Firm behaviour in price-quantity oligopolies: An experimental study of the mixed strategy equilibrium

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Abstract

We study oligopoly games with firms competing in prices and quantities simultaneously. We systematically compare our experimental results to the theoretical predictions using the mixed strategy equilibria for linear demand functions. Subjects' price choices are mainly between marginal cost and monopoly level but do not follow the equilibrium distribution. Although average prices and profits are above theoretical values, we do not observe a high level of collusion as expected in the literature. By comparing simulations based on the mixed strategy equilibrium to our experimental outcomes, we conclude that in this game price setting can be explained by strategic reaction to preceding round results. In contrast to the equilibrium prediction, we observe a decrease in prices and negative average profits for the triopoly game.

Keywords: Price-Quantity Competition, Mixed Strategy Equilibrium, Experimental Economics, Learning Direction Theory

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1. Introduction

The first systematic analysis of economic games that include both prices and quantities as strategic market interaction variables was conducted by Shubik (1955). The study of this type of games was the logical enhancement of the intensive research on pure quantity competition (Cournot, 1838) and pure price competition (Bertrand, 1883) in the systematic investigation of oligopolistic competitions. The price-quantity (PQ) games are also known as games of price competition with perishable goods and production in advance and are characterised by the absence of pure strategy equilibria. The first mixed strategy equilibrium was presented in Levitan and Shubik (1978), where a game with linear demand and positive inventory carrying cost is studied. The mixed strategy equilibrium for PQ games with non-increasing production costs was established in Gertner (1986).

In contrast to the *classical* PQ game, Kreps and Scheinkman (1983) argue that prices are more flexible in the short run than quantities. Their model therefore contains a simultaneous capacity choice before a simultaneous price competition. They show that (under mild assumptions about demand) the unique equilibrium outcome in this game is identical to the Cournot outcome. Davidson and Deneckere (1986) study a similar game with more general rationing rules and derive that the equilibrium outcome is more competitive than Cournot with firms using mixed strategies to set prices.

Whilst the argument of Kreps and Scheinkman (1983) might be true for a number of goods, the PQ game is still applicable for markets with perishable goods that need to be produced in advance (Davis, 2013). For these markets the PQ game can be interpreted as a price competition with sufficiently large capacities.

Although the literature contains a number of experimental studies on oligopoly games (need examples here!), see Engel (2007) for a comprehensive overview, models with simultaneous quantity and price choices have hardly been analysed in the laboratory. One main exception is Brandts and Guillen (2007), where collusion in repeated duopoly and triopoly games with fixed groups is studied. They observe that markets tend to monopolistic prices as a consequence of either bankruptcy or collusion.

In this article, we study a PQ game, with constant marginal cost. Whilst the game is easy to understand, it comprises a non-trivial mixed strategy equilibrium (Gertner, 1986) and is therefore ideal for an experimental study. We analyse repeated duopoly and triopoly PQ games with a linear demand and without exogenous capacity constraints. In contrast to Brandts and Guillen (2007) we study downward sloping demand and thereby hope to contribute to the body of experimental literature on classical oligopolistic competitions. Additionally, we provide the first systematic comparison between experimental data and the mixed strategy equilibrium of the PQ game. In Cracau and Franz (2011) we have shown that an experimental analysis of this game is indeed appropriate, as the mixed strategy equilibrium of the discrete (experimental) game converges towards the equilibrium of the continuous game as the discretisation becomes finer.

Our experimental results indicate that benchmarks from the Cournot, the Bertrand and the mixed strategy equilibrium do not predict subjects' price choices satisfactory. Although subjects chose prices seemingly at random from a range between the monopoly price and marginal cost, the price distribution significantly differs from the mixed strategy equilibrium predicted in Gertner (1986). We additionally observe a difference between the price choices of preceding round winners and losers, with the latter tending to decrease their prices. Using simulations of the mixed strategy equilibrium and a regression analysis, we are able to show that this effect is significant and cannot be explained by the equilibrium.

The article is organised as follows: Section 2 introduces the basic model of the PQ game. The experimental procedure is presented in Section 3 and the experimental outcome follows in Section 4. In Section 5 we discuss these outcomes in comparison to the related literature and finally Section 6 briefly concludes and explores ideas for future investigations.

2. The model

In this section we present the general model used for the experiments along with theoretical results from the literature. We start by explaining the duopoly game, before stating some results for a general game with n (> 2) firms. Let us therefore initially consider a game of two firms ($i = 1, 2$) that decide simultaneously on their price p_i and their production level q_i. Products are assumed to be homogeneous between the firms and the market demand is a given function $D(p)$. The game follows the winner-takes-all-rule, i.e. the firm i with the lower price sells its full output q_i up to the market demand $D(p_i)$. The firm j ($j \neq i$) that decided on the higher price can now satisfy the residual demand, which is given through the efficient rationing rule

$$D(p_j|p_i) = D(p_j) - s_i \,,$$

where s_i is the amount sold by the lower-price competitor i.[1] For the case of equal prices ($p_1 = p_2$), the market demand is shared equally between the firms, as far as the production levels q_i allow. These rules can be summed up by the following equation for the sales s_i of firm i (Gertner, 1986),

$$s_i(p_1, q_1, p_2, q_2) = \begin{cases} \min[q_i, D(p_i)] & \text{, if } p_i < p_j \,, \\ \min\,[q_i, D(p_i) - s_j] & \text{, if } p_i > p_j \,, \\ \min\left[q_i, D(p_i) - \min\left\{q_j, \frac{D(p_j)}{2}\right\}\right] & \text{, if } p_i = p_j \,. \end{cases} \tag{1}$$

To find an expression for the payoff π_i of firm i we introduce the production cost $C(q)$, which is assumed to be equal for both firms. Using s_i as given in (1), the payoff π_i is given by

$$\pi_i = p_i s_i - C(q_i) \,.$$

Gertner (1986) explains that a pure strategy equilibrium does not exist in this game. Hence, we focus on a mixed strategy equilibrium, i.e. each of the firms' strategies can be described by the probability density function $g_i(p_i, q_i)$ that formally states the probability of firm i to play the strategy (p_i, q_i). If we denote by G_i the probability distribution function related to g_i, then,

[1] Davidson and Deneckere (1986) discuss different rationing rules. In general, the choice of the rationing rule can have a major impact on the equilibrium of an oligopoly game. For the model presented here, however, Gertner (1986) shows that the results are not affected by choosing efficient rationing instead of proportional rationing.

