Pr(\phi_+|\phi) = 1$. [26]

A final remark on the specification of screening technology adopted in this paper concerns with what can be termed as *conditionally independent test* (CIT) assumption, made popular by pioneering work of Broecker (1990). CIT says that, conditional on a given borrower type θ, test results of all banks, i.e., ϕ^i from T^i across $i \in N$, are statistically mutually independent. To clarify somehow in advance, screening process is structured in a sequential manner in my setup, thus no explicit assumption is made about the statistical property of correlation pattern across T^i. In particular, given a multi-bank *sequential-test* setup, Assumption 3 should not be interpreted, or perhaps more relevant to the point, criticized as alluding *any* specific statistical property across T^i that would hold were T^i be conducted simultaneously upon a given borrower, unless for the intended case of a revisited borrower. On the contrary, in a multi-bank *simultaneous-test* setup, CIT assumption is clearly a convenient device to work with and imparts the renowned "winner's curse" effect into models of bank competition with screening albeit at the cost of necessitating mixed strategy equilibria in many setups.[27]

CIT assumption maintained in the multi-bank simultaneous-test setup has a clear root in the closely related common-value auction game, the analogy between which had already

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[26] I'm grateful to Yina Lu who informed me the wide use of statistical credit models among major commercial banks.

[27] The nonexistence of pure strategy equilibrium is caused by a combination of discrete signal space and the "winner's curse" effect, analogous to that of common value auction game (Broecker, 1990; Wang, 1991). If the signal space is continuous, then it is possible to purify the equilibrium (in the sense of Harsanyi 1973), as demonstrated by Riordan (1993); however the equilibrium strategy is still a complicated function of the continuum signal.

been acknowledged in the Broecker's paper. However, whereas CIT in a symmetric auction games deems an apparent plausibility as it's all about subjective valuation of anonymously (even randomly perhaps) gathered bidders, the same assumption is much less justifiable for the case of constantly interacting banks. Even if one were not convinced by the above argument favoring the assumption made in this paper based explicitly on technological considerations, one should not ground a counter-argument purely on the appealing theoretical implications imparted by CIT. In particular, the "winner's curse" effect, which can be derived easily from CIT assumption under simultaneous tests, should not be viewed as the only theoretical short-cut of adverse selection problems facing each bank. The reason is simple, that the "loser's curse" effect should be of an equal footing as the "winner's curse" effect,[28] i.e., missing good borrowers should be as plausible a consideration of each bank as getting bad ones.

CIT with sequential tests.

V.B Information Structure

Market segmentation a long the line of Dell'Ariccia and Marquez (2006) and direct information sharing, i.e., direct announcement of test results either fully or selectively.

Houston et al. (2010) for strong empirical evidence positive effects of information sharing among banks.

V.C Timing and Equilibrium Notion of the Static Game

Make a comment on the alternative specification on the lending process, i.e., approved borrower is forced by a pre-commitment to stay with the bank while only denied borrower can apply one more time, which leads to the same equilibrium outcome.

Make another comment on the competitive nature of the credit market which can be viewed as motivating the strategic complexity consideration.

Also make a comment on the competitive rational expectations perspective in interpreting the lending process.

V.D Alternative Forms of Contract and Competition

Commitment to screening intensity q and contract of the form $\langle R, q \rangle$. The proper measure of competitive intensity is by no means the level of price, i.e., interest rate in this case, but the percentage deviation of the actual profit rate (per unit product/loan) from the monopoly

[28] See Pesendorfer and Swinkels (1997) for a thorough analysis on the role played by "loser's curse" effect on bringing about informational efficiency in an auction game, and Holt and Sherman (1994) for experimental evidence on the prevalence of "loser's curse".

profit rate to be achieved under the same physical environment, where the only difference is the market structure.

VI Empirical Facts

VII Concluding Remarks

Appendix

A More on SLOOS Data

B Supplements to the Basic Setup

B.1 Properties of the Screening Technology

Various properties of the screening technology upon a fresh borrower are summarized in the following Claim, where $\nu^{\phi}(q,\mu) = \Pr(g|G,q,\mu)$ is defined in eq.(2) for $\phi \in \Phi = \{G, B\}$.

Claim B.1. $\forall\, (q,\mu) \in \mathbb{Q} \times \mathrm{Int}\,\mathcal{U}$ *and* $\phi \in \Phi$ *it follows that*

(*a*) $\nu^{\phi}(q,\mu)$ *is continuously differentiable,* $\nu^{\phi}\big(\tfrac{1}{2},\mu\big) = \mu$, $\nu^{G}(1,\mu) = 1$, $\nu^{B}(1,\mu) = 0$*;*

(*b*) $\partial_q \nu^{G}(q,\mu) > 0$, $\partial_q \nu^{B}(q,\mu) < 0$, $\partial_\mu \nu^{\phi}(q,\mu) > 0$*; and*

(*c*) $1 > \nu^{G}(q,\mu) > \mu > \nu^{B}(q,\mu) > 0\ \forall\, q \in \mathrm{Int}\mathbb{Q}$.

Proof: Part (*a*) is straightforward to verify given eq.(2), and part (*c*) follows from part (*b*). For (*b*), note that by eq.(2) there is $\nu^{G}(q,\mu) = \mu\big/\big(\mu + (1-\mu)\frac{1-q}{q}\big)$. Since $(1-q)/q = 1/q - 1$ is decreasing in q, $\nu^{G}(q,\mu)$ is increasing in q. For $\nu^{B}(q,\mu)$, note that $q/(1-q)$ is increasing in q, so that $\nu^{B}(q,\mu) = \mu\big/\big(\mu + (1-\mu)\frac{q}{1-q}\big)$ is decreasing in q. Analogous reasoning proves that $\nu^{\phi}(q,\mu)$ is strictly increasing in μ. *Q.E.D.*

For the clarity of exposition, fix the context: let ϕ be the class to which a θ-borrower is assigned when screened as a fresh borrower under prior $\mu \in \mathrm{Int}\,\mathcal{U}$ by a test with intensity q.

Claim B.2. *Suppose* $q < 1$*. Then* $\Pr(\theta|\phi_+,\phi) = \Pr(\theta|\phi)$ *for all* $\phi_+ = \phi \in \Phi \equiv \{G, B\}$ *and* $\theta \in \Theta \equiv \{g, b\}$ *if and only if* $\Pr(\phi_+|g,\phi) = \Pr(\phi_+|b,\phi)$ *for all* $\phi_+,\ \phi \in \Phi$.

Proof: When $\mu \in (0,1)$ and $q < 1$, both $\Pr(g|\phi),\ \Pr(b|\phi) > 0$ by eq.(2).

Necessity. By Bayes rule, for all $\phi_+,\ \phi$, and θ

$$\Pr(\phi_+|\theta,\phi) = \frac{\Pr(\phi_+,\theta|\phi)}{\Pr(\theta|\phi)} = \frac{\Pr(\theta|\phi_+,\phi)\Pr(\phi_+|\phi)}{\Pr(\theta|\phi)}. \tag{B.1}$$

Therefore, $\Pr(\phi_+|g,\phi) = \Pr(\phi_+|\phi) = \Pr(\phi_+|b,\phi)$ if $\phi_+ = \phi$. If $\phi_+ \neq \phi$, note that $\Pr(\phi_+|\theta,\phi) = 1 - \Pr(\Phi \setminus \phi_+|\theta,\phi)$.

