Multiple Experts Informing a Constrained Decision-maker

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Abstract

This paper studies how constraints on a decision-maker impede information transmission from multiple experts who share the same preferences with the decision-maker. The experts have no ex-ante conflicts of interests with each other since they can only observe some dimensions of a state where they are interested in and have their own areas of expertise. However, the constraints create strategic tensions hindering revelation of information that would otherwise be completely transmitted. I characterize (informative) prefect Bayesian equilibria under various forms of the constraints. First, I identify the extent of the constraints to which full information revelation is possible. Second, I show that there exist partially revealing equilibria irrespective of the constraint level. Finally, I analyze the effect of the constraint level on social welfare. Counterintuitively, the less binding constraints do not necessarily guarantee Pareto-superior outcomes.

JEL classification: D82; D83
Keywords: Strategic information transmission; Multidimensional cheap talk; Multiple senders

1 Introduction

Oslo's city council in Norway halted a project to construct a new museum for an internationally famous painter, Edvard Munch, in 2011. Even though the city already spent three years and \$19 million on the project, council members have shown reservations due to budget deficit. Oslo spent \$300 million on the FIS (Fédération Internationale de Ski, International Ski Federation) Nordic World Ski Championships in 2011 and it was far beyond an originally estimated cost in 2005 (Mohsin (2012)).

I use the previous episode to make an example to build a model that analyzes the effects of constraints on strategic information transmission. Suppose that you are a member of city council and need to make decisions about building a new art museum and funding an international sports event. Assuming that you are unfamiliar and disinterested with these projects, to make better decisions, you ask two experts to estimate the costs of projects and the economic benefits. Each expert has expertise only in one project and represents interested groups in that project. Given the city's available budget, the experts submit unverifiable reports respectively. Based on their statements you need to allocate the fixed budget. How do you know whether they reveal accurate estimates or overestimate the benefit and under-evaluate the cost? If they do not reveal their estimation precisely, how can you infer correctly from ambiguous reports?

Policymakers depend on experts when they need to make decisions based on factors that are unknown to them. However, interested experts may be apt to exaggerate a situation to pull decisions toward their own interests when their advices are unverifiable.[1] Anticipating overstatements, the policymakers, in turn, respond to the experts by underestimating their reports. As a result, the policymakers can obtain only equivocal information from the experts (Crawford and Sobel (1982)). The previous example illustrates that the introduction of constraints may make matters worse. The experts compete with each other for the limited budget in addition to conflicts of interests.

The purpose of this paper is to study how constraints on a decision-maker (receiver) impede information transmission from experts (senders) even if the decision-maker shares the same preferences with the experts.[2] Informational flow is measured by building an economic model that characterizes how much tensions between the decision-maker and the experts are induced according to the level of the constraints. In the model, the senders can observe only one dimension of a state and each sender's payoff is only determined in the observed dimension and independent of the other. The uninformed receiver consults with n senders who share common interests with her in each dimension and makes an n-dimensional decision that is limited by the constraints. The utility level of each agent is determined by both the state and the receiver's action. The receiver has the same initial preference as each sender in each dimension of the state.

Since the constraints impose a limit on the receiver's action, an optimal decision for the receiver might not be consistent any more with the senders. This inconsistency gives rise to an endogenous *interim bias* between each sender and the receiver, leading to uncertainty about their preferences. Consequently, each sender needs to take the other senders' preferences into account despite ex-ante independence of the others since a sender's payoff is ex-post related with the other senders through the receiver's action.

It is both the direction and the extent of the constraints that decide the level of information transmission from the senders to the receiver. Given a specific condition, e.g. if an

[1] There is a limit to the extent that a decision-maker can obtain information from experts even with verifiable messages. See Section 2 Related literaure.

[2] Hereafter, an adviser is referred to informer, expert, or sender while a policymaker is denoted by decision-maker, or receiver. Through the paper, I use male pronoun for a sender and female pronoun for the decision-maker.

action is bound only in one dimension, information can be wholly transferred irrespective of the level of the limit. Generally, as long as the constraint is *non-severe*, i.e. with some positive probabilities the receiver can take an action that satisfies all senders, full information revelation is possible since expected loss by message-manipulation is larger than expected gain.

Even if the constraint is so strict that it is impossible for information to be sent completely, the senders can still transfer meaningful messages to the receiver. Each dimension of the state is divided into segments and the senders reveal which segment the state belongs to.[3] Finer partitions imply more informative messages since information is partially transmitted in terms of partitions. In partially revealing equilibria, the less binding constraints do not necessarily guarantee more informative transmission; on the contrary, they may hinder more precise information transfer. This leads to a counterintuitive result that the less severe constraint might induce Pareto-inferior outcomes.

To focus primarily on the effect of the constraint on information transfer, I mainly study the common interest case where there is no ex-ante bias between the senders and the receiver. In Section 5, I extend the model to allow senders to differ from each other in preferences over decisions even without the constraint. It does not affect the qualitative nature of the results in partial equilibria. The receiver would better get advices from senders who have different preferences each other rather than senders who shares the same interests in some dimensions and irrelevant in others.

The paper is organized as follows. Related literature is discussed in Section 2. The model is presented in Section 3. The level of the constraints is defined into two categories to identify the conditions under which full information revelation is possible. In section 4, I show how the constraint causes an interim bias between agents and characterize fully revealing equilibrium. Section 5 demonstrates the existence of partially revealing equilibrium irrespective of the constraint level. I elucidate the main results with simplified examples. In Section 6, I extend the model to check robustness and discuss implication of modification.

2 Related literature

This paper is closely related with Ambrus and Takahashi (2008) where the shape of the state space decides the extent to which information is transmitted. Ambrus and Takahashi show that full information revelation is decided by the direction and magnitude of biases between agents induced by exogenously given constraints. I make a constraint endogenous so that I can show how information flow affected by the level of the constraint. I characterize all equilibria including partially revealing equilibria according to the level of constraints that shapes an action space while Ambrus and Takahashi focus only on possibility of a fully revealing equilibrium.

[3]The method is originally developed by Crawford and Sobel (1982) where a receiver get a partially revealing message from a sender who has a constant and exogeneous bias against the receiver

Che and Kartik (2009) study the same case as this paper in which experts and a decision-maker have no ex-ante conflicts. However, the experts and the decision-maker have different prior beliefs that lead to *an interim bias* that hinders information flows. The experts can acquire more precise information with a higher cost. In such cases, the decision-maker is better off getting advices from an expert who has a different prospect rather than the other who has similar view since the different opinions encourage an adviser to obtain accurate information at any cost. Kartik (2009) and Kartik, Ottaviani and Squintani (2007) approach the issue of the effect of costs on information transmission from a different perspective: intentional misreporting charges psychological costs of the sender. However, in this paper, the senders neither obtain information with costs nor suffer from manipulated reports directly. The senders' messages are purely cheap and the cost is only decided by the receiver's action regardless of the senders.

The paper is also similar to Alonso, Dessein and Matouschek (2008) in regard that the receiver makes a two-dimensional decision depending on uni-dimensionally informed senders. In their model the senders care about each other as well as the receiver since the senders partially share common interests induced by the receiver's decision. Each sender is directly affected by not only her own dimension but also the other dimension. However, in this paper, the senders are indifferent to different dimensional decisions.

This paper complements the literature on cheap talk, strategic information transmission, and persuasion. The key differences with the literature are as follows. First, in this model there are no ex-ante biases in the preferences between the senders and the receiver while in the literature the senders have either state-dependent or state-independent preference differences against the receiver.[4] The interested experts intentionally add deceptive noises into unverifiable messages to induce the decision-maker to take their preferred actions. Responding to purposefully ambiguous, the decision-maker exploits the tensions between opposing senders who want to pull the decision-maker toward their own directions (Gentzkow and Kamenica (2012)) and is fully informed as a consequence of rebuttals (Krishna and Morgan (2001)). If the messages are verifiable, the experts reveal the truth if and only if truth-telling is consistent with their own interests. Otherwise, they pretend to be ignorant (Shin (1994, 1998), Bhattacharya and Mukherjee (2011)). However, there is no ex-ante conflicts in this paper.