according to Shubik (1959) $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ form a mixed strategy equilibrium, if the integrals

$$\bar{V}_i = \int_0^\infty \int_0^\infty \pi_i(p_1, q_1, p_2, q_2) dG_j(p_j, q_j)\,,$$

are constant for all strategies (p_i, q_i) played with positive probability according to $G_i(p_i, q_i)$. Shubik (1959) refers to $\bar{V}_i$ as the value of the game for firm i, i.e. the maximum guaranteed profit it can achieve if the strategy of the opposition player is known. Note that in the case of the symmetric game considered here, the mixed strategy equilibrium is also symmetric, which means $G_1 \equiv G_2$. For our experiments we make the following simplifying assumptions of linear demand and cost curves:

$$D(p_i) = a - bp_i\,, \qquad C(q_i) = cq_i\,,$$

where a, b and c are non-negative constants. We are therefore considering a game with constant marginal cost, for which Gertner (1986) proved that all Nash equilibria satisfy $\bar{V}_i = 0$. The mixed strategy equilibrium derived in Gertner (1986) has the property that all strategies with positive probabilities are situated on the line $p = D(q)$, i.e. each firm always produces exactly the market demand $D(p_i)$ corresponding to the chosen price p_i. The probability distribution for the prices is given through the distribution function

$$F(p) = \begin{cases} 0\,, & \text{for } p < c\,, \\ 1 - c/p\,, & \text{for } c \leq p < a\,, \\ 1\,, & \text{for } p \geq a\,. \end{cases} \tag{2}$$

In particular, this implies that each firm has two options: (i) it can leave the market by choosing $p_i = a$ with a (non-zero) probability of c/a or (ii) it can stay in the market and choose a price from the interval $[c, a)$ using the distribution function $F(p)$ as given in (2). Looking at the probability density function corresponding to $F(p)$, we see that firms are more likely to play lower prices than higher prices. The lower price firm earns a positive profit, while the other firm faces losses equal to its production costs $C(q)$, but expected profits are equal to zero.

One can easily generalise the rules of the game for an arbitrary number ($n \geq 2$) of firms. The existence of a mixed strategy equilibrium can be generalised from the duopoly to the oligopoly game (Gertner, 1986). The distribution function related to the mixed strategy equilibrium in the oligopoly settings takes the form

$$F_n(p) = \begin{cases} 0\,, & \text{for } p < c\,, \\ 1 - (c/p)^{\frac{1}{n-1}}\,, & \text{for } c \leq p < a\,, \\ 1\,, & \text{for } p \geq a\,. \end{cases} \tag{3}$$

In particular, this implies that with increasing n the probability of market entry decreases, but the average price played in case of market entry increases. Similarly to the duopoly game, the expected profit for each of the firms is zero.

3. Experimental Procedure

Our experiment was designed to analyse the classical PQ game in a duopoly (*PQ2*) and triopoly (*PQ3*) treatment. At the beginning of the experiment, we randomly assigned subjects to groups of 2 or 3 that remained fixed for the rest of the experiment with each of the subjects in a group controlling one of the symmetric firms A, B (or C). Each treatment consists of a two-stage game. In the five rounds of the first stage, we let each firm act in a monopolistic market to allow the participants time to get used to the game. Afterwards, in the 20 rounds of the second stage, firms competed in a common (duopoly or triopoly) market.

We used the simplified linear demand function $D(p) = 100 - p$ ($a = 100, b = 1$) and the constant marginal production cost $c = 10$. Subjects now had to choose prices and production levels in the range $[0, 100]$.[2] For both price and quantity choices we allowed for 0.001 increments. This small increment was chosen, because we have shown that the mixed strategy equilibrium in our discrete PQ game converges to the one in the continuous game if the increment is sufficiently small (Cracau and Franz, 2011).

At the beginning of each stage, we gave subjects a what-if-calculator to help them get comfortable with the residual demand and profit calculation. After the subjects' simultaneous decisions, profits were calculated according to the model presented in Section 2. Then, all players were shown a summary with prices, production levels and profits. At the end of the experiment, subjects' total payoff consists of the sum of the payoff of all 25 rounds. Because subjects earned a starting budget from the monopoly stage, bankruptcy during the course of the game was not considered.[3] In the *PQ3* treatment, we provided a lump-sum payment of 3 Euro at the end of the experiment to compensate for low payoffs.

[2] For reasons of simplicity, production levels were limited to demand at the chosen price. Thus, choosing a price equal to 100 automatically corresponds to a market exit.

[3] For subjects with a negative total balance at the end of the second stage, we set earnings equal to zero (2 subjects in the *PQ3* treatment.)

We collected ten independent observations in the *PQ2* and nine independent observations in the *PQ3* treatment during three sessions in June 2011 and July 2012 at the MaXLab experimental laboratory at the University of Magdeburg. The experimental software was programmed using z-Tree (Fischbacher, 2007). Participants were mainly students from economic fields, recruited via ORSEE (Greiner, 2004). On average, the participants in the *PQ2* treatment earned 10.69 Euro and the participants in the *PQ3* treatment earned 9.18 Euro in a 45-minute session.

Table 1: Treatment parameter

	PQ2	*PQ3*
number of firms	2	3
number of observations	10	9

4. Results

In the first (monopolistic) stage of the game, subjects earned in total 88% (96% in the last round) of the possible monopoly profits. As this is in line with the literature (Potters et al., 2004), we conclude that all participants understood the experimental procedure and produced reliable observations.

Tables 2 and 3 summarise the experimental results for both treatments. As firms could sell quantities at different prices subject to the rationing rule, we follow Brandts and Guillen (2007) in presenting the average weighted market prices (AWP). Thereby, the prices at which units are sold are weighted by their respective market shares. Moreover, we present total production and total profits.

Table 2: *PQ2* — Summary of experimental results for all 20 rounds.

Obs.	AWP	Production	Profits	Collusion
PQ2-1	31.50	106.80	850.95	no
PQ2-2	17.88	103.20	192.52	no
PQ2-3	52.51	58.90	1838.92	yes
PQ2-4	23.44	134.26	309.70	no
PQ2-5	15.34	153.42	-256.30	no
PQ2-6	26.17	120.80	573.30	no
PQ2-7	24.57	105.91	527.25	no
PQ2-8	23.50	119.20	455.50	no
PQ2-9	18.95	114.50	107.46	no
PQ2-10	48.10	66.25	1757.50	yes
Av.	28.20	108.32	635.68	-

To evaluate these outcomes, we calculate benchmarks corresponding to the Cournot and Bertrand equilibrium, the mixed strategy equilibrium and the cooperative solution of the game. The benchmarks are presented in Table 4.

