Entry with Two Correlated Signals

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Abstract

We analyze industrial espionage extending the Milgrom and Roberts (1982) (hereafter MR) model. More precisely, we consider the case where a monopoly (M) is engaged in R&D trying to reduce his cost of production and deter a potential entrant (E) from entering the market. The R&D project may be successful or not and its outcome is a private information of M. The entrant has an access to an IS of a certain precision that generates a noisy signal on the outcome of the R&D project, and she decides whether or not to enter the market based on two signals: the price charged by M and the signal sent by the IS. It is assumed that the precision of the IS is exogenous and common knowledge.

We show that the separating equilibria of our model coincide with that of MR. The same result is obtained for pooling equilibria if the precision of the IS is sufficiently low to affect the decision of E of staying out. For the other extreme, if the IS is very accurate, then contrary to the MR model, pooling equilibrium does not exist. For intermediate values of the precision, the set of pooling equilibria is non-empty and E enters if the IS tells her the R&D project was not successful. Since in the MR model the entrant never enters in a pooling equilibrium, we show that the use of the IS by the potential entrant with high probability increases competition in pooling equilibrium.

JEL Classification: C72; D82; L10; L12

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1. Introduction.

The cost structure of the incumbent firm is a very important information for a firm contemplating market entry. Since this information is usually available in statements for internal use, the entrant firm could obtain it spying on the incumbent firm.

In this paper we deal with a monopoly, M, who is engaged in R&D activity with the aim to reduce his cost of production from the current cost $C_H(q)$ to $C_L(q)$, where q is the production level. The outcome of the R&D project is the private information of M. A potential entrant, E, assigns a certain probability, $\mu > 0$, that M fails to reduce his cost and probability $1-\mu > 0$ that the project was successful. If the project fails and E enters, she obtains positive profit. Otherwise, if the project succeeds and E enters, she will not be able to cover her entry cost and she will end up with negative profit.

The entrant has an access to an Intelligence System (IS) that allows her to collect (noisy) information about the cost structure of M. The IS sends one out of two signals. The signal h, which indicates that the investment was not successful (in which case we refer to M as having the type H), and the signal l, which indicates that the investment was successful (namely, M is of type L). The precision of the IS is α, $\frac{1}{2} \leq \alpha \leq 1$. That is, the signal sent by the IS is correct with probability α (for simplicity, whether the cost function is $C_H(q)$ or $C_L(q)$). The case where $\alpha = \frac{1}{2}$ is equivalent to the case where E does not use an IS. The case $\alpha = 1$ is the one where E knows exactly the outcome of the project.

It is assumed that the precision α of the IS is exogenously given. This would be the case if the entrant firm has already a spying technology before she considers entering the market where the incumbent firm is operating (e.g. she has the ability to plant a Trojan Horse in the computer system of the incumbent firm).

The entrant decides whether or not to enter the market based on a pair of signals: the price, p, that M charges for his product and the signal s (h or l) sent by the IS. If E enters the market, she competes with M (whether it is a Cournot or Bertrand competition, or any other mode of competition). It is assumed that the above is commonly known (including the precision α of the IS).

The interaction between E and M is described as a three stage game, $G(\alpha)$. In the first stage, M who knows the outcome of the R&D project, sets a price p and the IS sends a signal s (h or l). Based on the signals (p,s), E in the second stage decides whether or not to enter the market. If she decides to enter, then E in the third stage is engaged in a certain mode of competition with M.

The game $G(\alpha)$ is a game of incomplete information and, using Harsanyi's approach, we analyze it as a three player game, where the players are the two types, H and L, of M and the entrant, E. We analyze the sequential equilibria of $G(\alpha)$.

The case where $\alpha = \frac{1}{2}$, namely, where the IS has no value (and, therefore, can be ignored), is exactly the limit pricing model of Milgrom and Roberts (1982) (hereafter MR). Therefore, our model is an extension of the MR model where the entrant has an access to an intelligence system and it is only for $\frac{1}{2} < \alpha < 1$.

We distinguish two cases: the first one is the separating equilibrium where the two types of M charge different prices p_H and p_L, $p_H \neq p_L$; the second one is the pooling equilibrium case where $p_H = p_L$.

We show that the separating equilibria of our model coincide with that of MR and the IS makes no difference for either E or M. This is not very surprising since in a separating equilibrium E identifies the type of M with or without the use of the IS. Even though the off equilibrium behavior of E is affected by the signal of the IS, it does not affect the separating equilibria. The same result is obtained for pooling equilibria if the precision α of the IS is sufficiently low (close to $\frac{1}{2}$) to affect the decision of E. For the other extreme, if α is very accurate (close to 1), then contrary to the MR model, pooling equilibrium does not exist. In this case, E identifies with high probability the type of M and she will enter the market if the signal is h and she will stay out if the signal is l. The H type monopolist, who knows that his type is detected with high probability, has an incentive to deviate to his monopoly price, upsetting a pooling equilibrium. Let us next deal with the intermediate case, where α is bounded away from $1/2$ and 1. We show that the set of pooling equilibria is non-empty and the monopoly price of the L-type monopoly is the highest pooling equilibrium price. The decision of E is still entering if the signal is h and staying out if the signal is l. To compare this result with the result obtained in the MR model, suppose first that prior to the completion of the R&D project, the expected payoff of E from entering the market is positive. Then, contrary to our model, no pooling equilibrium exists in the MR model. Otherwise, E in a pooling equilibrium enters the market expecting positive profit and, hence, both types of M are best off with their monopoly prices, upsetting a pooling equilibrium. In the game $G(\alpha)$ where α is bounded away from 1, M of type H knows that with significant probability E will obtain the wrong signal l and will stay out. Hence, H succeeds to fool E about his type with significant probability. However, the precision α of the IS should not be too low. Otherwise, E will not trust the signal of the IS and she will enter the market whether the signal is h or l. In this case, the two types of M are best off with their monopoly prices, upsetting a pooling equilibrium.

Note that in the MR model the entrant never enters in a pooling equilibrium. Hence, the use of the IS with high probability increases competition in pooling equilibrium. The entrant enters the market for intermediate levels of α if the signal is h. This is true even when pooling equilibrium does not exist in the MR model. From this point of view, spying on incumbent firms increases competition with high probability.

This paper is related to Perea and Swinkels (1999) and Ho (2007, 2008) since they also consider espionage in the context of asymmetric information. However, in the present model the IS is not a decision maker who can act strategically as in Perea and Swinkels (1999) and Ho (2007, 2008). The paper is also related to Sakai (1985) since he considers two firms and one objective of the information gathering activity is, like in our model, the cost structure of the opponent firm. However, unlike us, the paper considers that both firms know neither the costs of their opponent nor their own costs.

Another related paper is Bagwell and Ramey (1988). They extend the MR model by allowing the incumbent to signal his costs with both price and advertisements. Hence, while in this paper both signals are sent by the incumbent, in our model he only signals his costs by the price, the other signal is generated by the IS operated by the entrant. Bagwell (2007) extends Bagwell and Ramey (1988) and considers a more general game in which the incumbent has two dimensions of private information, his costs and his level of patience[1].

The contribution of this paper is to extend the MR model to the case where the potential entrant has an access to an intelligence system to better detect the cost structure of the cost structure of the monopolist. Assuming that the precision α of the IS is common knowledge, we show that spying on incumbent firms increases competition with high probability.

The remainder of the paper is organized as follows. Section 2 sets out the model. The strategy of E is presented in Section 3. Section 4 analyzes separating equilibria of the game. Pooling equilibria is analyzed in Section 5. Section 6 concludes the paper. Most proofs of the results are presented in the Appendix.

2. The Model.

We start with the benchmark case of the limit price model of MR. Their game, which is denoted G_{MR}, consists of a monopoly M and a potential entrant E. The cost function of M is a private information and it can be of two types: L (low cost) and H (high cost). A potential entrant, E, assigns probability μ that M is of type H. In the first period M chooses a price as a function of his type. The price serves as a signal for E, who then decides whether to enter the market or stay out. If E enters, she incurs an entry cost K. In the second period, if E enters, E and M compete in the market.

[1] For other extensions of MR model see Albaek and Overgaard (1992a, 1992b), Bagwell (1992), Bagwell and Ramey (1990, 1991), Harrington (1986, 1987) and Linnemer (1998).

The form of competition (Cournot, Bertrand or other) is commonly known and once E enters, the outcome of the competition is assumed to be uniquely determined. By confining the analysis to sequential equilibria, the strategic interaction takes place only in the first period.

The strategy of the t-type monopoly is a first period price p_t for $t \in \{H, L\}$. The strategy of E is assumed to be of the form

$$\sigma_E(p) = \begin{cases} "Stay\,out", & p \le \overline{p} \\ "Enter", & p > \overline{p} \end{cases}$$

where the threshold $\overline{p}$ is the choice of E.

Let $Q(p)$ be the demand function and $C_t(q)$ be the cost function of the t-type monopoly.

Let D_H and D_L be the duopoly profits of the H-type and the L-type monopolists, respectively. For short we denote by H and L the H-type and the L-type monopolists, respectively. Let $\Pi_H(p)$ be the profit of H and let $\Pi_L(p)$ be the profit of L when they set the price p and when E does not enter. Denote by $D_E(H)$ and $D_E(L)$ the duopoly profits of E when she competes with H and L respectively. Denote by p_H^M and p_L^M the monopoly prices of H and L respectively (and by q_H^M and q_L^M the monopoly quantities). Finally, let $\hat{p}$ and p_0 be s.t.

$$\Pi_H(\hat{p}) = D_H \text{ and } \hat{p} < p_H^M$$

and

$$\Pi_L(p_0) = D_L \text{ and } p_0 < p_L^M .$$

See Figures 1 and 2 below.

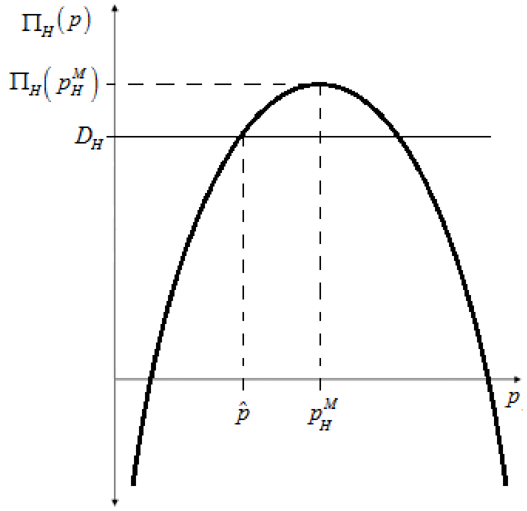

Figure 1

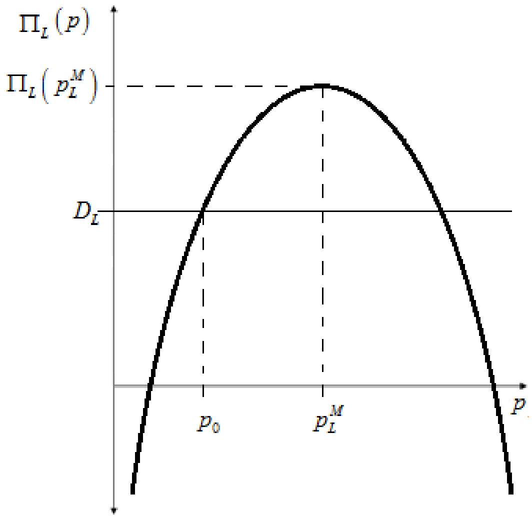

Figure 2

Assumptions

1. $D_E(L)-K \equiv \Delta_E(L)<0$ and $D_E(H)-K \equiv \Delta_E(H)>0$.
2. $\Pi_t(p)$, $t\in\{H,L\}$, is increasing in p whenever $p \le p_t^M$ and is decreasing in p whenever $p \ge p_t^M$.
3. $\Pi_L(p_L^M)-D_L > \Pi_H(p_H^M)-D_H$. Namely, L loses from entry more than H.
4. The cost functions $C_t(x)$, $t\in\{H,L\}$, are differentiable, $C_H'(q)>C_L'(q)$ and $C_H(0) \ge C_L(0)$.
5. $Q(p)$ is differentiable and $Q'(p)<0$ for all $p \ge 0$.

