Extended Abstract:
Stochastic Eventual Perfect Monitoring

Itai Arieli[*]and Yehuda (John) Levy[‡]

April 7, 2013

In this work, we study agents who interact repeatedly, and learn of each other's actions through private signals, which are chosen stochastically before each stage. The signalling structure is very general, and the joint distribution of the private signal profile at each stage can be conditioned on all past occurrences. We study signalling structures which allow players to "eventually learn their opponent's actions".

The work is divided into two parts. The first part is involved with expressing this learning condition in a formal way. We take several approaches:

- One approach defines, using the belief operators presented in Monderer and Samet (1989), stochastic eventual perfect monitoring (henceforth, SEMP) such that given that the players' behavioral strategies are known, and a particular event occurs, it becomes approximate knowledge with higher and higher probability as time goes on by all players.

- A similar approach defines SEMP to require that, given that an event occurs, it becomes approximate mutual belief with higher belief and of higher order as time goes on.

- A different approach generalizes the concept of merging, defined by Blackwell and Dubins (1962), and stipulates that given the true strategy profile and a strategy profile whose induced probably measure dominates the true measure and which the players believe, players' opinions of the future approaches the true opinion.

- Finally, a fourth condition takes a different approach and stipulates that whenever two strategy profile induce mutually singular measures on the sequences of actions, they also induce mutually singular measures on the

sequences of signals as well; this is a more analytic condition, and is the most versatile.

Our main result in this section of the paper is to show that these conditions are all equivalent.

The second part of the paper concerns applications. One application concerns determinacy in Borel games. Recently, Shmaya (2011) extended a classical result of Martin (1975) on games with Borel winning sets to games with eventual perfect monitoring (the deterministic counterpart of SEMP). We extend this result on the existence of a value in such games to the case of SEMP.

A further result concerns learning in repeated games. In this application, we extend the convergence to equilibrium under rational learning obtained by Kalai and Lehrer (1993) to the case of SEMP, in which case we get convergence to a weaker equilibrium concept (extensive-form correlated equilibria).

References

[1] BLACKWELL, D. AND DUBINS, L. (1962), Merging of Opinions with Increasing Information, *The Annals of Mathematical Statistics*, Vol. 33, 882–886.

[2] KALAI, E. AND LEHRER, E. (), Rational Learning Leads to Nash Equilibrium, *Econometrica*, 1019–1045.

[3] MARTIN, D. (1975), Borel Determinacy, *Annals of Mathematics* 102, 363–371

[4] MONDERER, D., AND SAMET, D. (1989), Approximating Common Knowledge with Common Beliefs, *Games Econ. Behav.*, 1, 170–190.

[5] SHMAYA, E. (2011) The Determinacy of Infinite Games with Eventual Perfect Monitoring, *Proc. Amer. Math. Soc.*, 139, 3665–3678.