For the *PQ2* treatment (Table 2, we see that observations PQ2-3 and PQ2-10 have lower production levels and higher average profits than the other observations. As these two observations are close to the cooperation benchmark, we identify them as collusive. Once both participants have agreed on a price at the or close to the monopoly level, the demand is shared equally between the parties. This yields high profits for both players. For the remaining eight observations, we do not observe cooperation, indicated by the

Table 3: *PQ3* — Summary of experimental results for all 20 rounds.

Obs.	AWP	Production	Profits	Collusion
PQ3-1	11.66	200.73	-983.93	no
PQ3-2	17.15	126.43	-200.94	no
PQ3-3	17.67	108.95	114.07	no
PQ3-4	12.53	219.40	-1159.28	no
PQ3-5	13.31	147.04	-374.99	no
PQ3-6	19.70	107.50	291.32	no
PQ3-7	16.87	183.22	-511.58	no
PQ3-8	12.63	132.58	-480.85	no
PQ3-9	15.19	129.82	-126.77	no
Av.	15.19	150.63	-381.44	-

low AWP and profits. In the *PQ3* treatment, we see no cooperation at all.

The average AWP for the competitive pairs in the *PQ2* treatment is 22.67 and thus between the Cournot and Bertrand prediction. Moreover, it seems to be close to the prediction of the mixed strategy equilibrium. However, Figure 1 illustrates that the distribution of prices differs visibly from the mixed strategy equilibrium prediction given in (2). A two-sample Kolmogorov-Smirnov test proves the cumulative density function of the observed prices to be significantly different from the equilibrium prediction ($p = 0.001$). We observe a greater fraction of prices in the range $[10, 55]$ (i.e. between marginal cost and the monopoly level) than predicted. In total, we only observe 18 out of 320 ($\approx 5.6\%$) prices above the monopoly / cartel price $p = 55$, compared to the 18.2% predicted by the mixed strategy equilibrium, which suggests that

Table 4: Theoretical benchmarks.

Prediction	AWP	Production	Profits
Cournot (duopoly)	40	60	1800
Cournot (triopoly)	32.5	67.5	1518.75
Bertrand	10	90	0
mixed strategy (duopoly)	19	134.95	0
mixed strategy (triopoly)	23.68	140.26	0
cooperation	55	45	2025

subjects perceive prices above the monopoly level as implausible. Whilst the equilibrium predicts 10% market exits, we do not observe these frequently. In particular, prices equal to 100 were not observed at all, but we observe 9 decisions with quantities equal to zero which we also denote as a market exit.

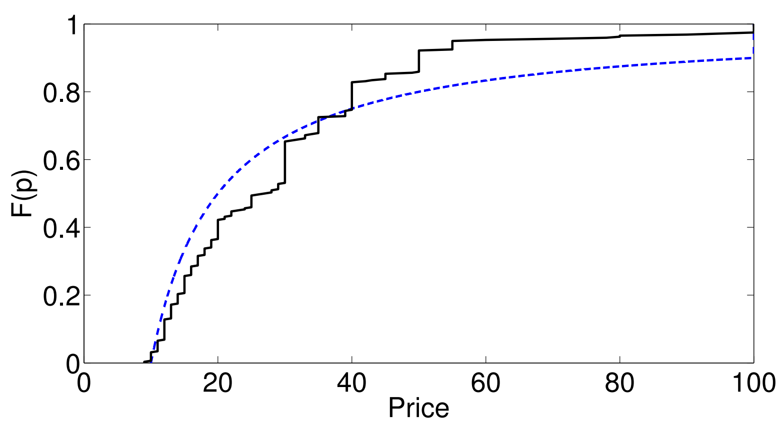


Figure 1: *PQ2* — Price distribution. Observations (solid line), prediction (dashed line).

In order to analyse various trends in the behaviour of the participants in the *PQ2* treatment, we now look at the time development of the market description values presented in Table 2. First, the evolution of the AWP over time is presented in Figure 2. We see no significant trend, except for a drop in prices during the first 3 rounds. This initial drop occurs, because the players were biased towards the monopoly price $p = 55$ from the first stage of the game. This bias, however, disappears quickly as the participants get used to the new situation and the AWP stays on the lower level.

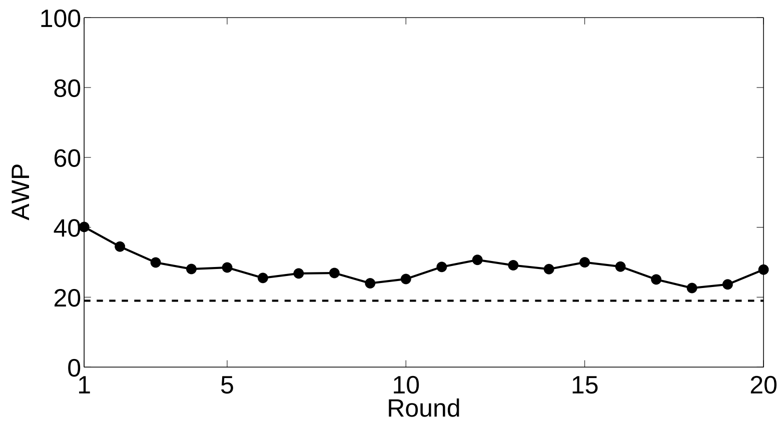


Figure 2: *PQ2* — Average weighted price. Observations (solid line), prediction (dashed line).

The development of profits over time in the *PQ2* treatment can be found in Figure 3. We, again, see no significant trend. We observe that the total payoff for participants in the non-collusive pairs is positive in 14 out of 16 cases. The positive average profit in the experiment (172.52) stands in contrast to the zero profit predicted by the equilibrium. Using a one-sided Wilcoxon Signed-Rank test, this difference proves to be significant ($p = 0.02$).

Figure 4 shows the evolution of the market production in the *PQ2* treatment, where we see no trend in time. The average market production for the competitive pairs is 119.76, which is below the expected value of the mixed

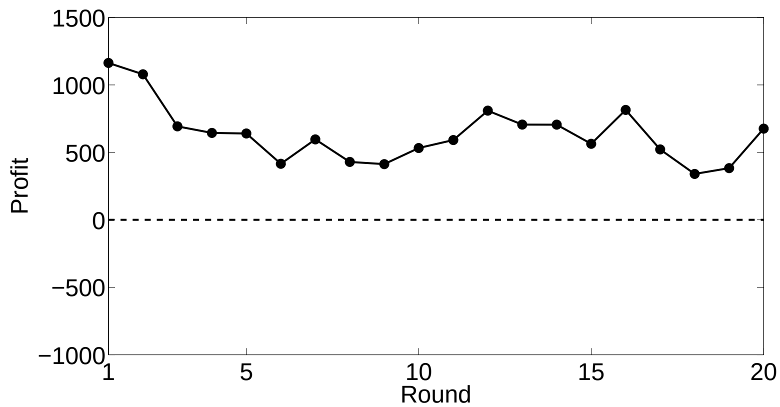


Figure 3: *PQ2* — Average profits. Observations (solid line), prediction (dashed line).