6. All the parameters of the model and the above five assumptions are commonly known.

<u>Lemma 1.</u> (i) $\Pi_L(p)-\Pi_H(p)$ decreases in p.

(ii) $p_H^M > p_L^M$.

(iii) $\hat{p} > p_0$.

<u>Proof:</u> Appears in the Appendix.

Let p_H and p_L be the equilibrium strategies of H an L respectively. In a separating equilibrium $p_H \neq p_L$, and in a pooling equilibrium $p_H = p_L = p^*$.

Propositions 1 and 2 below are due to Milgrom and Roberts (1982) and they characterize the separating and pooling equilibrium respectively for the case where E does not operate an IS on M.

<u>Proposition 1 (Milgrom and Roberts (1982)).</u> The set of sequential separating equilibria SSE_{MR} in G_{MR} is non-empty and

$$SSE_{MR} = \left\{ (p_H, p_L, \overline{p}) \middle| p_H = p_H^M, p_L = \overline{p}, p_0 \leq p_L \leq \min\left(p_L^M, \hat{p}\right) \right\}$$

<u>Remark:</u> By Lemma 1, $\hat{p} > p_0$ and SSE_{MR} is non-empty.

<u>Proposition 2 (Milgrom and Roberts (1982)).</u> The set of all sequential pooling equilibria in G_{MR}, $\sigma = (p_H, p_L, \overline{p})$, is characterized by

(i) $p_H = p_L = p^* = \overline{p}$

and

(ii) $\hat{p} \leq p^* \leq p_L^M$

Our goal is to extend the MR results to the case where E uses an Intelligence System (IS) to spy on M to better detect his type. Denote by $G(\alpha)$ the game that extends G_{MR} to allow espionage activity and where the Intelligence System operated by E is of precision α, $\frac{1}{2} \leq \alpha \leq 1$.

The game $G(\alpha)$ is a three-stage game. In the first stage M sets a price and the IS sends a signal, h or l. In the second stage, E who observes both the price set by M and the signal sent by the IS, decides whether or not to enter the market. Finally, in the third stage, if E enters, M and E compete in the market. As mentioned above, the third stage competition is assumed to generate a unique equilibrium outcome $\left(D_H, D_L, D_E(H), D_E(L)\right)$ if E enters and $\left(\Pi_H\left(p_H^M\right), \Pi_L\left(p_L^M\right)\right)$ if E does not enter.

It is assumed that Assumptions 1-6 hold and, in addition, α is commonly known.

3. The Strategy of E in $G(\alpha)$.

Given α, for every pair of signals (s, p), $s \in \{h, l\}$, $p \in \mathbb{R}_+$, let $Prob(H|s, p)$ and $Prob(L|s, p) = 1 - Prob(H|s, p)$ be the off equilibrium probability that E assigns to the event that M is of type H and of type L, respectively.

It is assumed that, conditional on the type of M, the signals are mutually independent. Namely, M chooses the price p independently of the choice of the IS. Nevertheless, the signals p and s are correlated. If E observes a very high price, then it is more likely that she will observe the signal h. If however E observes a low price, it is more likely that she will observe the signal l.

Hence, the off equilibrium probability that E assigns to the types of M is

$$Prob(H|h, p) = \frac{Prob(h, p|H)Prob(H)}{Prob(h, p|H)Prob(H) + Prob(h, p|L)Prob(L)}$$

$$= \frac{Prob(h|H)Prob(p|H)Prob(H)}{Prob(h|H)Prob(p|H)Prob(H) + Prob(h|L)Prob(p|L)Prob(L)}$$

Equivalently,

$$Prob(H|h, p) = \frac{\mu\alpha f(p|H)}{\mu\alpha f(p|H) + (1-\mu)(1-\alpha) f(p|L)} \tag{1}$$

Similarly,

$$Prob(H|l, p) = \frac{\mu(1-\alpha) f(p|H)}{\mu(1-\alpha) f(p|H) + (1-\mu)\alpha f(p|L)} \tag{2}$$

where $f(p|t)$ is the (density) probability that E assigns to the event that M of type t, $t \in \{H, L\}$ sends the signal p.

In a pure strategy equilibrium, if H assigns probability 1 to the event that $p = p_H$, then $f(p_H|H)=1$ and $f(p|H)=0$ if $p \neq p_H$. In this case, $f(p|H)$ is identified with the probability that H selects p. Similarly, $f(p_L|L)=1$ and $f(p|L)=0$, $\forall p \neq p_L$. Hence, for $p \neq p_H$ and $p \neq p_L$ (1) and (2) are not well defined for $p \notin \{p_H, p_L\}$ since the numerators and denominators are zero.

Using the notion of sequential equilibrium, we approach $f(p|t)$ by a sequence $(f_n(p|t))_{n=1}^{\infty}$, such that $f_n(p|t)>0$ and $\lim_{n\to\infty} f_n(p|t) = f(p|t)$ for all $p \in \mathbb{R}_+$. Let

$$Prob_n(H|h,p) \equiv \frac{\mu\alpha f_n(p|H)}{\mu\alpha f_n(p|H)+(1-\mu)(1-\alpha) f_n(p|L)} \tag{3}$$

$$Prob_n(H|l,p) \equiv \frac{\mu(1-\alpha) f_n(p|H)}{\mu(1-\alpha) f_n(p|H)+(1-\mu)\alpha f_n(p|L)} \tag{4}$$

Now $Prob_n(H|h,p)$ is well defined for all $p \in \mathbb{R}_+$ and (1) can be modified to be

$$Prob(H|h,p) \equiv \lim_{n\to\infty} \frac{\mu\alpha f_n(p|H)}{\mu\alpha f_n(p|H)+(1-\mu)(1-\alpha) f_n(p|L)}$$

We modify (2) in the same way. Note that different sequences of $(f_n(p|t))_{n=1}^{\infty}$ generate different conditional probabilities $Prob(t|s,p)$, $t \in \{H,L\}$, $s \in \{h,l\}$, $p \in \mathbb{R}_+$.

Let $\Pi_E(s,p)$ be the expected payoff of E given her on and off equilibrium beliefs, namely

$$\Pi_E(s,p) \equiv Prob(H|s,p)\Delta_E(H) + Prob(L|s,p)\Delta_E(L) \tag{5}$$

In a sequential equilibrium, if $\Pi_E(s,p)<0$, E will not enter the market and if $\Pi_E(s,p)>0$, E will enter. To simplify the analysis we assume that E stays out also when $\Pi_E(s,p)=0$. Namely, E stays out if and only if she observes (s,p) such that

$$\Pi_E(s,p) \equiv Prob(H|s,p)\Delta_E(H) + Prob(L|s,p)\Delta_E(L) \leq 0$$

<u>Assumption 7.</u>

(1) For each $t \in \{H,L\}$ and each n, $f_n(p|t)$ is differentiable in p for all $p \geq 0$.

(2) Let

$$g_n(p) = \frac{f_n(p|H)}{f_n(p|L)}$$

Then $g_n(p)$ is increasing in n for each p, and is increasing in p for each n. Furthermore, for every n, $\lim_{p\to 0} g_n(p) = 0$ and $\lim_{p\to\infty} g_n(p) = \infty$.

(3) Let $g(p) = \lim_{n\to\infty} g_n(p)$. Then, $g(p)$ is continuous in p.

<u>Lemma 2.</u> (i) For each $s \in \{h, l\}$ and $t \in \{H, L\}$, $Prob(t|s, p)$ is continuous in p and $Prob(H|s, p)$ is non-decreasing in p, $p \geq 0$.

(ii) For every $p \geq 0$, $Prob(H|h, p) > Prob(H|l, p)$.

(iii) Let $J_s = \{p \geq 0 | \Pi_E(s, p) \leq 0\}$. Then, J_s and $\mathbb{R}_+ \setminus J_s$ are both non-empty sets.

<u>Proof:</u>

(i) By (3),

$$Prob_n(H|h, p) = \frac{\mu\alpha \frac{f_n(p|H)}{f_n(p|L)}}{\mu\alpha \frac{f_n(p|H)}{f_n(p|L)} + (1-\mu)(1-\alpha)}$$

Hence,

$$Prob(H|h, p) = \frac{\mu\alpha g(p)}{\mu\alpha g(p) + (1-\mu)(1-\alpha)} \tag{6}$$

and by Assumption 7, $Prob(H|h, p)$ is continuous in p.

The proof that $Prob(H|l, p)$ is continuous is similarly derived by (4).

Since $Prob(L|s, p) = 1 - Prob(H|s, p)$, then $Prob(L|s, p)$ is also continuous.

Next note that $g(p)$ is non-decreasing in p since $g_n(p)$ is increasing in p for all n.

It is easy to verify by (6) that $\frac{\partial}{\partial p} Prob(H|h, p) \geq 0$ iff $g'(p) \geq 0$ and thus

$Prob(H|h,p)$ is non-decreasing in p. The proof that $Prob(H|l,p)$ is non-decreasing is similar.