Second, the senders are not comprehensive observers; they can partially observe the state-dimension in which they are interested while the senders in the literature can observe all dimensions of the state completely. Multidimensionality causes *informational spillover* between dimensions either leading to full information revelation even without sequential debate (Battaglini (2002)) or contrarily restricting revelation (Levy and Razin (2007)). Even the expert with state-independent preferences can persuade the receiver with unverifiable messages

[4] See Dziuda (2011), Li and Madarász (2008), Morgan and Stocken (2003) and Wolinsky (2003) for discussions about strategic information transmission in various points of view when a receiver is uncertain about a sender's preferences.

since multidimensionality makes cheap talk reliable (Chakraborty and Harbaugh (2010)). However, this spillover effect depends on the fact that the senders perfectly observe the realized state in every dimension. Consequently, the receiver in this paper cannot throughly exploit the strategic tensions in multiple dimensions.

Finally, the receiver cannot commit a mechanism before the senders report on the observed state. If the receiver can commit to a mechanism, she can elicit significantly more information than the senders would like to transfer (Wolinsky (2002)). Glazer and Rubinstein (2004, 2006) show that a mechanism is "credible", i.e. satisfying sequential rationality, if a state is finite and a decision is binary. Sher (2011) represents that the credibility in Glazer and Rubinstein (2004, 2006) is generalized depending on the assumption that the receiver's utility function is a concave transformation of the sender's. Kamenica and Gentzkow (2011) study a mechanism in which a sender commits to a signaling rule before informed in the view of the sender. However, there is neither a transfer nor a commitment in this paper.

3 Model

There are two informers, call them Sender 1 and Sender 2, and a decision-maker called the receiver.[5] A state, θ, is uniformly distributed on a state space $\Theta = [0,1] \times [0,1]$. After nature chooses a state, $\theta' = (\theta'_1, \theta'_2)$, each sender partially observes the state; Sender 1 observes θ'_1 while Sender 2 observes θ'_2. The senders simultaneously and privately send messages, $m_i \in M_i$ where M_i is any infinite set, to the receiver. After observing the messages, the receiver updates her belief β about the state and takes an action $a = (a_1, a_2) \in A = \Theta$. Denote the strategy for each sender as a function μ_i: $\Theta_i \mapsto M_i$ and the strategy for the receiver as a function $\alpha : M_1 \times M_2 \mapsto A$.

Sender i's payoff given by $U^{S_i}(a_i, \theta_i)$ depends only on the observed state, θ'_i, and the action, a_i, in the same dimension.[6] $U^{S_i}(a_i, \theta_i)$ is twice continuously differentiable and $U^{S_i}_{11} < 0 < U^{S_i}_{12}$ denoting partial derivatives by subscripts in the usual way. For simplicity, I assume that the maximum value of U^{S_i} is equal to zero. The receiver's payoff, given by $U^R(a, \theta)$, is decided by the difference between the state and the action in each dimension: $U^R = \sum U^{S_i}$. Since there is no preference bias between the senders and the receiver in each dimension, $U^{S_i}(\theta_i, \theta_i) = U^R(\theta, \theta) = 0$.

However, the receiver cannot always follow messages from the senders since her action is limited by a constraint. Suppose that the receiver's action needs consumption of good or service $x \in X = R^n_+$. Demanded X is matched by a correspondence Γ: $A \rightrightarrows X$. Let $p_i \geq 0$ denote the price of good or service x_i and $w \geq 0$ denote the receiver's available wealth. Then, a set $\{p_1, p_2, \ldots, p_n, w\}$ that is common knowledge constitutes a budget constraint: $\sum p_i x_i \leq w$. This budget constraint put limits on the receiver's choice set as follows: $\sum p_i \Gamma(A) \leq w$.

[5]For simplicity, I focus on the case of two senders and one receiver cases. In Section 6, I study the cases where there are more than two senders.

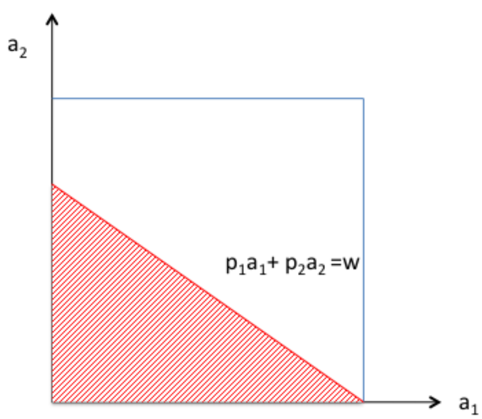
[6]Hereafter, superscripts of S and R denote a sender and a receiver.

From now on, let $\Gamma(a) = a$ for simplicity.[7]

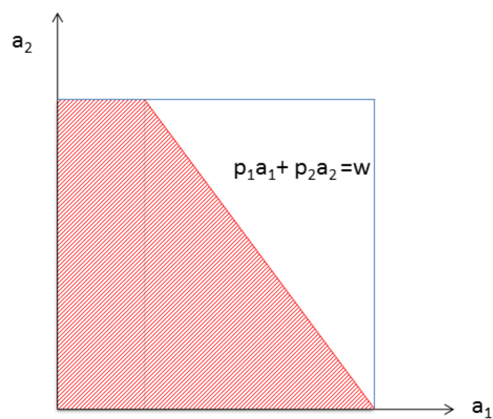
In following sections, I show how a constraint level affects the level of information transmission. Particularly, I identify necessary conditions on the constraint for fully revealing equilibria to exist in Section 4. The dichotomy of the constraint level is useful and convenient for the analysis.

Definition 1. *Given $\{p_1, p_2, w\}$, let $\bar{a}_i = w/p_i$ be a maximal action on each dimension. If min $\{\bar{a}_1, \bar{a}_2\} < 1$, then a constraint is severe. Otherwise, a constraint is non-severe.*

The constraint is classified by a boundary condition in an action space limited by itself. If the highest action that the receiver can take is on the upper bound in each dimension, i.e. $\bar{a}_i = 1$, then the constraint is non-severe. Otherwise, the constraint is severe (see Figure 1). As $w \to 0$, the constraint more *severely* restricts the action space.



(a) Severe constraint



(b) Non-severe constraint

Figure 1: Constraint level

4 Full information revelation

In this section, I characterize a fully revealing equilibrium. First, I show how the constraint creates divergences in interests between the senders and the receiver, even when preferences are ex-ante aligned. Then, I identify conditions under which full revelation is possible despite the constraint.

[7] I discuss this assumption in detail later in Section 6

4.1 Interim bias

Suppose that after Sender i observes his dimension of the state, θ'_i, he reveals the state truthfully. Given $(m_1, m_2) = (\theta'_1, \theta'_2)$, the receiver confronts the utility maximization problem: $\max_a U^R$ such that $\sum p_i a_i \leq w$. Figure 2 delineates how the *credulous* receiver responds to the message. If $\theta'_1 \leq (w - p_2\theta'_2)/p_1$, a point (θ'_1, θ'_2) is chosen. However, if $\theta'_1 \geq (w - p_2\theta'_2)/p_1$, the receiver chooses the point t which deviates from Sender 1(2)'s best point $a(b)$.

