A Computational Model of Conflict and Cooperation*

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Abstract

This paper considers the problem of 2-player conflict, which arises due to competition over resources. Each player represents a certain country and has some total resources/wealth. A player may either attack to get the resources or maintain peace. I present a computational game-theoretic model for such interaction between the players and show some key questions of interest to policy makers that can be answered efficiently. They are: (1) given the resources and the power of each player, is peace a stable situation? and (2) what can influence the likelihood of conflict? I propose one optimization algorithm to compute all the Nash Equilibria in the model and show that the peace efforts(negotiated settlements) may prove futile because one-sided defection is attractive. I also identify some material factors (size of the disputed item and the relative power e.g) and non-material factors(nationalism, audience costs, e.g) that can enter into the conflict/cooperation calculation and thereby influence the likelihood of conflict.

1 Introduction

The emergence of collective behavior and the evolution of cooperation is a fundamental question to any study on organizations or institutions. In the context of inter-state conflict, there has been little research on the origins of conflict from the lens of militarization[1]. When

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[1]In the context of the paper, I call increases in the military spending in response to potential conflicts "militarization".

facing potential conflicts, would countries necessarily militarize? Is cooperation possible under some circumstances? And what are those circumstances under which countries militarize and enter into conflict?

These questions manifest more complexity than a linear relationship that can be addressed by traditional models in econometrics and formal theory: potential conflicts do not have a constant effect on militarization; furthermore, even the direction of the effects need not be the same across all country-dyads. These potential manifestations of nonlinearities in the interrelationship preclude closed-form solutions and predict varying dynamics and multiple patterns, unarguably requiring a method that can sufficiently realize them (Marwala and Lagazio, 2011).

One major contribution of this paper is methodological, showing the advantages of mathematical modeling and its computer-based implementation in examining the issue of conflict and cooperation from a perspective of militarization. Without actually solving for single analytical solutions, this model uses Monte Carlo simulations and the accompanying graphical presentations that are sensitive to initial conditions and various parameters reveal the underlying dynamics in a compelling yet simplistic way.

Substantially, the model generates four findings, which not only help to understand the underlying complexity between the two variables but also suggest new theoretical and empirical terrains: (1) for any country dyad in bargaining over a divisible disputed item, its division will be proportionate to their relative power. (2) with double moral hazards, for any country dyad, their likelihood of conflict can not be eradicated and is a decreasing function of (the lower) gains from bargaining as a fraction of total wealth within the dyad. (3) some material factors like the value of the disputed item and the relative power contribute positively or negatively to this likelihood of conflict. (4) various situational, non-identifiable and non-material factors can also play a part in impacting this likelihood of conflict. The model therefore shows great flexibility in accommodating many of the theoretical claims in one single framework that have been made so far with regards to the onset and even outcome of conflict-to name a few, democratic peace theory, belligerent nationalism, audience costs theory and so forth. Most importantly, the model opens ground for a statistical test of the main theoretical claims that can be used for further work on interstate conflict analysis.

2 Literature Review

The closest literature to the theme of this paper would be the study by Nordhaus, Russett and Oneal on the effects of security environment on military spending (Nordhaus, Oneal and Russett, 2012). They first estimated the ex ante probability that a country will become involved in a fatal militarized interstate dispute using a model of dyadic conflict that incorporates key elements of liberal and realist theories of international relation. In a panel of 165 countries from 1950 to 2000, they found the prospectively generated estimate of the external threat to be a powerful variable in explaining military spending: a 1 percentage point increase in the aggregate probability of a fatal militarized dispute, as predicted by their liberal-realist model, leads to a 3 percent increase in a country's military expenditures.

Except for their study, few have extensively explored the relations between militarization and conflict. However, there has accumulated a vast literature on determinants of conflict in rationalist theories of international security and political economy. The expected utility theory constitutes the earliest attempt to rationalize and scientize the theory of conflict. Its main tenet is that decision-makers consider war and peace as the options of achieving desirable goals and choose to enter into conflict on the basis of the expected cost-benefit analysis (de Mesquita, 1980; De Mesquita and Lalman, 1986; Morrow, 1985, 1986).

In recent years, the rationalist school of conflict has undergone a sharp turn to bargaining theory(Chatterjee and Samuelson, 1987; Garfinkel and Skaperdas, 2000; Fearon, 1995; Fey and Ramsay, 2011; Filson and Werner, 2002; Leventoğlu and Tarar, 2008; Meirowitz and Sartori, 2008; Morrow, 1989; Powell, 2004; Skaperdas, 2006; Slantchev, 2003; Smith and Stam, 2004, 2006).Compared with expected utility theory, bargaining theory prides itself on two main aspects: first, it gets rid of the overly unrealistic and simplistic assumption of "fight to the finish" underlying the expected utility theory and can explain less extreme outcomes. The problem for the "fight to finish" assumption is that empirically very few conflicts end with the total defeat of one side; instead, the vast majority of conflicts end before one side is vanquished and even when both sides are still capable of fighting(Filson and Werner, 2002; Smith and Stam, 2004). This assumption excludes a priori any possibility that conflicts end short of a decisive military victory, while the bargaining theory models exactly the scenario which allows the existence of a negotiated deal acceptable to both sides, especially as the conflict draws on and one side's resources are being depleted(Filson and Werner, 2002; Smith and Stam, 2004); second, it again extends the empirical terrain by modeling not just the onset but also duration and outcome of conflict.(Filson and Werner, 2002)

The bargaining theory has two main explanations for conflict: the first argument is generalized as "bounded rationality and no common priors". Leaders may be unable to locate a mutually preferable negotiated settlement due to information asymmetry in terms of relative capabilities, resolve and incentives. Conflict could ensue out of misrepresentation of information (Chatterjee and Samuelson, 1987; Garfinkel and Skaperdas, 2000; Fearon, 1995), or out of overestimation of one's own strength or underestimation of the opponent's strength under mutual optimism(Fey and Ramsay, 2011).

The second argument is "commitment failure"(Fearon, 1995), which can explain both the onsets and duration and outcome of conflict. For conflict onset, rationally led states may be unable to arrange a settlement that both would prefer to war because one or more states would have incentives to renege on them. For conflict duration, communicational failures during bargaining within conflict influence the duration of conflict by prolonging the peace process. For conflict outcome, post-conflict bargaining mostly determines the way the conflict ends and could even breed a second conflict under certain circumstance. (Filson and Werner, 2002; Smith and Stam, 2004).

Recently Meirowitz and Sartori also proposed a third explanation—"strategic uncertainty" as a cause of conflict(Meirowitz and Sartori, 2008). They show that states have incentives to keep each other guessing about their exact levels of military capacity, even though doing so creates the risk of war. In other words, states even deliberately create information asymmetries that lead to war.

Regarding the duration and outcome of conflict, in a similar vein with Fearon(Fearon, 1995), Leventoglu and Slantchev provided a "armed peace" argument that though there exists specific windows of opportunity for players to terminate the war, the desirability of peace creates a commitment problem that undermines the likelihood of peace and prolongs fighting(Leventoğlu and Tarar, 2008). In a separate work, Slantchev argued that learning about each other's capabilities and resolve occurs when information is revealed by strategically manipulable negotiation and nonmanipulable battlefield outcomes(Slantchev, 2003); so warfare only ceases to be useful when it loses such informational content. For example, if one side would expect to have total victory or defeat, the information about relative capabilities and/or resolve ceases to be of any value and it is highly likely that war would end.

Despite the important contributions to the theory of conflict, the current international security literature has not satisfactorily unified militarization and conflict in one single framework. One reason for the deficiency in relevant literature might be that mainstream international security scholars have rarely situated the question of conflict in a resource-allocation context—namely, the trade-off among various usages (for example, guns or butter) to cope with potential conflicts. However, this context is necessary for exploring the determinants of military spending. Besides, it is also necessary for rigorously unifying militarization and conflict.

The microeconomic theory of conflict and cooperation (Skaperdas, 2006; Hirshleifer, 1988, 1994; Grossman and Kim, 1995) has successfully employed this resource allocation framework. It understands conflict as a universal human behavior and assumes that generally, individuals, groups, nations or any collectivity consisting of rational and self-interested entities, will allocate their resources for survival between two kinds of means of generating benefits or "income" for themselves: (1) production and exchange, which is peaceful; (2)"appropriative" acts, which are about plundering resources previously controlled by others (or to defend against such invasions), which is not peaceful (Hirshleifer, 1988). Whenever agents feel it optimal to invest in appropriative acts, conflict would probably arise.

There still exists a problem with the current microeconomic theory of conflict, which lays much emphasis on linear models and analytical solutions but leaves the innate complexity and nonlinearities insufficiently modeled. Notably, some scholars tried to extend the static microeconomic model of conflict to a dynamic one within a differential game setting and models the behaviors of agents facing a temporal trade-off between socially productive activities and appropriation(Eggert, Itaya and Mino, 2008). Instead, I argue that without going to the level of dynamic games, the computational approach illustrated in the paper is sufficient for uncovering the complexity that is much needed to capture real-world dynamics.

For uncovering the origins of conflict from militarization efforts of country dyads, I have developed and tested one computational model. Although computational models are not a panacea or always appropriate, I believe they would prove to be a useful methodology that has not been sufficiently exploited in the context of interstate conflict analysis.

3 The Workhorse Model

3.1 Model Set-Up

The unified computational model starts with a basic workhorse model of conflict. Here I model the standard and simplest pattern of conflict—a two-side interaction. Conflict of everyone against everyone, where all parties can be assumed to engage in the equivalent of "price-taking" behavior(Hirshleifer, 1988), captures the other extreme on the spectrum of conflict patterns.