(ii) Let

$$x_n(p) = \frac{Prob_n(H|h,p)}{Prob_n(H|l,p)}$$

By (3) and (4),

$$\begin{aligned}
x_n(p)-1 &= \frac{\frac{\alpha}{1-\alpha}\left[\mu(1-\alpha)f_n(p|H)+(1-\mu)\alpha f_n(p|L)\right]}{\mu\alpha f_n(p|H)+(1-\mu)(1-\alpha)f_n(p|L)}-1 \\
&= \frac{(1-\mu)\frac{\alpha^2}{1-\alpha}f_n(p|L)-(1-\mu)(1-\alpha)f_n(p|L)}{\mu\alpha f_n(p|H)+(1-\mu)(1-\alpha)f_n(p|L)} \\
&= \frac{(1-\mu)f_n(p|L)(2\alpha-1)}{(1-\alpha)\left[\mu\alpha f_n(p|H)+(1-\mu)(1-\alpha)f_n(p|L)\right]} \\
&= \frac{(1-\mu)(2\alpha-1)}{(1-\alpha)\left[\mu\alpha g_n(p)+(1-\mu)(1-\alpha)\right]}
\end{aligned}$$

Hence,

$$\lim_{n\to\infty}\left[x_n(p)-1\right] = \frac{(1-\mu)(2\alpha-1)}{(1-\alpha)\left[\mu\alpha g(p)+(1-\mu)(1-\alpha)\right]} > 0$$

Hence $\lim_{n\to\infty} x_n(p) > 1$ and, consequently, for every $p \ge 0$,

$$Prob(H|h,p) > Prob(H|l,p) \tag{7}$$

(iii) By (5),

$$\begin{aligned}
\Pi_E(s,p) &= Prob(H|s,p)\Delta_E(H)+Prob(L|s,p)\Delta_E(L) \\
&= Prob(L|s,p)\left[\frac{Prob(H|s,p)}{Prob(L|s,p)}\Delta_E(H)+\Delta_E(L)\right]
\end{aligned} \tag{8}$$

Let $s=h$. For every p,

$$\frac{Prob(H|h,p)}{Prob(L|h,p)} = \lim_{n\to\infty}\frac{\mu\alpha f_n(p|H)}{(1-\mu)(1-\alpha)f_n(p|L)} = \frac{\mu\alpha}{(1-\mu)(1-\alpha)}g(p)$$

We claim that $g(p) \to 0$ as $p \to 0$. This follows by Dini's theorem, as $g_n(p)$ is increasing in n, $g_n(p)$ is continuous in p and $g(p)$ is also continuous. Hence, for every $\delta > 0$, $\lim_{n\to\infty} g_n(p) = g(p)$ uniformly on $[0,\delta]$. Since for every n, $g_n(p) \to 0$ as $p \to 0$, we have $g(p) \to 0$ as $p \to 0$. Consequently,

$$\lim_p \frac{Prob(H|h,p)}{Prob(L|h,p)} = 0 \text{, as } p \to 0 \tag{9}$$

Inequality (9) holds also when h is replaced by l (the proof is similar).

Next, let us show that $Prob(L|h,p) > 0$ for small p.

$$\begin{aligned} Prob_n(L|h,p) &= \frac{(1-\mu)(1-\alpha) f_n(p|L)}{\mu\alpha f_n(p|H) + (1-\mu)(1-\alpha) f_n(p|L)} \\ &= \frac{1}{1 + \dfrac{\mu\alpha g_n(p)}{(1-\mu)(1-\alpha)}} \end{aligned} \tag{10}$$

Again, since $g_n(p) \to g(p)$ as $n \to \infty$ uniformly in any interval $[0,\delta]$, $\delta > 0$, and since $g(p) \to 0$ as $p \to 0$,

$$Prob(L|h,p) = \lim_n Prob_n(L|h,p) \to 1 \text{, as } p \to 0$$

In particular, $Prob(L|h,p) > 0$ for p sufficiently small. In a similar way, we can prove that $Prob(L|l,p) > 0$ for p sufficiently small.

Now, (8), (9) and the fact that $\Delta_E(L) < 0$ and $Prob(L|s,p) > 0$ for small p, imply that for sufficiently small p, $\Pi_E(s,p) < 0$ and $J_s \neq \varnothing$.

Let us show that for p sufficiently large, $\Pi_E(s,p) > 0$. We use the following claim.

Claim 1. $\lim_p Prob(L|s,p) = 0$ as $p \to \infty$.

Proof: Let $n = 1$ and $s = h$. By Assumption 7.2, $\lim \frac{f_1(p|H)}{f_1(p|L)} = \infty$. By (10),

$$Prob_1(L|h,p) \to 0 \text{ as } p \to \infty$$

Hence, for every $\varepsilon > 0$, there exists P s.t. for all $p > P$,

$$Prob_1\left(L\middle|h,p\right)<\varepsilon$$

By (3),

$$Prob_n\left(H\middle|h,p\right)=\frac{\mu\alpha}{\mu\alpha+(1-\mu)(1-\alpha)\dfrac{f_n\left(p\middle|L\right)}{f_n\left(p\middle|H\right)}}$$

By Assumption 7.2, $Prob_n\left(H\middle|h,p\right)$ is increasing in n and, hence, $Prob_n\left(L\middle|h,p\right)$ is decreasing in n for every p. Thus, for all $p>P$,

$$Prob_n\left(L\middle|h,p\right)<Prob_1\left(L\middle|h,p\right)<\varepsilon$$

Hence, for every $\varepsilon>0$ and for all $p>P$,

$$Prob\left(L\middle|h,p\right)=\lim_{n\to\infty}Prob_n\left(L\middle|h,p\right)\le\varepsilon$$

implying that

$$\lim_{p\to\infty}Prob\left(L\middle|h,p\right)=0$$

The proof that $\underset{p}{Prob}\left(L\middle|l,p\right)=0$, as $p\to\infty$ is similarly derived.

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Claim 1 together with (5) imply that for p sufficiently large, $\Pi_E\left(s,p\right)>0$, and the proof of Lemma 2 is completed.

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By part (i) of Lemma 2 and by (5), $\Pi_E\left(s,p\right)$ is continuous and non-decreasing in p (this follows from the fact that $Prob\left(H\middle|s,p\right)$ is continuous and non-decreasing in p, $\Delta_E\left(H\right)>0$, $Prob\left(L\middle|s,p\right)=1-Prob\left(H\middle|s,p\right)$ and $\Delta_E\left(L\right)<0$).

By part (iii) of Lemma 2, $\Pi_E\left(s,p\right)<0$ for small p and $\Pi_E\left(s,p\right)>0$ for sufficiently large p. Let

$$p_h=\max\left\{p\ge0\middle|\Pi_E\left(h,p\right)\le0\right\}$$

$$p_l=\max\left\{p\ge0\middle|\Pi_E\left(l,p\right)\le0\right\}$$

By the continuity of $\Pi_E\left(s,p\right)$ in p,

$$\Pi_E(h, p_h) = \Pi_E(l, p_l) = 0 \quad (11)$$

and E enters the market iff she observes either (h, p) s.t. $p > p_h$ or (l, p) s.t. $p > p_l$.

By (7) it is easy to verify that

$$\Pi_E(h, p) > \Pi_E(l, p) \quad (12)$$

By (11) and (12)

$$\Pi_E(l, p_l) = \Pi_E(h, p_h) > \Pi_E(l, p_h)$$

and since $\Pi_E(s, p)$ is non-decreasing in p, we have $p_l > p_h$.

We conclude that the decision rule of E when she observes the pair of signals (s, p) is given by Figure 3 below.

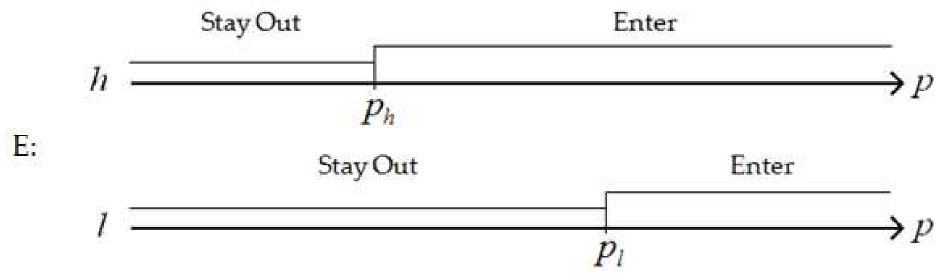

Figure 3

We summarize the above in the following lemma.

Lemma 3. Suppose that Assumption 1 holds. Then, any beliefs of E which satisfy Assumption 7, uniquely determine p_h and p_l, $p_h < p_l$, s.t. in every sequential equilibrium with these beliefs, E enters the market iff she observes the signal (h, p) with $p > p_h$ or the signal (l, p) with $p > p_l$.

Our next goal is to characterize the sequential equilibrium of $G(\alpha)$ given the above decision rule of E. We start with separating equilibria.

4. Separating Equilibria.

In a separating equilibrium $p_H \neq p_L$ and E identifies with probability 1 the type of M. Hence, E enters the market when observing the price p_H irrespective of the signal of the IS, and E stays out when observing p_L, again irrespective of s. Therefore, $p_H > p_l$ and $p_L \leq p_h$.

Notation: Let $\tilde{p}_t(\alpha)$ be the (unique) solution in p of the following equation,

$$\Pi_t(p) = \alpha\Pi_t(p_t^M) + (1-\alpha)D_t,\ t \in \{H, L\}$$

(see Figure 4)

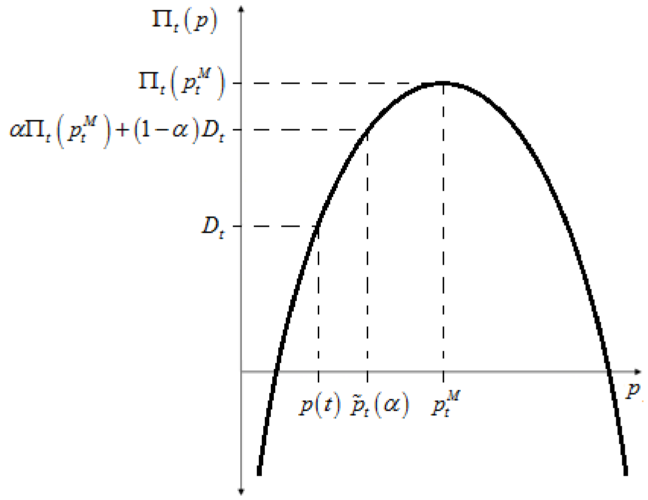

Figure 4

And let $\hat{p}_t(\alpha)$ be the unique solution in p of the following equation,

$$\Pi_t(p) = (1-\alpha)\Pi_t(p_t^M) + \alpha D_t$$

(see Figure 5)

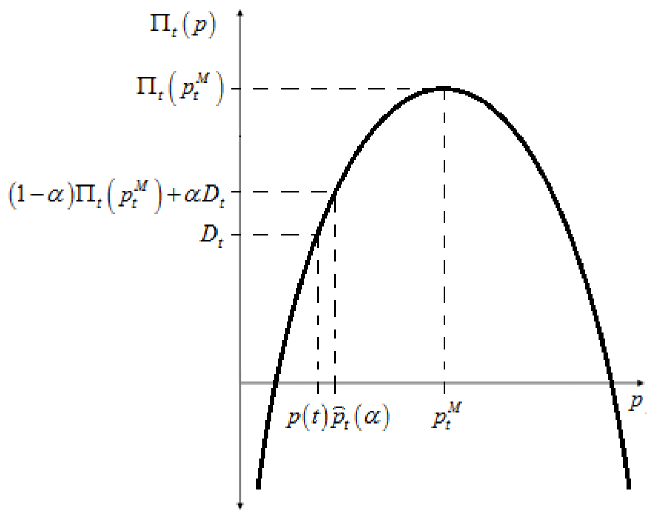

Figure 5

where $p(t) = \hat{p}$ if $t = H$ and $p(t) = p_0$ if $t = L$.

The following proposition characterizes the sequential separating equilibrium of $G(\alpha)$.