Lemma 1. *Suppose that the senders tell the truth. When a constraint is non-binding,* $\sum p_i m_i \leq w$*, the receiver takes the action indicated by the message pair sent by the senders. Otherwise, the receiver takes an action on the boundary of the constraint.*

Proof. See Appendix □

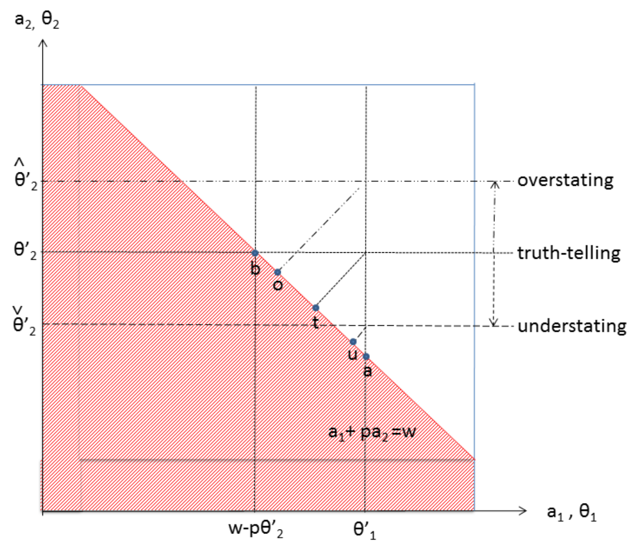


Figure 2: Receiver's response to Sender 2's messages

Since the receiver shares common interests with each sender in each dimension, she takes the action suggested by the senders as long as the constraint is non-binding. However, when the constraint is binding, the receiver optimizes by deviating from the point suggested by the senders, inducing an *interim bias*. Observe that due to the direction of the constraint, the interim biases of the senders are in a sense analogous to an upward biases.

4.2 Fully revealing equilibrium

In the previous section, I show that the senders share common interests with the receiver only if the constraint is not binding. How do the senders respond to the receiver without knowing whether the constraint is binding or not? It depends on each sender's prior belief on the others.

First, I identify whether each sender can get benefit from truth-telling regardless of the others. Without loss of generality, I focus on Sender 2. For simplicity, let $p_1 = 1$ and $p_2 = p$. Suppose that the constraint is non-severe. Given (θ_1', θ_2'), Sender 2 expects Sender 1 to report θ_1 truthfully to the receiver but does not know the value of θ_1'. Therefore, Sender 2's expected payoff depends on his belief about the other dimension of the state θ_1':

$$E(U_t^{S_2}) = \int_0^{w-p\,\theta_2'} U^{S_2}(\theta_2', \theta_2')d\theta_1 + \int_0^{1-w+p\,\theta_2'} U^{S_2}(\theta_2' - \frac{p}{p^2+1}\theta_1, \theta_2')d\theta_1 \tag{1}$$

Equation (1) represents Sender 2's expected utility from truth-telling. The first term in (1) denotes the expected payoff when the realized state is unbound and the second term indicates expected payoff when bounded. Observe that $\frac{p}{p^2+1}\theta_1$ reflects the interim bias between Sender 2 and the receiver and that it increases as θ_1' rises.

If Sender 2 believes that the state is bound, he may want to manipulate message m_2. As shown in Figure 2, by over-reporting the state, $\hat{\theta}_2' > \theta_2'$, Sender 2 persuades the receiver to choose the point o that is closer to his best point b than point t. However, under-reporting the state, $\check{\theta}_2' < \theta_2'$, induces a worse outcome, the point u, that reduces sender 2's payoff. Depending on the state, overstating has counteracting effects on the expected utility. Sending an overstated message that is ϵ higher than the real state, the sender 2 expects

$$E(U_d^{S_2}) = \int_0^{w-p\,\theta_2'-p\,\epsilon} U^{S_2}(\theta_2' + \epsilon, \theta_2')\, d\theta_1 + \int_0^{1-w+p\,\theta_2'+p\,\epsilon} U^{S_2}(\theta_2' + \frac{p^2}{p^2+1}\epsilon - \frac{p}{p^2+1}\theta_1, \theta_2')\, d\theta_1 \tag{2}$$

The first term in (2) shows the negative effect on the expected payoff due to the interest divergence induced by an ϵ higher message when the state is unbound. The second term represents the positive effect on the expected payoff by decreasing the interest gap by $\frac{p^2}{p^2+1}\epsilon$ when the state is bound. These two effects show the ex-ante trade-off of overstating the state in an unbound and a bound interval. Proposition 1 identifies the condition for revealing full information. Hereafter, I refer only to perfect Bayesian equilibrium.

Proposition 1. *There exists a fully revealing equilibrium if and only if a constraint is non-severe.*

Proof. See Appendix □

Overstating becomes more compelling as θ_2' increases since the probability that θ_1' lies in an unbound interval decreases and the positive effect prevails over the negative effect. However, as long as the constraint is non-severe, a misrepresentation loss is always larger than the associated gain irrespective of θ_2'. Therefore, the senders sincerely report the states; the receiver updates her belief based on truthful messages and makes a decision. This constitutes a perfect Bayesian equilibrium. The existence of fully revealing equilibrium with a given wealth level guarantees full information revelation with the higher wealth level.

Proposition 2. *Suppose that given $w = \underline{w}$ there exists a fully revealing equilibrum. Then, there exist other fully revealing equilibria for $\forall \hat{w}$ such that $\underline{w} \leq \hat{w}$.*

Proof. Without loss of generality, I focus on Sender 2. From (1) and (2)

$$\frac{d}{dw}(E(U^{S_2,t}) - E(U^{S_2,d})) = -U^{S_2}(0, -\epsilon)$$

By assumption,

$$-U^{S_2}(\theta_2' + \epsilon, \theta_2') > 0$$

$\square$

Before the constraint puts a limit on the receiver's choice set, the preferences between the senders and the receiver are ax-ante aligned; given prices, more wealth means a less binding constraint. It is intuitive that if full revelation is possible under a more strict condition, then full revelation is also attainable under a less strict condition.

Finally, suppose that an action is costless in one dimension. At least one sender is perfectly aligned with the receiver in interests. It keeps the other sender from overstating that an interim bias against the receiver is independent of the level of the constraint. Consequently, information is completely transmitted.

5 Partial information revelation

In this section, I shows that information can be partially transmitted even though a constraint is so severe that full revelation is impossible. First, I characterize partially revealing equilibria. Then, examples are provided to elucidate the main results of the paper explicitly. Particularly, I focus on a discontinuity in the number of steps between full and partial revelation. Finally, the key characteristic results are used for welfare analysis.

5.1 Partially revealing equilibrium

Crawford and Sobel, hereafter referred to as CS, transform unidimensional state space into partitions to characterize equilibria. Following CS, let $\rho(n) \equiv (\rho^0(n), \ldots, \rho^n(n))$ where $\rho^0(n) = 0 < \rho^1(n) < \cdots < \rho^n(n) = 1$ denote a partition with n steps in one dimension of the state space $[0,1]^2$. A point $\rho_i^j(n)$ is the j-th point in an n-step message of Sender i. Given $(\rho_1(n_1), \rho_2(n_2))$, the state space transforms into a rectilinear grid of $n_1 \times n_2$ cells. If Sender 1 sends a message $m_1 \in (\rho_1^j, \rho_1^{j+1})$, $j = 0, \ldots, n_1 - 1$, and Sender 2 sends a message $m_2 \in (\rho_1^k, \rho_1^{k+1})$, $k = 0, \ldots, n_2 - 1$, respectively, then the receiver identifies a cell $((\rho_1^j, \rho_2^k), (\rho_1^{j+1}, \rho_2^{k+1}))$ in the rectilinear grid where the state must lie. Given the grid, for each message pair (m_1, m_2), the receiver updates her belief and takes an action to maximize her expected utility as follows:

$$(a_1, a_2) \in \text{argmax} - \int_{\rho_1^j}^{\rho_1^{j+1}} \int_{\rho_2^k}^{\rho_2^{k+1}} U^R(a_1, a_2 | m_1, m_2) d\theta_2 d\theta_1 \quad s.t \;\; p_1 a_1 + p_2 a_2 \leq w$$

If the constraint is not binding, the receiver takes an action such that $a = E(\theta \mid m)$; otherwise, the receiver takes an action on the boundary of the constraint. The following theorem characterizes a partially revealing equilibrium when messages are transmitted in terms of partitions.

