Centralization or Decentralization in Multi-Agency Contracting Games?*

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February 4, 2013

Abstract

I explore whether two major contracting procedures, decentralized menu design and centralized mechanism design, are strategically equivalent in multi-agency contracting games. Unlike single-agency, multi-agency suggests strategic behaviors of the agents and the interrelated impacts of different agents' asymmetric information on the principal's objective. I find that centralization can take advantage of joint-base mechanisms, as a useful communication device related to relative information evaluation, to better deal with the interrelated information asymmetry in general multi-agency games with ex post implementation. Technically, my main contribution is to show that ex post menu design is merely strategically equivalent to individual-based ex post mechanism design *rather than* joint-based ex post mechanism design, which actually makes the principal better off than the former two. My findings support the rationale of centralized economics design and associated information communication from another perspective.

Keywords: multi-agency, ex post equilibrium, mechanism design, menu design, revelation principle, delegation principle, relative information evaluation

JEL Classification: C79 D82 D86

*For their helpful comments and stimulating remarks, I gratefully acknowledge Frank Page, Seungjin Han, Kim-Sau Chung, Yongchao Zhang, Ronald M. Giammarino, Michael Koss, Robert Becker, Filomena Garcia, Haomiao Yu, Ran Shao, and the participants/audiences at PET annual meeting 2012, Indiana University Microeconomics Workshop Fall 2012, and St Louis Fed/Missouri Economics Conference 2012. I am solely responsible for any errors.

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1 Introduction

Contracting games with a single agent are solved in either of two strategically equivalent ways – by mechanism design (a "centralized" contracting procedure), or menu design (a "decentralized" contracting procedure). The purpose of this paper is to explore whether the decentralized contracting procedure is still strategically equivalent to the centralized contracting procedure in multi-agency contracting games. Here multi-agency denotes that a single principal contracts with multiple agents. This paper focuses on one-shot pure-strategy multi-agency contracting games. The meaning of *strategic equivalence* between mechanism design and menu design is as follows: if there is an optimal mechanism solving the mechanism design problem, there also exists an optimal menu solving the menu design problem, vice versa, and solving these two problems brings the same (expected) payoff to the principal. If they are strategically equivalent, there will be no loss of generality for the principal to use either of them. Otherwise, it is important to know which one will be superior under what conditions.

It is also very important to concern about multi-agency beyond single-agency. Multi-agency situations are frequently observed in principal-agent relationships. Compared with the single-agency environment, the multi-agency environment implies that the impacts of different agents' asymmetric information on the principal's objective will be interrelated. First, the agents are allowed to be not only heterogeneous but also "fully interdependent." Each agent has a heterogeneous payoff normally depending on not only his own type, contract specification, and action but also those of other agents. Second, the principal's payoff will jointly depend on all the agents' types, contract specifications, and actions. Moreover, the agents will strategically interact with each other. The advancement of modern society makes such interrelated impacts and strategic behaviors of the agents turn out to be fairly commonplace. Meanwhile, these facts also result in a significant structural difference in multi-agency contracting problems.

To resolve the adverse selection and moral hazard problems, the principal needs to find a way to link a profile of outcome-contingent contracts and action-suggestions for the agents with each profile of their private information, so as to create incentives for the agents to enter into a contract-action selection profile in her[1] best interest. There are two major classes of contracting procedures to implement this goal in consideration of the fact that specifying contracts and actions for the agents, as a key decision in contracting games, can be centralized or decentralized. First, mechanism design can be viewed as a centralized

[1] Throughout the paper, masculine pronouns refer to the agents, whereas feminine pronouns apply to the principal.

contracting procedure. The principal will require the agents to report some messages after proposing to the agents a communication mechanism associating contract-action pair profiles with report profiles. Second, menu[2] design can be viewed as a decentralized contracting procedure. The principal proposes to the agents a joint menu, namely, a set of contract profiles. The agents are entitled to directly choose a contract within the pre-offered menu and an action in the known action set. This implies decentralization of the decision rights to specify contracts and actions.

Many researchers study the menu design as a different self-contained contracting procedure from mechanism design.[3] Because menu design is more straightforward and can skip information communication in specifying contracts and actions, people have been interested in whether centralized mechanisms are decentralizable by menus, that is, whether decentralized menu design are strategically equivalent to centralized mechanism design. Previous studies demonstrate that such a strategic equivalence can always be achieved in the single-agency situations. This fact is called the *delegation principle*. Page (1992) and Carlier (2001) characterize the incentive compatible direct mechanisms for single-principal single-agent contracting problems via the sets of contracts. Peters (2001), Martimort and Stole (2002), and Page and Monteiro (2003) study the strategic equivalence of menu design and mechanism design in multi-principal single-agent contracting problems.

Nonetheless, this paper surprisingly shows that the strategic equivalence of centralization and decentralization in contracting cannot simply be guaranteed in the general multi-agency environment. Centralization of the right to specify contracts and actions and corresponding communication between the principal and the agents will make the principal better off in general multi-agency contracting games. A loss of generality may be imposed if one still takes decentralization for granted in multi-agency contracting.

The interrelated impacts of the agents' asymmetric information and strategic behaviors among the agents make the structures of multi-agency contracting problems much more complicated. In classic mechanism design theory, the standard mechanisms of interest for centralization in multi-agency are *"joint-based"* mechanisms, which associate each contract and action suggestion for an individual agent with the joint report of all agents. If we directly compare joint-based mechanism design with menu design, it is difficult to find a clue about the basis of such comparison between centralization and decentralization immediately by extending the delegation principle for single-agency. Indeed, I find that *"individual-based"* mechanisms, which associate each contract and action suggestion for an

[2] Some authors use the term "catalogs" instead.

[3] See Martimort and Stole (2002), Page(1992), Page and Monteiro (2003), Carmona and Fajardo(2007) among many others.

individual agent merely with his individual report,[4] can serve as an important bridge for the comparison between centralization and decentralization in multi-agency. My further contribution is the clean formulation of relevant multi-agency contracting problems based on a generalized model setting. This provides the solid first step for the detailed analysis of this paper.

First recall that the multi-agency contracting game is essentially a two-stage sequential game. The principal's strategy is to design optimal mechanisms or menus for the agents. Each pre-offered mechanism or joint menu actually defines a non-cooperative subgame for all the agents to play simultaneously. In this paper, *Ex Post Equilibrium (EPE)* is of interest as the solution concept of such a subgame. EPE is a practical and robust solution concept, as it does not require each agent's probability assessment about the others' private information to be common knowledge. It has been increasingly studied in mechanism design theory.[5] In mechanism design, the principal seeks an optimal mechanism from a class of pre-offered mechanisms inducing a subgame for the agents in which some particular participation strategy profile of the agents will be achieved as EPE. Such mechanisms are called *ex post mechanisms*. In menu design, the principal seeks an optimal joint menu from a class of pre-offered joint menus inducing a subgame for the agents in which some particular contract-action selection strategy profile of the agents will be achieved as EPE. Such joint menus are called *ex post (joint) menus*. In this respect, the contracting games over either ex post mechanisms or ex post menus are referred to as *contracting games with ex post implementation*. Moreover, my analysis also permits certain feasible constraints to be imposed on the joint contract set, i.e. the set of contract profiles for all agents. This is consistent with many realistic situations, such as optimal auction design, budget constraint, etc. In that case, the subgames induced by mechanisms or menus are related to the "*constrained games*."[6]

My main finding is that ex post mechanism design will make the principal better off than ex post menu design in a general multi-agency situation. The revelation principle for ex post implementation allows people to restrict attention to *ex post incentive compatible* (EPIC) mechanisms. Technically, my main contribution is the delegation principle for ex

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[4] A related concept is *"reduced form mechanism,"* which is studied in the context of optimal auction design. See Matthews (1984) and Border (1991) among many others.

[5] See Dasgupta and Maskin(2000), Perry and Reny (2002), Bergemann and Valimaki (2002), Bergemann and Morris (2008), Chung and Ely (2006), and Bikhchandani (2006) among many others.

[6] Constrained games are also called generalized games, shared constraint games, games with coupled constraints, or abstract economies. The systematic description of "constrained games" in the normal form with complete information appears in the work of Rosen (1965) and Ponstein (1966). Since EPE is the strategy profile under which every action profile is just a Nash equilibrium at every type profile, the existing studies on Nash equilibria of "constrained games" can still be connected to this context.

post implementation, which indicates that ex post menu design are merely strategically equivalent to individual-based EPIC mechanism design *rather than* joint-based EPIC mechanism design. The set of individual-based EPIC mechanisms is actually equivalent to a subset of joint-based EPIC mechanisms. So joint-based EPIC mechanism design makes the principal better off than individual-based EPIC mechanism design and ex post menu design. However, these three ways of design must be all payoff-equivalent for the principal when the impacts of different agents' asymmetric information on the principal's objective are independent and separate. The main results of this paper suggest that centralization in multi-agency contracting can take advantage of joint-base mechanisms, as a useful communication device, to incorporate more comprehensive information and use relative information evaluation to deal with the interrelated information asymmetry problem. This fact indeed supports the rationale of centralized economics design and associated information communication from another perspective. In addition, I apply the main results to incentive-based financial regulation with interrelated banks, and discuss why the regulatory contracting cannot be decentralized without loss of generality.

Hammond (1979) and Han (2006) also address the issue of decentralization in multi-agency games with pure adverse selection, private valuations, and independent feasible contract sets of individual agents.[7] Hammond suggests that a dominant strategy incentive compatible mechanism is decentralizable by a profile of some particular contract sets indexed by type profiles. Han constructs a multi-principal multi-agent "bilateral contracting" environment restricting feasible mechanisms. The sets of messages that individual agents can report to individual principal are homogeneous and identical. All "bilateral" mechanisms the principals can offer to each agent are just the functions from the single uniform message set to the independent set of contracts available to that agent. He shows that "bilateral" mechanism design is strategically equivalent to menu design under Perfect Bayesian Equilibrium in "bilateral contracting" games. Instead, the focus of this paper lies on the following aspects. First, part of my contribution is to extend the models of Hammond and Han to a much more general situation in multi-agency games. The agents can be fully interdependent, and feasible contract sets of individual agents can be heterogeneous and cross-constrained. The message sets of different agents can be heterogeneous or unidentical. The mechanisms are allowed to be joint-based. Moreover, decentralization or delegation in this paper is centered on self-contained menu design games in which menus are the sets of contracts *independent of any index*. This is more realistic in economic practices. Lastly, ex post implementation, instead of Bayesian implementation or

[7] It implies the feasible contract sets of individual agents are not cross-constrained.

dominant strategy implementation, is addressed in this paper.

My results are of more significance if there are economically interesting environments where the existence of the nonconstant (or nontrivial) EPIC mechanism and the non-singleton ex post menu can be ensured. It is a moot point to find very mild conditions for such existence in a general setting.[8] Nonetheless, the main results of this paper are still applicable even if such existence cannot be guaranteed. Note that some constant mechanism must be EPIC, and some singleton menu must be an ex post menu. If only constant individual-based mechanisms can be EPIC, every ex post menu must be a singleton by the delegation principle, vice versa. Yet there could still exist some nonconstant joint-based EPIC mechanism that makes the principal better off. On the contrary, if only constant (or nontrivial) joint-based mechanisms are EPIC, then it will make no extra sense to consider decentralization. In addition, my results will still hold when there is a degenerated form of "full interdependence" among the agents, that is, when each agent's payoff only depends on his own type,[9] on his own contract, or on his own action. In this case, the issue of existence would usually be alleviated.

The rest of the paper is organized as follows. Section 2 presents basic elements of the model and provides some discussion and several examples within the model's scope. Section 3 explores the ex post mechanism design problem and proves the related revelation principle. Section 4 investigates the menu design problem and shows the delegation principle for multi-agency. The comparative advantage of joint-based mechanisms is further analyzed in Section 5. Section 6 examines an application of the main results in financial regulation. Concluding remarks are given in section 7.

2 Preliminaries

There are one principal and n agents indexed by $i \in \mathcal{N} = \{1, \cdots, n\}$ in the multi-agency contracting game of interest. The principal moves first, and then the agents follow simultaneously and behave non-cooperatively. The principal needs the agents' participation to realize some economic objective of hers. Each agent has hidden type and hidden action that are only observable to him. They induce two categories of asymmetric information problems: adverse selection and moral hazard.

To save words, throughout this paper the symbol $\mathcal{B}(X)$ is reserved for Borel σ-field of a certain space X, and the symbol $\mathcal{M}(X, Y)$ is reserved for the set of all $(\mathcal{B}(X), \mathcal{B}(Y))$-

[8] A well-known counterexample is raised by Jehiel, Moldovanu, Meyer-Ter-Vehn, and Zame (2006).

[9] In this case, each agent has *private valuation*. EPE will also degenerate to dominant strategy equilibrium.

measurable functions from one space X to another space Y.