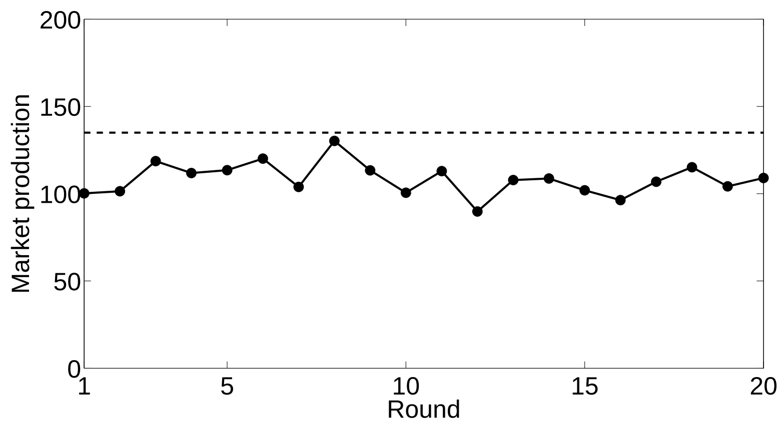


Figure 4: *PQ2* — Total production. Observations (solid line), prediction (dashed line).

strategy equilibrium but still above the Cournot and Bertrand predictions and even above total market size. We observe overproduction because firms had to decide on their production level before knowing their actual demand. The mixed strategy equilibrium predicts the production to be equal to the demand at the price in the same decision ($q = D(p)$). In the experiments, we see that only 46% of the production decisions satisfied this condition. On average the value of $q/D(p)$ in the *PQ2* treatment is 0.83, i.e. players are looking to satisfy 83% of the market demand at their chosen price. This resulted in a positive residual demand for the higher price firm in 27% of all rounds.[4]

We find no significant difference in profits between players who chose to produce the full market demand and players who did not. We therefore conclude that subjects had no disadvantage from deviating from the equilibrium condition $q = D(p)$. We observe a strong correlation between price and quantity choices (correlation coefficient = 0.67), which can be explained by the linear demand function.

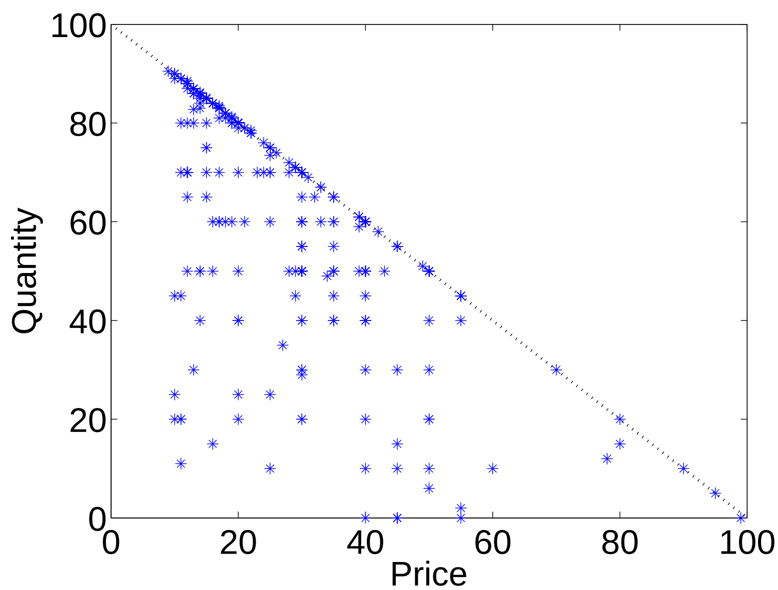


Figure 5: *PQ2* — PQ pairs. Observations (blue stars), prediction (dashed line).

[4]In the rest of the rounds, the price of the high price firm was too high to guarantee any residual demand.

Result 1. *In the duopoly treatment, the observed behaviour differs markedly from the equilibrium predictions, as can be seen in the different price distribution, the lower than expected production levels and the positive average profits.*

For the *PQ3* treatment, Figure 6 illustrates that the distribution of prices does not fit the mixed strategy equilibrium prediction given in (3) ($n = 3$). In contrast to the predicted 31.62% market exits, we only observe market exit decisions in 13 out of 540 choices in this treatment. As in our duopoly treatment, we observe the vast majority of prices in the range $[10, 55]$. In total, we only observe 35 out of the 580 ($\approx 6\%$) prices above the monopoly price $p = 55$.

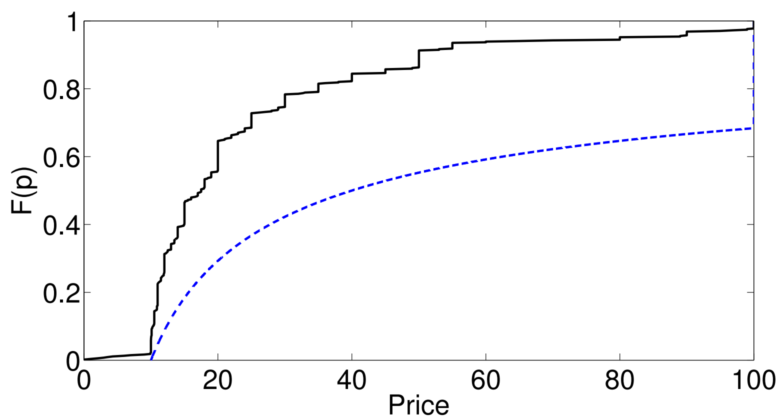


Figure 6: *PQ3* — Price distribution. Observations (solid line), prediction (dashed line).