Consider the following game. Suppose two countries, one is stronger and the other weaker, dispute over a fixed item Z external to both of them(for example, a territory or an oil reserve).[2], and decide whether to militarize and enter into conflict for it. Note that I specifically assume for this model of conflict that militarization equals actually entering into conflict

Let S_i denote the collection of player i's state in game, C the set of possible actions, and $C(S) \subseteq C$ the set of actions possible at state S. A strategy for player i is a function $s_i : S_i \mapsto A$ such that $s_i(S) \in C(S)$ for all $S \in S_i$.

The mathematical representation of the game is as follows(Mas-Colell et al., 1995):

- A fixed disputed item Z.
- A finite set of players $I = \{1, 2\}$: Country 1 and Country 2.
- A finite set of possible states of game S and a function $h : I \mapsto S$ assigning each state $s \in S$ to the country $i \in I$.
- A finite set of possible actions C and a function $p : S \mapsto C$ assigning each action $c \in C$ to the corresponding state $s \in S$.
- A collection of functions $A = \{\alpha_1(\cdot, \cdot, \cdot, \cdot), \alpha_2(\cdot, \cdot, \cdot, \cdot)\}$ assigning probabilities to the combination of actions and the combination of states, $\alpha_i : S \times S \times C \times C \mapsto [0, 1]$.
- A collection of payoff functions $U = \{u_1(\cdot), u_2(\cdot)\}$ assigning utilities for the players for each state that can be reached, $u_i : S \mapsto \mathbb{R}$. We take each $u_i(\cdot)$ to be a Bernoulli utility function.

[2] If we assume instead this fixed item is an integral part of one country, say the weaker country, the conclusions of the model will not change.

Thus, formally, the game is specified by the collection $\Gamma_1 = \{Z, I, S, C, h(\cdot), p(\cdot), A, U\}$

Definition 1. Let $Y, y \in S$; $F, f \in C$. The states are Country 1 and 2's total wealth, denoted as Y and y. And the actions are Country 1 and 2's militarization efforts, denoted as F and f.

Definition 2. *Contest success function*: Let $\alpha_1(Y, y, F, f) = \frac{F}{F+f}, \alpha_2(Y, y, F, f) = \frac{f}{F+f}$ be the probability of victory for each side.

The contest success function models the probability of winning and losing for each player. It has been used widely for modeling the relative success of countries in conflicts and has many forms. Its usage avoids a deterministic logic of winning and losing but instead reveals the innate uncertainties within conflict, which could even reverse the dominant advantages of one side. In this paper the linear form, $\frac{F}{F+f}$, is adopted.

Example 1. The most commonly used ones are two: one linear ($\frac{F}{F+f}$), and the other exponential ($\frac{e^F}{e^F+e^f}$)(Skaperdas, 1996). The linear form is relatively smooth while the exponential form could show more drastic patterns (see below).

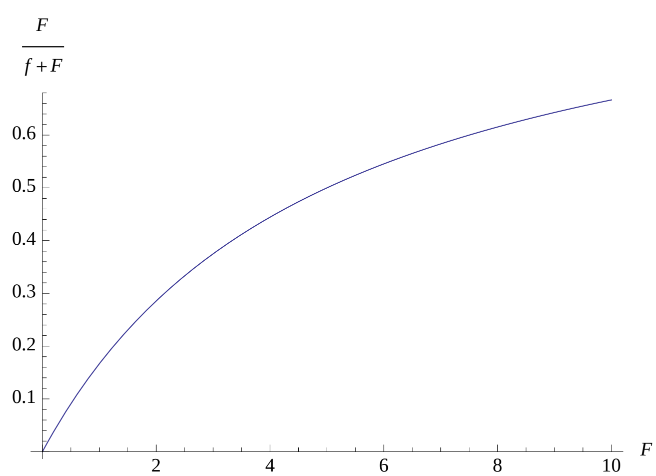

Figure 1: Linear Contest Success Function

Assumption 1. If either side decides to militarize, in other words, F(f) > 0, conflict would occur.

Though seemingly unrealistic because militarization would not readily be translated into conflict, it is a straightforward way of modeling the decision of entering into conflict. Besides,

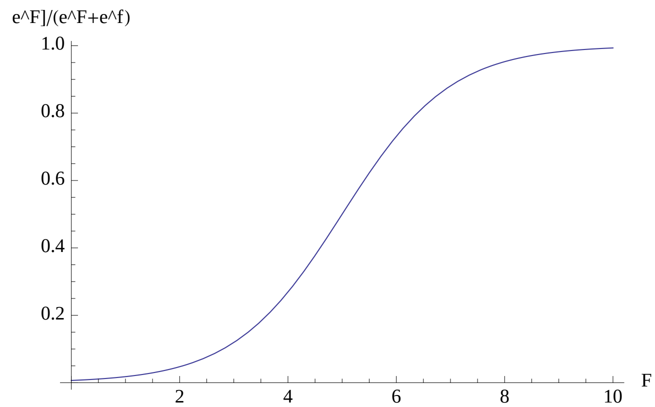

Figure 2: Nonlinear Contest Success Function

this is a model of conflict, which will be complicated in later sections with this assumption relaxed.

Assumption 2. Both countries maximize the expected utilities of entering into conflict, which are weighted by the probability of winning and losing(the contest success function).

Though people may challenge this assumption by arguing that the sides might just be maximizing the wealth of a certain class or coalition within the society, this assumption again directly models the rational behavior in conflict: both sides will determine how much *more* military expenditure is necessary to be expended than in the peace condition in order to secure a successful outcome, while directing the rest of their resources towards other needs.

Assumption 3. Both countries are risk averse.

Mathematically, this assumption means the expected utility function will be concave. A form of utility function $u_i(x) = \frac{x^{1-\theta}}{1-\theta}$ helps to model the expected utility of both sides(where x is the residuals in total resources) [3].

The enumerated assumptions above lead to the details of the model and the equilibrium analysis in the following section:

3.2 Equilibrium Analysis: Monte Carlo Simulation

The payoff structure is as below for the plunder game:

[3]For simplicity, with θ set to be 2 here, the utility function is log-form and the actors are thereby risk-averse

Table 1: Payoff Structure

	x(winning)	x(losing)
$Country1$	$Y - F + Z$	$Y - F$
$Country2$	$y - f + Z$	$y - f$

On the basis of the assumptions suggested previously, the expected utility of entering into the conflict for both sides is as below.

$$\begin{aligned} U1(Y, y, F, f) &= \frac{F}{F+f}\log(Y - F + Z) + \frac{f}{F+f}\log(Y - F) \\ U2(Y, y, F, f) &= \frac{f}{F+f}\log(y - f + Z) + \frac{F}{F+f}\log(y - f) \end{aligned}$$

Both sides simultaneously decide upon military spending and come to an equilibrium. The solution concept would be Nash Equilibrium. While the analytical solutions are hard to derive given the log forms of utility functions, they are free of corner solutions which are prevalent in linear-form utility functions(at least one side would possibly invest all of the total resources on conflicts), and are able to yield the same qualitative predictions as the linear form.

Algorithm 1: Finding F and f

Data: Y, y
Result: F, f
begin
  $F \leftarrow 0.1Y$;
  $f \leftarrow 0.1y$;
  repeat
    $F' \leftarrow F$;
    $f' \leftarrow f$;
    $F \leftarrow \text{NArgMax}(u_1(Y, y, F, f'), F)$;
    $f \leftarrow \text{NArgMax}(u_2(Y, y, F', f), f)$;
  until $|F - F'| < \epsilon \wedge |f - f'| < \epsilon$;

Monte Carlo simulations generate many pair-wise combinations of Y and y within the range of $(0, 8]$ (holding $y \leq Y$). Given the extreme complexity of the functional form, numerical computation is adopted to solve for the Nash Equilibriums for each pair(country dyad).

Theoretically, there will be four kinds of Nash Equilibriums with the features as follows, constituting the substance of this model of conflict:

1. $(F^*, f^*) = (0, 0)$. Both find it optimal not to militarize(enter into conflict) given the likelihood of conflict. The combinations of these equilibriums would constitute what I call here "the peace region."

2. $(F^*, f^*) = (+, +)$. Both would have incentives to enter into conflict.

3. $(F^*, f^*) = (0, +)$. The weaker side prefers conflict while the stronger one does not.

4. $(F^*, f^*) = (+, 0)$. The stronger side prefers conflict while the weaker one does not.

Situation 2, 3 and 4 will constitute "the conflict region."

On the basis of the typology of the Nash Equilibriums enumerated above I graph the conflict/peace regions of the four situations as below. The computation shows that of all the Nash Equilibriums, the $(0, +)$ and $(+, 0)$ cases do not exist.

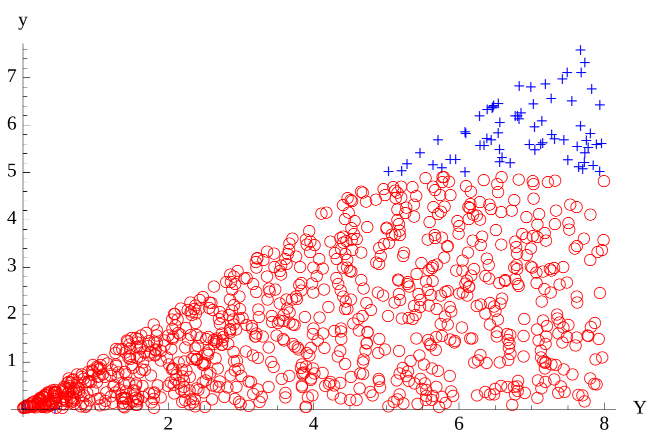

Figure 3: Conflict and Peace Region: $Z = 0.05$
The blue region ('+') denotes the "peace region" where the optimal military spendings for both will be exactly 0 and neither has any incentives to arm, while the red region ('$\circ$') captures the case in which both have incentives to militarize and conflict ensues. The optimal military spendings thereby will be $(+, +)$.