2.1 Agent Types and Actions

Agent i's type is $t_i \in T_i$, where T_i is a Borel space, i.e. a Borel subset of a Polish space.[10] We write $t = (t_i)_{i\in\mathcal{N}} \in T = \prod_{i=1}^{n} T_i$ and $t_{-j} = (t_i)_{i\in\mathcal{N}\setminus\{j\}} \in T_{-j} = \prod_{i\neq j}^{n} T_i$. Let μ_i be a probability measure defined on $\mathcal{B}(T_i)$ and μ be a probability measure on the associated product Borel σ-field $\mathcal{B}(T)$. Thus, T and T_{-i} are also Borel spaces. $(T, \mathcal{B}(T), \mu)$ is a probability measure space characterizing the common prior over the agents' private types. If the types of different agents are independent, $\mu = \mu_1 \times \cdots \times \mu_n$, i.e., μ is the product probability measure on $\mathcal{B}(T)$. Correlated types are also permitted.

Agent i's action is $a_i \in A_i$, where A_i is a compact metric space with Borel σ-field $\mathcal{B}(A_i)$ of A_i. We write $a = (a_i)_{i\in\mathcal{N}} \in A = \prod_{i=1}^{n} A_i$ and $a_{-i} = (a_j)_{j\in\mathcal{N}\setminus\{i\}} \in A_{-i} = \prod_{j\neq i}^{n} A_j$. $\mathcal{B}(A)$ is the associated product Borel σ-field of A. A_i contains an element a^0, which denotes "don't participate". Thus, A and A_{-i} are also compact metric spaces. Since a^0 is introduced, some agent with a certain type is permitted to voluntarily abstain from contracting. a^0 is commonly observable. All actions but a^0 chosen by agent i are only observable to i.

2.2 Outcomes

The end-of-period commonly observable outcome[11] is $\omega \in \Omega$, where Ω is a closed metric space. The outcomes may include not only monetary outcomes but also some non-monetary outcomes. If the outcomes can be separately observed across the agents, let $\omega = (\omega_1, \cdots, \omega_n) \in \Omega = \Omega_1 \times \cdots \times \Omega_n$, where $\omega_i \in \Omega_i$ is the observable outcomes associated to agent i. Let $\eta \in \mathcal{P}(\Omega)$ be a probability measure defined on $(\Omega, \mathcal{B}(\Omega))$, where $\mathcal{P}(\Omega)$ is equipped with the metrizable topology of weak convergence.

Given $(a,t) \in A \times T$, $p(\cdot|a,t)$ is a probability measure defined on the space $(\Omega, \mathcal{B}(\Omega))$. It represents the common belief concerning how actions and types of the agents stochastically determine observable outcomes. Denote the set of such probability measures by

$$\mathcal{P}(\Omega; A \times T) := \{p(\cdot|a,t) : (a,t) \in A \times T\} \subset \mathcal{P}(\Omega).$$

For each closed subset E of Ω, $p(E|\cdot,\cdot)$ is continuous on $A \times T$. Moreover, $\mathcal{P}(\Omega; A\times T) <<$

[10] A complete separable metric space is called Polish space.

[11] The observable outcome may be also called performance measurement, information system, or signals. Observability also implies verifiability throughout this paper.

η, that is, for each $(a,t) \in A \times T$, the probability measure $p(\cdot|a,t)$ is absolutely continuous with respect to the probability measure η.

2.3 Contracts

The end-of-period commonly observable allocation for agent i is $d_i \in D_i$, where D_i is a metric space. Allocation d_i stipulates a certain relationship of transfer of economic interests or wealth between the principal and the agent i. Allocation d_i can be the reward from the principal to agent i in the context of employee compensation, a product-price pair in the context of nonlinear pricing, or a social alternative-transfer pair in the context of social choice. Write $d = (d_i)_{i\in\mathcal{N}} \in D = \prod_{i=1}^{n} D_i$. Thus, D is also a metric space. The space $\mathcal{M}(\Omega, D_i)$ is equipped with the topology of pointwise convergence.

All possible contracts available to agent i is $f_i \in \mathcal{K}_i$, where $\mathcal{K}_i = \mathcal{M}(\Omega, D_i)$. The set of feasible joint contracts is $\mathcal{K} \subseteq \prod_{i=1}^{n} \mathcal{K}_i$. Its element is $f = (f_i)_{i\in\mathcal{N}} \in \mathcal{K}$. $\mathcal{K}$ may contain some realistic constraints on the contract profiles the principal can offer to the agents. Let $f(\omega) = (f_i(\omega))_{i\in\mathcal{N}}$ and $f_{-i}(\omega) = (f_j(\omega))_{j\in\mathcal{N}\setminus\{i\}}$. $\mathcal{K}$ is assumed to be a compact metric space. There are many examples fitting these assumptions on (joint) contract sets in multi-agency situations.

Example 1 *(Joint Contract Sets).*

(1) Finite contract sets: There are only finitely many contracts in each $\mathcal{K}_i$. $\mathcal{K}$ is a compact metric space as any subset of $\prod_{i=1}^{n} \mathcal{K}_i$.

(2) Product-price pairs with budget constraint: Each buyer i is offered a product-price pair (x_i, p_i). x_i is some product characteristics, such as quantity, quality, etc. p_i is the price the seller can charge for i. This setting is frequently observed in nonlinear pricing games. So the feasible joint contract set can be either

$$\mathcal{K} = \{(x_1, \cdots, x_n, p_1, \cdots, p_n) \in \mathbb{R}^n \times \mathbb{R}^n : 0 \leq \sum_i p_i x_i \leq W\},$$

where $W > 0$ is the budget upper bound of the seller, or

$$\mathcal{K} = \{(x_1, \cdots, x_n, p_1, \cdots, p_n) \in \mathbb{R}^n \times \mathbb{R}^n : 0 \leq p_i x_i \leq I_i, \text{ for each } i\},$$

where $I_i > 0$ is the budget upper bound of buyer i. In either case, $\mathcal{K}$ is a compact metric space.

(3) Contract sets for a single object: Each bidder i is offered a pair (x_i, p_i). x_i is i's

payment to the seller. p_i *is the probability that* i *gets the object. This setting is frequently observed in optimal auction games. So the feasible joint contract set can be*

$$\mathcal{K} = \{(x_1, \cdots, x_n, p_1, \cdots, p_n) \in \mathbb{R}^n \times \mathbb{R}^n : \sum_i p_i \leq 1, p_i \geq 0, 0 \leq x_i \leq I_i, \text{ for each } i\},$$

where $I_i > 0$ *is the wealth of bidder* i*. Obviously,* $\mathcal{K}$ *is a compact metric space.*

(4) Outcome-contingent contract sets: Assume that all the contracts are outcome-contingent.[12] *If for each* i*, (i)* $\mathcal{K}_i = \mathcal{M}(\Omega, \mathbb{R})$*,* $\mathcal{K} = \prod_{i=1}^{n} \mathcal{K}_i^s$*, where each* $\mathcal{K}_i^s$ *is a sequentially compact subset of* $\mathcal{K}_i$ *for the topology of pointwise convergence on* Ω*,(ii)* $\mathcal{K}_i^s$ *contains no redundant contracts, that is, if for any two* f_i *and* $\overline{f}_i$ *in* $\mathcal{K}_i^s$ *satisfying* $f_i(\omega') \neq \overline{f}_i(\omega')$ *for some* $\omega' \in \Omega$*,* $\eta(\{\omega \in \Omega : f_i(\omega) \neq \overline{f}_i(\omega)\}) > 0$*, and (iii)* $\mathcal{K}_i^s$ *is uniformly bounded, then by Proposition 1 in Tulcea (1973)* $\mathcal{K}_i^s$ *is compact and metrizable for the topology of pointwise convergence. So is* $\mathcal{K}$*.*

2.4 Payoffs

Let $u : \Omega \times D \times A \times T \to \mathbb{R}$ denote the principal's utility function over outcomes, allocations, actions, and types with respect to all agents. Assume that u is continuous on $\Omega \times D \times A$ and Borel-measurable on T. Let $v_i : D \times A \times T \to \mathbb{R}$ denote agent i's utility function defined over allocations, actions, and types with respect to all agents. Assume that v_i is continuous on $D \times A \times T$. Let $u(\cdot, f(\cdot), a, t)$ and $v_i(f(\cdot), a, t)$ be bounded for each $f \in \mathcal{K}$, $a \in A$, and $t \in T$. So they are η-integrable.

Given an agents' type profile t, a contract profile f for the agents, and an agents' action profile a, the principal's outcome-expected payoff function $\widehat{U} : \mathcal{K} \times A \times T \to \mathbb{R}$ is defined by $\widehat{U}(f, a, t) = \int_\Omega u(\omega, f(\omega), a, t) p(d\omega | a, t)$. Given an agents' type profile t, a contract profile f for the agents, and an agents' action profile a, the agent i's outcome-expected payoff function $\widehat{V}_i : \mathcal{K} \times A \times T \to \mathbb{R}$ is defined by $\widehat{V}_i(f, a, t) = \int_\Omega v_i(f(\omega), a, t) p(d\omega | a, t)$.

This generic setting permits the agents to be "fully interdependent." In detail, there are externalities in contracting among the agents because each agent's payoff depends on not only his own contract but also the other agents' contracts. Correlated types and interdependent valuations are allowed instead of private independent valuations, because each agent's outcome-expected payoff is based on not only his own type but also the other agents' types. Synergies[13] in agents' actions are also allowed since each agent's outcome-

[12] Page and Monterio (2003) raise a similar example for state-contingent contract sets in common agency.

[13] The term "synergy" is from Edmans, Goldstein and Zhu (2011).

expected payoff is based on not only his own action but also the other agents' actions. In addition, the principal's expected payoff is permitted to depend jointly on all the agents' types, contracts, and actions. Hence, the agents will behave strategically, and the impacts of their respective asymmetric information will be interrelated.

Furthermore, there are several continuity and measurability properties of the outcome-expected payoff functions.

Proposition 1 *(Continuity and measurability properties of outcome-expected payoff functions).*

(i) For each $i \in \mathcal{N}$*,* $\widehat{V}_i(\cdot,\cdot,\cdot)$ *is continuous on* $\mathcal{K} \times A \times T$*.*

(ii) for each $t \in T$*, then* $\widehat{U}(\cdot,\cdot,t)$ *is continuous on* $\mathcal{K} \times A$*, and for each* $(f,a) \in \mathcal{K} \times A$*,* $\widehat{U}(f,a,\cdot)$ *is* $B(T)$*-measurable.*

Proof. See Appendix. ■

In addition, we assume $\widehat{U}(f,a,\cdot)$ to be μ-integrable on T, for all $(f,a) \in \mathcal{K} \times A$.

2.5 Discussions and Examples

The basic model setup is intended to be as general as possible. The setup basically relies on general topological or metric structure and Borel measurability. The model does not entail order structure or linear algebraic structure. Although some natural orders or vector spaces may exist in many applications, no monotonicity or concavity of payoffs is assumed in the basic model setup. Differentiability or validity of First Order Approach is not necessarily required as well.

The model reduces to a pure moral hazard problem when T_i is a singleton for each i. It reduces to a pure adverse selection problem when for each i, A_i contains only two elements, "participate" and "don't participate." In some cases of pure adverse selection, e.g. some social choice scenarios, there are no outside options for the agents, or the agents always prefer "participate" to "don't participate." So A_i can even be reduced to contain only one element, "participate."

The following examples address a few significant economic problems in real life. They satisfy the assumptions proposed above.