Sufficiency. Again by Bayes rule, for all $\phi_+,\ \phi$

$$\begin{aligned}\Pr(\phi_+|\phi) = \frac{\Pr(\phi_+,\phi)}{\Pr(\phi)} &= \frac{\Pr(\phi_+|g,\phi)\Pr(g,\phi) + \Pr(\phi_+|b,\phi)\Pr(b,\phi)}{\Pr(\phi)} \\ &= \Pr(\phi_+|g,\phi)\Pr(g|\phi) + \Pr(\phi_+|b,\phi)\Pr(b|\phi). \end{aligned} \tag{B.2}$$

Since $\Pr(\phi_+|g,\phi) = \Pr(\phi_+|b,\phi)$ for $\phi_+ = \phi$, then $\Pr(\phi_+|\phi) = \Pr(\phi_+|\theta,\phi)$ for all θ, and by eq.(B.1), desired result follows. *Q.E.D.*

Claim B.3. *Suppose $q < 1$ and $\Pr(\phi_+|\phi) = 1$ for $\phi_+ = \phi \in \Phi$. Then $\Pr(\phi_+|g,\phi) = \Pr(\phi_+|b,\phi) = 1$ for all $\phi_+ = \phi \in \Phi$.*

Proof: As noted above, $\Pr(g|\phi),\ \Pr(b|\phi) > 0$ by eq.(2). Since $\Pr(\phi_+|\theta,\phi) \leq 1$, the desired result follows from eq.(B.2). *Q.E.D.*

B.2 Non-emptiness of the Parameter Space

Below we develop sufficient conditions that guarantee the non-emptiness of the parameter space, part of which is the space of the screening cost function $c(\cdot)$, under Assumption 1 and 4 (2–3 impose no independent restriction on the parameter space).

Let $\gamma = (\mu^0, X, \lambda, \theta^g, \theta^b, \underline{u}^g, \underline{u}^b, \mathfrak{q}) \in \Gamma$ denote the parameter vector with $\Gamma \subset \mathbb{R}^8$ the corresponding scalar-parameter space defined by model primitives (e.g., $\underline{u}^\theta > 0$) and Assumption 1. Denote $\mathrm{cl}(\Gamma)$ the closure of Γ under standard topology of $\mathbb{R}^8$. Essentially, Γ is a product space subject to several linear constraints enlisted in Assumption 1, so that $\mathrm{cl}(\Gamma)$ is a convex polyhedron with Γ the interior. A complete description of the parameter space of this model includes not only Γ, but also the space of function $c(\cdot)$. For this, let $\mathscr{C}_\mathfrak{q}(\mathbb{Q})$ denote the space of second-order differentiable functions such that part (*a*) in Assumption 4 is satisfied for a particular $\mathfrak{q} \in \left[\frac{1}{2}, 1\right]$, and $\mathscr{C}(\mathbb{Q}) = \cup_{\mathfrak{q}\in[1/2,1]}\mathscr{C}_\mathfrak{q}(\mathbb{Q})$. Clearly, the genuine *parameter space* of the model (for the static game) is

$$\mathscr{M} \equiv \{\langle \gamma, c(\cdot)\rangle \in \Gamma \times \mathscr{C}(\mathbb{Q}) \mid \text{Assumption 4 is satisfied by } \gamma \text{ and } c(\cdot)\},$$

which is a subset of $\Gamma \times \mathscr{C}(\mathbb{Q})$.

Recall that Assumption 4 (*b*)–(*c*) requires that (*i*) $c'(1) > \Delta_\eta(X, \mu^0)$, (*ii*) $c(1) < -(1-\mu^0)\eta^b(X)$, and (*iii*) $c''(q) \geq \mathfrak{C}$ over $\mathbb{Q}^1$. For a given $\gamma \in \Gamma$, we are interested in finding sufficient conditions on $c(\cdot) \in \mathscr{C}(\mathbb{Q})$ such that $\langle \gamma, c(\cdot)\rangle \in \mathscr{M}$. It turns out that a convenient way of identifying proper conditions is to focus on bounds of the second-order derivative of $c(\cdot)$. More specifically, for a given $c(\cdot) \in \mathscr{C}(\mathbb{Q})$, let $\underline{C} \leq c''(q) \leq \overline{C}$, $\forall\, q \in [\mathfrak{q}, 1]$, be some lower and upper bound of second-order derivative of $c(\cdot)$ over $[\mathfrak{q}, 1]$. Given $c(\mathfrak{q}) = c'(\mathfrak{q}) = 0$, basic calculus implies that

$$\underline{C}(1-\mathfrak{q}) \leq c'(1) = \int_\mathfrak{q}^1 c''(x)\mathrm{d}x \leq \overline{C}(1-\mathfrak{q}),$$
$$\frac{1}{2}\underline{C}(1-\mathfrak{q})^2 \leq c(1) = \int_\mathfrak{q}^1\int_\mathfrak{q}^q c''(x)\mathrm{d}x\mathrm{d}q \leq \frac{1}{2}\overline{C}(1-\mathfrak{q})^2.$$

Thus, if $\underline{C}$ is too big, (*ii*) will be violated for sure; whereas if $\overline{C}$ is too small, (*i*) can not hold for sure. In light of this, a set of *sufficient* conditions for (*i*)–(*iii*) to hold reads as follows

$$\mathfrak{V}(\gamma) \equiv \frac{\Delta_\eta(X,\mu^0)}{1-\mathfrak{q}} < \underline{C} < \overline{C} < \frac{-2(1-\mu^0)\eta^b(X)}{(1-\mathfrak{q})^2} \equiv \mathfrak{W}(\gamma), \tag{B.3}$$

$$\underline{C} > \mathfrak{C} \equiv \mathfrak{C}(\gamma) = \max\left\{\mathfrak{C}^1(\gamma), \mathfrak{C}^2(\gamma)\right\}, \tag{B.4}$$

where expressions of $\mathfrak{C}^1(\gamma)$ and $\mathfrak{C}^2(\gamma)$ are given in part (*c*) of Assumption 4. Note that if we can find $\underline{C}(\gamma)$, for a given $\gamma \in \Gamma$, such that

$$\mathfrak{V}(\gamma) < \underline{C}(\gamma) < \mathfrak{W}(\gamma) \text{ and } \underline{C}(\gamma) > \mathfrak{C}(\gamma) \tag{B.5}$$

hold simultaneously, then we can always find $\overline{C}(\gamma) < \mathfrak{W}(\gamma)$ such that eq.(B.3) is satisfied, thereby $\langle \gamma, c(\cdot)\rangle \in \mathscr{M}$ for any $c(\cdot) \in \mathscr{C}(\mathbb{Q})$ satisfying $\underline{C}(\gamma) \le c''(\cdot) \le \overline{C}(\gamma)$. The preceding discussion also suggests that the non-emptiness of $\mathscr{M}$ depends crucially whether there exists at least some subset of Γ such that for any γ in this subset we can find a $\underline{C}(\gamma)$ satisfying eq.(B.5).

By far it remains possible that, because restrictions put by (*i*)–(*iii*) are so severe, for no $\gamma \in \Gamma$ there exists a $\underline{C}(\gamma)$ satisfying eq.(B.5), which renders $\mathscr{M}$ empty. In what follows, we shall prove $\mathscr{M}$ is non-empty by constructing a particular lower bound (function) $\underline{C}(\gamma)$ and a particular subset $\Gamma_0 \subset \Gamma$ such that $\forall\, \gamma \in \Gamma_0$, $\underline{C}(\gamma)$ satisfies eq.(B.5). By choosing a proper $\overline{C}(\gamma)$, which is always possible given the particular $\underline{C}(\gamma)$, the following non-empty set

$$\mathscr{M}_0 = \{\langle \gamma, c(\cdot)\rangle \in \Gamma_0 \times \mathscr{C}(\mathbb{Q}) | \underline{C}(\gamma) \le c''(q) \le \overline{C}(\gamma) \text{ over } [\mathfrak{q}, 1]\}$$

becomes a subspace of $\mathscr{M}$, hence $\mathscr{M}$ itself is non-empty.