3.3 Comparative Statics: What Influences Conflict Onset

Since both F and f are endogenously determined given the fixed values of Y and y, we need to determine if the parameters in the model, Z(the size of the fixed dispute item), could influence militarization for certain country dyads and thereby "shape" the relative sizes of the "peace" and "conflict" regions.

Proposition 1. The value of the disputed item varies positively with the size of the conflict region.

Graphical representation: Varying Z could impact the relative possibilities of peace and conflict — as Z becomes higher, which means the potential gains from gaining the disputed item go down, the conflict region shrinks while the peace region expands.[4]

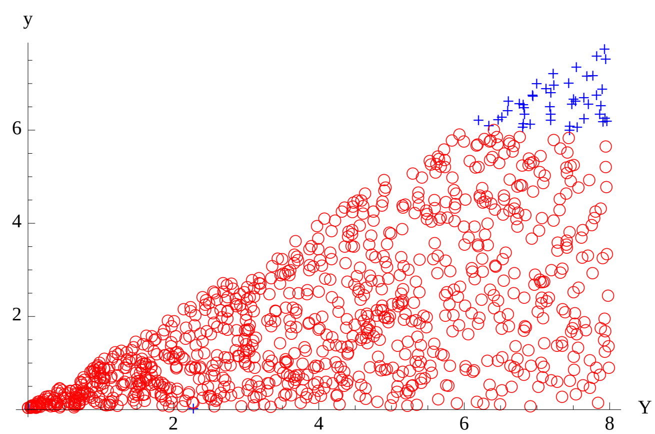

Figure 4: Conflict and Peace Region: Z=.06

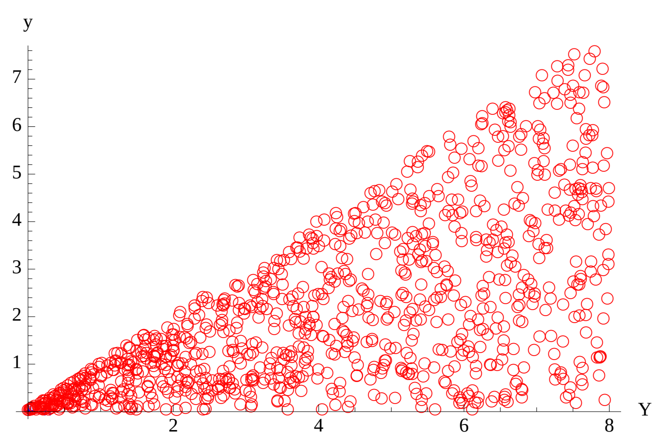

Figure 5: Conflict and Peace Region: Z=.1

[4]Please refer to Table 2, 3 and 4 for optimal F and f for selected dyads.

Proof: In the extreme case in which $Z = 0$ and the potential gains will be 0, the expected utility function, u_1 for example, will be reduced to $\log(Y - F)$. It is obvious to see the optimal F^* and f^* will be exactly 0 and peace will prevail. Generalizing this finding, the less valuable the disputed item, the greater the possibilities of bilateral peace. A more intuitive way to think about this would be that in the small Z case, both sides would see winning as not so desirable in the sense that the gains from winning might not be even enough to compensate the militarization costs. In this case cooperation would be preferred to conflict, which is the reason why a much more expanded peace region is observed in the small Z case. Conversely, a larger Z would elicit more aggressive behaviors.□

Graphical representation:

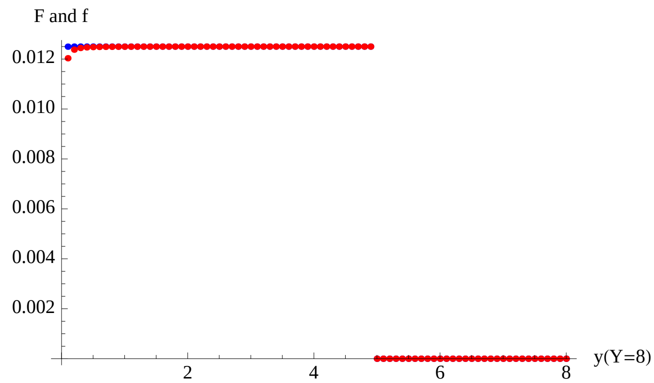

Figure 6: Optimal $F^* and f^* : Z = .05, y < Y = 8$

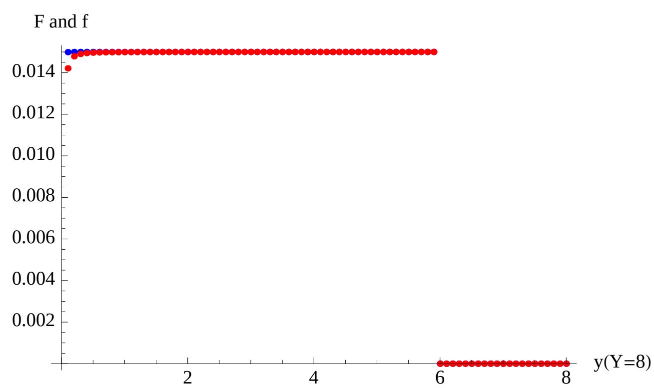

Figure 7: Optimal $F^* and f^* : Z = .06, y < Y = 8$

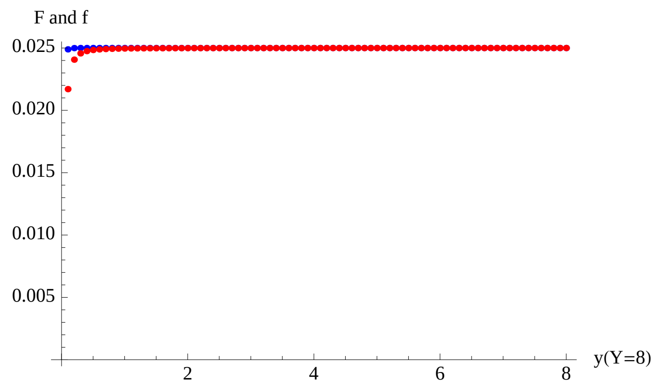

Figure 8: Optimal $F^* and f^* : Z = .1, y < Y = 8$

4 Extension of the Workhorse Model: From Conflict to Nash Bargaining

4.1 Model Set-Up

The graphical presentation of the conflict and peace region in the basic workhorse model is not convincing, because we do not observe as many conflicts in real world at a given time point. This raises a related question: since the dispute can be costly and inefficient, why should the countries not strike a negotiated settlement to avoid the conflict? This leads to an extension of the above model of conflict that incorporates the pre-conflict bargaining scenarios.

Consider the new game: suppose the disputed item is divisible. On account of the expected payoffs of entering into conflict calculated in the workhorse model, both players would engage in a bargaining process over the division of this external disputed item—a certain proportion would go to one country while the rest goes to the other.

Let d be the threat point, the value the players can expect to receive if negotiations break down. Let S be a feasible set, a closed convex subset of $\mathbb{R}^2$, the elements of which are interpreted as agreements. Set S is convex because an agreement could take the form of a correlated combination of other agreements.

The mathematical representation of the bargaining game is below:

- A set of players I = {1, 2}
- A set of utilities given by the threat point d.

- A set of utilities given by bargaining outcomes S compact and convex in $\mathbb{R}^2$, $B = \{B_1(\cdot), B_2(\cdot)\}$.
- A Nash bargaining problem (S, d).
- A function M that satisfies the six axioms of Nash Bargaining and gives a $s \in S$ such that $S > d$.

Thus, formally, the game is specified by the collection $\Gamma_2 = \{Z, I, S, d, U, M\}$

Definition 3. Let the utilities given by the threat point d be $U = \{u_1(\cdot), u_2(\cdot)\}$ solved previously from the model of conflict.

Definition 4. Let the bargaining outcome S be a set of the incidence of the disputed item, $S = \{S_1, S_2\} = \{\lambda, (1-\lambda)\}$. By this definition, λZ goes to Country 1 while $(1-\lambda)Z$ is for Country 2.

Assumption 4. Both countries bargain for an incidence of the disputed item, which gives utilities larger than those obtained under the threat point.

4.2 Equilibrium Analysis: Monte Carlo Simulations

In this game,

$$\begin{aligned}
u_1(Y, y, F, f) &= \frac{F}{F+f}\log(Y - F + Z) + \frac{f}{F+f}\log(Y - F) \\
u_2(Y, y, F, f) &= \frac{f}{F+f}\log(y - f + Z) + \frac{F}{F+f}\log(y - f) \\
B_1(Y, y, F, f) &= \log(Y + \lambda Z) \\
B_2(Y, y, F, f) &= \log(y + (1-\lambda)Z)
\end{aligned}$$

The Nash Product is below:

$$M(\lambda, Y, y, F, f) = [B_1(Y, y, F, f) - u_1(Y, y, F, f)][B_2(Y, y, F, f) - u_2(Y, y, F, f)]$$

Proposition 2. With both being risk-averse, both sides will always bargain instead of militarizing and entering into conflict.