Example 2 *(Pure moral hazard). As Holmstrom (1982) presents, n workers engage in a team production. For each worker i, T_i is a singleton. Monetary reward $d_i \in D_i = [\underline{d}, \overline{d}]$. Agent i's effort is $a_i \in A_i = [0,1]$. The joint production set $\Omega = [\underline{\omega}, \overline{\omega}]$. Output is generated by the parameterized probability $p(\cdot|a)$ with an associated distribution $F(\omega|a)$,* *where $F(\cdot|a)$ is continuous in a. Then let the principal's utility $u(\omega, d, a, t) = \omega - \sum_i d_i$. Let $v_i(\omega, d, a, t) = h_i(d_i) - c_i(a_i)$ in which worker i's utility is separable in money, $h_i(\cdot)$ and cost of effort, $c_i(\cdot)$. Functions h_i and c_i are both continuous.*

Example 3 *(Pure adverse selection). Consider n agents in the standard social choice mechanism design problem. Each agent i has his own private type $t_i \in T_i = \mathbb{R}$. $\mu(t)$ denotes the common prior on the type profiles. Allocation $y \equiv (x, s) \in Y = (X, S)$, where x is the agents' consumption choice and s is the agents' transfer to the principal. Set $X = \prod_{i=1}^{n} [\underline{x}_i, \overline{x}_i]$ and $S = \prod_{i=1}^{n} [\underline{s}_i, \overline{s}_i]$. Agent i's action is $a_i \in A_i = \{participate, don't participate\}$. Agent i's quasilinear payoff is $v_i(x, t, a_i, \theta) = h_i(x, \theta_i) + t_i$, where h_i is a concave function. The "virtual" social planner's utility may be $u(x, t, a, \theta) = \sum_{i=1}^{n} v_i(x, t, a_i, \theta)$.*

Example 4 *(Moral hazard and adverse selection). A general contractor hires n subcontractors to perform a project. Each subcontractor i has his own private cost parameter $t_i \in T_i = \mathbb{R}$. Let $\mu(t)$ denote the common prior on type profiles $t = (t_i)_{i \in \mathcal{N}}$. Agent i's effort is $a_i \in A_i = [0, 1]$. Monetary reward for each i is $d_i \in D_i = [\underline{d}, \overline{d}]$. Their performance (outcomes) can be measured separately. Agent i's performance is $\omega_i \in \Omega_i = [\underline{\omega}_i, \overline{\omega}_i]$. Joint performance is $\omega = (\omega_i)_{i \in \mathcal{N}} \in \Omega = \prod_{i=1}^{n} \Omega_i$. Let $p(\cdot|a, t)$ be the associated parameterized probability distribution over Ω. Assume $p(\cdot|a, t)$ to be continuous in a and t. Agent i's utility is $v_i(\omega, d, a, t) = h_i(d_i) - c_i(a_i, t_i)$, which is separable in money, $h_i(\cdot)$, and cost of effort, $c_i(\cdot)$. Functions v_i and c_i are both continuous. The general contractor's payoff is $u(\omega, d, a, t) = \sum_i \omega_i - \sum_i d_i$.*

3 Mechanism Design Problem and the Revelation Principle

To deal with asymmetric information and attain second-best solutions, the first way is centralized mechanism design. My analysis will basically follow the terminology and paradigm in the work of Myerson (1982) and Kadan et al. (2011).[14]

[14] Myerson (1982) formulates a generalized Bayesian model setup of the principal-agent problem with both adverse selection and moral hazard in multi-agency cases. Based on topological structure and Borel measurability, Kadan, Reny and Swinkels (2011) also present a general formulation of the direct mechanism design problem with both adverse selection and moral hazard in single-principal single-agent cases.

3.1 General Mechanisms and Direct Mechanisms

Agent i's report to the principal is $r_i \in R_i$, where R_i is a Borel space. The message that the principal sends to agent i is $m_i \in M_i$, where M_i is a Borel space. Let $r = (r_i)_{i\in\mathcal{N}} \in R = \prod_{i=1}^{n} R_i$ and $m = (m_i)_{i\in\mathcal{N}} \in M = \prod_{i=1}^{n} M_i$. Let Borel measurable function $\mathbf{k}_i : R \to \mathcal{K}_i$ (resp. $\mathbf{m}_i : R \to M_i$) specify a contract (resp. a message) to agent i for each report profile of all agents. Let Borel measurable function $\overline{\mathbf{k}}_i : R_i \to \mathcal{K}_i$ (resp. $\overline{\mathbf{m}}_i : R_i \to M_i$) specify a contract (resp. a message) to agent i for each report of single agent i.

Definition 1 *A* ***"joint-based" general mechanism*** *is a pair of functions* $(\mathbf{k}, \mathbf{m}) \in \mathcal{F}(R, \mathcal{K}, M)$*, where* $\mathbf{k} = (\mathbf{k}_1, \cdots \mathbf{k}_n) : R \to \mathcal{K}$ *satisfying* $(\mathbf{k}_1(r), \cdots \mathbf{k}_n(r)) \in \mathcal{K}$ *for each* $r \in R$*, and* $\mathbf{m} = (\mathbf{m}_1, \cdots, \mathbf{m}_n) : R \to M$*. An* ***"individual-based" general mechanism*** *is a pair of functions* $(\overline{\mathbf{k}}, \overline{\mathbf{m}}) \in \overline{\mathcal{F}}(R, \mathcal{K}, M)$*, where* $\overline{\mathbf{k}} = (\overline{\mathbf{k}}_1, \cdots \overline{\mathbf{k}}_n) : R \to \mathcal{K}$ *satisfying* $(\overline{\mathbf{k}}_1(r_1), \cdots \overline{\mathbf{k}}_n(r_n)) \in \mathcal{K}$ *for each* $r \in R$*, and* $\overline{\mathbf{m}} = (\overline{\mathbf{m}}_1, \cdots, \overline{\mathbf{m}}_n) : R \to M$*.*

Definition 2 *A* ***report function*** *for agent* i *is a function* $\rho_i : T_i \to R_i$ *specifying agent* i*'s report given each type of* i*. A* ***final decision function*** *for agent* i *is a function* $\delta_i : M_i \times T_i \to A_i$ *specifying the final action that agent* i *takes after* i *learns his type* t_i *and receives some message* m_i*. Any pair* (ρ_i, δ_i) *is referred to as a* ***participation strategy*** *for* i*.*

Thus, each $(\mathbf{k}, \mathbf{m})$ in $\mathcal{F}(R, \mathcal{K}, M)$ is still Borel measurable. So is each $(\overline{\mathbf{k}}, \overline{\mathbf{m}})$ in $\overline{\mathcal{F}}_{\mathcal{K}}(R, \mathcal{K}, M)$. Thus, the agent i's participation strategy is $(\rho_i, \delta_i) \in \Gamma_i \times \Delta_i$. Let $\Gamma_i = \mathcal{M}(T_i, R_i)$ and $\Delta_i = \mathcal{M}(M_i \times T_i, A_i)$. We write:

$$\rho = (\rho_i)_{i\in\mathcal{N}} \in \Gamma = \prod_{i=1}^{n} \Gamma_i, \delta = (\rho_i)_{i\in\mathcal{N}} \in \Delta = \prod_{i=1}^{n} \Delta_i.$$

$$\rho_{-i} = (\rho_j)_{j\in\mathcal{N}\setminus\{i\}} \in \Gamma_{-i} = \prod_{j\neq i}^{n} \Gamma_j, \delta_{-i} = (\rho_j)_{j\in\mathcal{N}\setminus\{i\}} \in \Delta_{-i} = \prod_{j\neq i}^{n} \Delta_j,$$

$$\rho(t) = (\rho_i(t_i))_{i\in\mathcal{N}}, \rho_{-i}(t_{-i}) = (\rho_j(t_j))_{j\in\mathcal{N}\setminus\{i\}}, \delta(m, t) = (\delta_i(m_i, t_i))_{i\in\mathcal{N}}.$$

After the joint-based (resp. individual-based) mechanism $(\mathbf{k}, \mathbf{m})$ (resp. $(\overline{\mathbf{k}}, \overline{\mathbf{m}})$) is offered, each agent i with type t_i sends a report $\rho_i(t_i)$ to the principal. Under the joint-based (resp. individual-based) mechanism, the principal is then committed to offering a contract $\mathbf{k}_i(\rho(t))$ (resp. $\overline{\mathbf{k}}_i(\rho_i(t_i))$) and sending a message $\mathbf{m}_i(\rho(t))$ (resp. $\overline{\mathbf{m}}_i(\rho_i(t_i))$) to each i. Next, i with type t_i will take an action $\delta_i(\mathbf{m}_i(\rho(t)), t_i)$ (resp. $\delta_i(\overline{\mathbf{m}}_i(\rho_i(t_i)), t_i)$).

A particular class of general mechanisms with $R_i = T_i$ and $M_i = A_i$ for each i are called **direct mechanisms**. In addition, $\mathbf{m}$ (resp. $\overline{\mathbf{m}}$) will be replaced with $\mathbf{a}$ (resp. $\overline{\mathbf{a}}$) for distinction in direct mechanisms.

3.2 Contracting Game over Ex Post Mechanisms

There are five stages in the principal-agent contracting game over mechanisms: At Stage 1, the principal proposes a mechanism, which is commonly observable, to the agents. At Stage 2, each agent unilaterally learns his true type, and the agents simultaneously send reports to the principal. At Stage 3, through the pre-offered mechanism, the principal assigns contracts and sends messages to the agents according to their reports. At Stage 4, after receiving the respective contracts and messages, the agents simultaneously decide to stay or leave and choose unobservable actions if participating. At Stage 5, outcomes are realized and the contracts are implemented.

Some notations are further simplified as below:

$$\mathbf{k}(\rho(t)) = (\mathbf{k}_i(\rho(t)))_{i\in\mathcal{N}}, \mathbf{m}(\rho(t)) = (\mathbf{m}_i(\rho(t)))_{i\in\mathcal{N}}, \mathbf{m}_{-i}(\rho(t)) = (\mathbf{m}_j(\rho(t)))_{j\in\mathcal{N}\setminus\{i\}},$$

$$\overline{\mathbf{k}}(\rho(t)) = (\overline{\mathbf{k}}_i(\rho_i(t_i)))_{i\in\mathcal{N}}, \overline{\mathbf{k}}_{-i}(\rho_{-i}(t_{-i})) = (\overline{\mathbf{k}}_j(\rho_j(t_j)))_{j\in\mathcal{N}\setminus\{i\}},$$

$$\overline{\mathbf{m}}(\rho(t)) = (\overline{\mathbf{m}}_i(\rho_i(t_i)))_{i\in\mathcal{N}}, \overline{\mathbf{m}}_{-i}(\rho_{-i}(t_{-i})) = (\overline{\mathbf{m}}_j(\rho_j(t_j)))_{j\in\mathcal{N}\setminus\{i\}},$$

$$\delta(\mathbf{m}(\rho(t)), t) = (\delta_i(\mathbf{m}_i(\rho(t)), t_i))_{i\in\mathcal{N}}, \delta_{-i}(\mathbf{m}_{-i}(\rho_{-i}(t_{-i})), t_{-i}) = (\delta_j(\mathbf{m}_j(\rho(t)), t_j))_{j\in\mathcal{N}\setminus\{i\}},$$

$$\delta(\overline{\mathbf{m}}(\rho(t)), t) = (\delta_i(\overline{\mathbf{m}}_i(\rho_i(t_i)), t_i))_{i\in\mathcal{N}}, \delta_{-i}(\overline{\mathbf{m}}_{-i}(\rho_{-i}(t_{-i}), t_{-i}) = (\delta_j(\overline{\mathbf{m}}_j(\rho_j(t_j)), t_j))_{j\in\mathcal{N}\setminus\{i\}}$$

Each mechanism offered by the principal induces a simultaneous-moved subgame of the agents in which ex post equilibrium is of interest as the solution concept. There are several relevant definitions as below.

Definition 3 *The agents' participation strategy profile* (ρ, δ) *is said to be an* ***ex post equilibrium (EPE)*** *under a joint-based general mechanism* $(\mathbf{k}, \mathbf{m})$ *if for each* $i \in \mathcal{N}$ *and each* $t \in T$,

$$\begin{aligned} & \widehat{V}_i(\mathbf{k}(\rho(t)), \delta(\mathbf{m}(\rho(t)), t), t) \\ \geq\ & \widehat{V}_i(\mathbf{k}(\rho_i^{'}(t_i), \rho_{-i}(t_{-i})), \delta_i^{\prime}(\mathbf{m}_i(\rho_i^{\prime}(t_i), \rho_{-i}(t_{-i}), t_i), \delta_{-i}(\mathbf{m}_{-i}(\rho_i^{\prime}(t_i), \rho_{-i}(t_{-i}), t_{-i}), t), \quad (1) \end{aligned}$$

for all $(\rho_i^{\prime}, \delta_i^{\prime}) \in \Gamma_i \times \Delta_i$. *Moreover, such pair* $(\mathbf{k}, \mathbf{m})$ *is referred to as a joint-based ex post general Mechanism.*

Definition 4 *The agents' participation strategy profile* (ρ, δ) *is said to be an* ***ex post equilibrium*** *under an individual-based general contracting mechanism* $(\overline{\mathbf{k}}, \overline{\mathbf{m}})$ *if for each*

$i \in \mathcal{N}$ and each $t \in T$,

$$\begin{aligned} & \widehat{V}_i(\overline{\mathbf{k}}(\rho(t)), \delta(\overline{\mathbf{m}}(\rho(t)), t), t) \\ \geq \ & \widehat{V}_i(\overline{\mathbf{k}}_i(\rho'_i(t_i)), \overline{\mathbf{k}}_{-i}(\rho_{-i}(t_{-i})), \delta'_i(\overline{\mathbf{m}}_i(\rho'_i(t_i), t_i), \delta_{-i}(\overline{\mathbf{m}}_{-i}(\rho_{-i}(t_{-i}), t_{-i}), t), \end{aligned} \tag{2}$$

for all $(\rho'_i, \delta'_i) \in \Gamma_i \times \Delta_i$. Moreover, such a pair $(\overline{\mathbf{k}}, \overline{\mathbf{m}})$ is referred to as an individual-based ex post general mechanism.