The market dynamics in the *PQ3* treatment are comparable to those in the duopoly treatment. Figure 7 illustrates the development of the AWP over time. Similar to the *PQ2* treatment, we see a drop in prices during the first rounds but no further significant trend. Overall, the AWP stays on a significantly lower level than in the duopoly treatment (MWU test, $p < 0.001$).[5]

Result 2. *In contrast to the equilibrium prediction, prices in the triopoly treatment were lower than in the duopoly treatment.*

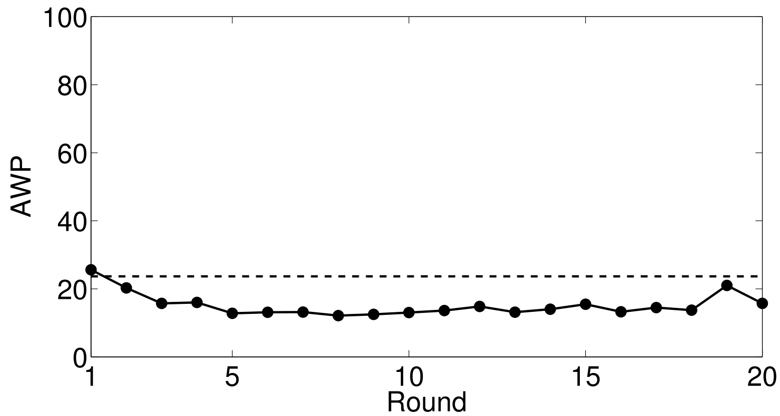


Figure 7: *PQ3* — Average weighted price. Observations (solid line), prediction (dashed line).

Figure 8 shows the profits over the 20 rounds in the *PQ3* treatment. Except for the first round, profits are negative. This difference to the equilibrium prediction of zero profits is significant (one-sided Wilcoxon Signed-Rank test $p = 0.02$). We see a slight positive trend with profits seeming to converge to zero. Overall, we observe that the total payoff is negative for 20 out of 27 participants.

Figure 9 shows the evolution of the market production in the *PQ3* treatment. We see a negative trend after the first periods. Total production is

[5]We observe the same effect for individual price choices, with the average price in the *PQ2* treatment (28.74) being higher than in the *PQ3* treatment (22.92).

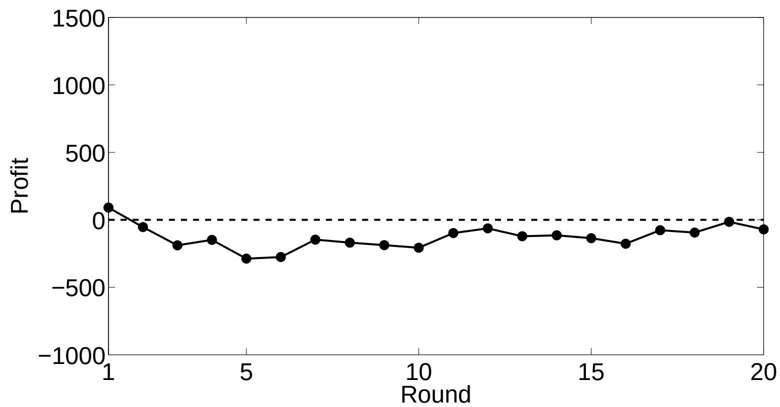


Figure 8: *PQ3* — Average profits. Observations (solid line), prediction (dashed line).

above market size but close to the mixed strategy equilibrium prediction. In this treatment only 28% of the decisions satisfied the equilibrium condition $q = D(p)$. In this treatment the average value of $q/D(p)$ is 0.65 and therefore lower than in the *PQ2* treatment.

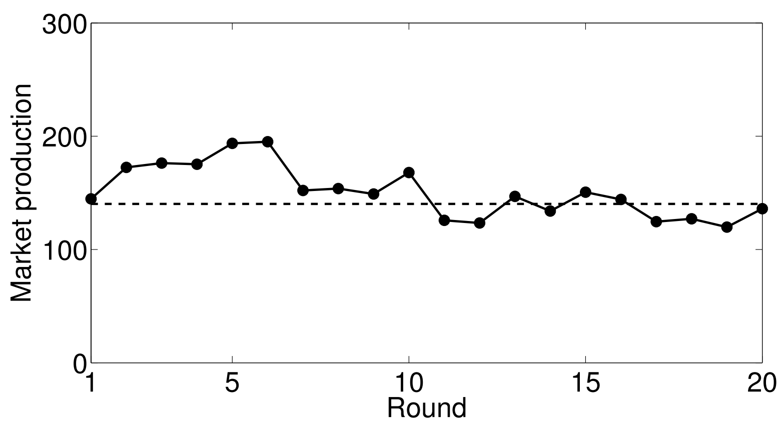


Figure 9: *PQ3* — Total production. Observations (solid line), prediction (dashed line).

Result 3. *In the triopoly treatment, the observed behaviour differs markedly from the equilibrium predictions, as can be seen in the different price distri-*

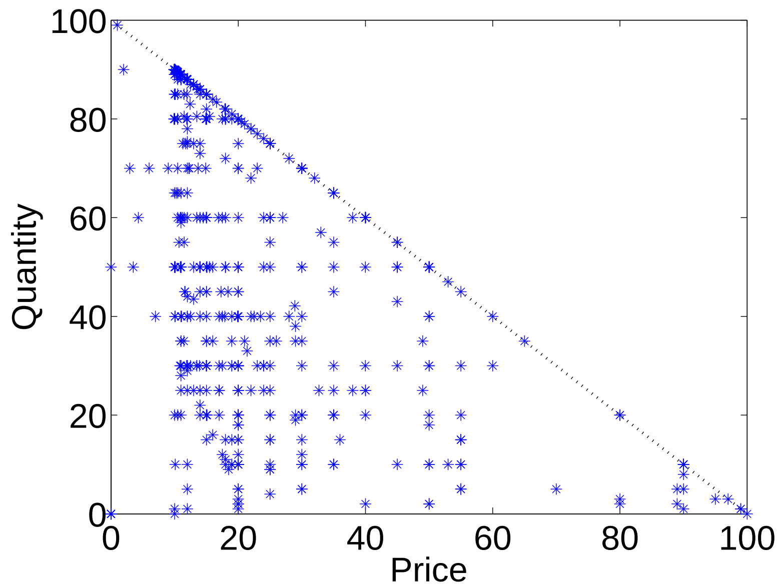


Figure 10: *PQ3* — PQ pairs. Observations (blue stars), prediction (dashed line).

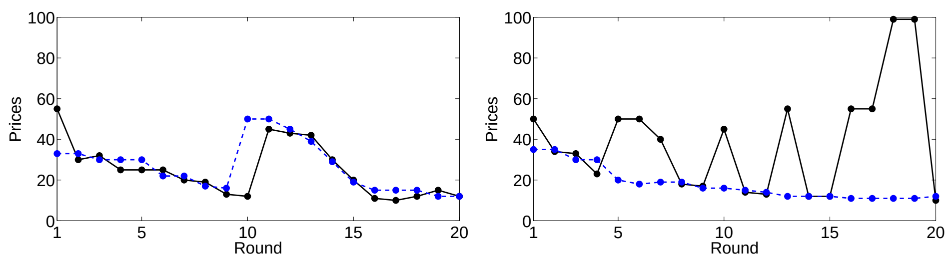
bution, the low production levels and the negative average profits.