Proof: Given the concavity of $u_1(Y, y, F, f)$,

Algorithm 2: Finding λ

```
Data: Y, y
Result: F, f, λ
begin
    F ← 0.1Y;
    f ← 0.1y;
    repeat
        F' ← F;
        f' ← f;
        F ← NArgMax(u1(Y, y, F, f'), F);
        f ← NArgMax(u2(Y, y, F', f), f);
    until |F − F'| < ϵ ∧ |f − f'| < ϵ;
    λ ← NArgMax(M(λ, Y, y, F, f), λ);
```

$u_1(Y, y, F, f) = \frac{F}{F+f}\log(Y-F+Z)+\frac{f}{F+f}\log(Y-F) \leq \log[\frac{F}{F+f}(Y-F+Z)+\frac{f}{F+f}(Y-F)] = \log[Y-F+\frac{F}{F+f}Z]$

$\exists\lambda = \frac{F}{F+f} \in [0,1]$ s.t $u_1(Y, y, F, f) \leq log(Y-F+\lambda Z) \leq log(Y+\lambda Z)$,

where the equality holds iff $F = 0$.

Symmetrically, $u_2(Y, y, F, f) = \frac{f}{F+f}\log(y-f+Z)+\frac{F}{F+f}\log(y-f) \leq \log[\frac{f}{F+f}(y-f+Z)+\frac{F}{F+f}(y-f)] = \log[y-f+\frac{f}{F+f}Z]$

Note λZ will be given to the stronger side while $(1-\lambda)Z$ will go to the weaker side.

So $\exists\lambda = \frac{F}{F+f} \in [0,1]$ s.t

$u_1(Y, y, F, f) \leq log(Y-F+\lambda Z) \leq log(Y+\lambda Z)$, $u_2(Y, y, F, f) \leq log[y-f+(1-\lambda)Z] \leq log[y+(1-\lambda)Z]$,

where the where the equality holds iff $F = 0$.

We thus conclude that the risk-averse dyad will always bargaining for incidence of the item instead of entering into conflict. □

Proposition 3. The stronger side in bargaining always gets the bigger share of the disputed item.

Corollary 1. The relative proportion of incidence $\frac{\lambda}{1-\lambda}$ varies positively with their relative power(Y/y).[5]

Please see Figure 9 below for evidences of Corollary 1 and Proposition 3.

[5] Please refer to Table 5 for $(\lambda, 1-\lambda)$ for selected dyads.

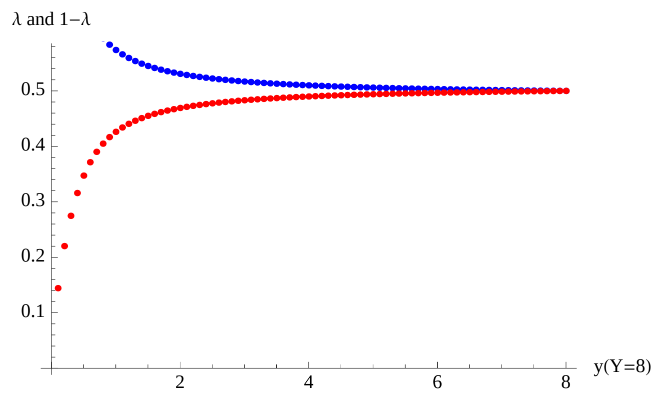

Figure 9: Bargaining Outcome: Z = 1, y < Y = 8

Proposition 4. When binding contracts are unavailable, there always exists the moral hazard issue where the players can defect, which means that bargaining cannot eradicate the possibility of conflict.

Proof. One-sided defection for Country 1 is profitable since it would only incur minor military spending to obtain Z since Country 2 has not militarized: $Y - \varepsilon + Z > Y + \lambda Z$

One-sided defection for Country 2 is also profitable since it would only incur minor military spending to obtain Z since Country 1 has not militarized: $y - \varepsilon + Z > y + (1 - \lambda)Z$

So one-sided defection dominates striking a negotiation for either side. The bargaining equilibrium is not stable. □

And from the above table, the bargaining model indicates gains from bargaining for the stronger side are not large(Gains1 < Gains2). Then we can hypothesize that it is more likely for the stronger country to defect than for its weaker counterpart.

5 What Influences the Likelihood of Conflict?

Proposition 4 indicates that even though for any given dyad the choice of bargaining for an incidence of the disputed item Pareto-dominates the choice of entering into conflict, the issue of double moral hazards is considerable. Double moral hazards would arise especially when the effort levels of the principal and the agent(specifically in my model, the two sides after bargaining) are not fully transparent to each other due to informational asymmetries. In other words, neither side can observe or verify the other's activities and a Cournot-Nash type of non-cooperative behavior emerges. So possibilities of conflict are not eradicated as a result of the negotiated settlement; instead, they always exist because one-sided defection is highly likely.

Given this likelihood of conflict which is actually endogenous to the extended workhorse model of conflict and cooperation, we now need to analyze its components. The basic idea is that the lower the gains from bargaining, the more likely various kinds of observable or unobservable "costs" from non-fighting will reverse the current situation, thereby making fighting even desirable. I will show below some of those identifiable and material factors and propose several theoretical postulations for the non-identifiable and non-material influences on the likelihood of conflict. Given data availabilities for both the dependent variable(conflict onset) and even some independent variables, this section would pave way for a computable general equilibrium(CGE) model that can eventually be used for empirical testing of the main argument on the determinants of conflict initiation.

Claim 1. The likelihood of conflict is a decreasing function of (the lower) gains from bargaining as a fraction of total wealth within the dyad.

Y	y	F	f	v1	v2	λ	Gain1	Gain2
5.09087	4.33501	0.248564	0.247983	1.67137	1.51713	0.50277	0.0502622	0.0581826
0.0694552	0.0373712	0.0512233	0.0186856	-1.05713	-2.91126	0.700831	0.796133	1.82222
4.84971	3.3812	0.24848	0.246709	1.62501	1.28042	0.507292	0.0533979	0.0738433
0.957245	0.923302	0.222244	0.219618	0.124114	0.0880568	0.503658	0.25494	0.26235
4.42368	3.01779	0.248208	0.245909	1.53708	1.17283	0.508619	0.0587269	0.0825462
7.98112	0.746386	0.24782	0.196727	2.11333	-0.139775	0.60255	0.0365337	0.274162
7.28814	3.20781	0.249359	0.246277	2.01827	1.23026	0.514128	0.0361451	0.0763596
8.65438	8.46797	0.249475	0.249451	2.18503	2.1638	0.500202	0.0292284	0.0298373
3.38425	0.224515	0.208861	0.0783788	1.3545	-1.36121	0.75908	0.0669974	0.596431
5.9383	2.04384	0.249174	0.241144	1.82085	0.80632	0.526434	0.0455159	0.11691
8.29339	1.78154	0.249633	0.23843	2.14483	0.677862	0.536288	0.0332882	0.130957
3.44367	1.78251	0.247458	0.238783	1.3005	0.679461	0.522749	0.077361	0.135799
4.9116	4.13349	0.248466	0.247789	1.63696	1.47165	0.503105	0.0521534	0.0609882
1.73185	0.684579	0.243222	0.190816	0.685815	-0.21903	0.579945	0.152208	0.318544
4.34758	3.2329	0.248114	0.246435	1.52037	1.23801	0.506481	0.0594489	0.0774405

Figure 10: Gains as a Fraction of Total Wealth for Randomized Dyads: Z = 1

The table above shows that the gains from bargaining vary within the dyad as well as across the dyads. Since empirically war is often initiated by a challenger against a target, we can further postulate that the side with relatively lower gains from bargaining (as proven before, the side with lower gains from bargaining is the stronger country) is more likely than its opponent in committing the one-sided defection and initiating the conflict. Given this empirical regularity, we postulate that the likelihood of conflict takes the form as shown below in Claim 2:

Claim 2. The likelihood of conflict $Pr(Conflict) = 1 - \frac{min(\frac{e^{B_1-U_1}}{Y}, \frac{e^{B_2-U_2}}{y})}{\frac{e^{B_1-U_1}}{Y} + \frac{e^{B_2-U_2}}{y}}$, where $\frac{e^{B_1-U_1}}{Y}$ denotes the gains from bargaining as a fraction of the total wealth for the stronger country.(See the table below) [6]

Y	y	F	f	v1	v2	λ	LikelihoodOfConflict
2.85084	0.0626856	0.122688	0.017729	1.27648	-2.70484	0.891202	0.633759
4.11156	3.65799	0.247841	0.247227	1.46686	1.35533	0.50246	0.741164
2.48893	1.91569	0.244797	0.240602	0.994175	0.747902	0.510202	0.55412
4.59704	2.95792	0.248353	0.245732	1.57391	1.15386	0.509863	0.769783
8.59494	4.2711	0.249511	0.247865	2.17849	1.50271	0.509453	0.880077
5.57714	2.20515	0.249015	0.242362	1.76017	0.877462	0.522553	0.811867
6.52559	3.78148	0.249156	0.247315	1.91099	1.3866	0.508982	0.840528
4.6116	2.99384	0.248359	0.245832	1.57692	1.16526	0.509583	0.770559
4.75259	4.3091	0.248352	0.247971	1.60535	1.51144	0.501754	0.777978
6.04399	2.2578	0.249165	0.242682	1.83761	0.899503	0.522767	0.827034
6.97911	3.53998	0.249278	0.246935	1.97615	1.32378	0.511249	0.851244
6.91407	2.78842	0.249317	0.245117	1.96733	1.09785	0.517401	0.849706
1.21472	0.715512	0.235739	0.19743	0.361796	-0.167626	0.553952	-0.0140157

Figure 11: The Likelihood of Conflict for Randomized Dyads: Z = 1

Lemma 1. For all given country dyads, the likelihood of conflict $Pr(Conflict)$ is almost independent of(or varies negatively with, though the effects are small) the value of the disputed item, Z.[7]

Proof: Table 8 and 9 which shows the probability of conflict for some selected dyads generated by Monte Carlo Simulations in the appendix can prove the proposition above.