Definition 5 *A joint-based direct mechanism $(\mathbf{k}, \mathbf{a})$ is **ex post incentive compatible (EPIC)** if it induces truthful reporting and obedient acting as the EPE for all the agents, i.e. for each $i \in \mathcal{N}$ and each $t \in T$,*

$$\begin{aligned} & \widehat{V}_i(\mathbf{k}(t), \mathbf{a}(t), t) \\ \geq \ & \widehat{V}_i(\mathbf{k}(t'_i, t_{-i}), a'_i, \mathbf{a}_{-i}(t'_i, t_{-i}), t), \end{aligned} \tag{3}$$

for all $a'_i \in A_i, t'_i \in T_i$.

Definition 6 *An individual-based direct contracting mechanism $(\overline{\mathbf{k}}, \overline{\mathbf{a}})$ is **ex post incentive compatible** if it induces truthful reporting and obedient acting as the EPE for all the agents, i.e., for each $i \in \mathcal{N}$ and each $t \in T$,*

$$\begin{aligned} & \widehat{V}_i(\overline{\mathbf{k}}(t), \overline{\mathbf{a}}(t), t) \\ \geq \ & \widehat{V}_i(\overline{\mathbf{k}}_i(t'_i), \overline{\mathbf{k}}_{-i}(t_{-i}), a'_i, \overline{\mathbf{a}}_{-i}(t_{-i}), t), \end{aligned} \tag{4}$$

for all $a'_i \in A_i, t'_i \in T_i$.

Throughout this paper, when the term "EPIC mechanisms" is used, it actually refers to the EPIC direct mechanism. In EPE, if a true type profile is given, either the participation strategy profile or truthful reporting and obedient acting can actually be viewed as Nash equilibrium.

Thus, there are four principal's optimization problems to address contracting games over ex post mechanisms.

(P1) joint-based ex post general mechanism design problem

$$\begin{aligned} & \max_{(\mathbf{k},\mathbf{m}) \in \mathcal{F}(R,\mathcal{K},M)} \max_{(\rho,\delta) \in \Gamma \times \Delta} \int_T \widehat{U}(\mathbf{k}(\rho(t)), \delta(\mathbf{m}(\rho(t)), t), t)\mu(dt) \\ & \text{s.t. } (\rho, \delta) \text{ is the EPE under } (\mathbf{k}, \mathbf{m}). \end{aligned}$$

(P1′) individual-based ex post general mechanism design problem

$$\max_{(\overline{\mathbf{k}},\overline{\mathbf{m}})\in\overline{\mathcal{F}}(R,\mathcal{K},M)} \max_{(\rho,\delta)\in\Gamma\times\Delta} \int_T \widehat{U}(\overline{\mathbf{k}}(\rho(t)),\delta(\overline{\mathbf{m}}(\rho(t)),t),t)\mu(dt)$$
$$\text{s.t. } (\rho,\delta) \text{ is the EPE under } (\overline{\mathbf{k}},\overline{\mathbf{m}}).$$

(P2) joint-based EPIC mechanism design problem

$$\max_{(\mathbf{k},\mathbf{a})\in\mathcal{F}(T,\mathcal{K},A)} \int_T \widehat{U}(\mathbf{k}(t),\mathbf{a}(t),t)\mu(dt)$$
$$\text{s.t. } (\mathbf{k},\mathbf{a}) \text{ is } \mathit{EPIC}.$$

(P2′) individual-based EPIC mechanism design problem

$$\max_{(\overline{\mathbf{k}},\overline{\mathbf{a}})\in\overline{\mathcal{F}}(T,\mathcal{K},A)} \int_T \widehat{U}(\overline{\mathbf{k}}(t),\overline{\mathbf{a}}(t),t)\mu(dt)$$
$$\text{s.t. } (\overline{\mathbf{k}},\overline{\mathbf{a}}) \text{ is } \mathit{EPIC}.$$

Remark 1 *The principal may also consider maximizing her ex-post payoff given any type profile t instead. But this will not bring structural change in the analysis of this paper.*

It is worth noting that the subgame induced by the pre-offered general mechanism may yield multiple equilibria. To solve the mechanism design problem, the principal needs to select a particular equilibrium for tie-breaking. She may have sufficient bargaining power to designate a particular equilibrium for the agents to play. Or, she can recommend the agents to play a particular equilibrium. Due to the focal-point effect, the agents will follow such a recommendation.[15] The principal considers the particular EPE (ρ, δ) that gives the principal the highest possible expected utility under each general mechanism. By choosing the optimal ex post general mechanism, the entire principal-agent contracting game can achieve an equilibrium. In direct mechanism design, the principal just sticks to truth-telling and obedient-acting as the particular EPE under each direct mechanism. But does it also lead to an equilibrium for the entire principal-agent game by choosing the optimal EPIC mechanisms? The revelation principle will provide the answer.

[15] Myerson (1988) also talks about multiple equilibria and equilibrium selection in mechanism design in a survey.

3.3 The Revelation Principle for Ex Post Mechanisms

Chung and Ely (2006) state a version of the revelation principle for *ex post* incentive compatibility together with *ex post* individual rationality in the pure adverse selection environment. In parallel with Myerson's (1982) synthesis of the revelation principle for generalized randomized strategy Bayesian games with adverse selection and moral hazard, the revelation principle in this paper is substantially proved for the class of generalized pure strategy ex post mechanism games with both adverse selection and moral hazard. Its implication is that one can restrict attention to the EPIC mechanisms out of the ex post general mechanisms.

Theorem 1 *(The revelation principle for ex post mechanisms)*

Given any EPE (ρ, δ) of the subgame played by the agents under any joint-based ex post general mechanism $(\mathbf{k}, \mathbf{m}) \in \mathcal{F}(R, \mathcal{K}, M)$ in ***(P1)****, there exists a joint-based EPIC mechanism $(\widetilde{\mathbf{k}}, \widetilde{\mathbf{a}}) \in \mathcal{F}(T, \mathcal{K}, A)$ in* ***(P2)*** *in which the principal obtains the same expected payoff as in the EPE (ρ, δ) of the given ex post general mechanism $(\mathbf{k}, \mathbf{m})$. Furthermore, the optimal joint-based EPIC mechanism solving* ***(P2)*** *is also optimal in the class of all joint-based ex post mechanisms. These results above also apply to individual-based ex post general mechanisms and individual-based EPIC mechanisms.*

Proof. See Appendix. ■

Referring to the setup of the four mechanism design problems in section 3.2, this revelation principle also suggests that it actually gives the principal the highest possible expected payoff under each given EPIC mechanism for the principal to pick truth-telling and obedient-acting as the EPE for the agents. By choosing the optimal EPIC mechanism, the entire principal-agent game also achieves an equilibrium. Anyway, from now on one can simply study the EPIC mechanism design problem instead of the ex post general mechanism design problem without any loss of generality.

4 Menu Design Problem and the Delegation Principle

4.1 Menus

An alternative way for the principal to seek a second best solution in contracting games is decentralized menu design. The principal does not need to process decentralized information or have the agents send messages to specify contracts for the agents. Instead she can design a joint menu, i.e. a subset of the feasible joint contract set, for the agents and allow them to pick the contracts from the joint menu on their own accord.

The possible contract menu for agent i is $C_i \in P(\mathcal{K}_i)$, where $P(\mathcal{K}_i)$ is the power set of $\mathcal{K}_i$. In view of feasible constraints in the joint contract set, the joint feasible menu is $C \in P_f(\mathcal{K}) \subseteq \{(C_1, \cdots, C_n) | (C_1, \cdots, C_n) \subseteq \mathcal{K}\}$, where $P_f(\mathcal{K})$ is a collection of nonempty, closed subsets of $\mathcal{K}$.

Because $\mathcal{K}$ is a compact metric space, $P_f(\mathcal{K})$ equipped with the Hausdorff metric is also a compact metric space (see Theorem 3.85 in Aliprantis and Border (2006)).

4.2 Contracting Game over Ex Post Menus

There are three stages in the principal-agent contracting game over menus: At Stage 1, the principal proposes a joint contract menu, which is commonly observable, to the agents. At Stage 2, each agent unilaterally learns his true type. The agents simultaneously select the contracts from the pre-offered joint menu and decide to stay or leave and choose unobservable actions if participating. At Stage 3, outcomes are realized and the contracts are implemented.

Each agent i's strategy is a pair of functions $\widetilde{f}_i : T_i \to \mathcal{K}_i$ and $\widetilde{a}_i : T_i \to A_i$. Agent i's contract selection according to his type is denoted by $\widetilde{f}_i \in \mathcal{F}_i$, and i's action selection according to his type is denoted by $\widetilde{a}_i \in \mathcal{A}_i$. Let

$$\widetilde{f} = (\widetilde{f}_i)_{i \in \mathcal{N}}, \widetilde{f}(t) = (\widetilde{f}_i(t_i))_{i \in \mathcal{N}}, \widetilde{f}_{-i}(t_{-i}) = (\widetilde{f}_j(t_j))_{j \in \mathcal{N} \setminus \{i\}},$$

$$\widetilde{a} = (\widetilde{a}_i)_{i \in \mathcal{N}}, \widetilde{a}(t) = (\widetilde{a}_i(t_i))_{i \in \mathcal{N}}, \widetilde{a}_{-i}(t_{-i}) = (\widetilde{a}_j(t_j))_{j \in \mathcal{N} \setminus \{i\}}.$$

Each menu offered by the principal induces a simultaneous-moved subgame played by the agents. Under a joint menu C, $\widetilde{f}(t) \in C$ for each $t \in T$. This actually imposes a constraint on the strategy profiles of the agents. Let $\widetilde{f} \in \mathcal{F}_c = \{\widetilde{f} | \widetilde{f}(t) \in C$ for each $t \in T\}$ and $\widetilde{a} \in \mathcal{A} = \prod_{i=1}^{n} \mathcal{A}_i$. EPE is regarded as the solution concept of this constrained subgame.

Definition 7 *A contract-action selection profile $(\widetilde{f}(\cdot), \widetilde{a}(\cdot))$ is said to be an* ***ex post equilibrium*** *under a joint menu C if for each $i \in \mathcal{N}$ and each $t \in T$,*

$$\widehat{V}_i(\widetilde{f}(t), \widetilde{a}(t), t) \geq \widehat{V}_i(\widetilde{f}_i'(t_i), \widetilde{f}_{-i}(t_{-i}), \widetilde{a}_i'(t_i), \widetilde{a}_{-i}(t_{-i}), t), \tag{5}$$

for all $\widetilde{f}_i' \in \mathcal{F}_i$ satisfying $(\widetilde{f}_i'(t_i), \widetilde{f}_{-i}(t_{-i})) \in C$ and all $\widetilde{a}_i' \in \mathcal{A}_i$. Moreover, such a joint menu C is referred to as an ***ex post (joint) menu****.*

In EPE, if a true type profile is given, the realized contract-action pair is a Nash equilibrium of the (constrained) subgame defined by an ex post menu. The principal can deduce that the agents will have the EPE contract-action selection profile in the subgame. The principal will hence have an optimization problem to address this contracting game over ex post menus.

(P3) Ex post menu design problem:
$\max_{C \in P_f(\mathcal{K})} \int_T \max_{\widetilde{f} \in \mathcal{F}_c, \widetilde{a} \in \mathcal{A}} \widehat{U}(\widetilde{f}(t), \widetilde{a}(t), t)\mu(dt)$
s.t. $(\widetilde{f}, \widetilde{a})$ is the EPE under C.

Again, in view of multiple equilibria, for tie-breaking the principal may designate or recommend $(\widetilde{f}, \widetilde{a})$ in her best interest for the agents with type profile t to follow. Hence, the principal can link one contract-action pair with each type profile t.

4.3 The Delegation Principle for Ex Post Implementation

Is there any connection between the menu design problem and the mechanism design problem? In fact we can completely characterize all ex post menus for contracting games using individual-based EPIC mechanisms. This helps establish the strategic equivalence between the individual-based EPIC mechanism design problem and the ex post menu design problem. These are summarized by the *delegation principle for ex post implementation*. In this sense, all individual-based EPIC mechanisms can be decentralized via ex post menus.

Consider the set-valued mapping $\Phi : T \times P_f(\mathcal{K}) \twoheadrightarrow \mathcal{K} \times A$ defined by

$$\Phi(t, C) := \{(\widetilde{f}(t), \widetilde{a}(t)) \in C \times A : (\widetilde{f}, \widetilde{a}) \text{ is the EPE under } C\}.$$

It is deducible by the principal and represents the t-type-profile agents' joint ex post equilibrium response to any menu offer C.