Theorem 1. *Suppose that the constraint is severe. Then, there exists an $n_1 \times n_2$-grid equilibrium for $\forall$ integers n_i such that $1 \leq n_i \leq n_i^*(p, w) < \infty$ if and only if the following conditions hold.*

$(I.T.)$ $\rho_1^0(n_1) = \rho_2^0(n_2) = 0$ *and* $\rho_1^{n_1}(n_1) = \rho_2^{n_2}(n_2) = 1$

(A) *For* $\quad j = 1, \ldots, n_1 - 1$

$$\sum_{k=0}^{n_2-1} [U^{S_1}(a((\rho_1^j, \rho_1^{j+1}), (\rho_2^k, \rho_2^{k+1})), \rho_1^j) - U^{S_1}(a(\rho_1^{j-1}, \rho_1^j), (\rho_2^k, \rho_2^{k+1})), \rho_1^j)](\rho_2^{k+1} - \rho_2^k) = 0$$

For $\quad k = 1, \ldots, n_2 - 1$

$$\sum_{j=0}^{n_1-1} [U^{S_2}(a((\rho_1^j, \rho_1^{j+1}), (\rho_2^k, \rho_2^{k+1})), \rho_2^k) - U^{S_2}(a(\rho_1^j, \rho_1^{j+1}), (\rho_2^{k-1}, \rho_2^k)), \rho_2^k)](\rho_1^{j+1} - \rho_1^j) = 0$$

(C) $p_1\, \rho_1^{n_1-1} + p_2\, \rho_2^{n_2-1} \leq w$

An $n_1 \times n_2$-grid equilibrium consists of μ_i, α and β such that

$\mu_1(\theta_1') = m_1 \in (\rho_1^j, \rho_1^{j+1})$ *if* $\theta_1' \in (\rho_1^j, \rho_1^{j+1})$

$\mu_2(\theta_2') = m_2 \in (\rho_2^k, \rho_2^{k+1})$ *if* $\theta_2' \in (\rho_2^k, \rho_2^{k+1})$

Given $m_1 \in (\rho_1^j, \rho_1^{j+1})$ *and* $m_2 \in (\rho_2^k, \rho_2^{k+1})$, *for* $\forall m \in m_1 \times m_2$

$$\alpha(m) \equiv \arg\max_a \int_{\rho_2^k}^{\rho_2^{k+1}} \int_{\rho_1^j}^{\rho_1^{j+1}} U^R(a, \theta)\beta(\theta_1, \theta_2 \mid (m_1, m_2))d\theta_1 d\theta_2 \quad \text{s.t} \quad p_1 a_1 + p_2 a_2 \leq w$$

Given $\bar{\Theta} = \{(\theta_1, \theta_2) \mid \mu_i(\theta_i) = m_i\}$

$$\beta(\theta \mid m) = \begin{cases} \frac{f(\theta)}{\int_{\bar{\Theta}} f(\theta) d\theta} & \text{if } \bar{\Theta} \neq \emptyset \text{ where } f \text{ is a uniform pdf} \\ 0 & \text{otherwise} \end{cases}$$

Proof. See appendix □

In Theorem 1, (I.T.) identifies both initial and terminal conditions for an $n_1(n_2)$-step partition for Sender 1(2). (A) shows an arbitrage condition decided by each boundary point between two adjacent intervals; when a state is realized at one of the boundaries, $\theta' = \rho^j$, the sender i needs to be indifferent between sending higher message $m_i \in (\rho^j, \rho^{j+1})$, and lower message $m_i \in (\rho^{j-1}, \rho^j)$. Finally, (C) is an auxiliary condition to guarantee that, given $\theta' \in (\rho^{n_i-1}, \rho^{n_i})$, the sender i does not send a message such that $m_i \in (\rho^{j-1}, \rho^j)$ for $j \neq n_i$.

Section 3.2 showed that full information revelation is impossible if a constraint is severe. By Proposition 1, given $p_1 = p_2 = 1$, there does not exist a fully revealing equilibrium when

$w < 1$. Now, I provide the simplest example of partial revelation for this case.

Example 1. Suppose that utilities are quadratic. For simplicity, let $p_1 = p_2 = 1$ and $w < 1$. I assume that each sender uses a *symmetric 2-step strategy*; there is a point, $\bar{\theta}$, such that if the realized state, θ', is lower than $\bar{\theta}$, sender i sends a low message, l, and otherwise sends a high message h. Sequentially, a state space is partitioned into 4 message cells as shown in Figure 3(a). After receiving each message pair $(m_1, m_2) = (s_1, s_2)$ for $s_i \in \{l, h\}$, the receiver updates her belief and takes an action to maximize her expected utility. Table 1 summarizes induced actions by each message pair.

Table 1: Induced action by message pair

message pair (m_1, m_2)	induced action (a_1, a_2)
(l, l)	$(\frac{\bar{\theta}}{2}, \frac{\bar{\theta}}{2})$
(h, l)	$(\frac{w}{2} + \frac{\bar{\theta}}{4}, \frac{w}{2} - \frac{\bar{\theta}}{4})$
(l, h)	$(\frac{w}{2} - \frac{\bar{\theta}}{4}, \frac{w}{2} + \frac{\bar{\theta}}{4})$
(h, h)	$(\frac{w}{2}, \frac{w}{2})$

For $\theta' = \bar{\theta}$, the senders must be indifferent between sending l and h yielding an arbitrage condition:

$$\bar{\theta} = \frac{4}{7}w + \frac{9}{28} - \frac{1}{7}\sqrt{2w^2 + 4w + \frac{81}{16}} \quad \text{for} \quad 0 \leq w < 1 \tag{3}$$

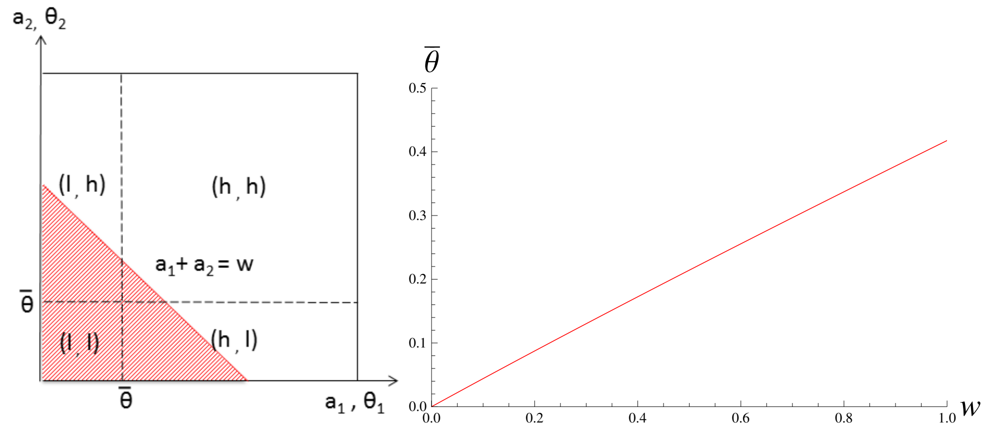
Figure 3(b) outlines Equation (3) showing a *monotonic* relationship between the arbitrage point and the wealth level in equilibrium. As w increases, $\bar{\theta}$ moves closer to the mid point of the state space [0, 1]; the length of the low interval becomes shorter while that of the high interval becomes longer. Consequently, the senders are ex-ante more likely to send a low message.

Theorem 1 indicates the possibility of multiple equilibria with grids composed of various numbers of cells. However, there exists an upper limit on the number of intervals, n^*, that can be sustained in equilibrium. I elucidate characteristics of equilibrium partitions to prove that n^* not be infinite.