To construct the particular Γ_0 and $\underline{C}(\gamma)$, first let $\hat{\gamma}$ denote a particular parameter vector such that $\hat{\mu}^0 > \frac{1}{2}$, $1 > \hat{\theta}^g > \hat{\theta}^b > 0$, $\widehat{X} > 1 > \hat{\lambda} > 0$, $\underline{\hat{u}}^b > 0 = \underline{\hat{u}}^g$, $\hat{\mathfrak{q}} = \frac{1}{2}$, and $\hat{\gamma}$ satisfies

$$\mathbb{E}\big[\mathrm{NPV}^\theta\big]\big|_{\gamma=\hat{\gamma}} = \hat{\mu}^0 \hat{\eta}^g\big(\widehat{X}\big) + \big(1-\hat{\mu}^0\big)\hat{\eta}^b\big(\widehat{X}\big) = 0, \tag{B.6}$$

where $\hat{\eta}^g\big(\widehat{X}\big) = \hat{\theta}^g \widehat{X} + \big(1-\hat{\theta}^g\big)\hat{\lambda} - 1 > 0$, and $\hat{\eta}^b\big(\widehat{X}\big) < 0$ is defined analogously. Existence of such a $\hat{\gamma}$ is of no problem. Also, observe that $\hat{\gamma} \notin \Gamma$ because of eq.(B.6), but $\hat{\gamma} \in \mathrm{cl}(\Gamma)$.

Claim B.4. *Suppose* $\underline{C}(\gamma) = 6\mu^0\eta^g(X)$. *Then eq.(B.5) holds at* $\gamma = \hat{\gamma}$.

Proof: (*a*) $\underline{C}(\hat{\gamma}) > \mathfrak{C}(\hat{\gamma})$. First note that eq.(B.6) implies $\Delta_\eta(X,\mu^0)|_{\gamma=\hat{\gamma}} = 2\hat{\mu}^0\hat{\eta}^g\big(\widehat{X}\big)$, therefore $\overline{q}|_{\gamma=\hat{\gamma}} = \frac{1}{2}$. It then follows that $\mathfrak{C}^2(\hat{\gamma}) < 0$, as $c'(\overline{q})|_{\gamma=\hat{\gamma}} = 0$, and

$$\mathfrak{C}(\hat{\gamma}) = \mathfrak{C}^1(\hat{\gamma}) = 2\Big(\hat{\mu}^0 - \big(1-\hat{\mu}^0\big)\hat{\theta}^b/\hat{\theta}^g\Big)\hat{\eta}^g\big(\widehat{X}\big).$$

Evidently, $\mathfrak{C}(\hat{\gamma}) < 2\hat{\mu}^0\hat{\eta}^g(\widehat{X}) < 6\hat{\mu}^0\hat{\eta}^g(\widehat{X}) = \underline{C}(\hat{\gamma})$.

(*b*) $\mathfrak{V}(\hat{\gamma}) < \underline{C}(\hat{\gamma})$. Since $\Delta_\eta(X,\mu^0)|_{\gamma=\hat{\gamma}} = 2\hat{\mu}^0\hat{\eta}^g(\widehat{X})$, we have $\mathfrak{V}(\hat{\gamma}) = 4\hat{\mu}^0\hat{\eta}^g(\widehat{X}) < 6\hat{\mu}^0\hat{\eta}^g(\widehat{X}) = \underline{C}(\hat{\gamma})$.

(*c*) $\underline{C}(\hat{\gamma}) < \mathfrak{W}(\hat{\gamma})$. From eq.(B.6), we have $\hat{\mu}^0\hat{\eta}^g(\widehat{X}) = -(1-\hat{\mu}^0)\hat{\eta}^b(\widehat{X})$, thus $\mathfrak{W}(\hat{\gamma}) = 8\hat{\mu}^0\hat{\eta}^g(\widehat{X}) > 6\hat{\mu}^0\hat{\eta}^g(\widehat{X}) = \underline{C}(\hat{\gamma})$. *Q.E.D.*

Corollary B.5. *There exists a non-empty set $\mathcal{M}_0$ which is a subspace of $\mathcal{M}$. Thus $\mathcal{M}$ is non-empty.*

Proof: Evidently, $\underline{C}(\gamma)$, $\mathfrak{C}(\gamma)$, $\mathfrak{V}(\gamma)$, and $\mathfrak{W}(\gamma)$ are continuous at $\gamma = \hat{\gamma}$, hence there exists $\epsilon > 0$ such that eq.(B.5) holds for all $\gamma \in \mathcal{B}(\hat{\gamma},\epsilon)$, where $\mathcal{B}(\hat{\gamma},\epsilon) \subset \mathbb{R}^8$ denotes the open ball centering at $\hat{\gamma}$ with radius ϵ calculated under the standard Euclidean norm. Since $\hat{\gamma} \in \mathrm{cl}(\Gamma)$, $\Gamma_0 \equiv \Gamma \cap \mathcal{B}(\hat{\gamma},\epsilon)$ is non-empty. Because $\underline{C}(\gamma)$ satisfies eq.(B.5) over Γ_0 by construction, we can always choose $\overline{C}(\gamma)\ \forall\,\gamma \in \Gamma_0$ such that eq.(B.3) and eq.(B.4) are satisfied. Consequently, $\mathcal{M}_0$ is a subspace of $\mathcal{M}$, and $\mathcal{M}$ is thereby non-empty. *Q.E.D.*

Parameter vectors lying in Γ_0 possess some intuitive implications in terms of the intuitions we are capturing by this model. First, because of requirement imposed by eq.(B.6), payoff from lending to all borrowers indiscriminately, as bounded above by $\eta^{ND}(X)|_{\gamma\in\Gamma_0} = \mathbb{E}^0[\mathrm{NPV}^\theta]|_{\gamma\in\Gamma_0}$, is *small*, relative to $\mu^0\eta^g(X)|_{\gamma\in\Gamma_0} \approx \hat{\mu}^0\hat{\eta}^g(\widehat{X}) > 0$, i.e., payoff from exerting a non-negligible effort in screening borrowers. Next, as $\underline{\hat{u}}^g = 0$, $\underline{u}^g|_{\gamma\in\Gamma_0}$ is also small, which is the consistent with our modeling intention that the private benefit accruing to borrowers be small. Lastly, $\hat{\mathfrak{q}} = \frac{1}{2}$ implies that $\mathfrak{q}|_{\gamma\in\Gamma_0}$ is close to $\frac{1}{2}$, so that the region of costless yet informative screening intensity is small, which is in accordance with our formulation preference.

C PROOFS OF RESULTS IN THE MAIN TEXT

Proof of Lemma 1: Monotonicity of $\eta^\phi(\cdot)$ follows from those of $\eta^\theta(R)$ and $\nu^\phi(q,\mu)$ (see Claim B.1 above). For the existence of $\underline{a}(R) \in \mathrm{Int}\,\mathcal{U}$, observe that $\forall\ R \in (\underline{R}, X]$, $\eta^g(R) > 0 > \eta^b(R)$, and $f(R,a) \equiv a\eta^g(R) + (1-a)\eta^b(R)$ is linear in $a \in \mathcal{U}$.