Also comparing the graphs below which are exemplar of the many simulations from varying the value of Z, we can gain an understanding into how the value of the disputed item could impact the likelihood of conflict.

The simulations indicate that the likelihood of conflict $Pr(Conflict)$ varies negatively with the value of the disputed item, Z.

Increases in Z lead to an enlarged lower gains from bargaining, which means the stronger side

[6]Please take Table 6 as reference for the gains from bargaining as a fraction of total wealth for selected dyads.
[7]Please take Table 7, 8 and 9 as reference for every $Pr(Conflict)$ under selected dyads.

will find the negotiated settlement more desirable than entering into conflict.

Since the stronger side is always the one with the lower gains from bargaining and the higher likelihood of committing one-sided defection, $Pr(Conflict)$ is thereby reduced. □

Graphical representation:

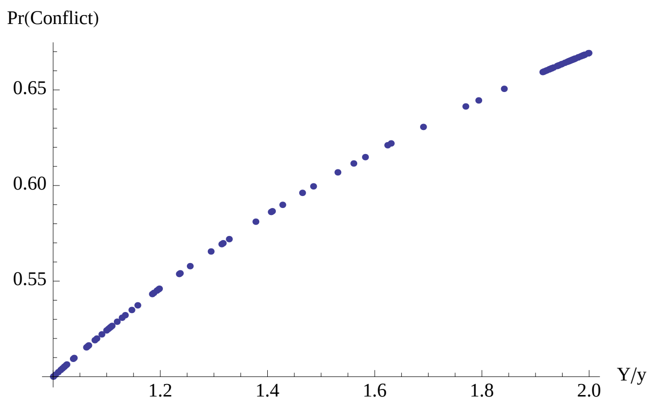

Figure 12: Pr(Conflict): Lower Z(Z $= \frac{y}{10}$), y $<$ Y $= 8$

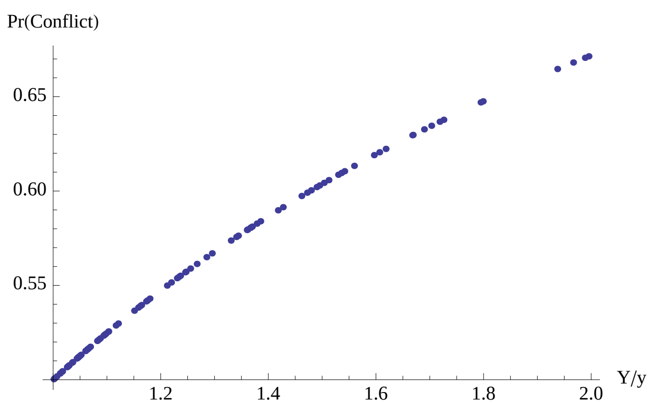

Figure 13: Pr(Conflict): Higher Z(Z $= \frac{y}{5}$), y $<$ Y $= 8$

Lemma 2. Given a fixed disputed item, the likelihood of conflict $Pr(Conflict)$ varies positively with the relative power($\frac{Y}{y}$). In other words, holding all the other factors equal, the more power preponderance the stronger country has, the more likely the conflict.

Proof: From the graphs below, we can see as $\frac{Y}{y}$ goes down, the of probability of conflict also goes down.

On the basis of the mathematical proof previously sketched, the stronger side would have a marginally lower probability of winning[8] and relative fewer gains from entering into conflict than from in the case of a higher relative power ratio, which makes bargaining in the lower relative power ratio case more desirable. Given our postulated form of the likelihood of conflict, $Pr(Conflict)$ is therefore reduced. □

Graphical representation:

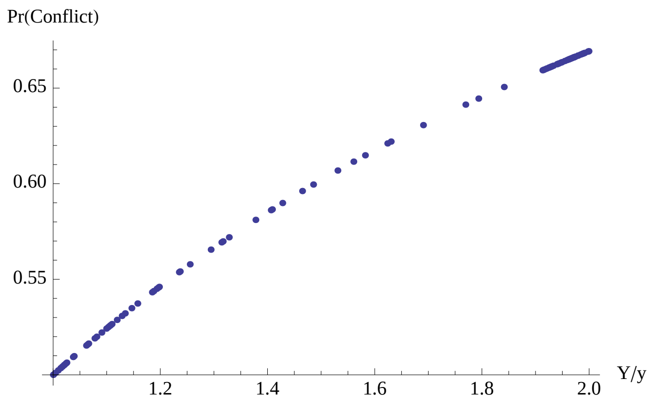

Figure 14: Pr(Conflict) and $\frac{Y}{y}$, y < Y = 8, $Z = \frac{y}{10}$

The above analysis on the influences of the material factors such as the fixed disputed item Z and the relative power on the likelihood of conflict is by no means complete in explaining conflict onset. Inevitably some non-material factors, many of which are situational on a case-by-case basis, and even external shock to the interaction could also play a part in the scenario.

Some salient non-material variables can also be used for explaining conflict onset. In the international relations literature, many scholars have suggested theoretical mechanisms operating behind the unobservable factors. One is democratic peace. For example, contemporary scholars have found there are several empirical regularities regarding the relationship between democracy and war-proneness: for instance, first, democracies do not generally fight with each other (Bremer, 1993; Maoz and Russett, 1993; Oneal and Russet, 1997); second, democracies are not immune from wars with nondemcoracies (Maoz and Abdolali, 1989); third, democracies tend to win a disproportionate share of the wars (Lake, 1992; Reiter and Stam III, 1998); fourth, democracies

[8] The contest function shows that the relative probability of winning for each side is a function of the relative conflict investments, which are directly determined by the relative power. And the likelihood of winning for both will be exactly $\frac{1}{2}$ if both sides are equal in power.

are more likely to initiate wars against autocracies than are autocracies against democracies (Oneal and Russet, 1997; Bennett and Stam, 1998).

Relatedly, another could be the impacts of audience costs. For instance, political leaders' political survival in some undemocratic countries may not face the electoral constraints that many of the leaders in democratic countries do. So for autocratic regimes, entering into conflict will not generate much audience costs, which can jeopardize the leaders' political future(Fearon, 1994; Schultz, 1999; Weeks, 2008, 2012). As a result, whimsies of the political leaders could matter more than they should in the political calculation, which will contribute to an enlarged likelihood of conflict.

What can be highly possible is that nationalism can enter into the calculation, carefully controlled and manipulated by the political leaders to serve their own ends. The theories on the impacts of radical or belligerent nationalism on war fighting can just be cases in point: when the constraints on mobilization are political, not material, authoritarian countries would have more advantages exactly because such constraints are relatively less binding for them. Consider the US-Vietnam War. The maximum number of U.S. troops in Vietnam in 1968 amounted to less than one quarter of one percent of the American population while Vietnam mobilized most of its civilians into the war(Mack, 1975). Conflicts tend to generate within democratic countries the kind of political constraints operating against full mobilization of the forces in the society, while the autocratic regimes are more adroit in manipulating the tools of belligerent nationalism(Mack, 1975).

One important non-material factor is strategic miscalculation during the interaction. One example is provided by de Figueiredo, Rackove and Weingast(de Figueiredo Jr, Rakove and Weingast, 2006; Coe, 2012). Undeniably humans have imperfect foresight and inaccurate understandings of the world within which they act; however, they argue instead how it could be rational for each side to pursue certain behavior, though when agents have incomplete information about the others they are interacting with. As a result, both will be surprised by the other's irrational reaction and still believe they are acting reasonably while the other side is acting unreasonably (de Figueiredo Jr, Rakove and Weingast, 2006; Coe, 2012). For example, preceding the American Revolution, the colonists were unaware that Britain's philosophy in 1765 had shifted from imperial federalism to absolute parliamentary sovereignty over the 18th century. And as far as the common interest of opposing France was concerned, Britain treated the colonists lightly. The colonists thus believed that they were playing a kind of repeated prisoner's dilemma with Britain, in which mutual cooperation would only be made possible by both the colonists' loyalty to the empire and Britain's non-intervention in their internal affairs. When the French threat was removed, Britain's interests changed so it imposed what it still saw as cooperative taxes. This, however, led the colonists to incorrectly believe that Britain still supports imperial federalism and view this as an one-sided defection. So they reciprocated with defection by declaring independence. Unaware of the false beliefs held by the colonists and surprised by their defection, Britain responded with real defection, which was oppression(de Figueiredo Jr, Rakove and Weingast, 2006; Coe, 2012).

Even external shock to the game can reverse the situation. It could be the role of allies. For example, Olson and Zeckhauser(Olson and Zeckhauser, 1966) indicated that alliances coordinate offensive policies in the alliance relationship. With the military assistance from the allies, entering into conflict without incurring too many costs upon itself, in certain circumstances, would be a wise choice. Or it could be many situational factors like the climate or geography of fighting that could reverse the desirability of peace over conflict.