Lemma 2. *Let $\delta^i(n)$ denote the length of the i-th segment, i.e. $\mid \rho^i(n) - \rho^{i-1}(n) \mid$, in a n-step partition. Given a severe constraint, an equilibrium partition $\rho^*(n)$ satisfies the following condition: $\delta^{i*} > \delta^{i-1*}$ for $i \in \{1, \ldots, n\}$.*

Proof. Suppose that $\delta^i \leq \delta^{i-1}$. Then $\delta_i^{n_i} \leq 1/n_i$. This contradicts the assumption that ρ^* is an equilibrium since it fails to satisfy Theorem 1 (C). □

High segments in each partition constitutes northeastern cells that are more limited by a constraint. In equilibrium, northeastern cells need to be larger than southwestern cells to be



(a) 2-step symmetric messages (b) Arbitrage condition

Figure 3: Symmetric 2×2 grid equilibrium

balanced at each arbitrage point. Especially, a northeasternmost cell is directly controlled by Theorem 1 (C). Figure 4 illustrates what happens if Lemma 2 and Theorem 1 (C) are not satisfied. Suppose that Sender i sends a message $m_i^{n-1} = [\rho_i^{n-2}, \rho_i^{n-1}]$ if $\theta_i \in [\rho_i^{n-2}, \rho_i^{n-1}]$ and $m_i^n = [\rho_i^{n-1}, \rho_i^n]$ if $\theta_i \in [\rho_i^{n-1}, \rho_i^n]$. Given m_i^n, the receiver takes $a_i^n \in [a_i^{n-1}, \rho_i^{n-1}]$ instead of $a_i'^n \in [\rho_i^{n-1}, \rho_i^n]$ due to a constraint. Therefore, there exist $\theta_i \in m_i' \subset m_i^{n-1}$ where Sender i would rather send m_i^n than m_i^{n-1}. As a result, a lower segment cannot be longer than a higher segment. This sufficiently satisfies *monotonicity* condition in CS.

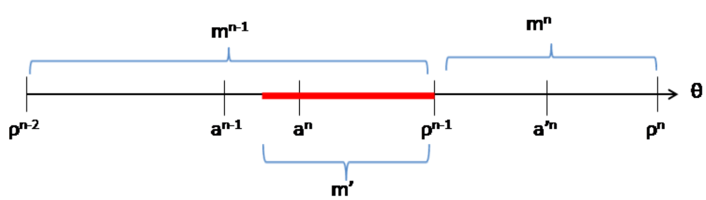


Figure 4: Non-equilibrium message parition

Lemma 3. *Suppose that there exist n^* informative equilibria. Then, Equilibrium partitions satisfy a monotonicity condition:*
Given equilibrium partitions $\rho^*(\hat{n})$ and $\rho^*(\tilde{n})$ with $n^* \geq \hat{n} > \tilde{n}$, $\rho^{*i}(\hat{n}) < \rho^{*i}(\tilde{n})$ for $1 \leq i \leq \tilde{n}$.

Proof. See appendix □

Note that to satisfy monotonicity we do not depend on additional assumptions about utilities that are needed by CS's Theorem 2.

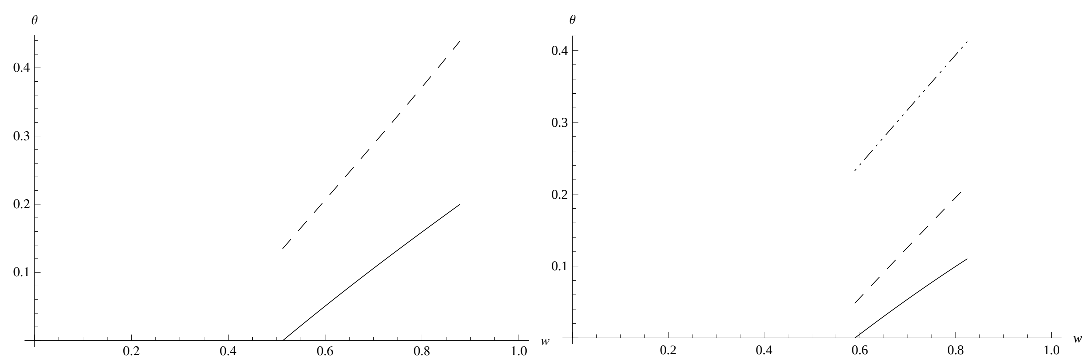
Lemma 4. *The maximum number of partition n^* is not infinite in equilibrium.*

Proof. See appendix □

Now, I show that n_i^* can not be infinite. Suppose that there exists an $n_i \times n_j$-grid equilibrium. As n_i increases, the length of the highest segment δ^{n_i} decreases according to Lemma 2. However, the length of the highest interval cannot be shorter than the threshold level determined by Theorem 1 (C). If n_i^* is infinite, it is contradicted by Lemma 2 and Theorem 1 (C).

Let us return to Example 1 to see what happens if the senders increase steps in their partitions.

Example 1. (Continued) Each sender uses a symmetric 3-step strategy. Figure 5 (a) depicts how an equilibrium message partition is divided into three segments. Each line delineates the movements of each arbitrage point $\rho^{i=1,2}(3)$ between intervals as w changes. Given w, the vertical distances between lines represent $\delta^{i=1,2}(3)$, the lengths of each interval.



(a) 3-step equilibrium case

(b) 4-step equilibrium case

Figure 5: Constraint severity and equilibrium partitions

Now, suppose that the senders adopt a symmetric 4-step strategy. Given w, each segment in the 4-step message is smaller than the corresponding segment in the 3-step message by Lemma 3 (see Figure 5 (b)). Monotonicity leads the result that $\rho^i(3) > \rho^i(4)$ for $1 \leq i \leq 3$.

Before increasing the strategy step above four, I compare equilibrium partitions in the model with those in the CS model. Given quadratic utilities, an equilibrium partition in the CS model satisfies the following condition:

$$(\rho_{cs}^i - \rho_{cs}^{i-1}) - (\rho_{cs}^{i-1} - \rho_{cs}^{i-2}) = 4b > 0 \tag{4}$$

where b is the bias between the sender and the receiver and the maximum number of steps is

$$n_{cs}^* = \left\lceil -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{1}{b}} \right\rceil$$

where $\lceil y \rceil$ denotes the smallest integer greater than or equal to y. Even though this model shares the important characteristic with the CS model such that $(\rho^i - \rho^{i-1}) > (\rho^{i-1} - \rho^{i-2})$, n^* is not identified in an explicit form. Note that the arbitrage conditions in Theorem 1 are not second-order linear difference equations like Equation (4), which represents an arbitrage condition in the CS model. With an $n \times n$-grid, the arbitrage conditions in this model are merged into a polynomial of degree $2n-1$. Therefore, to identify n^*, we have nothing to do but numerically calculate the value since there is no general algebraic solution for polynomial equations of degree five or higher by the Abel-Ruffini theorem (Knapp (2006)).

5.2 Discontinuity between partial and full revelation

The maximum number of steps, $n^*(p, w)$, is a function of the constraint that endogenously leading to an interim bias. In the previous section, I show that there exist a limit to the extent that the senders can increase message segments in their partitions. What happens to $n^*(p, w)$ if the constraint is slightly less than the feasible level for full revelation, i.e. $w = 1-\epsilon$ when $p_i = 1$? One might expect that as $\epsilon \to 0$, the constraint is less binding, so the senders can use more finely gridded message spaces. However, counterintuitively, only the least fine grid, i.e. a 2×2-grid, is feasible as $\epsilon \to 0$.