To prove (*a*), first observe that for $\mu \geq \underline{a}(R)$, $\nu^G(q,\mu) \geq \mu \geq \underline{a}(R)$, therefore $\eta^G(q,R,\mu) = f(R,\nu^G(q,\mu)) \geq 0$, $\forall\, q \in \mathbb{Q}$, where equality holds only if $\mu = \underline{a}(R)$ and $q = \frac{1}{2}$ so that $\nu^G(\frac{1}{2},\underline{a}(R)) = \underline{a}(R)$. Second, observe that $\eta^B(q,R,\mu) = f(R,\nu^B(q,\mu))$ which is strictly decreasing in q. Recall that $\nu^B(\frac{1}{2},\mu) = \mu$ and $\nu^B(1,\mu) = 0$, so that $\eta^B(\frac{1}{2},R,\mu) = f(R,\mu) \geq 0$ and $\eta^B(1,R,\mu) = f(R,0) < 0$, which implies the existence of a unique $q^B(R,\mu) \in \mathbb{Q}$. Moreover, since $\partial_R\eta^B,\ \partial_\mu\eta^B > 0$, it follows that $q^B(R,\mu)$ is strictly increasing in R and μ. Analogous reasoning establishes (*b*). *Q.E.D.*

Proof of Lemma 3: Notice that for $R \in (\underline{R}, X]$, $\eta^g(R) > 0$ and $\eta^b(R) < 0$, therefore $\Delta_\eta(R, \mu) > 0 \; \forall \, \mu \in \mathcal{U}$. For part (*a*) where $\mu \geq \underline{a}(R)$, first notice that when $q \leq q^B(R, \mu)$, $\eta^\phi(q, R, \mu) \geq 0 \; \forall \, \phi \in \Phi$, which results in

$$\begin{aligned}
\eta(q, R, \mu) &= \Pr(G|q, \mu)\eta^G(q, R, \mu) + \Pr(B|q, \mu)\eta^B(q, R, \mu) \\
&= q\mu\eta^g(R) + (1-q)(1-\mu)\eta^b(R) + (1-q)\mu\eta^g(R) + q(1-\mu)\eta^b(R) \\
&= \mu\eta^g(R) + (1-\mu)\eta^b(R) \\
&= \eta^{ND}(R, \mu).
\end{aligned}$$

Alternatively, when $q > q^B(R, \mu)$, $\eta^G(q, R, \mu) > 0$ and $\eta^B(q, R, \mu) < 0$, which result in

$$\begin{aligned}
\eta(q, R, \mu) &= \Pr(G|q, \mu)\eta^G(q, R, \mu) \\
&= q\mu\eta^g(R) + (1-q)(1-\mu)\eta^b(R) \\
&= q\Delta_\eta(R, \mu) + (1-\mu)\eta^b(R).
\end{aligned}$$

Moreover, the definition of $\underline{a}(R)$ is such that $a\eta^g(R) + (1-a)\eta^b(R) = 0$, so $\eta^{ND}(R, \mu) > 0$ when $\mu > \underline{a}(R)$. Proof for part (*b*) follows almost identical derivations. *Q.E.D.*

Proof of Proposition 1: To circumvent complications due to the kink of $\eta(q, R, \mu)$, let

$$\begin{aligned}
\widetilde{\eta}(q, R, \mu) &= q\Delta_\eta(R, \mu) + (1-\mu)\eta^b(R) \quad \forall \, q \in \mathbb{Q} \\
\widetilde{\pi}(R, \mu) &= \max_{q \in \mathbb{Q}} \widetilde{\eta}(q, R, \mu) - c(q)
\end{aligned} \tag{C.1}$$

$\forall \, (R, \mu) \in (\underline{R}, X] \times \mathcal{U}$ and it is evident that $\widetilde{\eta}(\cdot)$ is continuously differentiable over $\mathbb{Q} \times (\underline{R}, X] \times \mathcal{U}$. From Lemma 3, it can be easily verified that

$$\pi(R, \mu) = \max\left\{\widetilde{\pi}(R, \mu), \eta^{ND}(R, \mu), 0\right\}, \tag{C.2}$$

Thus the main task becomes investigating properties of $\widetilde{\pi}(R, \mu)$ and $\widetilde{\eta}(q, R, \mu)$. Note that $\widetilde{\eta}(q, R, \mu)$ is a linear function with slope $\Delta_\eta(R, \mu) > 0$ over $\mathbb{Q}$, therefore $\widetilde{\eta}(q, R, \mu) - c(q)$ is convex over $\mathbb{Q}$ and strictly convex over $\mathbb{Q}^1$, hence Kuhn-Tucker first-order necessary condition is also sufficient. The FOC takes the form

$$c'(q^*) \leq \Delta_\eta(R, \mu),$$

where the inequality holds only if $q^* = 1$ and $c'(q^*) < \Delta_\eta(R, \mu)$. It follows that optimal screening intensity $q^*(R, \mu) \; \forall \, (R, \mu) \in (\underline{R}, X] \times \mathcal{U}$ can be expressed as in eq.(7). Moreover, since $R > \underline{R}$, $\Delta_\eta(R, \mu) > 0 \; \forall \, \mu \in \mathcal{U}$, we have $q^*(R, \mu) > \mathfrak{q}$ (see the Remark following Proposition 1) over $(\underline{R}, X] \times \mathcal{U}$.

It also follows that

$$\widetilde{\pi}(R,\mu) = \widetilde{\eta}\big(q^*(R,\mu), R, \mu\big) - c\big(q^*(R,\mu)\big) - \zeta^*(R,\mu)(q^*(R,\mu) - 1)$$

is differentiable in R and μ, where $\zeta^*(R,\mu)$ is the multiplier in the optimum associated with constraint $q \leq 1$. Then by Envelope Theorem, there is

$$\partial_\mu \widetilde{\pi}(R,\mu) = q^*(R,\mu)\eta^g(R) - (1 - q^*(R,\mu))\eta^b(R).$$

For any $R \in (\underline{R}, X]$, since $q^*(R,\mu) \in (\mathrm{q}, 1]$ and $\eta^g(R) > 0 > \eta^b(R)$, $\partial_\mu \widetilde{\pi}(R,\mu) > 0$, therefore $\widetilde{\pi}(R,\mu)$ is strictly increasing in μ over $\mathcal{U}$.[29]

Following two claims are important for the rest of the proof of the proposition.

Claim C.1. $\forall\, R \in (\underline{R}, X]$, $\widetilde{\pi}(R, 0) < 0 < \widetilde{\pi}(R, \underline{a}(R))$ *where* $\underline{a}(R)$ *is given in Lemma 1.*

Proof: When $\mu = 0$, $\widetilde{\eta}(q, R, 0) = -\eta^b(R)q + \eta^b(R)$, thus $\widetilde{\eta}(q, R, 0) < 0$ over $\mathbb{Q}$ except for $q = 1$. It follows that $\widetilde{\eta}(q, R, 0) - c(q) < 0$ over $\mathbb{Q}$, and consequently $\widetilde{\pi}(R, 0) < 0$. Recall that when $\mu = \underline{a}(R)$, $\eta^{ND}(R, \underline{a}(R)) = 0$, so that $\widetilde{\eta}\big(\frac{1}{2}, R, \underline{a}(R)\big) - c\big(\frac{1}{2}\big) = \frac{1}{2}\eta^{ND}(R, \underline{a}(R)) = 0$. As $\partial_q\big(\widetilde{\eta}\big(\frac{1}{2}, R, \underline{a}(R)\big) - c\big(\frac{1}{2}\big)\big) = \Delta_\eta(R, \underline{a}(R)) > 0$, there is $\widetilde{\pi}(R, \underline{a}(R)) > 0$. *Q.E.D.*

Claim C.2. $\forall\, R \in (\underline{R}, X]$, $0 < \widetilde{\pi}(R, 1) < \eta^g(R)$.

