Price-Quantity Competition with Risk-averse Firms

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Abstract

In this paper we consider the market for a homogeneous good that is produced by two firms. The market demand is linearly decreasing in the price of the good. The marginal cost of production is constant and equal across firms. Firms engage in price-quantity competition: they simultaneously determine both the price and the production level of the good. Production level corresponds to *actual* production, it is not only a capacity constraint in the sense that production must be implemented at the chosen level, firms may not produce less. Since firms make their decisions simultaneously, a firm may end up with unsold products: when the production level of the firm that charges the highest price exceeds the residual demand the firm faces, then it can sell at most a part of its products.

It is known from the literature of price-quantity competition that there exists no pure-strategy Nash equilibrium in this model when firms are risk neutral and they maximize their (expected) profit (see e.g. Levitan and Shubik, 1978). The assumptions of risk neutrality and that firms can perfectly observe the price and production level of the other firm are crucial for this result. In this paper we show that a Nash equilibrium in pure strategies may exist when firms are risk averse and they hold noisy conjectures about the action of the other firm. We analyze how the symmetric pure-strategy Nash equilibrium depends on the degree of risk aversion and the amount of uncertainty in the conjectures.

We introduce uncertainty in the model in the conjectures of firms. Namely, firm i believes that the price and production level of firm j consists of a baseline price/production level plus a random error. We use a truncated standard normal distribution for the error term such that both the price and the production level are assumed to be normally distributed on a certain interval, with the mode of the distribution being equal to the baseline price/production level. We assume that firms have mean-variance preferences, that is they maximize their expected profit minus a weighted value of the variance of the profit. The weight in the objective function describes the risk attitude of firms. If the weight is

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zero, then firms are risk neutral whereas they are risk averse for positive weights. The larger the weight is, the higher is the degree of risk aversion.

We analytically derive the first-order conditions of the optimization problem of firms and we use these equations to characterize the symmetric pure-strategy Nash equilibria of the model. The equilibrium conditions are quite complex and they cannot be solved analytically, therefore we use numerical methods for finding a solution. Then we investigate numerically whether the resulting solution is the best response to the action of the other firm (i.e. the global maximum of the objective function is reached at that point, given the action of the other firm). The analysis shows that there is a unique solution to the first order conditions but it does not always correspond to the global maximum. In order to shed more light on this issue, we analyze how the existence of the symmetric Nash equilibrium in pure strategies depends on important parameters of the model such as the degree of risk aversion and the amount of uncertainty in the conjectures about the action of the other firm. We numerically characterize the parameter region for which the equilibrium exists and we find that, for a fixed degree of risk aversion, there exists an equilibrium if the uncertainty in the conjectures is sufficiently high. The more risk averse the firms are, the lower is the minimal amount of uncertainty for which the equilibrium exists.

Having established the existence of a unique Nash equilibrium in pure strategies, we analyze numerically how this equilibrium depends on the aforementioned parameters. Contrary to the mixed-strategy Nash equilibrium under risk neutrality and complete information (see Gertner, 1986), each firm now produces less than the market demand at the equilibrium price. Aggregate production, however, may be larger as well as smaller than the market demand, depending on the preferences and beliefs of the firms. We find that the more risk averse the firms are, the less they produce and the higher price they ask in equilibrium. Aggregate production exceeds market demand for low degrees of risk aversion and firms have some unsold products. As firms become more risk averse, aggregate production will not satisfy the demand in equilibrium. Thus, given the beliefs of firms, there exists an optimal level of risk aversion for which aggregate production equals equilibrium demand so no consumer or producer will be rationed. We also find that firms react differently to price and production uncertainty. When the amount of price uncertainty increases, equilibrium price increases, equilibrium production decreases and the aggregate production/market demand ratio slightly increases. In contrast, when production uncertainty increases, equilibrium price decreases while production may increases as well as decrease, depending on the other parameters. The aggregate production/market demand ratio typically decreases.