Corollary 1. *In equilibirum, the maximum number of cells in a grid increases as a constraint become less severe. However, it decreases beyond a threshold of the constraint.*

Sketch of proof. Section 4.1 shows that when n^* is infinite, it is contradicted Lemma 2 and Theorem 1 (C). Corollary 1 can be interpreted in the same context. When w is small, the monotonicity condition controls $n^*(w)$ for the senders; it requires a sufficiently high level of w that $\delta^{i*} > \delta^{i-1*}$ for $i \in \{1, \ldots, n\}$. On the contrary, when w is large, $n^*(w)$ is restrained by an incentive compatibility condition: $p_1\, \rho_1^{n_1-1} + p_2\, \rho_2^{n_2-1} \leq w$. Given w, $n^*(w)$ is decided by whichever is the stronger.

As the senders increase steps in messages, cells in the state grid multiply while size of each one shrinks. The monotonicity condition requires cells to be more densely packed in the southwest and less so in the northeast of the grid. However, by the incentive compatibility condition, the northeasternmost cell must be bigger than a threshold size settled on by the senders' steps. These two conditions squeeze the feasible level of the constraint for the $n_1 \times n_2$ grid from both the low and the high bound of the state as the senders increase message steps.

Example 1. (Revisited) Figure 3 (b) shows that a 2×2-grid equilibrium is possible as long as $w > 0$ while a 3×3-grid equilibrium exists only if $0.51 < w < 0.88$ as shown in Figure 5 (a). The range of w necessary for the existence of a 4×4-grid equilibrium is even smaller than that for a 3×3-grid equilibrium: $0.58 < w < 0.83$ (see Figure 5 (b)).[8] For $\forall n \geq 2$, $\bar{\delta}^n(n) = 1/2$ is the threshold of the highest segment length in the partition satisfying Theorem 1 (C). Given

[8]The numerical values are approximate values calculated by Wolfram *Mathematica* 8. The program code can be provided by a request.

$w = 1 - \epsilon$, only the 2×2-grid equilibrium is consistent with the condition $\delta^n(n) > 1/2$ as $\epsilon \to 0$ since $\delta^j(j) > \delta^k(k)$ for $2 \leq j < k$ by Lemma 2. Figure 6 shows that the maximum feasible number of equilibrium partitions does not changes monotonically. The range of w for more segments in the partition is truncated from both ends as the segment number increases. Consequently, the maximum number increases up to $n^*(w)$, then decreases as w rises. Recall that $n^*(p, w)$ is not infinite.

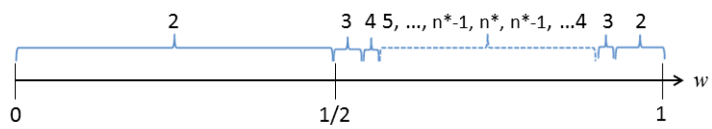


Figure 6: Maximum number of partitions in equilibrium

5.3 Welfare analysis

As the senders increase the number of steps in partitions, more finely divided cells provide the receiver with more precise information. Since the senders share the same interests with the receiver in non-binding cells, the increased accuracy in those cells raises the expected payoffs to the senders. However, it also clarifies conflicts with the receiver in binding cells. The remaining question is whether increased efficiency in the non-binding cells exceeds the interest divergence in their binding counterparts as the message space is more finely partitioned.

Proposition 3. *The Sender (Receiver) always strictly prefers the most informative equilibrium where an equilibrium message space consists of the maximum number of steps (cells): given $\{p_1, p_2, w\}$, EU^{*i} with a $n_1^* \times n_2^*$-grid $>$ EU^i with a $n_1 \times n_2$-grid for $n_1^* \times n_2^* > n_1 \times n_2$ and $i = R, S^1, S^2$.*

Proof. See appendix □

Proposition 3 shows that social welfare is maximized with the most finely divided grid in equilibrium. The benefit of more cells prevails over the cost since the cells are densely clustered below the constraint by a monotonicity condition. Both the senders and the receiver can benefit from this increased precision.

Preferences for more cells induce counterintuitive outcomes when combined with the non-monotone characteristics of maximum steps in equilibrium. I show how equilibrium welfare changes as a constraint becomes more severe. For comparison, I only consider the most informative equilibrium outcome for a given constraint level.

Theorem 2. *Given $\{p_1, p_2, w\}$, let $G(p_1, p_2, w)$ denote the maximum number of cells among equilibrium grids. Suppose that $i \in \{R, S^1, S^2\}$ gets $U^{i*}(p, w)$ in the most informative equilibrium. Fix p_1 and p_2, then $U^{i*}(p_1, p_2, w)$ increases in $w \in [0, w^*]$ and decreases in $w \in [w^*, max\{p_1, p_2\})$ where $G(p_1, p_2, w^*) > G(p_1, p_2, w)$.*

Proof. It holds by Corollary 1 and Proposition 3. □

Theorem 2 states that social welfare changes non-monotonically as a constraint level becomes less binding. The intuition behind Theorem 2 is clear once we recall that the number of equilibrium partitions reaches a peak at an intermediate level rather than at a low level of the constraint. The less binding constraints do not necessarily guarantee Pareto-superior results.

Figure 7 illustrates that given $p_1 = p_2 = 1$, how the expected utility chagnes as w increases. A solid line represents the expected utility in a 2×2-grid equilibrium and a dashed line represents the expected utility in a 3×3-grid equilibrium. Recall that a 3×3-grid equilibrium exists in a limited interval. There is a discountinuity at $w = 1$ since a fully revelation equilibrim exists when $w \geq 1$. The maximum expected utility in a severely constrained interval can not be larger than the maximum expected utility in a non-severely constrained interval since n^* is not infinite.

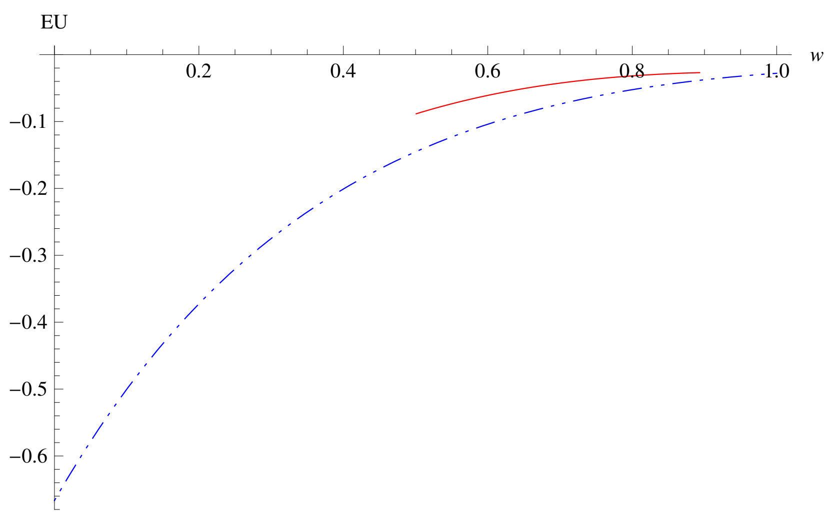


Figure 7: Maximum expected utility

6 Extension and discussion

Until now, I simplify the model to focus only on the effect of the constraint on information transmission. This section modifies and extends the model. I discuss implication of the variations in turn. Before discussion, I address two key assumptions that are fundamental to the basic model and further modification.

Uniformly distributed prior beliefs and Hyperplane constraints. Nonuniformity in prior belief with a linear constraint is in effect equivalent to uniform prior with a nonlinear constraint.

Then, what happens if constraints are not hyperplanes but any compact subsets of a state space. We can only answer a dichotomous question, the existence of a fully revealing equilibrium, already discussed by Ambrus and Takahashi who show that both direction and magnitude of biases decide the the existence. However, this model is hinged on the fact that the direction and magnitude of biases are not exogenously given but endogenously determined by the constraints. Note that the consistent directions of biases are only necessary for a partially revealing equilibrium.

Now, I modify the model one by one.