Proof: Observe that when $\mu = 1$, $\widetilde{\eta}\big(\frac{1}{2}, R, 1\big) = \frac{1}{2}\eta^g(R) > 0$, so that similar argument as for $\widetilde{\pi}(R, \underline{a}(R))$ proves that $\widetilde{\pi}(R, 1) > 0$. Next, since $q^*(R, 1) \in (\mathrm{q}, 1]$ and $\widetilde{\eta}(q, R, 1)$ is strictly increasing in q, it follows that $\widetilde{\pi}(R, 1) < \widetilde{\eta}(q^*(R, 1), R, 1) < \widetilde{\eta}(1, R, 1) = \eta^g(R)$. *Q.E.D.*

Return to the proof of the proposition. For any given $R \in (\underline{R}, X]$, $\widetilde{\pi}(R,\mu)$ is continuous and strictly increasing in μ over $\mathcal{U}$. First, $\widetilde{\pi}(R, 0) < 0 < \widetilde{\pi}(R, \underline{a}(R))$, there is a unique $\underline{\mu}(R) \in (0, \underline{a}(R))$ such that $\widetilde{\pi}\big(R, \underline{\mu}(R)\big) = 0$ and $\widetilde{\pi}(R,\mu) > 0$ iff $\mu \in \big(\underline{\mu}(R), 1\big]$. Next, observe that $\eta^{ND}(R,\mu)$ is strictly increasing in μ and

$$\eta^{ND}(R, 0) = \eta^b(R) < \eta^{ND}(R, \underline{a}(R)) = 0 < \eta^{ND}(R, 1) = \eta^g(R),$$

[29] Actually, $\widetilde{\pi}(R,\mu)$ is strictly convex in μ so long as $\eta^g(R) + \eta^b(R) \neq 0$ and $c'(1) \geq \Delta_\eta(R,\mu)$:

$$\begin{aligned}\partial^2_\mu \widetilde{\pi}(R,\mu) &= \big(\eta^g(R) + \eta^b(R)\big)\partial_\mu q^*(R,\mu) = \big(\eta^g(R) + \eta^b(R)\big)\frac{\partial_\mu \Delta_\eta(R,\mu)}{c''\big(q^*(R,\mu)\big)} \\ &= \frac{\big(\eta^g(R) + \eta^b(R)\big)^2}{c''\big(q^*(R,\mu)\big)} > 0,\end{aligned}$$

in which the differentiability of $q^*(R,\mu) = (c')^{-1}\big(\Delta_\eta(R,\mu)\big)$ is guaranteed by $c'(1) \geq \Delta_\eta(R,\mu)$ (see the discussion following eq.(7)).

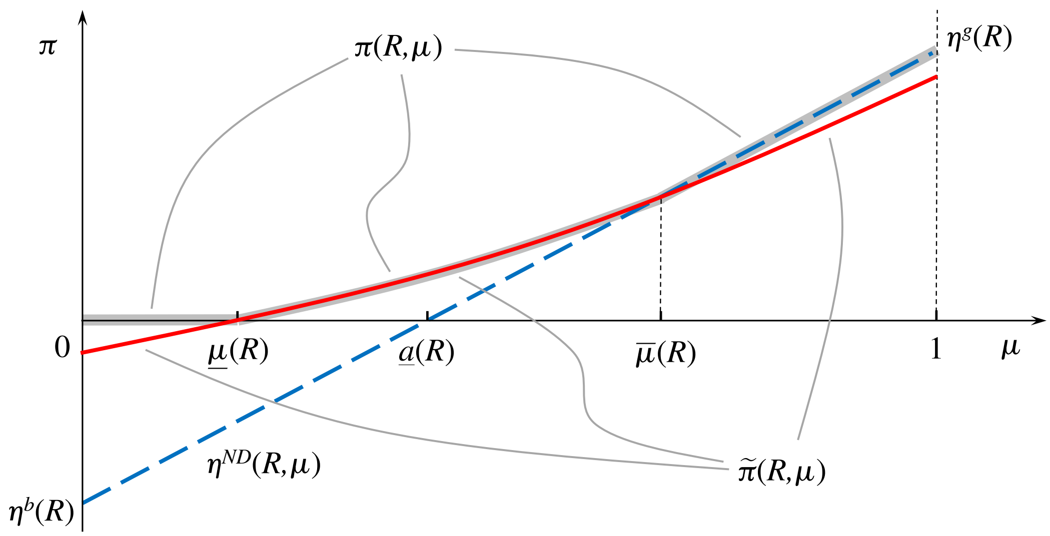

FIGURE 5: Illustration of $\widetilde{\pi}(R,\mu)$, $\eta^{ND}(R,\mu)$, and $\pi(R,\mu)$ as functions of $\mu \in \mathcal{U}$.
Notes: The grey shaded line denotes the profit function $\pi(R,\mu)$, which equals to $\max\{\widetilde{\pi}(R,\mu), \eta^{ND}(R,\mu), 0\}$.

it follows that: (*i*) $\eta^{ND}(R,\mu) < \widetilde{\pi}(R,\mu)$ over $\mu \in [\underline{\mu}(R), \underline{a}(R)]$; and (*ii*) there is a unique $\overline{\mu}(R) \in (\underline{a}(R), 1)$ such that $\widetilde{\pi}(R, \overline{\mu}(R)) = \eta^{ND}(R, \overline{\mu}(R))$ and $\widetilde{\pi}(R,\mu) < \eta^{ND}(R,\mu)$ iff $\mu \in (\overline{\mu}(R), 1]$. The preceding argument is illustrated in Figure 5.

Claim C.3. *$\forall\, R \in (\underline{R}, X]$,*

(*a*) *$q^G(R,\mu) > \mathfrak{q}$ if $\mu \in [\overline{\mu}(R), 1]$ and $q^B(R,\mu) > \mathfrak{q}$ if $\mu \in [0, \underline{\mu}(R)]$;*

(*b*) *$q^*(R,\mu) > q^G(R,\mu)$ if $\mu \in [\underline{a}(R), \overline{\mu}(R)]$; and*

(*c*) *$q^*(R,\mu) > q^B(R,\mu)$ if $\mu \in [\underline{\mu}(R), \underline{a}(R)]$.*

Proof: For part (*a*), note that $\mu \geq \overline{\mu}(R)$ iff $\eta^{ND}(R,\mu) \geq \widetilde{\pi}(R,\mu)$. Suppose $q^G(R,\mu) \leq \mathfrak{q}$, then $\widetilde{\eta}(\mathfrak{q}, R, \mu) \geq \eta^{ND}(R,\mu)$. As $\partial_q\big(\widetilde{\eta}(\mathfrak{q}, R, \underline{a}(R)) - c(\mathfrak{q})\big) > 0$, $\widetilde{\pi}(R,\mu) > \widetilde{\eta}(\mathfrak{q}, R, \mu) \geq \eta^{ND}(R,\mu)$, which leads to a contradiction. The same reasoning establishes that $q^B(R,\mu) > \mathfrak{q}$. To see part (*b*), note that in this case

$$\begin{aligned}\widetilde{\eta}\big(q^G(R,\mu), R, \mu\big) = \eta^{ND}(R,\mu) &\leq \widetilde{\eta}(q^*(R,\mu), R, \mu) - c(q^*(R,\mu)) \\ &< \widetilde{\eta}(q^*(R,\mu), R, \mu),\end{aligned}$$

thus $q^*(R,\mu) > q^G(R,\mu)$ as $\widetilde{\eta}(q, R, \mu)$ is increasing in q. To see part (*c*), note that in this case

$$\widetilde{\eta}(q^*(R,\mu), R, \mu) = c(q^*(R,\mu)) > 0 \geq \widetilde{\eta}\big(q^B(R,\mu), R, \mu\big),$$

and as $\widetilde{\eta}(q, R, \mu)$ is increasing in q, the result follows. *Q.E.D.*