Definition 8 *Given $C \in P_f(\mathcal{K})$, $\Phi(\cdot, C)$ is said to be **well-defined** if for each t, $\Phi(t, C)$ is nonempty.*

A well-defined $\Phi(\cdot, C)$ implies that the agents with each given type profile t can possess at least one Nash equilibrium contract-action pair under the joint menu C. In other words, there exists at least one ex post menu C. Moreover, the continuity of each $\widehat{V}_i$ will imply that well-defined $\Phi(\cdot, C)$ has a closed graph. Hence, Lemma 17.51 (Aliprantis and

Border 2006) implies that the corresponding set of Nash equilibria[16] is a closed subset of $C \times A$ and then is compact. In other words, $\Phi(\cdot, C)$ is compact-valued. Moreover, Borel measurability of $\Phi(\cdot, C)$ can be ensured.

Lemma 1 *For any $C \in P_f(\mathcal{K})$, if $\Phi(\cdot, C)$ is well-defined, $\Phi(\cdot, C)$ has a closed graph.*

Proof. See Appendix. ∎

Proposition 2 *For any $C \in P_f(\mathcal{K})$ satisfying $\Phi(\cdot, C)$ is well-defined, $\Phi(\cdot, C)$ is a Borel-measurable set valued function from T to $C \times A$.*

Proof. See Appendix. ∎

The following theorem describes the complete characterization of all individual-based EPIC mechanisms via ex post menus.

Theorem 2 *(The delegation principle version 1).*

Given a contracting mechanism $(\overline{\mathbf{k}}, \overline{\mathbf{a}}) \in \overline{\mathcal{F}}(T, \mathcal{K}, A)$, the following statements are true: (i) If $(\overline{\mathbf{k}}(\cdot), \overline{\mathbf{a}}(\cdot))$ is EPIC, then there exists a joint menu $C \in P_f(\mathcal{K})$ such that $\Phi(\cdot, C)$ is well-defined, and $(\overline{\mathbf{k}}(\cdot), \overline{\mathbf{a}}(\cdot))$ is a Borel-measurable selection from $\Phi(\cdot, C)$, that is, $(\overline{\mathbf{k}}(t), \overline{\mathbf{a}}(t)) \in \Phi(t, C)$ for all $t \in T$. (ii) If there exists a joint menu $C \in P_f(\mathcal{K})$ such that $\Phi(\cdot, C)$ is well-defined, and $(\overline{\mathbf{k}}(\cdot), \overline{\mathbf{a}}(\cdot))$ is a Borel-measurable selection from $\Phi(\cdot, C)$, then $(\overline{\mathbf{k}}(\cdot), \overline{\mathbf{a}}(\cdot))$ is EPIC.

Proof. (i) Assume that $(\overline{\mathbf{k}}(\cdot), \overline{\mathbf{a}}(\cdot)) \in \overline{\mathcal{F}}(T, \mathcal{K}, A)$ is EPIC. Define

$$C = \prod_{i=1}^{n} cl\{(\overline{\mathbf{k}}_i(t_i) : t_i \in T_i\} \cap \mathcal{K}, \text{ where } cl \text{ denotes the closure.}$$

First claim that $(\overline{\mathbf{k}}(t), \overline{\mathbf{a}}(t)) \in \Phi(t, C)$ for all $t \in T$, that is, for each $i = 1, \cdots, n$, and each $t \in T$,

$$\widehat{V}_i(\overline{\mathbf{k}}(t), \overline{\mathbf{a}}(t), t) \geq \widehat{V}_i(\widetilde{f}_i'(t_i), \overline{\mathbf{k}}_{-i}(t_{-i}), \widetilde{a}_i'(t_i), \overline{\mathbf{a}}_{-i}(t_{-i}), t),$$

for all $\widetilde{f}_i' \in \mathcal{F}_i$ satisfying $(\widetilde{f}_i'(t_i), \overline{\mathbf{k}}_{-i}(t_{-i})) \in C$ and all $\widetilde{a}_i' \in \mathcal{A}_i$. Suppose not. Then for some agent j, some agents' types profile $t' = (t_j', t_{-j}')$, some $\widetilde{f}_j' \in \mathcal{F}_j$ satisfying $(\widetilde{f}_j'(t_j'), \overline{\mathbf{k}}_{-j}(t_{-j}')) \in C$, and some $\widetilde{a}_j' \in \mathcal{A}_j$,

$$\widehat{V}_j(\overline{\mathbf{k}}(t'), \overline{\mathbf{a}}(t'), t') < \widehat{V}_j(\widetilde{f}_j'(t_j'), \overline{\mathbf{k}}_{-j}(t_{-j}'), \widetilde{a}_j'(t_j'), \overline{\mathbf{a}}_{-j}(t_{-j}'), t').$$

[16] Even if the game is constrained, classic fixed point theorems related to Nash equilibria can still apply. One can refer to Rosen (1965) and Ponstein (1966).

Because of the definition of C, any section of C is still closed. Thus, for any t_{-j}, there exists a sequence of type $\{t_{j,l}\}_l$ in T_j such that $(\overline{\mathbf{k}}_j(t_{j,l}), \overline{\mathbf{k}}_{-j}(t'_{-j})) \to (\widetilde{f}'_j(t'_j), \overline{\mathbf{k}}_{-j}(t'_{-j}))$ in C, as $l \to \infty$. But since $\widehat{V}_j(\overline{\mathbf{k}}(t'), \overline{\mathbf{a}}(t'), t') < \widehat{V}_j(\widetilde{f}'_j(t'_j), \overline{\mathbf{k}}_{-j}(t'_{-j}), \widetilde{a}'_j(t'_j), \overline{\mathbf{a}}_{-j}(t'_{-j}), t')$, by the continuity of $\widehat{V}_j(\cdot, \cdot, t')$, the fact that $(\overline{\mathbf{k}}_j(t_{j,l}), \overline{\mathbf{k}}_{-j}(t'_{-j})) \to (\widetilde{f}'_j(t'_j), \overline{\mathbf{k}}_{-j}(t'_{-j}))$ in C, as $l \to \infty$, implies that for l large enough,

$$\widehat{V}_j(\overline{\mathbf{k}}(t'), \overline{\mathbf{a}}(t'), t') < \widehat{V}_j(\overline{\mathbf{k}}_j(t_{j,l}), \overline{\mathbf{k}}_{-j}(t'_{-j}), \widetilde{a}'_j(t'_j), \overline{\mathbf{a}}_{-j}(t'_{-j}), t').$$

This contradicts the fact that $(\overline{\mathbf{k}}(\cdot), \overline{\mathbf{a}}(\cdot))$ is EPIC.

Therefore, $(\overline{\mathbf{k}}(\cdot), \overline{\mathbf{a}}(\cdot))$ is a Borel-measurable selection from $\Phi(\cdot, C)$. Clearly, $\Phi(\cdot, C)$ is well-defined.

(ii) Assume that $(\overline{\mathbf{k}}(t), \overline{\mathbf{a}}(t)) \in \Phi(t, C) \subseteq C \times A$ for all $t \in T$. For all $i \in \mathcal{N}$, all $t \in T$, all $\widetilde{f}_i{}' \in \mathcal{F}_i$ satisfying $(\widetilde{f}_i{}'(t_i), \overline{\mathbf{k}}_{-i}(t_{-i})) \in C$, and all $\widetilde{a}'_i \in \mathcal{A}_i$,

$$\widehat{V}_i(\overline{\mathbf{k}}(t), \overline{\mathbf{a}}(t), t) \geq \widehat{V}_i(\widetilde{f}_i{}'(t_i), \overline{\mathbf{k}}_{-i}(t_{-i}), \widetilde{a}'_i(t_i), \overline{\mathbf{a}}_{-i}(t_{-i}), t).$$

Since there are some $\widetilde{f}_i{}'$ satisfying $\widetilde{f}_i{}'(t_i) = \overline{\mathbf{k}}_i(t'_i)$ for any $t'_i \in T_i$ and some $\widetilde{a}'_i$ satisfying $\widetilde{a}'_i(t_i) = a'_i$ for any $a'_i \in A_i$, we have

$$\widehat{V}_i(\overline{\mathbf{k}}(t), \overline{\mathbf{a}}(t), t) \geq \widehat{V}_i(\overline{\mathbf{k}}_i(t'_i), \overline{\mathbf{k}}_{-i}(t_{-i}), a'_i, \overline{\mathbf{a}}_{-i}(t_{-i}), t),$$

for all $t'_i \in T_i$ and all $a'_i \in A_i$. Thus, $(\overline{\mathbf{k}}(\cdot), \overline{\mathbf{a}}(\cdot))$ is EPIC. ■

Furthermore, the ex post menu design problem **(P3)** can be rewritten in a compact way as long as $\Phi(\cdot, C)$ is well-defined for some $C \in P_f(\mathcal{K})$:

$$\max_{C \in P_f(\mathcal{K})} \int_T \max_{(\widetilde{f}(t), \widetilde{a}(t)) \in \Phi(t, C)} \widehat{U}(\widetilde{f}(t), \widetilde{a}(t), t)\mu(dt).$$

Now define the feasible individual-based EPIC mechanism set

$$\mathcal{IC}^I := \left\{(\overline{\mathbf{k}}(\cdot), \overline{\mathbf{a}}(\cdot)) \in \overline{\mathcal{F}}(T, \mathcal{K}, A) : (\overline{\mathbf{k}}(\cdot), \overline{\mathbf{a}}(\cdot)) \text{ is EPIC.}\right\}$$

The individual-based ex post direct mechanism game can also be stated compactly as

$$\max_{(\overline{\mathbf{k}}, \overline{\mathbf{a}}) \in \mathcal{IC}^I} \int_T \widehat{U}(\overline{\mathbf{k}}(t), \overline{\mathbf{a}}(t), t)\mu(dt).$$

Besides, define the equivalent mechanism set induced by a joint feasible menu C

$$\Sigma_\Phi(C) := \left\{(\overline{\mathbf{k}}(\cdot), \overline{\mathbf{a}}(\cdot)) \in \overline{\mathcal{F}}(T, \mathcal{K}, A) : (\overline{\mathbf{k}}(t), \overline{\mathbf{a}}(t)) \in \Phi(t, C) \text{ for all } t \in T\right\}.$$

It denotes the set of all measurable selections from $\Phi(\cdot, C)$ in $\overline{\mathcal{F}}(T, \mathcal{K}, A)$ for a given menu $C \in P_f(\mathcal{K})$. Next, define the overall equivalent mechanism set induced by the joint feasible menu set

$$\Sigma_\Phi := \bigcup_{C \in P_f(\mathcal{K})} \Sigma_\Phi(C).$$

Indeed, the delegation principle version 1 implies that

$$\mathcal{IC}^I = \Sigma_\Phi.$$

Let us first see a very useful lemma.

Lemma 2 *For each $C \in P_f(\mathcal{K})$ satisfying $\Phi(\cdot, C)$ is well-defined, there exists some $(\overline{\mathbf{k}}, \overline{\mathbf{a}}) \in \Sigma_\Phi(C)$ such that*

$$\widehat{U}(\overline{\mathbf{k}}(t), \overline{\mathbf{a}}(t), t) = \max_{(\widetilde{f}(t), \widetilde{a}(t)) \in \Phi(t, C)} \widehat{U}(\widetilde{f}(t), \widetilde{a}(t), t),$$

for all $t \in T$. Moreover, the function $t \mapsto \max_{(f,a) \in \Phi(t,C)} \widehat{U}(f, a, t)$ is Borel measurable.

Proof. See Appendix. ■

Now we are ready to see the main result on the strategic equivalence of individual-based EPIC mechanisms design and ex post menu design.

Theorem 3 *(The delegation principle version 2).*

(i) If $(\overline{\mathbf{k}}^*(\cdot), \overline{\mathbf{a}}^*(\cdot))$ solves the contracting problem over individual-based EPIC mechanisms given by

$$\max_{(\overline{\mathbf{k}}, \overline{\mathbf{a}}) \in \mathcal{IC}^I} \int_T \widehat{U}(\overline{\mathbf{k}}(t), \overline{\mathbf{a}}(t), t) \mu(dt),$$

then $C^* = \prod_{i=1}^{n} cl\{(\overline{\mathbf{k}}_i^*(t_i) : t_i \in T_i\} \cap \mathcal{K}$ solves the contracting problem over ex post menus given by

$$\max_{C \in P_f(\mathcal{K})} \int_T \max_{(\widetilde{f}(t), \widetilde{a}(t)) \in \Phi(t, C)} \widehat{U}(\widetilde{f}(t), \widetilde{a}(t), t) \mu(dt).$$

(ii) If C^* solves the contracting problem over ex post menus given by

$$\max_{C \in P_f(\mathcal{K})} \int_T \max_{(\widetilde{f}(t), \widetilde{a}(t)) \in \Phi(t, C)} \widehat{U}(\widetilde{f}(t), \widetilde{a}(t), t) \mu(dt),$$

then $(\overline{\mathbf{k}}^*(\cdot), \overline{\mathbf{a}}^*(\cdot)) \in \Sigma_\Phi(C^*)$ *satisfying*

$$(\overline{\mathbf{k}}^*(t), \overline{\mathbf{a}}^*(t)) \in \underset{(\widetilde{f}(t),\widetilde{a}(t))\in\Phi(t,C^*)}{\text{argmax}} \widehat{U}(\widetilde{f}(t), \widetilde{a}(t), t)$$

for all $t \in T$*, solves the contracting problem over individual-based EPIC mechanisms given by*

$$\max_{(\overline{\mathbf{k}},\overline{\mathbf{a}})\in\mathcal{IC}^I} \int_T \widehat{U}(\prod_{i=1}^n \overline{\mathbf{k}}_i(t_i), \prod_{i=1}^n \overline{\mathbf{a}}_i(t_i), t)\mu(dt).$$

In both cases, the optimal objective values of the two problems are equal.