The above discussion paves way for the final claim of this paper, which is a statistical formulation of the theoretical model in the paper. This opens a new avenue for future research on calibrating the parameters in the statistical model below with real data and testing the theoretical claims made so far from the computational model.

Claim 3. $Pr(Conflict) = \alpha_0 + \alpha_1 Z + \alpha_2 \log(\frac{Y}{y}) + \beta X + \epsilon$,
where Z denotes the value of the item under dispute, $\log(\frac{Y}{y})$ the relative power, X the list of non-material factors influencing conflict onset and ϵ the external shock to the game; α_0, α_1, α_2 and β are parameters to be calibrated with statistical data.

Concluding this part, success of bargaining may not mean peace; the likelihood of conflict depends on first and foremost on the relative desirability of conflict over peace (proxied by the value of the disputed item Z) and then on the relative power within the dyad and many other complicated factors in specific situations.

6 Conclusion

In this paper, I presented a game-theoretic model for 2-player conflict driven by competition over resources, which is a combination of the resource-allocation framework with the bargaining framework. In order to predict the outcome, I also propose an optimization algorithm to compute all the Nash Equilibria. In particular, I showed that the model can be used to answer the questions such as: whether some peace efforts, such as negotiated settlement, will help to maintain peace, and what can influence the likelihood of conflict. The findings the model generates that can answer the questions efficiently are: (1) the peace efforts(negotiated settlements) may prove futile because one-sided defection is attractive. (2) some material factors (size of the disputed item and the relative power e.g) and non-material factors(nationalism, audience costs, e.g) that can enter into the conflict/cooperation calculation and influence the likelihood of conflict.

For future work, there are a number of interesting directions. From an empirical perspective, besides running the algorithm to predict outcomes and conduct comparative statics in different scenarios, it would be interesting to test my theoretical propositions with real-world data(just as mentioned previously). Also, to directly model the strategic miscalculation would be helpful for convincingly accounting for some real world cases, for instance, Hitler's miscalculation of the resolve and the willingness to fight of the Soviet army when invading Stalingrad(Tooze, 2008).

From a game-theory perspective, it would also be interesting to extend the model to a multi-player game, where a myriad of possibilities can further arise. First, coalition formation is possible and can be incorporated in such a multi-player game setting(Shapley, 1971; Rozen, 2012). Improved computational capabilities would facilitate the simulation of multi-actor systems with a sufficiently high degree of complexity. (Epstein and Axtell, 1997; Gaylord and d'Andria, 1998) Second, the interaction can even take place within a nonlinear dynamic system which embraces many other forms of complexity(Scheffran, 2003). Nonlinear dynamical systems, which have been partly transferred to the analysis of some socio-economic interactions(Scheffran, 2003; Weidlich and Huebner, 2008), can also be expected to work reasonably well in modeling conflict and cooperation.

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A Appendix

Table 2: Optimal (F^*, f^*) for selected pairs of (Y, y); $Z = 0.05$

Y	y	F^*	f^*	Y	y	F^*	f^*
8	0.1	0.0124962	0.0120332	8	4.1	0.0124999	0.0124997
8	0.2	0.0124998	0.012377	8	4.2	0.0124999	0.0124997
8	0.3	0.0124999	0.0124443	8	4.3	0.0124999	0.0124997
8	0.4	0.0124999	0.0124684	8	4.4	0.0124999	0.0124997
8	0.5	0.0124999	0.0124797	8	4.5	0.0124999	0.0124997
8	0.6	0.0124999	0.0124858	8	4.6	0.0124999	0.0124998
8	0.7	0.0124999	0.0124896	8	4.7	0.0124999	0.0124998
8	0.8	0.0124999	0.012492	8	4.8	0.0124999	0.0124998
8	0.9	0.0124999	0.0124937	8	4.9	0.0124999	0.0124998
8	1.	0.0124999	0.0124949	8	5.	0	0
8	1.1	0.0124999	0.0124957	8	5.1	0	0
8	1.2	0.0124999	0.0124964	8	5.2	0	0
8	1.3	0.0124999	0.0124969	8	5.3	0	0
8	1.4	0.0124999	0.0124974	8	5.4	0	0
8	1.5	0.0124999	0.0124977	8	5.5	0	0
8	1.6	0.0124999	0.012498	8	5.6	0	0
8	1.7	0.0124999	0.0124982	8	5.7	0	0
8	1.8	0.0124999	0.0124984	8	5.8	0	0
8	1.9	0.0124999	0.0124986	8	5.9	0	0
8	2.	0.0124999	0.0124987	8	6.	0	0
8	2.1	0.0124999	0.0124988	8	6.1	0	0
8	2.2	0.0124999	0.0124989	8	6.2	0	0
8	2.3	0.0124999	0.012499	8	6.3	0	0
8	2.4	0.0124999	0.0124991	8	6.4	0	0
8	2.5	0.0124999	0.0124992	8	6.5	0	0

Table 2: (continued)

Y	y	F^*	f^*	Y	y	F^*	f^*
8	2.6	0.0124999	0.0124992	8	6.6	0	0
8	2.7	0.0124999	0.0124993	8	6.7	0	0
8	2.8	0.0124999	0.0124993	8	6.8	0	0
8	2.9	0.0124999	0.0124994	8	6.9	0	0
8	3.	0.0124999	0.0124994	8	7.	0	0
8	3.1	0.0124999	0.0124995	8	7.1	0	0
8	3.2	0.0124999	0.0124995	8	7.2	0	0
8	3.3	0.0124999	0.0124995	8	7.3	0	0
8	3.4	0.0124999	0.0124996	8	7.4	0	0
8	3.5	0.0124999	0.0124996	8	7.5	0	0
8	3.6	0.0124999	0.0124996	8	7.6	0	0
8	3.7	0.0124999	0.0124996	8	7.7	0	0
8	3.8	0.0124999	0.0124996	8	7.8	0	0
8	3.9	0.0124999	0.0124997	8	7.9	0	0
8	4.	0.0124999	0.0124997	8	8.	0	0

Table 3: Optimal (F^*, f^*) for selected pairs of (Y, y); $Z = 0.06$

Y	y	F^*	f^*	Y	y	F^*	f^*
8	0.1	0.0149907	0.0142109	8	4.1	0.0149999	0.0149995
8	0.2	0.0149995	0.0147897	8	4.2	0.0149999	0.0149995
8	0.3	0.0149999	0.0149045	8	4.3	0.0149999	0.0149995
8	0.4	0.0149999	0.0149457	8	4.4	0.0149999	0.0149995
8	0.5	0.0149999	0.014965	8	4.5	0.0149999	0.0149996
8	0.6	0.0149999	0.0149756	8	4.6	0.0149999	0.0149996
8	0.7	0.0149999	0.014982	8	4.7	0.0149999	0.0149996
8	0.8	0.0149999	0.0149862	8	4.8	0.0149999	0.0149996

Table 3: (continued)

Y	y	F^*	f^*	Y	y	F^*	f^*
8	0.9	0.0149999	0.0149891	8	4.9	0.0149999	0.0149996
8	1.	0.0149999	0.0149911	8	5.	0.0149999	0.0149996
8	1.1	0.0149999	0.0149927	8	5.1	0.0149999	0.0149997
8	1.2	0.0149999	0.0149938	8	5.2	0.0149999	0.0149997
8	1.3	0.0149999	0.0149947	8	5.3	0.0149999	0.0149997
8	1.4	0.0149999	0.0149955	8	5.4	0.0149999	0.0149997
8	1.5	0.0149999	0.014996	8	5.5	0.0149999	0.0149997
8	1.6	0.0149999	0.0149965	8	5.6	0.0149999	0.0149997
8	1.7	0.0149999	0.0149969	8	5.7	0.0149999	0.0149997
8	1.8	0.0149999	0.0149972	8	5.8	0.0149999	0.0149997
8	1.9	0.0149999	0.0149975	8	5.9	0.0149999	0.0149997
8	2.	0.0149999	0.0149978	8	6.	0.	0.
8	2.1	0.0149999	0.014998	8	6.1	0.	0.
8	2.2	0.0149999	0.0149982	8	6.2	0.	0.
8	2.3	0.0149999	0.0149983	8	6.3	0.	0.
8	2.4	0.0149999	0.0149984	8	6.4	0.	0.
8	2.5	0.0149999	0.0149986	8	6.5	0.	0.
8	2.6	0.0149999	0.0149987	8	6.6	0.	0.
8	2.7	0.0149999	0.0149988	8	6.7	0.	0.
8	2.8	0.0149999	0.0149989	8	6.8	0.	0.
8	2.9	0.0149999	0.0149989	8	6.9	0.	0.
8	3.	0.0149999	0.014999	8	7.	0.	0.
8	3.1	0.0149999	0.0149991	8	7.1	0.	0.
8	3.2	0.0149999	0.0149991	8	7.2	0.	0.
8	3.3	0.0149999	0.0149992	8	7.3	0.	0.
8	3.4	0.0149999	0.0149992	8	7.4	0.	0.
8	3.5	0.0149999	0.0149993	8	7.5	0.	0.

Table 3: (continued)

Y	y	F^*	f^*	Y	y	F^*	f^*
8	3.6	0.0149999	0.0149993	8	7.6	0.	0.
8	3.7	0.0149999	0.0149993	8	7.7	0.	0.
8	3.8	0.0149999	0.0149994	8	7.8	0.	0.
8	3.9	0.0149999	0.0149994	8	7.9	0.	0.
8	4.	0.0149999	0.0149994	8	8	0.	0.