Proposition 3. Consider the game $G(\alpha)$ for $\frac{1}{2} < \alpha < 1$, and let SSE be the set of all sequential separating equilibrium points of $G(\alpha)$. Let SSE_t be the set of all equilibrium prices of the *t*-type monopolist in SSE. Then,

(1) $SSE_L = \left\{ p_L \middle| p_0 \le p_L \le \min\left(p_L^M, \hat{p}\right)\right\}$ and $SSE_H = \left\{ p_H^M \right\}$.

(2) Let $p_L \in SSE_L$. If $p_L < p_L^M$, then $p_L = p_h$. If $p_L = p_L^M$, then $p_L^M \le p_h$.

(3) The set SSE coincides with SSE_{MR}, the set of all sequential separating equilibrium points of G_{MR}.

(4) Let $p_L \in SSE_L$ and suppose that $p_L < p_L^M$. Let p_h and $\overline{p}$ be the equilibrium cutoff price for entry in $G(\alpha)$ and in G_{MR} respectively. Then, $\overline{p} = p_h$.

(5) Let $p_L \in SSE_L$ and suppose that $p_L < p_L^M$. Then the equilibrium strategy of E in $G(\alpha)$ coincides with the equilibrium strategy of E in G_{MR} for all $p_L \notin (p_h, p_l]$. If $p_L \in (p_h, p_l]$, then E in $G(\alpha)$ enters the market with positive probability (which is α if M is of type H and $1-\alpha$ if M is of type L) and stays out for sure in G_{MR}.

<u>Proof:</u> Appears in the Appendix.

Part (5) of Proposition 3 asserts that in $G(\alpha)$ E is less inclined to enter the market. For all prices below $\overline{p} = p_h$ E stays out of the market in both games G_{MR} and $G(\alpha)$. For prices above p_l, E enters for sure in both these games. But for prices p, $p_h < p \le p_l$, E in $G(\alpha)$ enters the market iff the signal sent by the IS is h. In contrast, E in this region enters the market for sure in the game G_{MR}. The difference between $G(\alpha)$ and G_{MR} with regard to sequential separating equilibrium is only in the behavior of E off the equilibrium path.

5. **Pooling Equilibrium.**

By pooling equilibrium we refer to triples of the form $\sigma = (s_E, p_H, p_L)$ where s_E is the strategy of E and $p_H = p_L \equiv p^*$.

Given the signal l of the IS, the expected payoff of E is

$$\Pi_E(l|\alpha) \equiv Prob(H|l)\Delta_E(H) + Prob(L|l)\Delta_E(L)$$

Equivalently,

$$\Pi_E(l|\alpha) = \frac{\mu(1-\alpha)}{\mu(1-\alpha)+(1-\mu)\alpha}\Delta_E(H) + \frac{(1-\mu)\alpha}{\mu(1-\alpha)+(1-\mu)\alpha}\Delta_E(L)$$

Hence, if the IS sends the signal l, E does not enter the market when observing the price p^* iff

$$\Pi_E(l|\alpha) \leq 0 \tag{13}$$

Let

$$\bar{\alpha}_l = \frac{\mu\Delta_E(H)}{\mu\Delta_E(H)-(1-\mu)\Delta_E(L)} \tag{14}$$

Note that (13) holds and E does not enter iff $\alpha \geq \bar{\alpha}_l$.

Since $\Delta_E(L) < 0$, $0 < \bar{\alpha}_l < 1$ and $\bar{\alpha}_l < 1/2$ iff

$$\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0 \tag{15}$$

Thus, for $1/2 < \alpha < 1$, E does not enter iff (15) holds.

Suppose next that the IS sends the signal h. Then the expected payoff of E is

$$\Pi_E(h|\alpha) \equiv Prob(H|h)\Delta_E(H) + Prob(L|h)\Delta_E(L)$$

Equivalently,

$$\Pi_E(h|\alpha) = \frac{\mu\alpha}{\mu\alpha+(1-\mu)(1-\alpha)}\Delta_E(H) + \frac{(1-\mu)(1-\alpha)}{\mu\alpha+(1-\mu)(1-\alpha)}\Delta_E(L)$$

Hence, if the IS sends the signal h, E does not enter the market when observing the price p^* iff

$$\Pi_E(h|\alpha) \leq 0$$

Let

$$\bar{\alpha}_h = \frac{-(1-\mu)\Delta_E(L)}{\mu\Delta_E(H)-(1-\mu)\Delta_E(L)} \tag{16}$$

Note that $\Pi_E(h|\alpha) \le 0$ iff $\alpha \le \bar{\alpha}_h$.

Since $\Delta_E(L) < 0$, $0 < \bar{\alpha}_h < 1$. Note that $\bar{\alpha}_h > 1/2$ iff (15) holds.

<u>Corollary 1.</u> Suppose that $1/2 < \alpha < 1$ and

$$\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$$

Then E stays out iff she observes the signal l or if $\alpha \le \bar{\alpha}_h$.

In other words, if (15) holds, the entrant enters the market if and only if the signal is h and $\alpha > \bar{\alpha}_h$. Let

$$\delta = \frac{\Pi_H(p_L^M) - D_H}{\Pi_H(p_H^M) - D_H} = \frac{\Pi_H(p_L^M) - \Pi_H(\hat{p})}{\Pi_H(p_H^M) - \Pi_H(\hat{p})} \tag{17}$$

Clearly $0 < \delta < 1$.

The following proposition characterizes the pooling equilibria of the game $G(\alpha)$.

<u>Proposition 4.</u> Consider the game $G(\alpha)$, where $1/2 < \alpha < 1$. Let $SPEP$ be the set of all sequential pooling equilibrium prices and SPE the set of all sequential pooling equilibria of $G(\alpha)$.

(1) If $p_L^M \le \hat{p}$, then $SPE = \emptyset$, unless $p_L^M = \hat{p}$ and $\alpha \le \bar{\alpha}_h$. In this case $SPEP = \{p_L^M\}$.

(2) Suppose that $p_L^M > \hat{p}$ and $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$. Then,

(i) For $\alpha \le \bar{\alpha}_h$, in every equilibrium in SPE, E stays out irrespective of the signal s and $SPEP = [\hat{p}, p_L^M]$. The set $SPEP$ coincides with the set $SPEP_{MR}$ of all sequential pooling equilibrium prices of the game G_{MR}.

(ii) If $\bar{\alpha}_h < \delta$, then for all α, $\bar{\alpha}_h < \alpha \le \delta$, E enters iff $s = h$, $SPE \ne \emptyset$ and $SPEP = [\max(\tilde{p}_H(\alpha), \bar{p}_L(\alpha)), p_L^M]$.

(iii) For $\alpha > \delta$, $SPE = \varnothing$.

(3) Suppose that $p_L^M > \hat{p}$ and $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) > 0$. Then,

(i) For $\alpha < \bar{\alpha}_l$, $SPE = \varnothing$.

(ii) If $\bar{\alpha}_l \leq \delta$, then for all α, $\bar{\alpha}_l \leq \alpha \leq \delta$, E enters iff $s = h$, $SPE \neq \varnothing$ and $SPEP = \left[\max\left(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)\right), p_L^M\right]$.

(iii) For $\alpha > \delta$, $SPE = \varnothing$.

(4) Suppose that $\delta < \max\left(\bar{\alpha}_l, \bar{\alpha}_h\right)$. Then $SPE = \varnothing$ [2].

<u>Proof:</u> Appears in the Appendix.

Proposition 4 asserts that sequential pooling equilibrium does not exist if either $p_L^M < \hat{p}$ or if $\alpha > \delta$. The first condition, $p_L^M < \hat{p}$, implies that the cost function of H is significantly higher than that of L. Even the duopoly price $\hat{p}$, when H competes with E, is above the monopoly price of L. In this case, it is too costly for H to mimick L and to fool E about his type. The other condition, $\alpha > \delta$, means that the IS is sufficiently accurate so that when E observes the signal h, she knows that the true type of M is H with high probability, and she is best off entering the market. In this case, H, who knows that his type is detected with high probability, has no reason to pool and he is best off charging the monopoly price p_H^M, upsetting the pooling equilibrium.

For intermediate values of α ($\bar{\alpha}_h < \alpha \leq \delta$ or $\bar{\alpha}_l \leq \alpha \leq \delta$), the set of pooling equilibria is non-empty and the decision of E is to enter the market if and only if the signal sent by the IS is h. In this case, M of type H knows that α is sufficiently low so with significant probability E will obtain the wrong signal l and will stay out. However, we also need the precision α to be not too low since, otherwise, E will not trust the signal and she will enter whether the signal is h or l. But then, the two type monopolists are best off with their monopoly prices, upsetting a pooling equilibrium.

Note that when $\alpha = \delta$, then $\max\left(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)\right) = \tilde{p}_H(\alpha) = p_L^M$ and $SPEP = \left\{p_L^M\right\}$.

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[2] It is easy to verify that if $\delta = \bar{\alpha}_h$, then $SPE = \varnothing$.

Proposition 4 also asserts that if $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)>0$ (in which case $\bar{\alpha}_h<\bar{\alpha}_l$) and if α is relatively small ($\alpha<\bar{\alpha}_l$), then $SPE=\varnothing$. Without the use of the IS, when the expected profit of the entrant is positive, pooling equilibrium does not exist since E will enter the market and both types of M are best off deviating to their monopoly price. Hence, the use of a relatively not accurate IS has no impact on this result.

The relationship between δ and $\bar{\alpha}_h$ or $\bar{\alpha}_l$ is not obvious and in general it is quite complex. But in light of part (3) of Proposition 4 it is important to shed a light on this relationship. We next analyze this relationship for the linear demand and linear cost functions case, assuming a Cournot competition if E enters the market.

Suppose that $p=a-Q$ is the total demand function and suppose that the cost functions are given by

$$C_L(q)=C_E(q)=c_Lq$$

$$C_H(q)=c_Hq$$

where $c_L<c_H$. Proposition 5 summarizes the results of this linear model.

<u>Proposition 5.</u> Consider the linear model and assume that, if entry occurs, E and M are engaged in a Cournot competition. (1) if $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$, there exists $\bar{K}$ s.t. if $\frac{a-c_L}{c_H-c_L}>\frac{5}{2}$ and if $K<\bar{K}$, then $\delta>\bar{\alpha}_h$ and for every $\alpha\in(\bar{\alpha}_h,\delta]$ the set $SPEP$ is non-empty and contains p_L^M; (2) if $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)>0$, there exists $\tilde{K}$ s.t. if $\frac{a-c_L}{c_H-c_L}>\frac{5}{2}$ and if $K\geq\tilde{K}$, then $\delta\geq\bar{\alpha}_l$ and for every $\alpha\in[\bar{\alpha}_l,\delta]$ the set $SPEP$ is non-empty and contains p_L^M.

<u>Remark:</u> The nature of Proposition 5 essentially does not change if we replace Cournot competition by Bertrand competition.

<u>Proof:</u> Appears in the Appendix.

Proposition 5 asserts that in the linear model, if α is not very accurate and the demand is not too small (it is sufficient that the demand intensity, a, exceeds $2.5c_H$), then

$\delta > \max(\bar{\alpha}_l, \bar{\alpha}_h)$ and p_L^M is a sequential pooling equilibrium price. In particular $SPEP \neq \varnothing$.