Multidimensional space. It does not change any key characteristics of the main results only to extend the state space from *two* to n dimension if the union of each sender's message set is equal to the state space and the intersection is the empty set, i.e., $\cup_i M_i = \Theta$ and $\cap_i M_i = \emptyset$. Conflicts of interests between the senders happen only through the receiver taking account of the constraint that causes interim biases whose directions are constant. The uniform prior belief leads to monotonicity in message partitions in a partially revealing equilibrium.

Overlapping dimensions. One might think if messages are overlapped in some dimensions, $\cap_i M_i \neq \emptyset$, the receiver can benefit from informational reaffirmation in those dimensions. However, interim biases lead into different directions senders who share the same interests in some dimensions but different preferences in others. As a result, the receiver gets different messages about even the overlapped dimensions.

Heterogenous preferences. Exogenous biases, b_i for Sender i, keep senders from revealing the state truthfully. However, the receiver can exploit the tensions between senders who have inconsistent interests but the same message space as in Krishna and Morgan. As opposed to the previous common interest cases, the receiver can get more information from more senders since some interim biases are aligned in the opposite directions.

In summary, a partially revealing equilibrium exists as long as prior beliefs are uniformly distributed and an action space is limited by a hyperplane. Given the constraint, the receiver would better get advices from senders who have different preferences each other rather than senders who shares the same interests in some dimensions and irrelevant in others.

Appendix

Proof of Lemma 1. U^R is concave since the Hessian matrix of the receiver's utility, $H(U^R)$ is negative semi-definite:

$$H(U^R) = \begin{bmatrix} U_{11}^R & U_{12}^R \\ U_{21}^R & U_{22}^R \end{bmatrix} = \begin{bmatrix} U_{11}^{S_1} & 0 \\ 0 & U_{22}^{S_2} \end{bmatrix}$$

If $\sum p_i \theta_i' \le w$, given $(m_1, m_2) = (\theta_1', \theta_2')$, the receiver takes an action, $(a_1^*, a_2^*) = (\theta_1', \theta_2')$ since $U_1^R((\theta_1', \theta_1'), (\theta_1', \theta_1')) = U_1^{S_1}(\theta_1', \theta_1') + U_1^{S_2}(\theta_2', \theta_2') = 0$. Otherwise, $a^* = \max_a U^R$ such that $\sum p_i a_i \le w$. Then, there exists interior solution a^* satisfying

$$\frac{U_2^R}{U_1^R}\bigg|_{a=a^*} = \frac{p_2}{p_1} \quad \text{and} \quad \sum p_i a_i^* = w$$

Otherwise, there is a corner solution.

Proof of Proposition 1. Without loss of generality, I focus on Sender 2. First, I show that a fully revealing equilibrium (FRE) exists $\Leftarrow$ a constraint is non-severe. Suppose that there does not exist a FRE. Then,

$$\begin{aligned} E(U_t) - E(U_d) = & \underbrace{-\int_0^{w-p\,\theta_2'-p\,\epsilon} U^{S_2}(\theta_2'+\epsilon, \theta_2')\, d\theta_1}_{>0} \\ & + \int_0^{1-w+p\,\theta_2'} [U^{S_2}(\theta_2' - \frac{p}{p^2+1}\theta_1, \theta_2') - U^{S_2}(\theta_2' + \frac{p^2}{p^2+1}\epsilon - \frac{p}{p^2+1}\theta_1, \theta_2')] d\theta_1 \\ & \underbrace{-\int_{1-w+p\,\theta_2'}^{1-w+p\,\theta_2'+p\,\epsilon} U^{S_2}(\theta_2' + \frac{p^2}{p^2+1}\epsilon - \frac{p}{p^2+1}\theta_1, \theta_2')\, d\theta_1}_{>0} \\ & \le 0 \end{aligned} \tag{5}$$

The first and the third terms on the right-hand side are positive by the assumption. To guarantee that the second term is negative, $1 - w + p\,\theta_2' > 0$ for $\forall\,\theta_2'$. Since $\theta_2' \ge 0$, $w < 1$. This contradicts that the constraint is non-severe.

Using contrapositive, I show that a FRE exists $\Rightarrow$ a constraint is non-severe. If a constraint is severe, $\exists\,\theta_2' > \frac{w}{p}$. In such cases, there does not exist a FRE since

$$\begin{aligned} E(U_t) - E(U_d) = & \int_0^1 [U^{S_2}(\theta_2' - \frac{p}{p^2+1}\theta_1, \theta_2') - U^{S_2}(\theta_2' + \frac{p^2}{p^2+1}\epsilon - \frac{p}{p^2+1}\theta_1, \theta_2')] d\theta_1 \\ & < 0 \end{aligned} \tag{6}$$

Proof of Theorem 1. A sufficient condition is easily proved by contrapositive. I show a necessary condition by following the proof of Theorem 1 of Crawford and Sobel (1982). The partition, ρ, is determined by (I.T), (A), and (C); (I.T.) are initial and terminal conditions. (A) is an arbitrage condition where the senders are indifferent between induced actions by the receiver for each partition at boundaries between partitions. Since Sender i considers uncertainty in an observation of the other sender, (A) does not form a second-order nonlinear difference equation as in CS arbitrage condition. (C) is an auxiliary condition to guarantee that senders follow the signaling rule defined by the theorem, especially at the highest interval. Let

$$M(\rho^1) \equiv \max\{j : \exists \rho \text{ s.t } 0 < \rho^1 < \rho^2 < \cdots < \rho^j \leq 1 \text{ satisfying (A) and (C)}\}$$

Since $\rho^{j+2} - \rho^j$ is bounded above zero for any solution (A) satisfying (C), $M(\rho^1)$ is finite and uniformly bounded. Let $n^* \equiv M(\rho^1_*) = \sup_{0<\rho^1\leq 1} M(\rho^1)$ for $\forall \rho^1 \in (0,1]$, then n^* is the maximum number of steps. Given $n = M(\rho^1)$, $M(\rho^1)$ is continuous at ρ^1 if ρ^n is less than unity since ρ^n varies continuously with ρ^1. Since $M(\rho^1)$ discontinuously varies by one and $M(\rho^1 = 1) = 1$, $1 \leq n \leq n^*$. Therefore, if $M(\rho^1) = n$ and $M(\rho^1)$ is discontinuous at ρ^1, ρ satisfies (I.T.), (A), and (C).

Second, the receiver's strategy is itself the best response to the message pair (m_1, m_2)

Finally, we show that the sender i follows the signaling rule in which $m_i^k \in (\rho_i^k, \rho_i^{k+1})$ is the best response to the receiver's action strategy for Sender i whose observation, θ_i', is located between ρ_i^k and ρ_i^{k+1}. For $0 \leq j \leq k \leq l \leq n$ and $\theta_i^k \in (\rho^k, \rho^{k+1})$

$$
\begin{aligned}
&\sum_{h=0}^{n_{-i}-1} [U^{S_i}(\alpha((\rho_i^k, \rho_i^{k+1}), (\rho_{-i}^h, \rho_{-i}^{h+1})), \theta_i^k) - U^{S_i}(\alpha(\rho_i^j, \rho_i^{j+1}), (\rho_{-i}^h, \rho_{-i}^{h+1})), \theta_i^k)](\rho_{-i}^{h+1} - \rho_{-i}^h) \\
&\geq \sum_{h=0}^{n_{-i}-1} [U^{S_i}(\alpha((\rho_i^k, \rho_i^{k+1}), (\rho_{-i}^h, \rho_{-i}^{h+1})), \rho_i^k) - U^{S_i}(\alpha(\rho_i^j, \rho_i^{j+1}), (\rho_{-i}^h, \rho_{-i}^{h+1})), \rho_i^k)](\rho_{-i}^{h+1} - \rho_{-i}^h) \\
&\geq 0 \\
&\sum_{h=0}^{n_{-i}-1} [U^{S_i}(\alpha((\rho_i^k, \rho_i^{k+1}), (\rho_{-i}^h, \rho_{-i}^{h+1})), \theta_i^k) - U^{S_i}(\alpha(\rho_i^l, \rho_i^{l+1}), (\rho_{-i}^h, \rho_{-i}^{h+1})), \theta_i^k)](\rho_{-i}^{h+1} - \rho_{-i}^h) \\
&\geq \sum_{h=0}^{n_{-i}-1} [U^{S_i}(\alpha((\rho_i^k, \rho_i^{k+1}), (\rho_{-i}^h, \rho_{-i}^{h+1})), \rho_i^k) - U^{S_i}(\alpha(\rho_i^l, \rho_i^{l+1}), (\rho_{-i}^h, \rho_{-i}^{h+1})), \rho_i^k)](\rho_{-i}^{h+1} - \rho_{-i}^h) \quad \geq 0
\end{aligned}
$$