5. Discussion with respect to the literature

For the very first time in experimental economics, we conducted an experiment with simultaneous price and quantity choice and linear demand. Previous studies of the PQ game have not involved linear demand (Brandts and Guillen, 2007; Davis, 2013). Brandts and Guillen (2007) analyse the PQ game in a dynamic setting with inelastic demand. Their results of markets with two and three firms show a price development coming close to the monopoly level due to either collusion or bankruptcy. Davis (2013) conducts an experiment to evaluate the effect of advance production in Bertrand-Edgeworth duopolies with step-wise demand. He concludes that the introduction of advance production reduces profits, which can be partially explained by the reduction of tacit collusion. Note that in economic terms, advance production is comparable to the costly production in the PQ game. Therefore, this finding relates to our duopoly treatment yielding lower levels of collusion and lower profits than a standard Bertrand experiment, see for example Dufwenberg and Gneezy (2000) and Muren (2000).

We are also the first to consider the exact formulation of the underlying mixed strategy equilibrium prediction and compare it to the experimental outcomes. Although we find a dispersion of prices and a mixture of positive and negative profits in both treatments, subjects' price and quantity choices do not match the distributions predicted by the equilibrium. In contrast to the equilibrium prediction, prices in the *PQ3* treatment are significantly lower than in the duopoly treatment. The finding that markets with more firms yield stronger competition is well aligned with the experimental literature, see for example Dolbear et al. (1968) and Huck et al. (2004) for quantity competitions, Dufwenberg and Gneezy (2000) and Abbink and Brandts (2008) for price competitions as well as Brandts and Guillen (2007) for a PQ competition.

Overall, we argue that the mixed strategy equilibrium does not adequately describe the price choices made by the players in our experiment. This finding is in line with Palacios-Huerta and Volij (2008) who show that subjects inexperienced in real life tasks with mixed strategy equilibria fail to play even simple mixed strategies. The weak prediction power of the mixed strategy equilibrium in oligopoly games was also shown in Davis and Holt (1994) and Brown-Kruse et al. (1994). The latter study contains a capacity constrained Bertrand-Edgeworth game. Prices tend to fall in the first periods and then show a dispersion. This dispersion in prices can be better explained by the Bertrand-Edgeworth cycle theory than by mixed strategies. We identify an Edgeworth cycle in one of our observations (PQ2-2, see Figure 11(a)), whilst the pricing behaviour in the remaining observations cannot be explained satisfactorily by this theory (e.g. PQ2-3, see Figure 11(b)).



(a) Edgeworth-cycle in PQ2-3

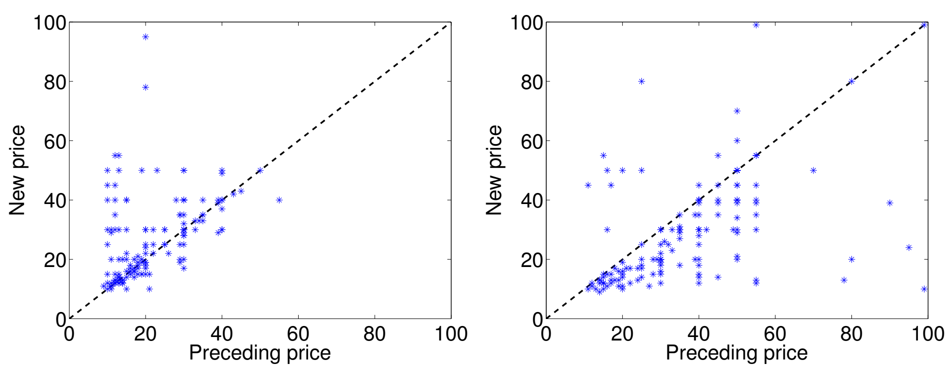
(b) Fight in PQ2-2

Figure 11: *PQ2* — Fight and Edgeworth-cycle price behaviour. Price choice of player A (solid line) and player B (dashed line).

To gather deeper understanding of the choices observed in the experiment, we suggest that subjects may react to the result of the previous rounds rather than choosing independently at random from a mixed strategy. We support this by differentiating subjects' price changes in the *PQ2* treatment from one round to the next in dependence of the outcome of the previous round.[6] Figure 12 contrasts the price decisions of preceding round's winners and losers for this treatment. One can clearly see that preceding round's winners tend to increase their prices, whereas the majority of losers decrease prices. On average, winners increase their prices by 5.14 while losers decrease their prices

[6]The following analysis concentrates mainly on price choices, but similar effects can be observed for the quantity choices due to the high correlation between these.

by 7.22. A χ^2 test of independence - based on the absolute frequencies as shown in Table 5 - proves this finding to be highly significant ($p < 0.0001$). This result is in line with previous findings, see for example Neugebauer and Selten (2006) or Ockenfels and Selten (2005) for first-price sealed-bid auctions and Bruttel (2009) for a Bertrand duopoly. In Table 6 we study the dependence on previous rounds outcome using regression analysis. We model subjects' price choices in dependence of the previous round price choices and a dummy, $LOSS$, that is 1 if the subject lost the previous round and 0 otherwise. We can see in Table 6 that preceding round losers *ceteris paribus* chose significantly lower prices than preceding round winners. The same reasoning holds for the triopoly treatment (see Figure 13 and Tables 7 and 8. From this we conclude that winning / losing the previous round has a major impact on the price choice of a participant. This result contributes to the learning direction theory (Selten and Stoecker, 1986 and Selten and Buchta, 1999).



(a) Preceding round's winner (b) Preceding round's loser

Figure 12: *PQ2* — Price reactions depending on the preceding round's outcome.