Finally, suppose that $\frac{a-c_L}{c_H-c_L} \leq \frac{1+3\sqrt{14}}{5}$. Then it can be verified that, if $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$, then $SPEP=\varnothing$ for all $\alpha > \bar{\alpha}_h$. Also if $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)>0$, then $SPEP=\varnothing$ for all $\alpha \geq \bar{\alpha}_l$.

6. Conclusion.

In this paper we analyzed industrial espionage when a potential entrant, E, does not observe the outcome of the R&D project carried out by an incumbent monopolist with the aim to reduce his cost of production and deter E from entering the market. E develops an Intelligence System (IS) of precision α that allows her to collect noisy information about the cost structure of M. Based on this information and the price that M charges for his product, E decides whether or not to enter the market. We assumed that α is exogenously given and commonly known by both firms.

We showed that the separating equilibria of our model are not affected by the spying activity of E. This is not very surprising since in a separating equilibrium E identifies the type of M with or without the use of the IS. The same result is obtained for pooling equilibria if the precision α of the IS is sufficiently low to affect E's decision of staying out. If α is very accurate, then pooling equilibrium does not exist. For intermediate values of α we find that pooling equilibrium exists and E enters the market if the IS tells her the cost of M is high. Hence, the use of the IS with high probability increases competition in pooling equilibrium. And, from this point of view, spying on incumbent firms increases competition with high probability.

An interesting suggestion for further research might be to analyze the more realistic scenario where α is the private information of E.

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Appendix

Proof of Lemma 1.

(i)

$$\Pi_L(p)-\Pi_H(p)=C_H(Q(p))-C_L(Q(p))$$

$$\frac{\partial}{\partial p}\left[\Pi_L(p)-\Pi_H(p)\right]=Q'(p)\left[C_H'(Q(p))-C_L'(Q(p))\right]$$

By Assumptions 4 and 5 the right side is negative.

(ii)

$$p_L^M q_L^M - C_L(q_L^M) \geq p_H^M q_H^M - C_L(q_H^M)$$
$$p_H^M q_H^M - C_H(q_H^M) \geq p_L^M q_L^M - C_H(q_L^M)$$

Adding the two inequalities we have

$$C_H(q_L^M)-C_L(q_L^M) \geq C_H(q_H^M)-C_L(q_H^M)$$

By Assumption 4 we have that $q_L^M \geq q_H^M$ and hence $p_L^M \leq p_H^M$.

Let us show that $p_L^M < p_H^M$. If not, then $p_L^M = p_H^M$. Since the First Orden Condition (FOC) for M of type *t* is

$$\frac{\partial \Pi_t}{\partial p}(Q(p))=0 \leftrightarrow C_t'(Q(p))=p+\frac{Q(p)}{Q'(p)}$$

the solution does not depend on *t*, namely $C_L'(Q(p_L^M))=C_H'(Q(p_L^M))$. But this contradicts Assumption 4.

(iii)

By Assumption 3,

$$\Pi_L(p_L^M)-D_L > \Pi_H(p_H^M)-D_H$$

Note that $D_L=\Pi_L(p_o)$ and $D_H=\Pi_H(\hat{p})$. Hence this inequality can be written as

$$\Pi_L(p_L^M)-\Pi_L(p_0) > \Pi_H(p_H^M)-\Pi_H(\hat{p})$$

Thus,

$$\Pi_L(p_L^M)-\Pi_H(p_L^M)+\underbrace{\Pi_H(p_L^M)-\Pi_H(p_H^M)}_{<0} > \Pi_L(p_0)-\Pi_H(\hat{p})$$

Hence,

$$\Pi_L\left(p_L^M\right)-\Pi_H\left(p_L^M\right)>\Pi_L\left(p_0\right)-\Pi_H\left(\hat{p}\right) \tag{A1}$$

Since $p_0 \le p_L^M$, we have by section (i) of Lemma 1

$$\Pi_L\left(p_0\right)-\Pi_H\left(p_0\right)>\Pi_L\left(p_L^M\right)-\Pi_H\left(p_L^M\right)$$

This together with (A1) imply that

$$\Pi_H\left(\hat{p}\right)>\Pi_H\left(p_0\right)$$

But $p_0 < p_H^M$ and $\hat{p} < p_H^M$ and by Assumption 2 $p_0 < \hat{p}$.

■

Proof of Proposition 3.

The H-type monopoly, knowing that entry will occur is best off choosing the price p_H^M. Thus $SSE_H = \left\{p_H^M\right\}$ and E enters for sure when she observes the price p_H^M. In particular, $p_H^M > p_l$.

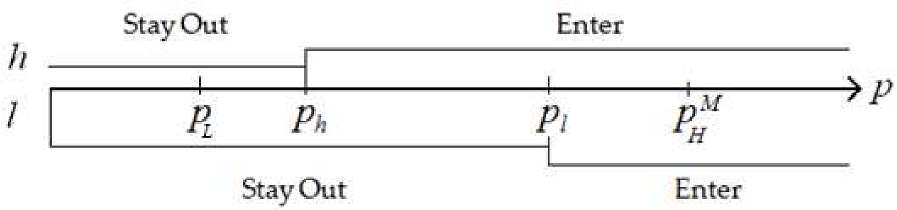

Figure 6

Next let us show that $SSE_L = \left\{p_L \middle| p_0 \le p_L \le \min\left(p_L^M, \hat{p}\right)\right\}$ for all α, $1/2 < \alpha < 1$. We consider two cases.

Case 1: Suppose first that $p_L^M \le \hat{p}$. We show that $p_L = p_L^M$ can be supported as a separating equilibrium price. Let p_h and p_l be s.t.

$$p_L = p_L^M \le p_h \le \hat{p} < p_l \le \tilde{p}_H\left(\alpha\right) < p_H^M \tag{A2}$$

To make sure that H has no incentive to deviate to either p_h or p_l, the following two inequalities should hold

$$\Pi_H\left(p_H^M\right)+D_H \ge \Pi_H\left(p_h\right)+\Pi_H\left(p_H^M\right) \tag{A3}$$

and

$$\Pi_H\left(p_H^M\right)+D_H \ge \Pi_H\left(p_l\right)+\alpha D_H+\left(1-\alpha\right)\Pi_H\left(p_H^M\right) \tag{A4}$$

These two are equivalent to

$$\Pi_H\left(\hat{p}\right)=D_H \geq \Pi_H\left(p_h\right)$$

and

$$\Pi_H\left(p_l\right)\leq \alpha\Pi_H\left(p_H^M\right)+(1-\alpha)D_H = \Pi_H\left(\tilde{p}_H\left(\alpha\right)\right)$$

(see Figure 4). Thus $p_h \leq \hat{p}$ and $p_l \leq \tilde{p}_H\left(\alpha\right)$.

By (A2) the two incentive compatibility constraints of H are satisfied and hence $p_L^M \in SSE_L$

Next let p_L be s.t. $p_0 \leq p_L < p_L^M$. Let us show that we can support p_L as a separtaing equilirium price. Let p_h and p_l be s.t.

$$p_L = p_h < p_l < p_L^M \leq \hat{p} < \tilde{p}_H\left(\alpha\right) < p_H^M \tag{A5}$$

Similarly to the previous case (A3) and (A4) must hold and thus $p_h \leq \hat{p}$ and $p_l \leq \tilde{p}_H\left(\alpha\right)$.

Since $p_L^M \leq \hat{p} < \tilde{p}_H\left(\alpha\right)$, by (A5) the two incentive compatibility constraints of H hold.

Next, since $p_L^M > p_l$, there are two relevant incentive compatibility constraints for L

$$\Pi_L\left(p_L\right)+\Pi_L\left(p_L^M\right)\geq \Pi_L\left(p_L^M\right)+D_L \tag{A6}$$

and

$$\Pi_L\left(p_L\right)+\Pi_L\left(p_L^M\right)\geq \Pi_L\left(p_l\right)+\alpha\Pi_L\left(p_L^M\right)+(1-\alpha)D_L \tag{A7}$$

(A6) and (A7) are equivalent to

$$\Pi_L\left(p_L\right)\geq D_L = \Pi_L\left(p_0\right) \tag{A8}$$

and

$$\Pi_L\left(p_L\right)\geq \Pi_L\left(p_l\right)-(1-\alpha)\left[\Pi_L\left(p_L^M\right)-D_L\right] \tag{A9}$$

Since $p_h = p_L \geq p_0$, (A8) holds. As for (A9), it holds for every $\alpha < 1$, provided that p_h is sufficiently close to p_l. Hence (A5) guarantees that, for all $1/2 < \alpha < 1$, $p_L \in SSE_L$ provided that $p_L = p_h$, p_l is sufficiently close to p_h and $p_l < p_L^M$.

Case 2: Suppose next that $\hat{p} < p_L^M$ and let p_L be s.t. $p_0 \le p_L \le \hat{p}$. We will show that for all α, $1/2 < \alpha < 1$, $p_L \in SSE_L$. Let p_h and p_l be s.t.

$$p_L = p_h \le \hat{p} < p_l < \min\left(p_L^M, \tilde{p}_H(\alpha)\right) < p_H^M \quad \text{(A10)}$$

As in (A3) and (A4), the incentive compatibility constraints of H are equivalent to $p_h \le \hat{p}$ and $p_l \le \tilde{p}_H(\alpha)$. By (A10) the two incentive compatibility constraints of H are satisfied.

Next, since $p_L^M > p_l$, in order for L not to deviate (A8) and (A9) must hold. Since $p_h = p_L \ge p_0$, (A8) holds. Similar to Case 1, for every $\alpha < 1$, (A9) holds if p_l is sufficiently close to p_h. Hence (A10) guarantees that, for all $1/2 < \alpha < 1$, $p_L \in SSE_L$ provided that $p_l - p_h$ is sufficiently small and $p_l < \min\left(p_L^M, \tilde{p}_H(\alpha)\right)$.

Cases 1 and 2 prove that any price $p_L \in SSE_L$ if $p_0 \le p_L \le \min\left(p_L^M, \hat{p}\right)$. Finally, we need to show that if $p_L \notin \left[p_0, \min\left(p_L^M, \hat{p}\right)\right]$, then $p_L \notin SSE_L$.

Let $Q = \left[p_0, \min\left(p_L^M, \hat{p}\right)\right]$.

Case A: $p_L^M \le \hat{p}$.

Subcase A.1. Suppose that $p_L^M \le \hat{p} < p_h$. There is no separating equilibrium in this case since by (A3) $p_h \le \hat{p}$, a contradiction.

Subcase A.2. Suppose that $p_L^M \le p_h \le \hat{p}$. Then by Assumption 2 L is best off choosing $p_L = p_L^M$ and $p_L^M \in Q$.

Subcase A.3. Suppose that $p_h < p_L^M \le p_l$. Since $p_L \le p_h < p_L^M$, by Assumption 2 L is best off choosing $p_L = p_h < p_L^M$.

From the incentive compatibility constraint of L we have

$$\Pi_L(p_h) + \Pi_L\left(p_L^M\right) \ge \Pi_L\left(p_L^M\right) + \alpha\Pi_L\left(p_L^M\right) + (1-\alpha)D_L \quad \text{(A11)}$$

or

$$\Pi_L(p_h) \ge \alpha\Pi_L\left(p_L^M\right) + (1-\alpha)D_L = \Pi_L\left(\tilde{p}_L(\alpha)\right)$$

(see Figure 4). Thus $p_h \geq \tilde{p}_L(\alpha)$. Consequently,

$$p_0 < \tilde{p}_L(\alpha) \leq p_L = p_h < p_L^M$$

and hence $p_L \in Q$.