Proof of Lemma 3. There are two equilibrium partitions, $\rho(n)$ and $\rho(k)$ s.t $n > k \geq 1$. Suppose that $\delta^i(k) \leq \delta^i(n)$. Since a sum of each segment is equal to one in each partition,

$$\delta^1(n) + \delta^2(n) + \cdots + \delta^n(n) = 1 \tag{7}$$

$$\delta^1(k) + \delta^2(k) + \cdots + \delta^k(k) = 1 \tag{8}$$

From (5)-(4)

$$(\delta^1(k) - \delta^1(n)) + (\delta^2(k) - \delta^2(n)) + \cdots + (\delta^k(k) - \delta^k(n)) = \delta^{k+1}(n) + \cdots + \delta^n(n) \qquad (9)$$

The left side of the equation is not positive while the right side is not negative. Therefore, $\rho^i(n) < \rho^i(k)$ since $\delta^i(k) > \delta^i(n)$

Proof of Lemma 4. Given $\{p_1, p_2, w\}$ a space is separated by a budget constraint $p_1\rho_1^{n_1-1} + p_2\rho_2^{n_2-1} = w$. For simplicity, I focus on Sender 2. By monotonicity of Lemma 3, $\rho_2^{n_2-1}(n_2)$ is increasing function of n_2. Then, there exists n_2^* satisfying that $p_1\rho_1^{n_1-1} + p_2\rho_2^{n_2^*-1} \leq w$ and $p_1\rho_1^{n_1-1} + p_2\rho_2^{n_2^*} > w$.

Proof of Proposition 3. Let $\rho_x \equiv (\rho_x^0, a_x^1, \ldots, a_x^{n_1+1})$ be the partition that satisfies (A) for $j = 2, \ldots, n_1$ with $a_x^0 = 0, a_x^{n_1} = x$, and $\rho_x^{n_1+1} = 1$. If $x = \rho^{n_1-1}(n_1)$, then $\rho_x = 0$, and if $x = \rho^{n_1}(n_1+1)$ then $\rho_x = \rho(n_1+1)$ and (A) is satisfied for all $j = 1, \ldots, n_1$. Let ρ_y have same characteristics for $k = 2, \ldots, n_2$. Then, the expected utility of Sender 1 is

$$EU^{S_1}(x,y) = \sum_{j=0}^{n_1}\sum_{k=0}^{n_2}\int_{\rho_x^j}^{\rho_x^{j+1}} U^{S_1}(\alpha((\rho_x^j, \rho_x^{j+1}), (\rho_y^k, \rho_y^{k+1})), \theta_1)(\rho_y^{k+1} - \rho_y^k)d\theta_1$$

$$\begin{aligned}\frac{dEU^{S_1}(x)}{dx} &= \sum_j\sum_k \frac{d\rho_x^j}{dx}[U^{S_1}(\alpha((\rho_x^{j-1}, \rho_x^j), (\rho_y^k, \rho_y^{k+1})), \rho_x^j) - U^{S_1}(\alpha((\rho_x^j, \rho_x^{j+1}), (\rho_y^k, \rho_y^{k+1})), \rho_x^j)](\rho_y^{k+1} - \rho_y^k) \\ &\quad + \sum_{j=0}^{n_1}\sum_{k=0}^{n_2}\frac{d\alpha}{dx}\int_{\rho_x^j}^{\rho_x^{j+1}} U_1^{S_1}(\alpha((\rho_x^j, \rho_x^{j+1}), (\rho_y^k, \rho_y^{k+1})), \theta_1)(\rho_y^{k+1} - \rho_y^k)d\theta_1\end{aligned}$$

The first term on the right-hand side is positive by definition of ρ and Lemma 2. The second term is nonnegative since $d\alpha/dx > 0$ by Lemma 2, and the integral expressions are all nonnegative by our assumption that $U_{12}^{S_1} > 0$. By the first-order conditions that determine the receiver's optimal choice of $\alpha(\rho_x^j, \rho_x^{j+1})$. Without loss of generality, we could derive the same result for Sender 2. The expected utility for the receiver is

$$EU^R(x,y) = \sum_{j=0}^{n_1}\sum_{k=0}^{n_2}\int_{\rho_y^k}^{\rho_y^{k+1}}\int_{\rho_x^j}^{\rho_x^{j+1}} U^R(\alpha((\rho_x^j, \rho_x^{j+1}), (\rho_y^k, \rho_y^{k+1})), (\theta_1, \theta_2))d\theta_1 d\theta_2$$

By the envelope theorem

$$\begin{aligned}\frac{d^2EU^R(x,y)}{dydx} = \sum_j\sum_k f_{1,2}(\rho_x^j, \rho_y^k)\frac{d\rho_x^j}{dx}\frac{d\rho_y^k}{dy}[&U^R(\alpha((\rho_x^{j-1}, \rho_x^j), (\rho_y^{k-1}, \rho_y^k)), (\rho_x^j, \rho_y^k)) \\ &- U^R(\alpha((\rho_x^j, \rho_x^{j+1}), (\rho_y^k, \rho_y^{k+1})), (\rho_x^j, \rho_y^k))]\end{aligned}$$

By definition of ρ,

$$\frac{d\rho_x^j}{dx} \geq 0 \text{ and } \frac{d\rho_y^k}{dy} \geq 0$$

By assumption

$$
\begin{aligned}
&U^R(\alpha((\rho_x^{j-1}, \rho_x^j), (\rho_y^{k-1}, \rho_y^k)), (\rho_x^j, \rho_y^k)) - U^R(\alpha((\rho_x^j, \rho_x^{j+1}), (\rho_y^k, \rho_y^{k+1})), (\rho_x^j, \rho_y^k)) \\
&= [U^{S_1}(\alpha_1((\rho_x^{j-1}, \rho_x^j), (\rho_y^{k-1}, \rho_y^k)), \rho_x^j) - U^{S_1}(\alpha_1((\rho_x^j, \rho_x^{j+1}), (\rho_y^k, \rho_y^{k+1})), \rho_x^j)] \\
&+ [U^{S_2}(\alpha_2((\rho_x^{j-1}, \rho_x^j), (\rho_y^{k-1}, \rho_y^k)), \rho_y^k) - U^{S_2}(\alpha_2((\rho_x^j, \rho_x^{j+1}), (\rho_y^k, \rho_y^{k+1})), \rho_y^k)]
\end{aligned}
$$

By Lemma 2,

$$
U^{S_1}(\alpha_1((\rho_x^{j-1}, \rho_x^j), (\rho_y^{k-1}, \rho_y^k)), \rho_x^j) \geq U^{S_1}(\alpha_1((\rho_x^j, \rho_x^{j+1}), (\rho_y^k, \rho_y^{k+1})), \rho_x^j)
$$

$$
U^{S_2}(\alpha_2((\rho_x^{j-1}, \rho_x^j), (\rho_y^{k-1}, \rho_y^k)), \rho_y^k) \geq U^{S_2}(\alpha_2((\rho_x^j, \rho_x^{j+1}), (\rho_y^k, \rho_y^{k+1})), \rho_y^k)
$$

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