To further support the idea of the learning direction theory, we simu-

Table 5: *PQ2* — Price reactions (absolute frequencies).

	price increase	price decrease	no price change	$\sum$
preceding winner	76	53	20	149
preceding loser	13	111	25	149
preceding ties	6	10	66	82
$\sum$	95	174	111	380

Table 6: *PQ2* — Regression results

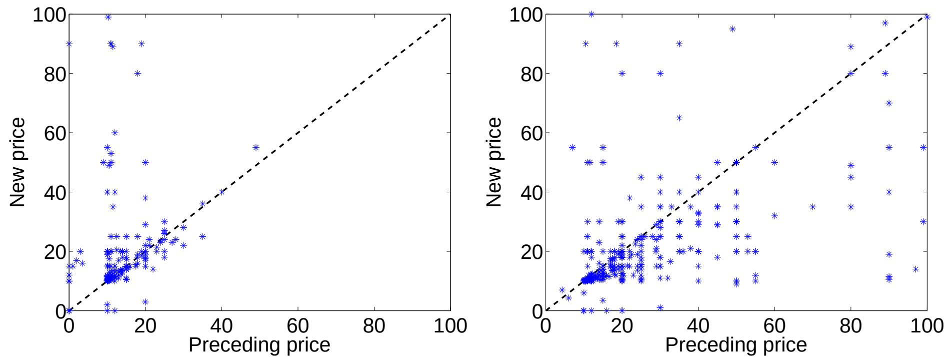
Fixed-effects regression with price as dependent variable.

($F = 17.77$, $p < 0.01$)

independent variable	coefficient	standard error	t	$P > \|t\|$
Constant	19.36297	1.79147	10.81	< 0.001
Preceding price	0.36927	0.06240	5.92	< 0.001
LOSS	−3.78175	1.83629	−2.06	0.040

Table 7: *PQ3* — Price reactions (absolute frequencies).

	price increase	price decrease	no price change	$\sum$
preceding winner	91	63	27	181
preceding loser	58	220	51	329
preceding ties	1	0	2	3
$\sum$	150	283	80	513



(a) Preceding round's winner (b) Preceding round's loser

Figure 13: *PQ3* — Price reactions depending on the preceding round's outcome.

Table 8: *PQ3* — Regression results.

Fixed-effects regression with price as dependent variable.

($F = 31.78$, $p < 0.01$)

independent variable	coefficient	standard error	t	$P > \|t\|$
Constant	16.62901	1.37816	12.07	< 0.001
Preceding price	0.36942	0.04634	7.97	< 0.001
LOSS	−4.39027	1.60825	−2.73	0.007

late eight pairs of agents playing the mixed strategy equilibrium for the PQ duopoly game with our game parameters from the experiment. Using this data, we can precisely identify to what extent the pricing pattern we observe is a behavioural effect. A contrary explanation would be that losers have inherently played a higher-than-average price and are therefore more likely to decrease their choice in the next round.

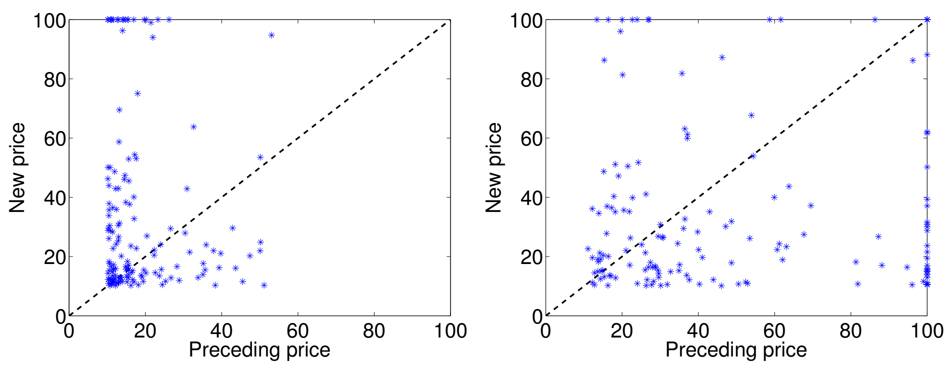
Proposition 1. *In the mixed strategy equilibrium of the simple PQ game, preceding round winners will relatively more often increase their prices com-*

Table 9: Price reactions (absolute frequencies for simulated agents).

	price increase	price decrease	no price change	∑
preceding winner	97	55	0	152
preceding loser	50	102	0	152
preceding ties	0	0	0	0
∑	147	157	0	304

pared to preceding round losers. Preceding round losers will relatively more often decrease their prices compared to preceding round winners.

Proof. See Appendix A. □



(a) Preceding round's winner (b) Preceding round's loser

Figure 14: Price reactions depending on the preceding round's outcome (simulated agents).

Figure 14 and Table 9 summarise the price changes in the simulation. We find a great dispersion of prices. Following Proposition 1, we also find preceding round winner to increase prices relatively more often than preceding round loser and vice versa. We calculate that in this simulation, preceding

Table 10: Regression results (simulated agents).

Fixed-effects regression with price as dependent variable.

($F = 0.12$, $p = 0.8854$)

independent variable	coefficient	standard error	t	$P > \|t\|$
Constant	34.45275	2.77028	12.44	< 0.001
Preceding price	0.03482	0.07056	0.49	0.622
LOSS	−1.08095	4.06313	−0.27	0.790

round winners increase prices on average by 8.60 while preceding round losers decrease their prices on average by 8.60. We thus see that the effect of winning or losing on price changes is stronger than in our experiment. However, our regression results in Table 10 show that this effect is due to the random choice from the probability distribution rather than due to behavioural effects.[7]

Result 4. *In the experimental data, players react directly to the outcome of a previous round. On average, winners increase their prices while losers decrease prices, with the absolute price change by losers being stronger. We identify this result as a behavioural patter, rather than an effect inherently caused by drawing prices independently from a random distribution.*

Finally, only two of our ten pairs in the duopoly treatment revealed collusion. For the remaining eight pairs we observe competitive behaviour with no tendency to cooperation or tacit collusion. This finding fits with Fonseca

[7]Note that we used the regression model for reasons of comparability only. For other purposes, the regression model is not appropriate.

and Normann (2008) who study a capacity-constrained Bertrand-Edgeworth game. As in our experiment, subjects neither follow the mixed strategy equilibrium in their game nor agree on a certain price level. For our data, we have seen that the development of the AWP does not follow any trend. This result stands in contrast to Brandts and Guillen (2007) who observe a high level of collusion in a similar framework. Their experimental design differs from ours mainly in three parameters: (i) the number of rounds was 50, (ii) they allowed for bankruptcies, and (iii) they used a box demand ($D = 100$). Whilst the higher number of repetitions might give subjects more time to establish collusion, Brandts and Guillen (2007) observe a dramatic price increase even in the first 20 rounds of their experiment. This stands in contrast to the constant AWP detected in our experiment and therefore the number of rounds does not seem to cause the lower level of collusion. The exclusion of bankruptcies, however, might lead to more competitive first rounds in our game, which might render subjects less likely to collude overall. Additionally, the lower prices in our experiment could be influenced by the fact that price undercutting is more attractive due to the linear demand. However, note that the one-shot mixed strategy equilibrium is the same for the game in Brandts and Guillen (2007) and ours.