Subcase A.4. Suppose that $p_l < p_L^M$. Similarly to the previous case, L is best off choosing $p_L = p_h < p_L^M$.

In order for L not to deviate, (A8) must hold. Equivalently, $p_h \geq p_0$. Hence

$$p_0 \leq p_L = p_h < p_L^M$$

and $p_L \in Q$.

Case B: $\hat{p} < p_L^M$.

Subcase B.1. Suppose that $p_L^M \leq p_h$. There is no separating equilibrium in this case since by (A3) $p_h \leq \hat{p}$, a contradiction.

Subcase B.2. Suppose that $p_h < p_L^M \leq p_l$. Then, L is best off choosing $p_L = p_h$.

By (A3), in order for H not to deviate, $p_h \leq \hat{p}$ must hold. By (A11), L has no incentive to deviate if $p_h \geq \tilde{p}_L(\alpha)$. Consequently,

$$p_0 < \tilde{p}_L(\alpha) \leq p_L = p_h \leq \hat{p}$$

and $p_L \in Q$.

Subcase B.3. Suppose that $p_l < p_L^M$. Again, L is best off choosing $p_L = p_h$ and $p_h \leq \hat{p}$ must hold. To guarantee that L has no incentive to deviate, $p_h \geq p_0$ must hold (see (A8)). Hence

$$p_0 \leq p_L = p_h \leq \hat{p}$$

and $p_L \in Q$.

■

Proof of Proposition 4.

Let $A_l = \{\alpha | \Pi_E(l|\alpha) \leq 0\}$ and $A_h = \{\alpha | \Pi_E(h|\alpha) \leq 0\}$.

Case 1. $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$.

In this case $\bar{\alpha}_l < \frac{1}{2} < \bar{\alpha}_h < 1$. Hence, $\alpha > \bar{\alpha}_l$ and by Corollary 1, $\alpha \in A_l \quad \forall \alpha$, $\frac{1}{2} < \alpha < 1$. Namely, if the IS sends the signal l, E does not enter the market when observing the price p^* irrespective the precision α of the IS.

Subcase 1.1. $\frac{1}{2} < \alpha \le \bar{\alpha}_h$.

In this case $\alpha \in A_l \cap A_h$. Namely, E does not enter the market when observing the price p^* irrespective of the signal sent by the IS.

Let us characterize the pooling equilibria in this case. We start with a lemma.

Lemma 4. Suppose that $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$ and $\frac{1}{2} < \alpha \le \bar{\alpha}_h$. Then in every pooling equilibrium

(i) $p_H^M > p_h$

(ii) $p^* \le p_l$

Proof: (i) Suppose to the contrary that $p_H^M \le p_h$. Then also $p_L^M < p_h$ and E stays out whether she observes p_L^M or p_H^M. Hence, both types of M will set their (different) monopoly prices, a contradiction.

(ii) Suppose to the contrary that $p^* > p_l$. Then at least one type of M has an incentive to deviate. Indeed if $p_L^M > p_l$ and $p^* = p_L^M$, the H-type monopoly is better off deviating to p_H^M or p_l depending on α and the parameters of the model. Similarly, if $p_L^M > p_l$ and $p^* = p_H^M$ the L-type monopoly is better off deviating to p_L^M or p_l depending on α and the parameters of the model. Finally, if $p_L^M \le p_l$, at least one type of M has an incentive to deviate to his monopoly price, a contradiction.

■

Lemma 5. Suppose that $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$, $\frac{1}{2}<\alpha\le\bar{\alpha}_h$ and $p_L^M<\hat{p}$. Then $SPE=\varnothing$.

Proof: Suppose to the contrary that $p^*\in SPEP$ and suppose that p_h and p_l are the equilibrium thresholds of E. We consider six cases.

Case 1. Suppose that $p_L^M\le p_h$. First note that in this case $p^*=p_L^M$. Next observe that the incentive compatibility constraint of H

$$\Pi_H\left(p_L^M\right)+\Pi_H\left(p_H^M\right)\ge\Pi_H\left(p_H^M\right)+D_H \tag{A12}$$

requiring that H has no incentive to deviate to p_H^M, is equivalent to

$$\Pi_H\left(p_L^M\right)\ge D_H=\Pi_H\left(\hat{p}\right)$$

and hence $p_L^M\ge\hat{p}$ (see Figure 4), a contradiction.

Case 2. Suppose that $p_h<p_L^M<p_H^M\le p_l$.

In this case $p^*\le p_h$ (otherwise, by Lemma 4, $p^*\in\left(p_h,p_l\right]$ and at least one type of M has an incentive to deviate to his monopoly price). Therefore $p^*=p_h$ must hold.

The incentive compatibility constraint of H is,

$$\Pi_H\left(p_h\right)+\Pi_H\left(p_H^M\right)\ge\Pi_H\left(p_H^M\right)+\alpha D_H+(1-\alpha)\Pi_H\left(p_H^M\right) \tag{A13}$$

Equivalently,

$$\Pi_H\left(p_h\right)\ge\alpha D_H+(1-\alpha)\Pi_H\left(p_H^M\right)=\Pi_H\left(\widehat{p}_H(\alpha)\right)$$

or $\widehat{p}_H(\alpha)\le p_h<p_H^M$ (see Figure 5). But $p_L^M<\hat{p}<\widehat{p}_H(\alpha)$ and in particular $p_L^M<p_h$, a contradiction.

Case 3. Suppose that $p^*\le p_h<p_L^M\le p_l<p_H^M$.

In this case again $p^*=p_h$. The incentive compatibility constraint of H is

$$\Pi_H\left(p_h\right)+\Pi_H\left(p_H^M\right)\ge\Pi_H\left(p_H^M\right)+D_H \tag{A14}$$

Equivalently,

$$\Pi_H\left(p_h\right)\ge D_H=\Pi_H\left(\hat{p}\right)$$

or $\hat{p} \le p_h$ (see Figure 4). But we deal with the case where $p_h < p_L^M < \hat{p}$, a contradiction.

Case 4. Suppose that $p_h < p_L^M \le p_l < p_H^M$ and $p^* \in (p_h, p_l]$.

In this case $p^* = p_L^M$. In order for H not to deviate from p_L^M to p_H^M, the inequality

$$\Pi_H(p_L^M) + \alpha D_H + (1-\alpha)\Pi_H(p_H^M) \ge \Pi_H(p_H^M) + D_H \qquad \text{(A15)}$$

should hold. Equivalently,

$$\alpha \le \frac{\Pi_H(p_L^M) - D_H}{\Pi_H(p_H^M) - D_H} = \frac{\Pi_H(p_L^M) - \Pi_H(\hat{p})}{\Pi_H(p_H^M) - \Pi_H(\hat{p})} \equiv \delta < 0$$

a contradiction.

Case 5. Suppose that $p^* \le p_h < p_l < p_L^M < p_H^M$.

In this case $p^* = p_h$ and (A14) guarantees that H has no incentive to deviate from p^* to p_H^M. By (A14), $\hat{p} \le p_h < p_L^M$, a contradiction.

Case 6. Suppose that $p_h < p_l < p_L^M < p_H^M$ and $p^* \in (p_h, p_l]$.

Clearly in this case $p^* = p_l$ and E follows the signal sent by the IS.

In order for H not to deviate from p_l to p_H^M, the inequality

$$\Pi_H(p_l) + \alpha D_H + (1-\alpha)\Pi_H(p_H^M) \ge \Pi_H(p_H^M) + D_H \qquad \text{(A16)}$$

must hold. Equivalently,

$$\Pi_H(p_l) \ge \alpha\Pi_H(p_H^M) + (1-\alpha)D_H = \Pi_H(\tilde{p}_H(\alpha))$$

and $\tilde{p}_H(\alpha) \le p_l < p_H^M$. But $p_L^M < \hat{p} < \tilde{p}_H(\alpha)$, a contradiction. We conclude that if $p_L^M < \hat{p}$, then $SPE = \varnothing$.

■

We next deal with the case where $\hat{p} < p_L^M$. We need to show that $SPEP = \{p^* | \hat{p} \le p^* \le p_L^M\}$ for all α, $1/2 < \alpha < 1$.

First we prove the following lemma.

Lemma 6. Suppose that $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$, $1/2<\alpha\le\bar{\alpha}_h$ and $\hat{p}<p_L^M$. Then

$$\left\{p^*\middle|\hat{p}\le p^*\le p_L^M\right\}\subseteq SPEP$$

Proof: We start by showing that $p_L^M\in SPEP$.

Let p_h and p_l be s.t.

$$p^*=p_h=p_L^M<p_l<p_H^M \tag{A17}$$

Clearly L is best off with p_L^M and has no incentive to deviate.

The two incentive compatibility constraints of H in this case are:

(i) H has no incentive to deviate from p_L^M to p_l. Namely,

$$\Pi_H\left(p_L^M\right)+\Pi_H\left(p_H^M\right)\ge\Pi_H\left(p_l\right)+\alpha D_H+(1-\alpha)\Pi_H\left(p_H^M\right) \tag{A18}$$

Equivalently,

$$\Pi_H\left(p_l\right)-\Pi_H\left(p_L^M\right)\le\alpha\left[\Pi_H\left(p_H^M\right)-D_H\right]$$

(ii) H has no incentive to deviate from p_L^M to p_H^M if (A12) holds.

Since $\hat{p}<p_L^M$, (A12) holds. As for (A18), it holds for every $1/2<\alpha<1$, provided that p_l is sufficiently close to p_L^M. Hence (A17) for p_l sufficiently close to p_L^M, guarantees that, for all $1/2<\alpha<1$, $p_L^M\in SPEP$.

Next let p_h and p_l be s.t.

$$\hat{p}\le p^*=p_h<p_l<p_L^M<p_H^M \tag{A19}$$

The incentive compatibility constraints of L in this case are two:

(i) L has no incentive to deviate from p_h to p_l.

$$\Pi_L\left(p_h\right)+\Pi_L\left(p_L^M\right)\ge\Pi_L\left(p_l\right)+\alpha\Pi_L\left(p_L^M\right)+(1-\alpha)D_L \tag{A20}$$

Equivalently,

$$\Pi_L\left(p_l\right)-\Pi_L\left(p_h\right)\le(1-\alpha)\left[\Pi_L\left(p_L^M\right)-D_L\right]$$

(ii) L has no incentive to deviate from p_h to p_L^M.

$$\Pi_L\left(p_h\right)+\Pi_L\left(p_L^M\right)\ge\Pi_L\left(p_L^M\right)+D_L \tag{A21}$$

Equivalently,

$$\Pi_L(p_h) \geq D_L = \Pi_L(p_0)$$

or $p_0 \leq p_h$ (see Figure 4).