Proof. (i) Since $(\overline{\mathbf{k}}^*(t), \overline{\mathbf{a}}^*(t)) \in \Phi(t, C^*)$, we have

$$\int_T \max_{(\widetilde{f}(t),\widetilde{a}(t))\in\Phi(t,C^*)} \widehat{U}(\widetilde{f}(t), \widetilde{a}(t), t)\mu(dt) \geq \int_T \widehat{U}(\overline{\mathbf{k}}^*(t), \overline{\mathbf{a}}^*(t), t)\mu(dt).$$

Thus, for all $(\overline{\mathbf{k}}, \overline{\mathbf{a}}) \in \mathcal{IC}^I$,

$$\int_T \widehat{U}(\overline{\mathbf{k}}^*(t), \overline{\mathbf{a}}^*(t), t)\mu(dt) \geq \int_T \widehat{U}(\overline{\mathbf{k}}(t), \overline{\mathbf{a}}(t), t)\mu(dt).$$

Then, by the delegation principle version 1, $\mathcal{IC}^I = \Sigma_\Phi = \bigcup_{C\in P_f(\mathcal{K})} \Sigma_\Phi(C)$. Hence, for all $(\overline{\mathbf{k}}, \overline{\mathbf{a}}) \in \bigcup_{C\in P_f(\mathcal{K})} \Sigma_\Phi(C)$, we have

$$\int_T \widehat{U}(\overline{\mathbf{k}}^*(t), \overline{\mathbf{a}}^*(t), t)\mu(dt) \geq \int_T \widehat{U}(\overline{\mathbf{k}}(t), \overline{\mathbf{a}}(t), t)\mu(dt). \tag{6}$$

Moreover, by Lemma 2, for each $C \in P_f(\mathcal{K})$, there exists some $(\overline{\mathbf{k}}', \overline{\mathbf{a}}') \in \Sigma_\Phi(C)$ such that $\widehat{U}(\overline{\mathbf{k}}'(t), \overline{\mathbf{a}}'(t), t) = \max_{(\widetilde{f}(t),\widetilde{a}(t))\in\Phi(t,C)} \widehat{U}(\widetilde{f}(t), \widetilde{a}(t), t)$ for all $t \in T$. Thus, by (6), for each $C \in P_f(\mathcal{K})$, we have

$$\begin{aligned} & \int_T \widehat{U}(\overline{\mathbf{k}}^*(t), \overline{\mathbf{a}}^*(t), t)\mu(dt) \\ \geq \; & \int_T \max_{(\widetilde{f}(t),\widetilde{a}(t))\in\Phi(t,C)} \widehat{U}(\widetilde{f}(t), \widetilde{a}(t), t)\mu(dt). \end{aligned}$$

Therefore, $\int_T \max_{(f,a)\in\Phi(t,C^*)} \widehat{U}(f, a, t)\mu(dt) \geq \int_T \max_{(\widetilde{f}(t),\widetilde{a}(t))\in\Phi(t,C)} \widehat{U}(\widetilde{f}(t), \widetilde{a}(t), t)\mu(dt)$ for all $C \in P_f(\mathcal{K})$. Hence, C^* solves the given contracting game over menus.

Clearly,

$$\max_{C\in P_f(\mathcal{K})}\int_T \max_{(\widetilde{f}(t),\widetilde{a}(t))\in\Phi(t,C)} \widehat{U}(\widetilde{f}(t),\widetilde{a}(t),t)\mu(dt)$$
$$= \int_T \widehat{U}(\overline{\mathbf{k}}^*(t),\overline{\mathbf{a}}^*(t),t)\mu(dt) = \max_{(\overline{\mathbf{k}},\overline{\mathbf{a}})\in\mathcal{IC}^I}\int_T \widehat{U}(\overline{\mathbf{k}}(t),\overline{\mathbf{a}}(t),t)\mu(dt).$$

(ii) For each $C\in P_f(\mathcal{K})$, by hypotheses,

$$\int_T \widehat{U}(\overline{\mathbf{k}}^*(t),\overline{\mathbf{a}}^*(t),t)\mu(dt) = \int_T \max_{(\widetilde{f}(t),\widetilde{a}(t))\in\Phi(t,C^*)} \widehat{U}(\widetilde{f}(t),\widetilde{a}(t),t)\mu(dt)$$
$$\geq \int_T \max_{(\widetilde{f}(t),\widetilde{a}(t))\in\Phi(t,C)} \widehat{U}(\widetilde{f}(t),\widetilde{a}(t),t)\mu(dt) = \int_T \widehat{U}(\overline{\mathbf{k}}(t),\overline{\mathbf{a}}(t),t)\mu(dt),$$

for all $(\overline{\mathbf{k}}(\cdot),\overline{\mathbf{a}}(\cdot))\in\Sigma_\Phi(C)$ satisfying $\widehat{U}(\overline{\mathbf{k}}(t),\overline{\mathbf{a}}(t),t) = \max_{(\widetilde{f}(t),\widetilde{a}(t))\in\Phi(t,C)} \widehat{U}(\widetilde{f}(t),\widetilde{a}(t),t)$ for any $t\in T$. The existence of such a $(\overline{\mathbf{k}}(\cdot),\overline{\mathbf{a}}(\cdot))$ is ensured by Lemma 2. It implies that

$$\int_T \widehat{U}(\overline{\mathbf{k}}^*(t),\overline{\mathbf{a}}^*(t),t)\mu(dt) = \max_{(\overline{\mathbf{k}},\overline{\mathbf{a}})\in\bigcup_{C\in P_f(\mathcal{K})}\Sigma_\Phi(C)}\int_T \widehat{U}(\overline{\mathbf{k}}(t),\overline{\mathbf{a}}(t),t)\mu(dt). \tag{7}$$

Also, by the delegation principle version 1,

$$\mathcal{IC}^I = \Sigma_\Phi = \bigcup_{C\in P_f(\mathcal{K})}\Sigma_\Phi(C). \tag{8}$$

Hence, by (7) and (8), we have

$$\max_{C\in P_f(\mathcal{K})}\int_T \max_{(\widetilde{f}(t),\widetilde{a}(t))\in\Phi(t,C)} \widehat{U}(\widetilde{f}(t),\widetilde{a}(t),t)\mu(dt) = \int_T \max_{(\widetilde{f}(t),\widetilde{a}(t))\in\Phi(t,C^*)} \widehat{U}(\widetilde{f}(t),\widetilde{a}(t),t)\mu(dt)$$
$$= \int_T \widehat{U}(\overline{\mathbf{k}}^*(t),\overline{\mathbf{a}}^*(t),t)\mu(dt) = \max_{(\overline{\mathbf{k}},\overline{\mathbf{a}})\in\bigcup_{C\in P_f(\mathcal{K})}\Sigma_\Phi(C)}\int_T \widehat{U}(\overline{\mathbf{k}}(t),\overline{\mathbf{a}}(t),t)\mu(dt)$$
$$= \max_{(\overline{\mathbf{k}},\overline{\mathbf{a}})\in\mathcal{IC}^I}\int_T \widehat{U}(\overline{\mathbf{k}}(t),\overline{\mathbf{a}}(t),t)\mu(dt).$$

Therefore, $(\overline{\mathbf{k}}^*(\cdot),\overline{\mathbf{a}}^*(\cdot))$ solves the given contracting game over mechanisms. ■

This subtle delegation principle suggests that ex post menu design is strategically equivalent to individual-based EPIC mechanism design *rather than* joint-based EPIC mechanism design. According to the delegation principle version 1, through the EPE behavior of the agents for each type profile in choosing a contract-action profile from the pre-offered joint menu, each ex post menu design links a contract-action profile for the agents to each true type profile of theirs. Such an EPE behavior implicitly defines an individual-based EPIC mechanism. The delegation principle version 2 further shows that two kinds of designs will eventually yield the same optimal objective values in contracting problems. It intimates that ex post menus only incorporate separate information as individual-based EPIC mechanisms do *rather than* relative information as joint-based EPIC mechanisms do in specifying contract-action pairs for each agent.

5 Comparative Advantage of Joint-based Mechanisms

An immediate concern after the revelation principle and delegation principle is whether joint-based mechanism can function better for the principal than individual-based mechanism design and menu design. Joint-based mechanism design can *make the principal better off* than individual-based mechanism design (resp. menu design) if the solution to the joint-based mechanism design problem yields at least as high an expected payoff to the principal as the solution to the individual-based mechanism design problem (resp. menu design problem). Throughout section 5, it suffices to study EPIC mechanisms out of general ex post mechanisms due to the revelation principle. Through the revelation principle and delegation principle, individual-based EPIC mechanism serves as a bridge for us to connect all the procedures in contracting games and compare them. The center is to compare the joint-based EPIC mechanism design problem (**P2**) with the individual-based EPIC mechanism design problem (**P2′**).

(**P2**) can be rewritten in a compact form as below:

$$\begin{aligned} &\max_{(\mathbf{k},\mathbf{a})\in\mathcal{IC}^J}\int_T \widehat{U}(\mathbf{k}(t),\mathbf{a}(t),t)\mu(dt) \\ \mathcal{IC}^J \quad : &= \{(\mathbf{k}(\cdot),\mathbf{a}(\cdot))\in\mathcal{F}(T,\mathcal{K},A):(\mathbf{k}(\cdot),\mathbf{a}(\cdot)) \text{ is EPIC.}\} \end{aligned}$$

(**P2′**) can be rewritten in a compact form as below:

$$\begin{aligned} &\max_{(\overline{\mathbf{k}},\overline{\mathbf{a}})\in\mathcal{IC}^I}\int_T \widehat{U}(\overline{\mathbf{k}}(t),\overline{\mathbf{a}}(t),t)\mu(dt) \\ \mathcal{IC}^I \quad : &= \left\{(\overline{\mathbf{k}}(\cdot),\overline{\mathbf{a}}(\cdot))\in\overline{\mathcal{F}}(T,\mathcal{K},A):(\overline{\mathbf{k}}(\cdot),\overline{\mathbf{a}}(\cdot)) \text{ is EPIC.}\right\} \end{aligned}$$

We need to utilize some manipulation to compare **(P2)** and **(P2′).** Let $\pi_i : T \to T_i$ be the projection function defined by $\pi_i(t_1, \cdots, t_n) = t_i$. Define the projective joint-based direct mechanism set

$$\mathcal{F}_\pi(T,\mathcal{K},A) := \left\{((\overline{\mathbf{k}}_i \circ \pi_i)_{i\in\mathcal{N}}, (\overline{\mathbf{a}}_i \circ \pi_i)_{i\in\mathcal{N}}) \in \mathcal{F}(T,\mathcal{K},A) : (\overline{\mathbf{k}}, \overline{\mathbf{a}}) \in \overline{\mathcal{F}}(T,\mathcal{K},A)\right\}.$$

Thus, $\mathcal{F}_\pi(T,\mathcal{K},A) \subseteq \mathcal{F}(T,\mathcal{K},A)$. Then it is safe to define the projective joint-based EPIC mechanism set

$$\mathcal{IC}_I^J := \left\{((\overline{\mathbf{k}}_i \circ \pi_i)_{i\in\mathcal{N}}, (\overline{\mathbf{a}}_i \circ \pi_i)_{i\in\mathcal{N}}) \in \mathcal{F}(T,\mathcal{K},A) : (\overline{\mathbf{k}}, \overline{\mathbf{a}}) \in \mathcal{IC}^I\right\}.$$

Observe that $\mathcal{IC}^I$ and $\mathcal{IC}_I^J$ are equivalent, i.e. they are in 1-1 correspondence with each other. Now consider a new mechanism design problem **(P2″)**:

$$\max_{(\mathbf{k},\mathbf{a})\in\mathcal{IC}_I^J} \int_T \widehat{U}(\mathbf{k}(t), \mathbf{a}(t), t)\mu(dt).$$

Obviously, **(P2′)** and **(P2″)** will achieve the identical maxima at optimal 1-1 corresponding optimal mechanisms $(\overline{\mathbf{k}}^*, \overline{\mathbf{a}}^*)$ and $(\mathbf{k}^*, \mathbf{a}^*) = ((\overline{\mathbf{k}}_i^* \circ \pi_i)_{i\in\mathcal{N}}, (\overline{\mathbf{a}}_i^* \circ \pi_i)_{i\in\mathcal{N}})$. In this sense, one can view **(P2′)** and **(P2″)** as essentially equivalent problems. Thus one can convert **(P2′)** to **(P2″)** and alternatively compare **(P2)** with **(P2″)** to check the comparison between joint-based mechanism design and individual-based mechanism design.