As a direct implication of this claim and Lemma 1, $\eta^{\phi}(q, R, \mu) > 0\,(< 0)$ over $\mathbb{Q}^0\ \forall\,\phi \in \Phi$ when $\mu \geq \overline{\mu}(R)$ $(\leq \underline{\mu}(R))$. Moreover, when $\mu > \overline{\mu}(R)$, $\pi(R,\mu) = \eta^{ND}(R,\mu)$, so that $\mathcal{BQ}(\pi(R,\mu)) = \mathbb{Q}^0$ as $q^G(R,\mu) > \mathfrak{q}$. Analogously, when $\mu < \underline{\mu}(R)$, $\pi(R,\mu) = 0 > \widetilde{\pi}(R,\mu)$, so that $\mathcal{BQ}(\pi(R,\mu)) = \mathbb{Q}^0$ as $q^B(R,\mu) > \mathfrak{q}$. When $\mu = \overline{\mu}(R)$, $\pi(R,\mu) = \eta^{ND}(R,\mu) = \widetilde{\eta}(q^*(R,\mu), R, \mu) - c(q^*(R,\mu))$, therefore $\mathcal{BQ}(\pi(R,\mu)) = \mathbb{Q}^0 \cup \{q^*(R,\mu)\}$; so for the case of $\mu = \underline{\mu}(R)$. Lastly, observe that when $\mu = \overline{\mu}(R)$, $q^*(R,\mu) > q^G(R,\mu) > \mathfrak{q}$, it follows that $\eta^{\phi}(q,R,\mu) > 0$ over $\mathbb{Q}^0\ \forall\,\phi \in \Phi$ except for $\eta^B(q,R,\mu) < 0$ at $q = q^*(R,\mu)$; and analogously, when $\mu = \underline{\mu}(R)$, $q^*(R,\mu) > q^B(R,\mu) > \mathfrak{q}$, it follows that $\eta^{\phi}(q,R,\mu) < 0$ over $\mathbb{Q}^0\ \forall\,\phi \in \Phi$ except for $\eta^G(q,R,\mu) > 0$ at $q = q^*(R,\mu)$. For interim value $\mu \in \big(\underline{\mu}(R), \overline{\mu}(R)\big)$, desired result follows from the fact that $q^*(R,\mu) > q^G(R,\mu)$ and $q^B(R,\mu)$. This completes the proof of Proposition 1. *Q.E.D.*

Proof of Lemma 5: Complete separating equilibrium is impossible, because all b-borrowers will be denied optimally by lending banks as they are separated from g-borrowers.

The remaining possibility is quasi separating. It suffices to focus on the situation with two distinct loans. To fix notation, suppose there are two loans, $\ell_1 = R_1 < \ell_2 = R_2$, with *arbitrary* priors (of banks) μ_1 and μ_2 respectively. Also, let $p_j^{\theta} = p^{\theta}(R_j, \mu_j) > 0\ \forall\,\theta \in \Theta$ (recall that, by Proposition 3 and the associating Remark, no borrower will choose a loan with $p^{\theta} = 0$), and $q_j^* = q^*(R_j,\mu_j) \in (\mathfrak{q}, 1) \subset \big(\frac{1}{2}, 1\big)$ if (R_j,μ_j) is such that $q_j^* > \mathfrak{q}$ ($q_j^* = 1$ implies $p_j^b = 0$, hence excluded from the discussion), for $j = 1, 2$. Observe that for ℓ_1 and ℓ_2 both being present in an equilibrium outcome, it has to be the case that g-borrowers are indifferent between ℓ_j, $j = 1, 2$; otherwise, some R_j attracts only b-borrowers, but this leads them to be denied for sure, i.e., $p_j^b = 0$. This observation indicates that $u^g(R_1)p_1^g = u^g(R_2)p_2^g$, so that $p_1^g < p_2^g$ as $R_1 < R_2$ implies $u^g(R_1) > u^g(R_2)$. According to eq.(8), there are thus two cases to consider:

- $p_2^g = 1$, hence $p_2^b = 1$, $p_1^g = q_1^* > \frac{1}{2}$, and $p_1^b = 1 - q_1^*$; and
- $p_2^g = q_2^* \in \big(\frac{1}{2}, 1\big)$, hence $p_2^b = 1 - q_2^*$, $p_1^g = q_1^* > \frac{1}{2}$, and $p_1^b = 1 - q_1^*$.

CASE 1: $p_2^g = 1$ For this to be an equilibrium outcome, it is necessary for b-borrowers not to strictly prefer ℓ_2 to ℓ_1, i.e., $u^b(R_1)(1 - q_1^*) \geq u^b(R_2)$; otherwise, all b-borrowers will choose ℓ_2 and μ_1 will become 1, which in turn implies $p_1^g = 1$. Rearranging g- and b-borrower's incentive constraints yields

$$f(\theta^b) \leq 1 - q_1^* < q_1^* = f(\theta^g),$$

where $f(\xi) \equiv \frac{\xi(X-R_2)+\underline{u}^{\theta}}{\xi(X-R_1)+\underline{u}^{\theta}}$ over $[0,1]$. However, since $R_1 < R_2$ and $\underline{u}^{\theta} > 0$, it is easily verified that $f(\xi)$ is strictly decreasing over $[0,1]$, therefore $f(\theta^b) > f(\theta^g)$ as $\theta^b < \theta^g$, which results in a contradiction.

CASE 2: $p_2^g < 1$ First, note that $q_2^* > q_1^*$ as $u^g(R_1)q_1^* = u^g(R_2)q_2^*$ and $u^g(R_1) > u^g(R_2)$. Second, for this case to be an equilibrium outcome, there must be $u^b(R_1)(1-q_1^*) = u^b(R_2)(1-q_2^*)$; otherwise either one of $p_j^g = 1$, for $j = 1, 2$. However, this is impossible since $u^b(R_1) > u^b(R_2) > 0$ as $R_1 < R_2$ and $1 - q_1^* > 1 - q_2^* > 0$. *Q.E.D.*

Proof of Lemma 7: The second half of Assumption 4 (b) is equivalent to

$$\begin{aligned}\eta^{ND}(X,\mu^0) &= \mu^0\eta^g(X) + (1-\mu^0)\eta^b(X)\\ &< \mu^0\eta^g(X) - c(1) = \eta(1,X,\mu^0) - c(1),\end{aligned}$$

therefore, by letting $\widetilde{\pi}(R) \equiv \widetilde{\pi}(R,\mu^0)$ where $\widetilde{\pi}(R,\mu)$ is defined in eq.(C.1), it follows that $\widetilde{\pi}(X) = \max_{q\in\mathbb{Q}^1}\widetilde{\eta}(q,X,\mu^0) - c(q) > \eta^{ND}(X,\mu^0) = \mathbb{E}^0\big[\mathrm{NPV}^\theta\big] > 0$ according to Assumption 1. Straightforward calculation shows that on the one hand $\widetilde{\pi}'(R) = \mu^0 q^*(R)\theta^g + (1-\mu^0)(1-q^*(R))\theta^b > 0$, since $\theta^g > \theta^b > 0$, $q^*(R) > \frac{1}{2}$, and $\mu^0 > \frac{1}{2}$ by Assumption 1; and on the other hand, $\partial_R\eta^{ND}(R,\mu^0) = \mu^0\theta^g + (1-\mu^0)\theta^b$.