The two incentive compatibility constraints of H are the one given by (A14) and

$$\Pi_H(p_h) + \Pi_H(p_H^M) \geq \Pi_H(p_l) + \alpha D_H + (1-\alpha)\Pi_H(p_H^M) \qquad \text{(A22)}$$

Equivalently,

$$\Pi_H(p_l) - \Pi_H(p_h) \leq \alpha\left[\Pi_H(p_H^M) - D_H\right]$$

(A14) and (A21) imply $\hat{p} \leq p_h$ and $p_0 \leq p_h$ respectively. By Lemma 1, $\hat{p} > p_0$. Hence, $\hat{p} \leq p_h$, which is consistent with (A19). (A20) and (A22) hold for every $1/2 < \alpha < 1$ provided that p_h is sufficiently close to p_l. Hence (A19) for $0 < p_l - p_h$ sufficiently small, guarantees that, for all $1/2 < \alpha < 1$, $p^* \in SPEP$, and the proof of Lemma 6 is complete.

■

<u>Lemma 7.</u> Suppose that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$ and $1/2 < \alpha \leq \bar{\alpha}_h$. Then, $SPEP \subseteq \left[\hat{p}, p_L^M\right]$.

<u>Proof:</u> Let $p^* \in SPEP$. By Lemma 4, $p^* \leq p_l$ and $p_H^M > p_h$.

Let $R = \left[\hat{p}, p_L^M\right]$.

The relevant cases are

Case 1. Suppose that $p_L^M \leq p_h$. Then $p^* = p_L^M \in R$.

Case 2. Suppose that $\hat{p} < p_h < p_L^M < p_H^M \leq p_l$.

Similarly to case 2 of Lemma 5, $p^* = p_h$. By the incentive compatibility constraint of H given by (A13), $\hat{p} < \widehat{p}_H(\alpha) \leq p^* = p_h < p_L^M$. Hence $p^* \in R$.

Case 3. Suppose that $p_h \leq \hat{p} < p_L^M < p_H^M \leq p_l$.

Similar to the previous case $\widehat{p}_H(\alpha) \leq p^* = p_h$. Hence, no pooling equilibrium exists in this case since $p_h \leq \hat{p} < \widehat{p}_H(\alpha)$.

Case 4. Suppose that $\hat{p} \leq p^* \leq p_h < p_L^M \leq p_l < p_H^M$. Then clearly $p^* \in R$.

Case 5. Suppose that $p^* \le p_h < \hat{p} < p_L^M \le p_l < p_H^M$.

Similarly to the previous case $p^* = p_h$, and by (A14) $\hat{p} \le p_h$, a contradiction. Hence, there exists no pooling equilibrium in this case.

Case 6. Suppose that $p_h < p_L^M \le p_l < p_H^M$ and $p^* \in (p_h, p_l]$.

Clearly in this case $p^* = p_L^M$ and $p_L^M \in SPEP$.

Case 7. Suppose that $\hat{p} \le p^* \le p_h < p_l < p_L^M < p_H^M$. Then $p^* \in R$.

Case 8. Suppose that $p^* \le p_h < \hat{p} < p_l < p_L^M < p_H^M$.

Similarly to case 5 above, there is no pooling equilibrium in this case.

Case 9. Suppose that $p_h < \hat{p} < p_l < p_L^M < p_H^M$ and $p^* \in (p_h, p_l]$.

Clearly in this case $p^* = p_l$. From the incentive compatibility constraint of H given by (A16), $\hat{p} < \tilde{p}_H(\alpha) \le p^* = p_l < p_L^M$. Hence $p^* \in R$.

Case 10. Suppose that $p_h < p_l \le \hat{p} < p_L^M < p_H^M$ and $p^* \in (p_h, p_l]$.

Similarly to the previous case $\tilde{p}_H(\alpha) \le p^* = p_l$. Hence, no pooling equilibrium exists in this case since $p_l \le \hat{p} < \tilde{p}_H(\alpha)$.

The above 10 cases prove that if $p^* \in SPEP$, then $p^* \in R$, as claimed.

■

Finally, let us show that if $p_L^M = \hat{p}$, then $SPEP = \{p_L^M\}$ for all $1/2 < \alpha < 1$.

__*Lemma 8.*__ Suppose that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$, $1/2 < \alpha \le \bar{\alpha}_h$ and $p_L^M = \hat{p}$. Then $p_L^M \in SPEP$.

<u>Proof:</u> Let p_h and p_l be s.t. (A17) holds. The incentive compatibility constraints of H are given by (A12) and (A18). Clearly (A12) holds since $p_L^M = \hat{p}$. But also (A18) holds for every $1/2 < \alpha < 1$, provided that p_l is sufficiently close to p_L^M. Hence, $p_L^M \in SPEP$ for all $1/2 < \alpha < 1$ is guaranteed by (A17) with p_l sufficiently close to p_L^M.

■

Lemma 9. Suppose that $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$, $\frac{1}{2}<\alpha\le\bar{\alpha}_h$, $p_L^M=\hat{p}$ and $p^*\in SPEP$. Then $p^*=p_L^M$.

Proof: We consider the same six cases as in proof of Lemma 5.

Case 1. Suppose that $p_L^M\le p_h$. Then $p^*=p_L^M$, as claimed.

Case 2. Suppose that $p_h<p_L^M<p_H^M\le p_l$.

Similarly to case 2 of Lemma 5, $p^*=p_h$. By (A13), $\hat{p}<\widehat{p}_H(\alpha)\le p_h<p_H^M$ must hold. But $p_h<p_L^M=\hat{p}$, a contradiction. Hence, there is no pooling equilibrium in this case.

Case 3. Suppose that $p^*\le p_h<p_L^M\le p_l<p_H^M$.

In this case again $p^*=p_h$. The incentive compatibility constraint of H is given by (A14) and implies $\hat{p}\le p_h$. But in this case $p_h<p_L^M=\hat{p}$. Consequently, no pooling equilibrium exists in this case.

Case 4. Suppose that $p_h<p_L^M\le p_l<p_H^M$ and $p^*\in(p_h,p_l]$.

In this case $p^*=p_L^M\in SPEP$, as claimed.

Case 5. Suppose that $p^*\le p_h<p_l<p_L^M<p_H^M$.

In this case $p^*=p_h$ and H has no incentive to deviate from p^* to p_H^M if (A14) holds, or equivalently, $\hat{p}\le p_h<p_L^M$, a contradiction. Hence, there is no pooling equilibrium in this case either.

Case 6. Suppose that $p_h<p_l<p_L^M<p_H^M$ and $p^*\in(p_h,p_l]$.

Clearly in this case $p^*=p_l$. From the incentive compatibility constraint of H given by (A16), $\tilde{p}_H(\alpha)\le p_l$ must hold. But $p_L^M=\hat{p}<\tilde{p}_H(\alpha)$. Consequently, no pooling equilibrium exists in this case.

■

Lemmas 8 and 9 establish the second part of part (1) of the proposition.

Subcase 1.2. $\bar{\alpha}_h < \alpha < 1$.

In this case $\alpha \in A_l \setminus \bar{A}_h$. Namely, E enters the market when observing the price p^* if the IS sends the signal h and does not enter if the IS sends the signal l. Hence, accordingly to the strategy of E defined in Lemma 3, $p_h < p^* \le p_l$.

Let us find pooling equilibria in this case.

Lemma 10. Suppose that $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$ and $\bar{\alpha}_h < \alpha < 1$. Then, in every pooling equilibrium $p_L^M > p_h$ and $p_H^M > p_l$.

Proof: Suppose to the contrary that $p_L^M \le p_h$ or $p_H^M \le p_l$. Then, at least one type of M has an incentive to deviate to his monopoly price.

■

Lemma 11. Suppose that $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$, $\bar{\alpha}_h < \alpha < 1$ and $p_L^M \le \hat{p}$. Then $SPE = \varnothing$.

Proof: Suppose to the contrary that $p^* \in SPEP$. We consider two cases.

Case 1. Suppose that $p_h < p_L^M \le p_l < p_H^M$. Note that in this case $p^* = p_l = p_L^M$.

In order for H not to deviate from p_L^M to p_H^M, (A15) should hold. Equivalently $\alpha \le \delta \le 0$, a contradiction.

Case 2. Suppose that $p_h < p_l < p_L^M < p_H^M$. Note that in this case $p^* = p_l$.

H has no incentive to deviate from p_l to p_H^M if (A16) holds. Equivalently, $p_l \ge \tilde{p}_H(\alpha)$ (see Figure 4). Since $p_l < p_L^M$, $p_L^M > \tilde{p}_H(\alpha)$ must hold. Equivalently, $\alpha < \delta \le 0$, a contradiction.

■

Note that Lemmas 5 and 11 establish the first part of part (1) of the proposition.

Lemma 12. Suppose that $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$, $\bar{\alpha}_h < \alpha < 1$ and $\hat{p} < p_L^M$. Then

$$SPEP = \left\{ p^* \middle| \max\left(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)\right) \le p^* \le p_L^M \right\}$$

and this set is non-empty if $\delta > \bar{\alpha}_h$ and for all α, $\bar{\alpha}_h < \alpha \le \delta$.

Proof: We start by showing that $p_L^M \in SPEP$.

Let p_h and p_l be s.t.

$$p_h < p^* = p_L^M = p_l < p_H^M \tag{A23}$$

In order for H not to deviate from p_L^M to p_H^M, (A15) should hold. Equivalently $\alpha \leq \delta$, where $0 < \delta < 1$ since $p_L^M > \hat{p}$.

H has no incentive to deviate from p_L^M to p_h, if

$$\Pi_H\left(p_L^M\right) + \alpha D_H + (1-\alpha)\Pi_H\left(p_H^M\right) \geq \Pi_H\left(p_h\right) + \Pi_H\left(p_H^M\right) \tag{A24}$$

holds. Equivalently,

$$\Pi_H\left(p_L^M\right) - \Pi_H\left(p_h\right) \geq \alpha\left(\Pi_H\left(p_H^M\right) - D_H\right)$$

The incentive compatibility constraint of L is given by

$$\Pi_L\left(p_L^M\right) + \alpha\Pi_L\left(p_L^M\right) + (1-\alpha)D_L \geq \Pi_L\left(p_h\right) + \Pi_L\left(p_L^M\right) \tag{A25}$$

Equivalently,

$$\Pi_L\left(p_L^M\right) - \Pi_L\left(p_h\right) \geq (1-\alpha)\left(\Pi_L\left(p_L^M\right) - D_L\right)$$

Note that (A24) and (A25) hold for p_h sufficiently small. Hence (A23) p_h sufficiently small and for $\delta > \bar{\alpha}_h$, guarantees that, for all $\bar{\alpha}_h < \alpha \leq \delta$, $p_L^M \in SPEP$.

Next let p_h and p_l be s.t.

$$p_h < \max\left(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)\right) \leq p^* = p_l < p_L^M < p_H^M \tag{A26}$$

H has no incentive to deviate from p_l to p_H^M if (A16) holds. Equivalently, $p_l \geq \tilde{p}_H(\alpha)$ (see Figure 4). Since $p_l < p_L^M$, $p_L^M > \tilde{p}_H(\alpha)$ must hold. Equivalently, $\alpha < \delta$. Note that $0 < \delta < 1$ since $p_L^M > \hat{p}$.