Apparently, $\mathcal{IC}_I^J \subseteq \mathcal{IC}^J$. With the same objective function, **(P2)** will hence yield a solution at least as good as **(P2′)** and also **(P3)** due to the delegation principle. This result is summarized in Theorem 4.

Theorem 4 *Assume that optimal solutions to **(P2)** and **(P2′)** exist. Joint-based EPIC mechanism design makes the principal better off than both individual-based EPIC mechanism design and ex post menu design.*

Compared with individual-based EPIC mechanisms, joint-based EPIC mechanisms suggest the relative information evaluation, that is, the principal's contract-action specification for each agent is on the basis of peer types (reports). She can refer to not only single agent's information communication but also all the other agents'. With the relative information evaluation, the principal can enhance her efficiency of decision making to deal with information asymmetry. The revelation principle suggests that this result can be extended from EPIC mechanisms to general mechanisms. This also interprets the reason why joint-based mechanism design is more frequently observed in practice and of more interest to researchers.

Although the individual-based EPIC mechanism is not frequently observed in practice, it serves as a hub in the examination of centralization versus decentralization in multi-agency contracting. This analysis suggests that the joint-based ex post mechanism design dominates the individual-based ex post mechanism design and also the ex post menu design in multi-agency environments. In this respect, centralization of the right to specify contracts and actions can take advantage of joint-based ex post mechanisms, in the sense of relative information evaluation, to make the principal better off than decentralization in contracting.

It is worth noting that interrelated impacts of the agents' asymmetric information on the principal's welfare are important for the comparative advantage of joint-based mechanisms. When the impacts of different agents' asymmetric information are independent and separate, the joint-based ex post mechanism design, individual-based ex post mechanism design and ex post menu design must be *payoff-equivalent*, that is, the solutions to all three design problems yield the same expected payoff for the principal.[17]

Proposition 3 *If (i)* $\mathcal{K} = \mathcal{K}_1^c \times \cdots \times \mathcal{K}_n^c$*, where* $\mathcal{K}_i^c$ *is a compact subset of* $\mathcal{K}_i$ *for each* $i \in \mathcal{N}$*, (ii) for each* i*,* $\widehat{V}_i(f, a, t) \equiv \widehat{V}_i(f_i, a_i, t_i)$*, and (iii)* $\int_T \widehat{U}(f, a, t)\mu(dt) \equiv \sum_{i=1}^{n} \int_{T_i} \widehat{U}_i(f_i, a_i, t_i)\mu_i(dt_i)$*, where* $\int_{T_i} \widehat{U}_i(f_i, a_i, t_i)\mu_i(dt_i)$ *is the expected payoff that the principal can get merely from agent* i*, then joint-based EPIC mechanism design is payoff-equivalent to both individual-based EPIC mechanism design and ex post menu design.*

Proof. The proof is straightforward. Under the hypotheses i)-iii), solving **(P2)** is equivalent to solving n independent problems simultaneously. The i-th problem is

$$\max_{(\overline{\mathbf{k}}_i, \overline{\mathbf{a}}_i) \in \overline{\mathcal{F}}(T_i, \mathcal{K}_i, A_i)} \int_{T_i} \widehat{U}(\overline{\mathbf{k}}_i(t_i), \overline{\mathbf{a}}_i(t_i), t_i)\mu_i(dt_i)$$
$$\text{s.t. } \widehat{V}_i(\overline{\mathbf{k}}_i(t_i), \overline{\mathbf{a}}_i(t_i), t_i) \geq \widehat{V}_i(\overline{\mathbf{k}}_i(t_i'), a_i', t_i), \text{for all } a_i' \in A_i \text{ and } t_i, t_i' \in T_i,$$

where $\overline{\mathcal{F}}(T_i, \mathcal{K}_i, A_i) = \mathcal{M}(T_i, \mathcal{K}_i) \times \mathcal{M}(T_i, \mathcal{A}_i)$. In this case, $(\overline{\mathbf{k}}, \overline{\mathbf{a}})$ will yield the sum of the optimal objective values equal to the optimal objective value of **(P2′)** and also **(P3).** ■

Hypothesis (i) suggests that feasible contract profiles for all agents are not cross-constrained. Hypothesis (ii) implies that the agents are fully independent, i.e., each agent's expected payoff function only depends on his own contract, action and type. Hypothesis (iii) means that the principal's expect payoff is additively separable with respect to individual agents.

[17] The delegation principle indicates that this equivalence occurs if some feasible constraint forces the feasible mechanisms to be individual-based. The "bilateral contracting" environment studied by Han (2006) can be viewed as a particular example in this situation.

In this situation, the principal can treat the multiple agents as separate and independent individuals in her viewpoint. Even if these agents are heterogeneous, individual-based ex post mechanism design or ex post menu design can still take the same effect as joint-based ex post mechanism design. It is also worth noting that ex post equilibria of the subgames played by the agents coincide with dominant strategy equilibria in this context, as each agent merely has private valuation.

6 An Application: Financial Regulation

As an aftermath of the recent financial crisis, more attention has been paid to economic regulation again. In most contexts, one regulator is facing many regulatees instead of only one regulatee. Nowadays, the impacts of different regulatees' asymmetric information have become more and more interrelated. As a result, severe exacerbation of asymmetric information has been observed in financial crisis. Giammarino et al. (1993) examine an incentive approach to banking regulation with respect to single agent versus single principal. They simply focus on the design of decentralized menus of regulatory options in the regulatory contracting process due to the delegation principle for single-agency. However, decentralization cannot function better than centralization in multi-agency. Let us see a simple application of the main results of this paper in banking regulation with interrelated impacts of asymmetric information.

One regulator engages in regulating two duopolistic banks $i = 1, 2$. We simply consider pure adverse selection. The innate quality of the bank i's loan portfolio, as i's private information, is $q_i \in \{L, H\}$, where L stands for low type and H stands for high type. The probability distribution μ of (q_1, q_2) is given as follows:

$$\mu(L, H) = \mu(H, L) = \frac{7}{16}; \mu(L, L) = \mu(H, H) = \frac{1}{16}.$$

This suggests that it is most likely that one bank is high type and the other is low type. The regulatory option is simply the level of equity financing required, denoted by $e_i \in \{1, 10\}$ for bank $i = 1, 2$.

Bank i's payoff function is $v_i(e_i, q_1, q_2)$. The banks' rankings of contracts for each type profile, given by the banks' payoff functions, are as below:

$v_1(10, L, H) = v_1(1, L, H), v_1(10, L, L) > v_1(1, L, L),$

$v_1(1, H, H) > v_1(10, H, H), v_1(1, H, L) > v_1(10, H, L);$

$v_2(10, H, L) = v_2(1, H, L), v_2(10, L, L) > v_2(1, L, L),$

$v_2(1, H, H) > v_2(10, H, H), v_2(1, L, H) > v_2(10, L, H).$

Assume that all the utilities are positive and the outside option is 0. This means that the individual rationality is automatically met.

The payoff function of the regulator $u(e_1, e_2, q_1, q_2)$ is specified as follows:

$u(10, 10, H, H) = 15, u(10, 1, H, H) = u(1, 10, H, H) = 13,$

$u(10, 1, L, H) = u(1, 10, H, L) = 12, u(1, 1, H, H) = 11,$

$u(10, 10, L, H) = u(10, 10, H, L) = 10, u(10, 10, L, L) = 8,$

$u(10, 1, L, L) = u(1, 10, L, L) = 6, u(10, 1, H, L) = u(1, 10, L, H) = 4,$

$u(1, 1, L, H) = u(1, 1, H, L) = 3, u(1, 1, L, L) = 1.$

The expected payoff of the regulator given any (e_1, e_2) is

$$\widetilde{U}(e_1, e_2) = \frac{7}{16}u(e_1, e_2, L, H) + \frac{1}{16}u(e_1, e_2, L, L) + \frac{7}{16}u(e_1, e_2, H, L) + \frac{1}{16}u(e_1, e_2, H, H).$$

The joint-based direct regulatory contracting mechanism is $\mathbf{E} = \{E_i(q_1, q_2)\}_{i=1,2}$. If the banks report their types (q_1', q_2') to the regulator, the level of equity financing required $E_i(q_1, q_2)$ will be specified to each bank i. All the available mechanisms can thus be viewed as all the combination of the ordered tuples (q_1', q_2', e_1, e_2). In view of the banks' rankings of contracts for each type profile, all individual-based EPIC mechanisms available to the regulator are:

$$\overline{\mathbf{E}}^1 = \{\overline{E}_i(H) = 1, \overline{E}_i(L) = 10\}_{i=1,2}, \overline{\mathbf{E}}^2 = \{\overline{E}_i(H) = \overline{E}_i(L) = 10\}_{i=1,2},$$

$$\overline{\mathbf{E}}^3 = \{\overline{E}_i(H) = \overline{E}_i(L) = 1\}_{i=1,2}, \overline{\mathbf{E}}^4 = \{\overline{E}_1(H) = \overline{E}_1(L) = 1, \overline{E}_2(H) = \overline{E}_2(L) = 10\},$$

$$\overline{\mathbf{E}}^5 = \{\overline{E}_1(H) = \overline{E}_1(L) = 10, \overline{E}_2(H) = \overline{E}_2(L) = 1\},$$

$$\overline{\mathbf{E}}^6 = \{\overline{E}_1(H) = \overline{E}_1(L) = 1, \overline{E}_2(H) = 1, \overline{E}_2(L) = 10\},$$

$$\overline{\mathbf{E}}^7 = \{\overline{E}_1(H) = \overline{E}_1(L) = 10, \overline{E}_2(H) = 1, \overline{E}_2(L) = 10\},$$

$$\overline{\mathbf{E}}^8 = \{\overline{E}_1(H) = 1, \overline{E}_1(L) = 10, \overline{E}_2(H) = \overline{E}_2(L) = 10\},$$

$$\overline{\mathbf{E}}^9 = \{\overline{E}_1(H) = 1, \overline{E}_1(L) = 10, \overline{E}_2(H) = \overline{E}_2(L) = 1\}.$$

The expected payoff of the regulator given EPIC mechanism $\mathbf{E}$ is

$$\begin{aligned} U(\mathbf{E}) &= \frac{7}{16}u(\mathbf{E}(L, H), L, H) + \frac{1}{16}u(\mathbf{E}(L, L), L, L) \\ &\quad + \frac{7}{16}u(\mathbf{E}(H, L), H, L) + \frac{1}{16}u(\mathbf{E}(H, H), H, H). \end{aligned}$$

If one restricts interest merely in individual-based mechanism design, the optimal objective value of $U(\mathbf{E})$ is equal to $\frac{187}{16}$. It will be achieved under a particular joint-based

EPIC mechanism $\mathbf{E}^1 = \{E_1(L, H \text{ or } L) = 10, E_1(H, H \text{ or } L) = 1, E_2(L \text{ or } H, H) = 1, E_2(L \text{ or } H, L) = 10\}$, which is equivalent to the individual-based EPIC mechanisms $\overline{\mathbf{E}}^1$.

However, consider another "pure" joint-based mechanism

$$\mathbf{E}^0 = \{E_1(L, L) = 10, E_1(H, H) = 10, E_1(H, L) = 1, E_1(L, H) = 10;$$

$$E_2(L, L) = 10, E_2(H, L) = 10, E_2(L, H) = 1, E_2(H, H) = 10\}.$$

Unlike $\mathbf{E}^1$, E^0 suggests that the low level of equity requirement is designated for only the relative high type instead of the nominally high type. Note that E^0 is a "perturbation" of $\mathbf{E}^1$ by modifying the function value only at (H, H). It is easy to verify that $\mathbf{E}^0$ is also EPIC according to the banks' rankings of contracts. In addition, $u(\mathbf{E}^0(H, H), H, H) > u(\mathbf{E}^1(H, H), H, H)$. So $\mathbf{E}^0$ wins over $\mathbf{E}^1$ for the regulator, since $U(\mathbf{E}^0) = \frac{191}{16} > \frac{187}{16} = U(\mathbf{E}^1)$. In such a finite environment, joint-based mechanism design must strongly dominate individual-based mechanism design.