Assumption 4 (b) also states that $c'(1) > \Delta_\eta(X,\mu^0)$, which implies that

$$c'(1) > \Delta_\eta(R,\mu^0), \quad \forall\, R,$$

since $\partial_R\Delta_\eta(R,\mu^0) = \mu^0\theta^g - (1-\mu^0)\theta^b > 0$ by Assumption 1, i.e., $\Delta_\eta(R,\mu^0)$ is increasing in R. It then follows that $q^*(R) < 1$ according to eq.(7). Given $q^*(R) \in \big(\frac{1}{2},1\big)$, it follows that $\widetilde{\pi}'(R) < \partial_R\eta^{ND}(R,\mu^0)$, hence $\widetilde{\pi}(R) > \eta^{ND}(R,\mu^0)$, $\forall\, R$. Observe that $\widetilde{\eta}(q,\underline{R},\mu^0) - c(q) = (1-\mu^0)(1-q)\eta^b(\underline{R}) - c(q) < 0\ \forall\, q \in \mathbb{Q}$, $\widetilde{\pi}(\underline{R}) < 0$, so we conclude that there exists a unique $R^0 \in (\underline{R}, X)$ such that $\widetilde{\pi}(R^0) = 0$, and $\widetilde{\pi}(R) > 0\ (< 0)$ iff $R > R^0$ $(< R^0)$. In addition, recall that by eq.(C.2), $\pi(R) = \max\big\{\widetilde{\pi}(R), \eta^{ND}(R,\mu^0), 0\big\}$, thus $\pi(R) = \widetilde{\pi}(R) > \eta^{ND}(R,\mu^0)$ over $[R^0, X]$. In viewing of Figure 5, this amounts to say that $\mu^0 \in \big(\underline{\mu}(R), \overline{\mu}(R)\big)\ \forall\, R \in (R^0, X]$ and $\mu^0 = \underline{\mu}(R^0) < \overline{\mu}(R^0)$, thus by Proposition 2 all B-borrowers are denied credit over $[R^0, X]$. Correspondingly, $\forall\, R \in [R^0, X]$ and the fixed μ^0, optimal screening decision entails $q^*(R) = q^*(R,\mu^0) = (c')^{-1}\big(\Delta_\eta(R,\mu^0)\big) \in (\mathrm{q}, 1)$. Observe that

$$\frac{\mathrm{d}q^*(R)}{\mathrm{d}R} = \frac{\partial_R\Delta_\eta(R,\mu^0)}{c''(q^*(R))} = \frac{\mu^0\theta^g - (1-\mu^0)\theta^b}{c''(q^*(R))} > 0,$$

we conclude that $\mathrm{d}q^*(R)/\mathrm{d}R > 0$. Note this is true over the entire domain $\mathscr{R}$, not only $[R^0, X]$, hence $q^*(R)$ is strictly increasing over $\mathscr{R}$. *Q.E.D.*

Proof of Lemma 8: It is useful to first develop several simple results regarding to the candidate interest rate in the second round lending process, $\overline{R}_+(R)$, defined as the unique solution of eq.(10) when $\mu = \mu^0$.

Claim C.4. *(i) Over $\mathscr{R}$, $\overline{R}_+(R)$ is strictly increasing in R and $\overline{R}_+(R) > R$; (ii) $\overline{R}_+(R) \le X$ iff $q^*(R) = (c')^{-1}\big(\Delta_\eta(R)\big) \le \overline{q} \equiv \frac{\mu^0 \eta^g(X)}{\Delta_\eta(X)}$, or equivalently, R is such that $\Delta_\eta(R) \le c'(\overline{q})$; and (iii) if $q^*(\underline{R}) \le \overline{q}$, then there exists a unique $R^1 \in [\underline{R}, X)$ such that $\overline{R}_+(R) \le X \ \forall\, R \in [\underline{R}, R^1]$; otherwise $\overline{R}_+(R) > X \ \forall\, R \in \mathscr{R}$.*

Proof: (*i*) Solving eq.(10) for $\overline{R}_+(R)$ yields

$$\overline{R}_+(R) = \frac{1-\lambda}{(\theta^g - \theta^b)\nu^B(R) + \theta^b} + \lambda, \tag{C.3}$$

where $\nu^B(R) \equiv \nu^B(q^*(R), \mu^0)$ is given by eq.(2). By Claim B.1, $\partial_q \nu^B(q, \mu^0) < 0$, and by Lemma 7, $q^*(R)$ is strictly increasing in R, thus $\overline{R}_+(R)$ is strictly increasing in R over $\mathscr{R}$. Note that when $R_+ = R$ in eq.(10), $\eta_+\big(R, \nu^B(q^*(R), \mu^0\big) = \eta^B\big(q^*(R), R, \mu^0\big) < 0$ by Proposition 2, so that $\overline{R}_+(R) > R$ over $\mathscr{R}$ as $\eta_+(\cdot)$ is strictly increasing in R_+.

(*ii*) Observe that $\overline{R}_+(R) \le X$ is equivalent to $\nu^B(R)\eta^g(X) + (1-\nu^B(R))\eta^b(X) \ge 0$, which in turn is equivalent to

$$\mu^0(1-q^*(R))\eta^g(X) + (1-\mu^0)q^*(R)\eta^b(X) \ge 0.$$

From this last inequality, we obtain $\overline{R}_+(R) \le X \iff q^*(R) \le \frac{\mu^0 \eta^g(X)}{\Delta_\eta(X)}$, and by eq.(7), we arrive at $(c')^{-1}\big(\Delta_\eta(R)\big) \le \overline{q}$, or equivalently, $\Delta_\eta(R) \le c'(\overline{q})$.

(*iii*) If $q^*(\underline{R}) \le \overline{q}$, $\overline{R}_+(\underline{R}) \le X$, hence there exists uniquely a required $R^1 \in [\underline{R}, X)$ since $\overline{R}_+(X) > X$ and $\overline{R}_+(R)$ is continuous and strictly increasing in R. Evidently, if $q^*(\underline{R}) > \overline{q}$, $\overline{R}_+(\underline{R}) > X$, thus $\overline{R}_+(R) > X$ over $\mathscr{R}$. *Q.E.D.*

Resuming the proof of the monotonicity of $U^\theta(R)$ over $\mathscr{R}$. We will first look at $U^g(R)$, then $U^b(R)$.

MONOTONICITY OF $U^g(R)$ By the preceding Claim, $\mathbb{1}_{\{\overline{R}_+(R) \le X\}} = 1$ iff $\overline{R}_+(\underline{R}) \le X$ and $R \in [\underline{R}, R^1]$. In this case, $\mathbb{1}_{\{\overline{R}_+(R) \le X\}}$ is weakly decreasing in R over $\mathscr{R}$; otherwise, $\mathbb{1}_{\{\overline{R}_+(R) \le X\}} = 0$ over $\mathscr{R}$, nonetheless, it is weakly decreasing. Also by the preceding claim, $\overline{R}_+(R)$ is strictly increasing in R, and $u^g\big(\overline{R}_+(R)\big)$ is thereby strictly decreasing in R. In addition, that $q^*(R)$ is strictly increasing implies $1 - q^*(R)$ is strictly decreasing. To sum up, the second term of $U^g(R)$, $U_2^g(R)$, is decreasing in R, and when it is not zero, it is strictly decreasing.

To prove the first term of $U^g(R)$, i.e., $U_1^g(R) = u^g(R)q^*(R)$, is decreasing in R, let's take derivative of $U_1^g(R)$ w.r.p. R, which yields

$$\frac{\mathrm{d}U_1^g(R)}{\mathrm{d}R} = -\theta^g q^*(R) + u^g(R)\frac{\mu^0\theta^g - (1-\mu^0)\theta^b}{c''(q^*(R))}.$$

Clearly, $\mathrm{d}U_1^g(R)/\mathrm{d}R < 0$ is equivalent to $c''(q^*(R)) > \big(\mu^0 - (1-\mu^0)\theta^b/\theta^g\big)u^g(R)/q^*(R)$. Recalling that $q^*(R) > \frac{1}{2}$, the RHS of the last inequality is less than $2\big(\mu^0 - (1-\mu^0)\theta^b/\theta^g\big)\cdot u^g(R)$, which in turn is less than $2\big(\mu^0 - (1-\mu^0)\theta^b/\theta^g\big)\big(\eta^g(X) + \underline{u}^g\big)$ as $u^g(R) \le u^g(\underline{R}) = \eta^g(X) + \underline{u}^g$. Part (c) of Assumption 4 implies that $c''(q^*(R)) > 2\big(\mu^0 - (1-\mu^0)\theta^b/\theta^g\big)\big(\eta^g(X) + \underline{u}^g\big)$, therefore, $U_1^g(R)$ is strictly decreasing in R. Hence we conclude that $U^g(R)$ is strictly decreasing in R.