In order for H not to deviate from p_l to p_h,

$$\Pi_H\left(p_l\right) + \alpha D_H + (1-\alpha)\Pi_H\left(p_H^M\right) \geq \Pi_H\left(p_h\right) + \Pi_H\left(p_H^M\right) \tag{A27}$$

should hold. Equivalently,

$$\Pi_H\left(p_l\right) - \Pi_H\left(p_h\right) \geq \alpha\left(\Pi_H\left(p_H^M\right) - D_H\right)$$

Next, let us consider the two incentive compatibility constraints of L.

(i) In order for L not to deviate from p_l to p_L^M,

$$\Pi_L\left(p_l\right) + \alpha\Pi_L\left(p_L^M\right) + (1-\alpha)D_L \geq \Pi_L\left(p_L^M\right) + D_L \tag{A28}$$

should hold. Equivalently, $p_l \geq \hat{p}_L(\alpha)$ (see Figure 5).

(ii) In order for L not to deviate from p_l to p_h,

$$\Pi_L(p_l) + \alpha\Pi_L(p_L^M) + (1-\alpha)D_L \geq \Pi_L(p_h) + \Pi_L(p_L^M) \tag{A29}$$

should hold. Equivalently,

$$\Pi_L(p_l) - \Pi_L(p_h) \geq (1-\alpha)\left(\Pi_L(p_L^M) - D_L\right)$$

(A16) and (A28) imply that $\max(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)) \leq p^* = p_l < p_L^M$, which is consistent with (A26), but it needs $\delta > \bar{\alpha}_h$ and $\bar{\alpha}_h < \alpha < \delta$. Note that (A27) and (A29) hold for p_h sufficiently small. Hence, (A26) for p_h sufficiently small and $\delta > \bar{\alpha}_h$, guarantees that, for all $\bar{\alpha}_h < \alpha < \delta$, $p^* \in SPEP$.

■

<u>Case 2.</u> $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) > 0$.

Note that in this case $\bar{\alpha}_h < 1/2 < \bar{\alpha}_l < 1$. Hence $\alpha > \bar{\alpha}_h$ and $\alpha \notin A_h$, $\forall\alpha$, $1/2 < \alpha < 1$. Namely, if the IS sends the signal h, E enters the market when observing the price p^* irrespective the precision α of the IS.

<u>Subcase 2.1.</u> $1/2 < \alpha < \bar{\alpha}_l$.

In this case $\alpha \notin A_l \cup A_h$. Namely, E enters the market when observing the price p^* irrespective of the signal sent by the IS and, therefore, both H and L should select the prices p_H^M and p_L^M, respectively. Since $p_L^M < p_H^M$, no pooling equilibrium exists in this case.

<u>Subcase 2.2.</u> $\bar{\alpha}_l \leq \alpha < 1$.

In this case $\alpha \in A_l \setminus \bar{A}_h$. Namely, E enters the market when observing the price p^* if the IS sends the signal h and does not enter if the IS sends the signal l. Hence, similarly to Subcase 1.2, if $p_L^M \leq \hat{p}$, then $SPE = \varnothing$. In particular, this together with Lemmas 5 and 11 establish the first part of part (1) of the proposition. If $\hat{p} < p_L^M$, then

$$SPEP = \left\{ p^* \middle| \max(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)) \leq p^* \leq p_L^M \right\}$$

and this set is non-empty if $\delta \geq \bar{\alpha}_l$ and for all α, $\bar{\alpha}_l \leq \alpha \leq \delta$.

■

Proof of Proposition 5.

In this linear model,

$$p_L^M = \frac{a+c_L}{2},\ p_H^M = \frac{a+c_H}{2}$$

$$\Pi_L\left(p_L^M\right) = \left(\frac{a-c_L}{2}\right)^2,\ \Pi_H\left(p_H^M\right) = \left(\frac{a-c_H}{2}\right)^2$$

$$D_L = \left(\frac{a-c_L}{3}\right)^2,\ D_E\left(L\right) = \left(\frac{a-c_L}{3}\right)^2$$

$$D_H = \left(\frac{a-2c_H+c_L}{3}\right)^2,\ D_E\left(H\right) = \left(\frac{a-2c_L+c_H}{3}\right)^2$$

Note that

$$c_L < c_H \leq \frac{a+c_L}{2} < a \tag{A30}$$

must hold and it is easy to verify that $p_L^M > \hat{p}$.

$$\Delta_E\left(L\right) = \left(\frac{a-c_L}{3}\right)^2 - K$$

$$\Delta_E\left(H\right) = \left(\frac{a-2c_L+c_H}{3}\right)^2 - K$$

Since $\Delta_E\left(L\right) < 0$ and $\Delta_E\left(H\right) > 0$,

$$\left(\frac{a-c_L}{3}\right)^2 < K < \left(\frac{a-2c_L+c_H}{3}\right)^2 \tag{A31}$$

must hold.

(1) Suppose that $\mu\Delta_E\left(H\right)+\left(1-\mu\right)\Delta_E\left(L\right) < 0$. Then,

$$K > \mu\left(\frac{a-2c_1+c_2}{3}\right)^2 + \left(1-\mu\right)\left(\frac{a-c_1}{3}\right)^2 \tag{A32}$$

(A31) together with (A32) imply that

$$\mu\left(\frac{a-2c_1+c_2}{3}\right)^2+(1-\mu)\left(\frac{a-c_1}{3}\right)^2<K<\left(\frac{a-2c_1+c_2}{3}\right)^2 \quad \text{(A33)}$$

We conclude that (A30) and (A33) must hold.

For the existence of a pooling equilibrium we need to have $\delta>\bar{\alpha}_h$ for every $\alpha\in(\bar{\alpha}_h,\delta]$. This holds iff

$$\frac{a-c_L}{c_H-c_L}>\frac{1+3\sqrt{14}}{5} \quad \text{(A34)}$$

and

$$\mu\left(\frac{a-2c_L+c_H}{3}\right)^2+(1-\mu)\left(\frac{a-c_L}{3}\right)^2<K<\bar{K} \quad \text{(A35)}$$

where $\bar{K}$ is the solution to $\delta=\bar{\alpha}_h$ (see (16) and (17)).

It can be shown that when (A34) holds,

$$\mu\left(\frac{a-2c_L+c_H}{3}\right)^2+(1-\mu)\left(\frac{a-c_L}{3}\right)^2<\bar{K}<\left(\frac{a-2c_L+c_H}{3}\right)^2$$

and thus, the set of all K's s.t. (A35) holds, is non-empty.

(2) Suppose that $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)>0$. Hence,

$$K<\mu\left(\frac{a-2c_L+c_H}{3}\right)^2+(1-\mu)\left(\frac{a-c_L}{3}\right)^2 \quad \text{(A36)}$$

(A36) together with (A31) imply

$$\left(\frac{a-c_L}{3}\right)^2<K<\mu\left(\frac{a-2c_L+c_H}{3}\right)^2+(1-\mu)\left(\frac{a-c_L}{3}\right)^2 \quad \text{(A37)}$$

Hence, we conclude that (A30) and (A37) must hold.

For the existence of a pooling equilibrium we need that $\delta\geq\bar{\alpha}_l$ for every $\alpha\in[\bar{\alpha}_l,\delta]$.

This holds iff (A34) holds and

$$\tilde{K}\leq K<\mu\left(\frac{a-2c_L+c_H}{3}\right)^2+(1-\mu)\left(\frac{a-c_L}{3}\right)^2 \quad \text{(A38)}$$

where $\tilde{K}$ is the solution to $\delta=\bar{\alpha}_l$ (see (14) and (17)).

It can be shown that when (A34) holds,

$$\left(\frac{a-c_L}{3}\right)^2 < \tilde{K} < \mu\left(\frac{a-2c_L+c_H}{3}\right)^2 + (1-\mu)\left(\frac{a-c_L}{3}\right)^2$$

and the set of all K's s.t. (A38) holds, is non-empty.

■

If we replace Cournot competition by Bertrand competition, we have

$$p_L^M = \frac{a+c_L}{2}, \; p_H^M = \frac{a+c_H}{2}$$

$$\Pi_L\left(p_L^M\right) = \left(\frac{a-c_L}{2}\right)^2, \; \Pi_H\left(p_H^M\right) = \left(\frac{a-c_H}{2}\right)^2$$

$$D_L = 0, \; D_E(L) = 0$$

$$D_H = 0, \; D_E(H) = (c_H - c_L)(a - c_H)$$

Note that $c_L < c_H < a$ must hold and $p_L^M > \hat{p}$ iff $c_H < \frac{a+c_L}{2}$. Hence

$$c_L < c_H < \frac{a+c_L}{2} < a \quad \text{(A39)}$$

must hold.

$\Delta_E(L) = 0 - K$

$\Delta_E(H) = (c_H - c_L)(a - c_H) - K$

Since $\Delta_E(L) < 0$ and $\Delta_E(H) > 0$,

$$0 < K < (c_H - c_L)(a - c_H) \quad \text{(A40)}$$

must hold.

(1) Suppose that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$. Then,

$$K > \mu(c_H - c_L)(a - c_H) \quad \text{(A41)}$$

(A40) together with (A41) imply that

$$\mu(c_H - c_L)(a - c_H) < K < (c_H - c_L)(a - c_H) \quad \text{(A42)}$$

We conclude that (A39) and (A42) must hold.

For the existence of a pooling equilibrium we need to have $\delta > \bar{\alpha}_h$ for every $\alpha \in (\bar{\alpha}_h, \delta]$. This holds iff

$$\frac{a-c_H}{c_H-c_L} > \sqrt{2} \tag{A43}$$

and

$$\mu\left(c_H-c_L\right)\left(a-c_H\right) < K < \bar{K}_1 \tag{A44}$$

where $\bar{K}_1$ is the solution to $\delta = \bar{\alpha}_h$ (see (16) and (17)).

It can be shown that when (A43) holds,

$$\mu\left(c_H-c_L\right)\left(a-c_H\right) < \bar{K}_1 < \left(c_H-c_L\right)\left(a-c_H\right)$$

and thus, the set of all K's s.t. (A44) holds, is non-empty.

(2) Suppose that $\mu\Delta_E\left(H\right)+\left(1-\mu\right)\Delta_E\left(L\right) > 0$. Hence,

$$K < \mu\left(c_H-c_L\right)\left(a-c_H\right) \tag{A45}$$

(A45) together with (A40) imply

$$0 < K < \mu\left(c_H-c_L\right)\left(a-c_H\right) \tag{A46}$$

Hence, we conclude that (A39) and (A46) must hold.

For the existence of a pooling equilibrium we need that $\delta \geq \bar{\alpha}_l$ for every $\alpha \in \left[\bar{\alpha}_l, \delta\right]$. This holds iff (A43) holds and

$$\tilde{K}_1 \leq K < \mu\left(c_H-c_L\right)\left(a-c_H\right) \tag{A47}$$

where $\tilde{K}_1$ is the solution to $\delta = \bar{\alpha}_l$ (see (14) and (17)).

It can be shown that when (A43) holds,

$$0 < \tilde{K}_1 < \mu\left(c_H-c_L\right)\left(a-c_H\right)$$

and the set of all K's s.t. (A47) holds, is non-empty.

■