Table 4: Optimal (F^*, f^*) for selected pairs of (Y, y); $Z = 0.1$

Y	y	F^*	f^*	Y	y	F^*	f^*
8	0.1	0.0248928	0.0217022	8	4.1	0.0249994	0.0249975
8	0.2	0.0249934	0.0240663	8	4.2	0.0249994	0.0249977
8	0.3	0.0249988	0.0245702	8	4.3	0.0249994	0.0249978
8	0.4	0.0249995	0.024754	8	4.4	0.0249994	0.0249979
8	0.5	0.0249996	0.0248409	8	4.5	0.0249994	0.024998
8	0.6	0.0249996	0.0248887	8	4.6	0.0249994	0.024998
8	0.7	0.0249995	0.0249178	8	4.7	0.0249994	0.0249981
8	0.8	0.0249995	0.0249368	8	4.8	0.0249994	0.0249982
8	0.9	0.0249995	0.0249499	8	4.9	0.0249994	0.0249983
8	1.	0.0249995	0.0249593	8	5.	0.0249994	0.0249983
8	1.1	0.0249994	0.0249663	8	5.1	0.0249994	0.0249984
8	1.2	0.0249994	0.0249717	8	5.2	0.0249994	0.0249985
8	1.3	0.0249994	0.0249758	8	5.3	0.0249994	0.0249985
8	1.4	0.0249994	0.0249791	8	5.4	0.0249994	0.0249986
8	1.5	0.0249994	0.0249818	8	5.5	0.0249994	0.0249986
8	1.6	0.0249994	0.024984	8	5.6	0.0249994	0.0249987
8	1.7	0.0249994	0.0249858	8	5.7	0.0249994	0.0249987
8	1.8	0.0249994	0.0249873	8	5.8	0.0249994	0.0249988

Table 4: (continued)

Y	y	F^*	f^*	Y	y	F^*	f^*
8	1.9	0.0249994	0.0249886	8	5.9	0.0249994	0.0249988
8	2.	0.0249994	0.0249897	8	6.	0.0249994	0.0249989
8	2.1	0.0249994	0.0249907	8	6.1	0.0249994	0.0249989
8	2.2	0.0249994	0.0249915	8	6.2	0.0249994	0.0249989
8	2.3	0.0249994	0.0249922	8	6.3	0.0249994	0.024999
8	2.4	0.0249994	0.0249928	8	6.4	0.0249994	0.024999
8	2.5	0.0249994	0.0249934	8	6.5	0.0249994	0.024999
8	2.6	0.0249994	0.0249939	8	6.6	0.0249994	0.024999
8	2.7	0.0249994	0.0249943	8	6.7	0.0249994	0.0249991
8	2.8	0.0249994	0.0249947	8	6.8	0.0249994	0.0249991
8	2.9	0.0249994	0.0249951	8	6.9	0.0249994	0.0249991
8	3.	0.0249994	0.0249954	8	7.	0.0249994	0.0249992
8	3.1	0.0249994	0.0249957	8	7.1	0.0249994	0.0249992
8	3.2	0.0249994	0.024996	8	7.2	0.0249994	0.0249992
8	3.3	0.0249994	0.0249962	8	7.3	0.0249994	0.0249992
8	3.4	0.0249994	0.0249964	8	7.4	0.0249994	0.0249992
8	3.5	0.0249994	0.0249966	8	7.5	0.0249994	0.0249993
8	3.6	0.0249994	0.0249968	8	7.6	0.0249994	0.0249993
8	3.7	0.0249994	0.024997	8	7.7	0.0249994	0.0249993
8	3.8	0.0249994	0.0249971	8	7.8	0.0249994	0.0249993
8	3.9	0.0249994	0.0249973	8	7.9	0.0249994	0.0249993
8	4.	0.0249994	0.0249974	8	8.	0.0249994	0.0249994

Table 5: Bargaining outcomes $(\lambda, 1-\lambda)$ for selected pairs of (Y, y); $Z = 1$

Y	y	λ	$1-\lambda$	Y	y	λ	$1-\lambda$
8	0.1	0.855668	0.144332	8	4.1	0.509559	0.490441
8	0.2	0.779803	0.220197	8	4.2	0.509088	0.490912
8	0.3	0.725231	0.274769	8	4.3	0.508639	0.491361
8	0.4	0.684125	0.315875	8	4.4	0.508211	0.491789
8	0.5	0.652738	0.347262	8	4.5	0.507802	0.492198
8	0.6	0.628592	0.371408	8	4.6	0.507412	0.492588
8	0.7	0.609839	0.390161	8	4.7	0.507038	0.492962
8	0.8	0.595091	0.404909	8	4.8	0.50668	0.49332
8	0.9	0.583319	0.416681	8	4.9	0.506337	0.493663
8	1.	0.573776	0.426224	8	5.	0.506008	0.493992
8	1.1	0.565922	0.434078	8	5.1	0.505692	0.494308
8	1.2	0.559367	0.440633	8	5.2	0.505388	0.494612
8	1.3	0.553826	0.446174	8	5.3	0.505096	0.494904
8	1.4	0.549088	0.450912	8	5.4	0.504815	0.495185
8	1.5	0.544959	0.455041	8	5.5	0.504544	0.495456
8	1.6	0.541391	0.458609	8	5.6	0.504284	0.495716
8	1.7	0.53824	0.46176	8	5.7	0.504032	0.495968
8	1.8	0.535455	0.464545	8	5.8	0.503789	0.496211
8	1.9	0.532976	0.467024	8	5.9	0.503555	0.496445
8	2.	0.530754	0.469246	8	6.	0.503328	0.496672
8	2.1	0.52875	0.47125	8	6.1	0.503109	0.496891
8	2.2	0.526935	0.473065	8	6.2	0.502897	0.497103
8	2.3	0.525282	0.474718	8	6.3	0.502692	0.497308
8	2.4	0.523771	0.476229	8	6.4	0.502494	0.497506
8	2.5	0.522384	0.477616	8	6.5	0.502302	0.497698
8	2.6	0.521107	0.478893	8	6.6	0.502115	0.497885

Table 5: (continued)

Y	y	λ	$1-\lambda$	Y	y	λ	$1-\lambda$
8	2.7	0.519927	0.480073	8	6.7	0.501934	0.498066
8	2.8	0.518833	0.481167	8	6.8	0.501759	0.498241
8	2.9	0.517817	0.482183	8	6.9	0.501589	0.498411
8	3.	0.51687	0.48313	8	7.	0.501423	0.498577
8	3.1	0.515986	0.484014	8	7.1	0.501263	0.498737
8	3.2	0.515158	0.484842	8	7.2	0.501107	0.498893
8	3.3	0.514381	0.485619	8	7.3	0.500955	0.499045
8	3.4	0.513652	0.486348	8	7.4	0.500807	0.499193
8	3.5	0.512965	0.487035	8	7.5	0.500664	0.499336
8	3.6	0.512316	0.487684	8	7.6	0.500524	0.499476
8	3.7	0.511704	0.488296	8	7.7	0.500388	0.499612
8	3.8	0.511125	0.488875	8	7.8	0.500255	0.499745
8	3.9	0.510576	0.489424	8	7.9	0.500126	0.499874
8	4.	0.510054	0.489946	8	8.	0.5	0.5

Table 6: Gains From Bargaining as a Fraction of Total Wealth ($\frac{e^{Gains1}}{Y}$, $\frac{e^{Gains2}}{y}$) for selected pairs of (Y, y); $Z = 1$

Y	y	$\frac{e^{Gains1}}{Y}$	$\frac{e^{Gains2}}{y}$	Y	y	$\frac{e^{Gains1}}{Y}$	$\frac{e^{Gains2}}{y}$
8	0.1	0.127783	22.2115	8	4.1	0.129134	0.259067
8	0.2	0.128702	9.20064	8	4.2	0.129128	0.252552
8	0.3	0.129182	5.49341	8	4.3	0.129123	0.246355
8	0.4	0.12944	3.81789	8	4.4	0.129118	0.240455
8	0.5	0.129571	2.88612	8	4.5	0.129113	0.234829
8	0.6	0.129627	2.30187	8	4.6	0.129109	0.22946
8	0.7	0.129642	1.90535	8	4.7	0.129104	0.22433
8	0.8	0.129634	1.6206	8	4.8	0.1291	0.219424

Table 6: (continued)