Monotonicity of $U^b(R)$ Similar reasoning as for $U_2^g(R)$ establishes that $U_1^b(R)$ is strictly decreasing in R. The remaining work is to show that $U_2^b(R)$ is also decreasing in R. Since $\mathbb{1}_{\{\overline{R}_+(R)\le X\}} = 0$ if $\overline{R}_+(\underline{R}) > X$, in which case the desired result trivially follows from $U^b(R) = U_1^b(R)$, we shall consider the case of $\overline{R}_+(\underline{R}) \le X$ in what follows.

Given that $\overline{R}_+(\underline{R}) \le X$, $\mathbb{1}_{\{\overline{R}_+(R)\le X\}} = 0$ for $R > R^1$ by the preceding Claim. Consequently, it suffices to show that $U^b(R) = u^b(R)(1-q^*(R)) + u^b\big(\overline{R}_+(R)\big)q^*(R)$ is decreasing over $[\underline{R}, R^1]$. Taking derivative w.r.p R yields

$$\frac{\mathrm{d}U^b(R)}{\mathrm{d}R} = -\theta^b(1-q^*(R)) - u^b(R)\frac{\mathrm{d}q^*(R)}{\mathrm{d}R} + \frac{\mathrm{d}}{\mathrm{d}R}\Big(u^b\big(\overline{R}_+(R)\big)q^*(R)\Big). \tag{C.4}$$

Plugging $\overline{R}_+(R)$ given by eq.(C.3) into $u^b\big(\overline{R}_+(R)\big)$ results in

$$u^b\big(\overline{R}_+(R)\big) = \theta^b(X-\lambda) + \underline{u}^b - \frac{1-\lambda}{1+(\theta^g/\theta^b - 1)\nu^b(R)},$$

substituting out $\nu^b(R) = \mu^0(1-q^*(R))\big/\big(\mu^0(1-q^*(R)) + (1-\mu^0)q^*(R)\big)$ and rearranging resulting expression yields

$$u^b\big(\overline{R}_+(R)\big) = \kappa + \frac{\Delta_\eta(\underline{R})\zeta/\xi}{q^*(R)\xi - \zeta},$$

where

$$\begin{aligned}
\kappa &= \theta^b(X-\lambda) + \underline{u}^b - (1-\lambda)(2\mu^0 - 1)/\xi,\\
\xi &= (1+\theta^g/\theta^b)\mu^0 - 1 > 0,\\
\zeta &= \mu^0\theta^g/\theta^b.
\end{aligned}$$

Note that R affects $u^b\big(\overline{R}_+(R)\big)$ only through its impact on $q^*(R)$. It follows that

$$\frac{\mathrm{d}}{\mathrm{d}R}\Big(u^b\big(\overline{R}_+(R)\big)q^*(R)\Big) = \left(\kappa - \frac{\Delta_\eta(\underline{R})\zeta^2/\xi}{(q^*(R)\xi - \zeta)^2}\right)\frac{\mathrm{d}q^*(R)}{\mathrm{d}R},$$

hence the second and third term on the RHS of eq.(C.4) can be consolidated into

$$\begin{aligned}
&\left(-u^b(R) + \kappa - \frac{\Delta_\eta(\underline{R})\zeta^2/\xi}{(q^*(R)\xi - \zeta)^2}\right)\frac{\mathrm{d}q^*(R)}{\mathrm{d}R}\\
&\qquad = \left(\theta^b(R-\lambda) - (1-\lambda)(2\mu^0 - 1)/\xi - \frac{\Delta_\eta(\underline{R})\zeta^2/\xi}{(q^*(R)\xi - \zeta)^2}\right)\frac{\mathrm{d}q^*(R)}{\mathrm{d}R},
\end{aligned}$$

and by using $\mathrm{d}q^*(R)/\mathrm{d}R = \big((1+\theta^g/\theta^b)\mu^0 - 1\big)\theta^b/c''(q^*(R)) = \xi\theta^b/c''(q^*(R))$, the last equation becomes

$$\left(\theta^b(R-\lambda)\xi - (1-\lambda)(2\mu^0-1) - \frac{\Delta_\eta(\underline{R})}{(q^*(R)\xi/\zeta-1)^2}\right)\frac{\theta^b}{c''(q^*(R))}.$$

Some algebra shows that $\Big(\theta^b(R-\lambda)\xi-(1-\lambda)(2\mu^0-1) = \Delta_\eta(R)$, thus eq.(C.4) simplifies into

$$\frac{1}{\theta^b}\frac{\mathrm{d}U^b(R)}{\mathrm{d}R} = -(1-q^*(R)) + \left(\Delta_\eta(R) - \frac{\Delta_\eta(\underline{R})}{(q^*(R)\xi/\zeta-1)^2}\right)\frac{1}{c''(q^*(R))}, \tag{C.5}$$

from which it can be easily seen that $\mathrm{d}U^b(R)/\mathrm{d}R < 0$ is equivalent to

$$\left(\Delta_\eta(R) - \frac{\Delta_\eta(\underline{R})}{(q^*(R)\xi/\zeta-1)^2}\right)\frac{1}{c''(q^*(R))} < 1-q^*(R). \tag{C.6}$$

Since we are considering the case of $R \in [\underline{R}, R^1]$, the preceding Claim implies that $q^*(R) \leq \overline{q}$ and $\Delta_\eta(R) \leq c'(\overline{q})$, thus a sufficient condition for eq.(C.6) is

$$\begin{aligned}
&\left(c'(\overline{q}) - \frac{\Delta_\eta(\underline{R})}{(q^*(R)\xi/\zeta-1)^2}\right)\frac{1}{c''(q^*(R))} < 1-\overline{q},\\
\iff \quad & c''(q^*(R)) > c'(\overline{q})/(1-\overline{q}) - \frac{\Delta_\eta(\underline{R})/(1-\overline{q})}{(q^*(R)\xi/\zeta-1)^2}.
\end{aligned} \tag{C.7}$$

Moreover, observe that $0 < \xi/\zeta = 1 - \frac{1-\mu^0}{\mu^0}\frac{\theta^b}{\theta^g} < 1$ and $\frac{1}{2} < q^*(R) < 1$, thus $0 > q^*(R)\xi/\zeta - 1 > \frac{1}{2}\xi/\zeta - 1$, and consequently

$$(q^*(R)\xi/\zeta-1)^2 < \frac{1}{4}(\xi/\zeta-2)^2 = \frac{1}{4}\left(1+\frac{1-\mu^0}{\mu^0}\frac{\theta^b}{\theta^g}\right)^2 < \frac{1}{4}\left(1+\frac{1-\mu^0}{\mu^0}\right)^2 = \frac{1}{(2\mu^0)^2}.$$

As a result, $-1/(q^*(R)\xi/\zeta-1)^2 < -(2\mu^0)^2$, and eq.(C.7) is thereby implied by

$$c''(q^*(R)) > \frac{c'(\overline{q}) - (2\mu^0)\Delta_\eta(\underline{R})}{1-\overline{q}},$$

which is precisely $\mathfrak{C}^2$ in part (*c*) of Assumption 4. *Q.E.D.*

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