Let us check the menu design next. The set of available contract menu for each bank i is $\{\{1\}, \{10\}, \{1, 10\}\}$. There are nine possible joint menus:

$$C^1 = (\{1, 10\}, \{1, 10\}), C^2 = (\{10\}, \{10\}), C^3 = (\{1\}, \{1\}),$$

$$C^4 = (\{1\}, \{10\}), C^5 = (\{10\}, \{1\}), C^6 = (\{1\}, \{1, 10\}),$$

$$C^7 = (\{10\}, \{1, 10\}), C^8 = (\{1, 10\}, \{10\}), C^9 = (\{1, 10\}, \{1\}).$$

They are all ex post (joint) menus according to the banks' rankings of contracts. One can easily compute the banks' EPE response $\Phi(q_1, q_2, C)$. For each $h = 1, 2, \cdots, 9$, we have

$$\Phi(q_1, q_2, C^h) = (\overline{E}_1^h(q_1), \overline{E}_2^h(q_2)).$$

Note that C^1 yields a (Nash) feasible contract-selection profile for each type profile as follows:

$$\Phi(L, H, C^1) = (10, 1), \Phi(H, H, C^1) = (1, 1),$$

$$\Phi(L, L, C^1) = (10, 10), \Phi(H, L, C^1) = (1, 10).$$

Apparently, this profile is directly corresponding to $\overline{\mathbf{E}}^1$ and $\mathbf{E}^1$, according to $\widetilde{U}$ and U. Among the nine ex post menus, only C^1 induces the optimal objective value for the regulator that is equal to $\frac{187}{16}$. Individual-based ex post mechanism design thus yields a strategically equivalent outcome as ex post menu design, because the delegation principle takes effect here. Moreover, joint-based ex post mechanism design strongly dominates ex post menu design. Therefore, it is more desirable for the bank regulatory contracting to be centralized in terms of joint-based mechanisms than to be decentralized in this multi-agency environment with interrelated impacts of asymmetric information.

7 Concluding Remarks

The difference between centralized mechanism design and decentralized menu design becomes more salient in multi-agency especially when the impacts of different agents' asymmetric information are interrelated. Centralized mechanism design can make the principal better off than decentralized menu design in contracting games with ex post implementation. Centralization of the right to specify contracts and actions and corresponding information communication are important in mitigating the interrelated information asymmetry problem. By using joint-based mechanisms (relative information evaluation), the principal can refer to each single agent's report relative to all others' reports in specifying contract and action recommendation for that agent, so as to incorporate more comprehensive information to better deal with the interrelated information asymmetry. These findings can be applicable in various multi-agency environments, such as business practices, organization management, public policy, etc.

The analysis of this paper is based on a world in which transaction costs are negligible. However, one must pay attention to the effect of transaction costs in the real world. Since mechanism design entails centralized communication, it may incur higher transaction costs than menu design. The transaction cost of using mechanisms may be remarkably high in some occasions. For instance, there may be too many agents for the principal to handle, or the principal may not have sufficient technological capacity to process mass data. In that case, the principal may still use menu design instead of joint-based mechanisms in practice. As long as the increment of transaction cost of using joint-based mechanisms relative to using menus is not beyond the benefit increment of using joint-based mechanisms relative to using menus, joint-based mechanisms are still of more significance than menus in contracting games with ex post implementation.

Moreover, a number of directions are expected for future research:

1. In case the information structure is fine enough, the agents are still able to play Bayesian Nash games. It is technically demanding to address this situation. But I expect that results similar to those of this paper still hold.

2. The general conditions for existence of ex post implementation or optimal solutions

to all contracting problems need to be examined precisely.

3. This analysis scrutinizes only the influences of different contracting procedures on the principal's welfare other than the agents' welfare. The related analysis will require more specific settings.

4. The results of this paper call for further empirical or experimental testing.

8 Appendix

Proof of Proposition 1

(i) The continuity of $\widehat{V}_i(\cdot,\cdot,\cdot)$ on $\mathcal{K}\times A\times T$ follows from Delbaen's Lemma(1974). The proof is similar to Proposition 3.1 in Page (1987).

(ii) The proof for the continuity of $\widehat{U}(\cdot,\cdot,t)$ is similar to (i).

Next note that $u(\cdot, f(\cdot), a, \cdot)$ is Borel-measurable on $\Omega\times\Omega\times T$, and $u(\cdot, f(\cdot), a, t)$ is bounded for each t. By Propositions 7.26 and 7.29 in Bertsekas and Shreve (1978), the mapping

$$t \mapsto \int_{\Omega} u(\omega, f(\omega), a, t)p(d\omega|a,t)$$

is $B(T)$-measurable, that is, $\widehat{U}(\cdot, f, a)$ is $B(T)$-measurable.■

Proof of Theorem 1

For any EPE (ρ,δ) of the subgame played by the agents under any joint-based general ex post mechanism $(\mathbf{k},\mathbf{m})\in\mathcal{F}(R,\mathcal{K},M)$ in **(P1)**, we have that for each $i\in\mathcal{N}$ and each $t\in T$,

$$\begin{aligned} & \widehat{V}_i(\mathbf{k}(\rho(t)), \delta(\mathbf{m}(\rho(t)),t),t) \\ \geq\ & \widehat{V}_i(\mathbf{k}(\rho_i'(t_i),\rho_{-i}(t_{-i})), \delta_i'(\mathbf{m}_i(\rho_i'(t_i),\rho_{-i}(t_{-i}),t_i), \\ & \delta_{-i}(\mathbf{m}_{-i}(\rho_i'(t_i),\rho_{-i}(t_{-i}),t_{-i}),t), \end{aligned} \tag{9}$$

for all $\rho_i'\in\mathcal{M}(T_i,R_i)$ and all $\delta_i'\in\mathcal{M}(M_i\times T_i, A_i)$.

Now pick a direct mechanism $(\widetilde{\mathbf{k}},\widetilde{\mathbf{a}})\in\mathcal{F}(T,\mathcal{K},A)$ such that

$$(\widetilde{\mathbf{k}}(t),\widetilde{\mathbf{a}}(t)) \equiv (\mathbf{k}(\rho(t)), \delta(\mathbf{m}(\rho(t)),t)), \tag{10}$$

for all $t\in T$. Observe that such a direct mechanism gives the same expected utility to the principal as the originally given general mechanism, since the probability distribution over decision vectors for any type vector is the same.

Next, we claim that this direct mechanism is EPIC. (9) implies that for each $i \in \mathcal{N}$ and each $t \in T$,

$$\begin{aligned} & \widehat{V}_i(\mathbf{k}(\rho(t)), \delta(\mathbf{m}(\rho(t)), t), t) \\ \geq\ & \widehat{V}_i(\mathbf{k}(\rho_i(t_i'), \rho_{-i}(t_{-i})), a_i', \delta_{-i}(\mathbf{m}_{-i}(\rho_i(t_i'), \rho_{-i}(t_{-i})), t_{-i}), t), \end{aligned} \tag{11}$$

for all $a_i' \in A_i$ and all $t_i' \in T_i$. This is because for each $a_i' \in A_i$ and each $t_i' \in T_i$ there always exist some $\rho_i' \in \mathcal{M}(T_i, R_i)$ and some $\delta_i' \in \mathcal{M}(M_i \times T_i, A_i)$ such that $\rho_i'(t_i) = \rho_i(t_i')$ and $\delta_i'(\mathbf{m}_i(\rho_i'(t_i), \rho_{-i}(t_{-i})), t_i) = \delta_i'(\mathbf{m}_i(\rho_i(t_i'), \rho_{-i}(t_{-i})), t_i) = a_i'$. (10) and (11) imply that for each $i \in \mathcal{N}$ and each $t \in T$,

$$\widehat{V}_i(\widetilde{\mathbf{k}}(t), \widetilde{\mathbf{a}}(t), t) \geq \widehat{V}_i(\widetilde{\mathbf{k}}(t_i', t_{-i}), a_i', \widetilde{\mathbf{a}}_{-i}(t_i', t_{-i}), t),$$

for all $a_i' \in A_i$ and all $t' \in T$. Therefore, this particular direct mechanism $(\widetilde{\mathbf{k}}, \widetilde{\mathbf{a}})$ is EPIC.

Furthermore, the optimal solution to **(P1)** will clearly bring the same expected utility to the principal as the optimal solution to **(P2)**. By similar argument, these results also apply to individual-based ex post general mechanisms and individual-based EPIC mechanisms.■

Proof of Lemma 1

Let $\varphi_C(\cdot) := \Phi(\cdot, C)$. We need to show that $Gr\varphi_C = \{(t, \widetilde{f}(t), \widetilde{a}(t)) \in T \times C \times A | (\widetilde{f}, \widetilde{a})$ is the EPE under $C\}$ is closed.

First fix $t \in T$. Pick any arbitrary sequence $\{(t^\alpha, \widetilde{f}(t^\alpha), \widetilde{a}(t^\alpha))\}$ in $Gr\varphi_C$ satisfying

$$\begin{aligned} (\widetilde{f}(t^\alpha), \widetilde{a}(t^\alpha)) &\in \varphi_C(t^\alpha), \\ \text{and } (t^\alpha, \widetilde{f}(t^\alpha), \widetilde{a}(t^\alpha)) &\to (t, \widetilde{f}(t), \widetilde{a}(t)), \text{as } \alpha \to \infty. \end{aligned}$$

Thus it suffices to show that $(\widetilde{f}(t), \widetilde{a}(t)) \in \varphi_C(t)$, that is, for each $i \in \mathcal{N}$,

$$\widehat{V}_i(\widetilde{f}(t), \widetilde{a}(t), t) \geq \widehat{V}_i(\widetilde{f}_i'(t_i), \widetilde{f}_{-i}(t_{-i}), \widetilde{a}_i'(t_i), \widetilde{a}_{-i}(t_{-i}), t),$$

for all $\widetilde{f}_i' \in \mathcal{F}_i$ satisfying $(\widetilde{f}_i'(t_i), \widetilde{f}_{-i}(t_{-i})) \in C$ and all $\widetilde{a}_i' \in \mathcal{A}_i$.

For each $i \in \mathcal{N}$,

$$\widehat{V}_i(\widetilde{f}(t^\alpha), \widetilde{a}(t^\alpha), t^\alpha) \geq \widehat{V}_i(f_i'(t_i^\alpha), f_{-i}^\alpha(t_{-i}^\alpha), \widetilde{a}_i'(t_i^\alpha), \widetilde{a}_{-i}(t_{-i}^\alpha), t^\alpha),$$

for all $\widetilde{f}_i' \in \mathcal{F}_i$ satisfying $(\widetilde{f}_i'(t_i), \widetilde{f}_{-i}(t_{-i})) \in C$ and all $\widetilde{a}_i' \in \mathcal{A}_i$. Then by joint continuity of $\widehat{V}_i$,

$$\widehat{V}_i(\widetilde{f}(t), \widetilde{a}(t), t) \geq \widehat{V}_i(\widetilde{f}_i'(t_i), \widetilde{f}_{-i}(t_{-i}), \widetilde{a}_i'(t_i), \widetilde{a}_{-i}(t_{-i}), t),$$

for all $\widetilde{f}_i' \in \mathcal{F}_i$ satisfying $(\widetilde{f}_i'(t_i), \widetilde{f}_{-i}(t_{-i})) \in C$ and all $\widetilde{a}_i' \in \mathcal{A}_i$.■

Proof of Proposition 2

Note that T, C and A are all Borel spaces. The graph of $\Phi(\cdot, C)$ is closed in $T \times C \times A$ by Lemma 1. $\Phi(\cdot, C)$ is compact-valued by Lemma 17.51 (Aliprantis and Border 2006). Thus, by Theorem 3 in Himmelberg, Parthasarathy and Van Vleck (1976), $\Phi(\cdot, C)$ is Borel-measurable. ■

Proof of Lemma 2

Note that T and $\mathcal{K} \times A$ are Borel space. By Proposition 2, for each $C \in P_f(\mathcal{K})$ satisfying $\Phi(\cdot, C)$ is well-defined, $\Phi(\cdot, C)$ is Borel-measurable and compact-valued. By Proposition 1, $\widehat{U}$ is Borel-measurable and $\widehat{U}(\cdot, \cdot, t)$ is continuous.

Hence, by Theorem 2 in Himmelberg, Parthasarathy and Van Vleck (1976), there exists some $(\overline{\mathbf{k}}, \overline{\mathbf{a}}) \in \Sigma_\Phi(C)$ such that $\widehat{U}(\overline{\mathbf{k}}(t), \overline{\mathbf{a}}(t), t) = \max_{(\widetilde{f}(t), \widetilde{a}(t)) \in \Phi(t, C)} \widehat{U}(\widetilde{f}(t), \widetilde{a}(t), t)$ for all $t \in T$. Moreover, the function $t \mapsto \max_{(\widetilde{f}(t), \widetilde{a}(t)) \in \Phi(t, C)} \widehat{U}(\widetilde{f}(t), \widetilde{a}(t), t)$ is also Borel measurable.■

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