Y	y	$\frac{e^{Gains1}}{Y}$	$\frac{e^{Gains2}}{y}$	Y	y	$\frac{e^{Gains1}}{Y}$	$\frac{e^{Gains2}}{y}$
8	0.9	0.129613	1.40722	8	4.9	0.129096	0.214727
8	1.	0.129587	1.2419	8	5.	0.129092	0.210227
8	1.1	0.129558	1.11036	8	5.1	0.129088	0.205911
8	1.2	0.129529	1.00338	8	5.2	0.129084	0.201768
8	1.3	0.129501	0.914781	8	5.3	0.129081	0.197788
8	1.4	0.129474	0.840266	8	5.4	0.129077	0.193962
8	1.5	0.129448	0.776783	8	5.5	0.129074	0.190281
8	1.6	0.129424	0.722057	8	5.6	0.12907	0.186736
8	1.7	0.129402	0.674428	8	5.7	0.129067	0.183321
8	1.8	0.129381	0.632608	8	5.8	0.129064	0.180028
8	1.9	0.129361	0.595607	8	5.9	0.129061	0.176852
8	2.	0.129343	0.562646	8	6.	0.129058	0.173785
8	2.1	0.129326	0.533102	8	6.1	0.129056	0.170822
8	2.2	0.12931	0.506476	8	6.2	0.129053	0.167959
8	2.3	0.129296	0.482358	8	6.3	0.12905	0.16519
8	2.4	0.129282	0.460413	8	6.4	0.129048	0.162511
8	2.5	0.129269	0.440361	8	6.5	0.129045	0.159917
8	2.6	0.129256	0.42197	8	6.6	0.129043	0.157404
8	2.7	0.129245	0.405042	8	6.7	0.129041	0.154969
8	2.8	0.129234	0.389411	8	6.8	0.129038	0.152608
8	2.9	0.129224	0.374933	8	6.9	0.129036	0.150318
8	3.	0.129214	0.361487	8	7.	0.129034	0.148095
8	3.1	0.129205	0.348966	8	7.1	0.129032	0.145937
8	3.2	0.129196	0.337279	8	7.2	0.12903	0.143842
8	3.3	0.129188	0.326345	8	7.3	0.129028	0.141805
8	3.4	0.12918	0.316094	8	7.4	0.129026	0.139825
8	3.5	0.129172	0.306465	8	7.5	0.129024	0.137899

Table 6: (continued)

Y	y	$\frac{e^{Gains1}}{Y}$	$\frac{e^{Gains2}}{y}$	Y	y	$\frac{e^{Gains1}}{Y}$	$\frac{e^{Gains2}}{y}$
8	3.6	0.129165	0.297402	8	7.6	0.129022	0.136026
8	3.7	0.129158	0.288858	8	7.7	0.129021	0.134203
8	3.8	0.129152	0.280788	8	7.8	0.129019	0.132428
8	3.9	0.129145	0.273156	8	7.9	0.129017	0.1307
8	4.	0.129139	0.265926	8	8.	0.129015	0.129015

Table 7: $Pr(Conflict)$ for selected pairs of (Y, y); $Z = 1$

Y	y	$Pr(Conflict)$	Y	y	$Pr(Conflict)$
8	0.1	0.872217	8	4.1	0.870866
8	0.2	0.871298	8	4.2	0.870872
8	0.3	0.870818	8	4.3	0.870877
8	0.4	0.87056	8	4.4	0.870882
8	0.5	0.870429	8	4.5	0.870887
8	0.6	0.870373	8	4.6	0.870891
8	0.7	0.870358	8	4.7	0.870896
8	0.8	0.870366	8	4.8	0.8709
8	0.9	0.870387	8	4.9	0.870904
8	1.	0.870413	8	5.	0.870908
8	1.1	0.870442	8	5.1	0.870912
8	1.2	0.870471	8	5.2	0.870916
8	1.3	0.870499	8	5.3	0.870919
8	1.4	0.870526	8	5.4	0.870923
8	1.5	0.870552	8	5.5	0.870926
8	1.6	0.870576	8	5.6	0.87093
8	1.7	0.870598	8	5.7	0.870933
8	1.8	0.870619	8	5.8	0.870936

Table 7: (continued)

Y	y	$Pr(Conflict)$	Y	y	$Pr(Conflict)$
8	1.9	0.870639	8	5.9	0.870939
8	2.	0.870657	8	6.	0.870942
8	2.1	0.870674	8	6.1	0.870944
8	2.2	0.87069	8	6.2	0.870947
8	2.3	0.870704	8	6.3	0.87095
8	2.4	0.870718	8	6.4	0.870952
8	2.5	0.870731	8	6.5	0.870955
8	2.6	0.870744	8	6.6	0.870957
8	2.7	0.870755	8	6.7	0.870959
8	2.8	0.870766	8	6.8	0.870962
8	2.9	0.870776	8	6.9	0.870964
8	3.	0.870786	8	7.	0.870966
8	3.1	0.870795	8	7.1	0.870968
8	3.2	0.870804	8	7.2	0.87097
8	3.3	0.870812	8	7.3	0.870972
8	3.4	0.87082	8	7.4	0.870974
8	3.5	0.870828	8	7.5	0.870976
8	3.6	0.870835	8	7.6	0.870978
8	3.7	0.870842	8	7.7	0.870979
8	3.8	0.870848	8	7.8	0.870981
8	3.9	0.870855	8	7.9	0.870983
8	4.	0.870861	8	8.	0.870985

Table 8: $Pr(Conflict)$ for selected pairs of (Y, y); $Z = \frac{y}{10}$

Y	y	$Pr(Conflict)$	Y	y	$Pr(Conflict)$
8	0.1	0.87496	8	4.1	0.873357

Table 8: (continued)

Y	y	$Pr(Conflict)$	Y	y	$Pr(Conflict)$
8	0.2	0.87492	8	4.2	0.873317
8	0.3	0.87488	8	4.3	0.873277
8	0.4	0.874839	8	4.4	0.873237
8	0.5	0.874799	8	4.5	0.873197
8	0.6	0.874759	8	4.6	0.873157
8	0.7	0.874719	8	4.7	0.873117
8	0.8	0.874679	8	4.8	0.873077
8	0.9	0.874638	8	4.9	0.873037
8	1.	0.874598	8	5.	0.872997
8	1.1	0.874558	8	5.1	0.872957
8	1.2	0.874518	8	5.2	0.872917
8	1.3	0.874478	8	5.3	0.872878
8	1.4	0.874438	8	5.4	0.872838
8	1.5	0.874398	8	5.5	0.872798
8	1.6	0.874357	8	5.6	0.872758
8	1.7	0.874317	8	5.7	0.872718
8	1.8	0.874277	8	5.8	0.872678
8	1.9	0.874237	8	5.9	0.872638
8	2.	0.874197	8	6.	0.872599
8	2.1	0.874157	8	6.1	0.872559
8	2.2	0.874117	8	6.2	0.872519
8	2.3	0.874077	8	6.3	0.872479
8	2.4	0.874037	8	6.4	0.872439
8	2.5	0.873997	8	6.5	0.8724
8	2.6	0.873957	8	6.6	0.87236
8	2.7	0.873917	8	6.7	0.87232
8	2.8	0.873876	8	6.8	0.87228

Table 8: (continued)

Y	y	$Pr(Conflict)$	Y	y	$Pr(Conflict)$
8	2.9	0.873836	8	6.9	0.872241
8	3.	0.873796	8	7.	0.872201
8	3.1	0.873756	8	7.1	0.872161
8	3.2	0.873716	8	7.2	0.872121
8	3.3	0.873676	8	7.3	0.872082
8	3.4	0.873636	8	7.4	0.872042
8	3.5	0.873596	8	7.5	0.872002
8	3.6	0.873556	8	7.6	0.871962
8	3.7	0.873516	8	7.7	0.871923
8	3.8	0.873476	8	7.8	0.871883
8	3.9	0.873436	8	7.9	0.871843
8	4.	0.873396	8	8.	0.871804

Table 9: $Pr(Conflict)$ for selected pairs of (Y, y); $Z = \frac{y}{5}$

Y	y	$Pr(Conflict)$	Y	y	$Pr(Conflict)$
8	0.2	0.874918	8	4.1	0.871639
8	0.2	0.874835	8	4.2	0.871558
8	0.3	0.874753	8	4.3	0.871477
8	0.4	0.87467	8	4.4	0.871395
8	0.5	0.874588	8	4.5	0.871314
8	0.6	0.874505	8	4.6	0.871233
8	0.7	0.874423	8	4.7	0.871152
8	0.8	0.874341	8	4.8	0.871071
8	0.9	0.874258	8	4.9	0.870989
8	1.	0.874176	8	5.	0.870908
8	1.1	0.874094	8	5.1	0.870827

Table 9: (continued)

Y	y	$Pr(Conflict)$	Y	y	$Pr(Conflict)$
8	1.2	0.874012	8	5.2	0.870746
8	1.3	0.873929	8	5.3	0.870665
8	1.4	0.873847	8	5.4	0.870584
8	1.5	0.873765	8	5.5	0.870503
8	1.6	0.873683	8	5.6	0.870422
8	1.7	0.873601	8	5.7	0.870341
8	1.8	0.873519	8	5.8	0.87026
8	1.9	0.873437	8	5.9	0.870179
8	2.	0.873355	8	6.	0.870099
8	2.1	0.873273	8	6.1	0.870018
8	2.2	0.873191	8	6.2	0.869937
8	2.3	0.873109	8	6.3	0.869856
8	2.4	0.873027	8	6.4	0.869776
8	2.5	0.872945	8	6.5	0.869695
8	2.6	0.872863	8	6.6	0.869614
8	2.7	0.872781	8	6.7	0.869534
8	2.8	0.8727	8	6.8	0.869453
8	2.9	0.872618	8	6.9	0.869372
8	3.	0.872536	8	7.	0.869292
8	3.1	0.872455	8	7.1	0.869211
8	3.2	0.872373	8	7.2	0.869131
8	3.3	0.872291	8	7.3	0.86905
8	3.4	0.87221	8	7.4	0.86897
8	3.5	0.872128	8	7.5	0.86889
8	3.6	0.872047	8	7.6	0.868809
8	3.7	0.871965	8	7.7	0.868729
8	3.8	0.871884	8	7.8	0.868648

Table 9: (continued)

Y	y	$Pr(Conflict)$	Y	y	$Pr(Conflict)$
8	3.9	0.871802	8	7.9	0.868568
8	4.	0.871721	8	8.	0.868488