6. Concluding remarks

In this paper we present a systematic comparison between experimental outcomes and the mixed strategy equilibrium for a simple PQ game. Although some of our results are well-known characteristics of market experiments, our duopoly study incorporates two main contributions that had been not addressed by the existing literature. First, we found that firms' price and quantity choices do not follow the mixed strategy equilibrium. Second, we show that the observed behaviour can be explained using learning direction theory (Selten and Stoecker, 1986). In particular, we find that chosen prices depend on the result of the preceding round.

We repeated the experiments with a triopoly market and found similar results. Additionally, we found average prices in this treatment lower than in the duopoly market, another contradiction to the mixed strategy equilibrium (Gertner, 1986).

A typical caveat of oligopoly experiments with fixed pairs may be the existence of multiple equilibria, including cooperative strategies (Kreps et al., 1982). Because the PQ game has a unique one-shot equilibrium and it is finitely repeated, our repeated game has a unique subgame perfect equilibrium. We are aware of the fact that in economic experiments with fixed pairs competition tends to be lower than the theory predicts (Muren, 2000; Huck et al., 2004). Thus, applying a random matching procedure would be a natural variation of our experiment.

Overall, our results provide a good basis for further analysis of experimental oligopolies with price-quantity competition. Experimental designs are no longer limited to deciding between using price or quantity competition in classic oligopoly markets but may include more realistic bivariate decisions on prices and quantities. In particular, games with endogenous timing as in Hamilton and Slutsky (1990) or endogenous choice of the decision variable as suggested by Tasnádi (2006) can be studied experimentally allowing for simultaneous price-quantity choices.

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Appendix A. Proof of Proposition 1

Let us denote the price chosen by firm $i \in \{1, 2\}$ in round $t \in \mathbb{N}$ by p_i^t. Let us further denote the probability of an event X by $P(X)$ and the probability of an event X given that event Y occurred by $P(X|Y) = P(X \cap Y)/P(Y)$. Then Proposition 1 can be written in the form

$$P\left(p_i^t > p_i^{t-1} | p_i^{t-1} < p_j^{t-1}\right) > P\left(p_i^t > p_i^{t-1} | p_i^{t-1} > p_j^{t-1}\right) ,$$
$$P\left(p_i^t < p_i^{t-1} | p_i^{t-1} > p_j^{t-1}\right) > P\left(p_i^t < p_i^{t-1} | p_i^{t-1} < p_j^{t-1}\right) .$$

In order to simplify the calculation, we introduce the probability density function of prices in $[c, a)$ as the derivative of the distribution function given in (2):

$$f(p) = \frac{c}{p^2} , \quad p \in [c, a) .$$

We start by calculating the probability of winning a round $P(p_i^t < p_j^t) = P(p_i^{t-1} < p_j^{t-1})$:

$$P(p_i^t < p_j^t) = \int_c^a f(p_i^t)(1 - F(p_i^t))\mathrm{d}p_i^t = \frac{1}{2}\left(1 - \frac{c^2}{a^2}\right) .$$

Obviously, the probability of losing a round is identical to this probability and a tie only occurs if both players choose to exit the market and hence has a probability of c^2/a^2. Let us now calculate the probability of a winner decreasing their price:

$$\begin{aligned} P\left(p_i^t < p_i^{t-1} | p_i^{t-1} < p_j^{t-1}\right) &= \frac{P\left(p_i^t < p_i^{t-1} < p_j^{t-1}\right)}{P\left(p_i^{t-1} < p_j^{t-1}\right)} \\ &= \frac{\int_c^a f(p_i^t) \int_{p_i^t}^a f(p_i^{t-1})(1 - F(p_i^{t-1}))\mathrm{d}p_i^{t-1}\mathrm{d}p_i^t}{\frac{1}{2}\left(1 - \frac{c^2}{a^2}\right)} \\ &= \frac{1}{3}\left(1 - \frac{2c^2}{a(a+c)}\right) . \end{aligned}$$

Due to symmetry, this probability is equal to the probability of a loser increasing their price $P\left(p_i^t > p_i^{t-1} | p_i^{t-1} > p_j^{t-1}\right)$. The probability of a price increase by a winner can be calculated as one minus the probability of a price decrease by a winner, because the winning price must be smaller than a and maintaining the exact same price level therefore has a probability of zero. Hence:

$$P\left(p_i^t > p_i^{t-1} | p_i^{t-1} < p_j^{t-1}\right) = \frac{2}{3}\left(1 + \frac{c^2}{a(a+c)}\right) .$$

The missing probability we need to calculate is the probability of a price decrease by a loser. This is not equal to one minus the probability of the price decrease by a loser, because a loser could have chosen a price equal to a and can therefore maintain their current price level. We calculate the probability through:

$$\begin{aligned}
P\left(p_i^t < p_i^{t-1} | p_i^{t-1} > p_j^{t-1}\right) &= \frac{P\left(p_i^t < p_i^{t-1}, p - i^{t-1} > p_j^{t-1}\right)}{P\left(p_i^{t-1} < p_j^{t-1}\right)} \\
&= \frac{\int_c^a f(p_i^{t-1}) F(p_i^{t-1})^2 \mathrm{d}p_i^{t-1} + (1 - F(a))F(a)^2}{\frac{1}{2}\left(1 - \frac{c^2}{a^2}\right)} \\
&= \frac{2}{3}\left(1 - \frac{2c^2}{a(a+c)}\right) .
\end{aligned}$$

All that's left to do is to verify the inequalities:

$$\begin{aligned}
P\left(p_i^t < p_i^{t-1} | p_i^{t-1} > p_j^{t-1}\right) &= \frac{2}{3}\left(1 - \frac{2c^2}{a(a+c)}\right) \\
&> \frac{1}{3}\left(1 - \frac{2c^2}{a(a+c)}\right) \\
&= P\left(p_i^t < p_i^{t-1} | p_i^{t-1} < p_j^{t-1}\right) ,
\end{aligned}$$

where the inequality holds because $1 - \frac{2c^2}{a(a+c)}$ is always positive for $a > c$. The second inequality can be shown straightforwardly as follows:

$$\begin{aligned} P\left(p_i^t > p_i^{t-1} | p_i^{t-1} < p_j^{t-1}\right) &= \frac{2}{3}\left(1 + \frac{c^2}{a(a+c)}\right) > \frac{2}{3} > \frac{1}{3} \\ &> \frac{1}{3}\left(1 - \frac{2c^2}{a(a+c)}\right) \\ &= P\left(p_i^t > p_i^{t-1} | p_i^{t-1} > p_j^{t-1}\right) \ . \end{aligned}$$

□