ASYMMETRIC INFORMATION AND THE SUSTAINABILITY OF SOVEREIGN DEBT

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Abstract. This paper uses information asymmetry to provide a potential answer to two questions in international macroeconomics. First, why do countries repay unsecured sovereign debt? Bulow and Rogoff (1989) prove that, in a competitive financial market, the threat of credit exclusion cannot sustain repayment of uncollateralized debt, if the borrower cannot commit to repay. However, their argument relies on a key assumption: symmetric information. This paper show that that debt repayment is sustainable in any period in which the borrower has private information on a persistent shock. The intuition is that repayment of debt is a signal of a good persistent shock, which is correlated with a better income distribution in the future. This will translate to a cheaper insurance premium for the borrower who repays, and more expensive for the defaulter.

Second, the model provides an answer to an empirical question: while debt has a much higher interest rate than savings, why do many countries simultaneously issue debt and accumulate reserves? Finally, the paper delivers an interesting policy implication: if the lending and insurance market has free entry, then more information can mean less debt and can reduce welfare.

1. Introduction

This paper provides an answer to two questions in international macroeconomics: first, is the threat of credit exclusion sufficient to sustain repayment of sovereign debt, and second, why do countries simultaneously borrow at a high interest rate and save (accumulate reserves) at a lower interest rate?

Bulow and Rogoff (1989) formalizes the first question in the context of sovereign borrowing. A small country borrows and saves with competitive foreign investors to smooth its consumption and insure itself against income shocks. The investors are fully committed to honor any obligation, but the country cannot commit to repay.

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Due to weak international enforcements, debt has no collateral, and so the strongest punishment that investors can impose on a defaulting borrower is a ban from future credit. But because of free entry in the financial market, a defaulting country can always find some one who will be willing to offer it a payment-in-advance insurance contract (which involves no credit). This makes the punishment threat too weak to sustain any debt repayment. Thus uncollateralized debt is unsustainable in any (subgame perfect) equilibrium. Bulow and Rogoff (1989) establishes an important benchmark based on which a large literature on sovereign debt sustainability has emerged. Uncollateralized debt repayment has been an interesting a puzzle for a few decades.

This paper shows the impossibility result however relies on a crucial assumption: symmetric information. If information is asymmetric, even just for one period, then a positive level of debt is sustainable. Suppose there is one period in which the borrowing country has private information about a shock to its fundamentals (e.g. there is a political transition that privately changes the government's time-preferences for future economic outcomes). The country's income shocks are verifiable, and thus financial securities can be indexed to incomes, but there is no indexation for the unobservable shock. For simplicity suppose there are only two hidden states, good fundamental and bad fundamental, and a country with high fundamental will have a better probability distribution of future incomes. Would the representative government of the country have an incentive to repay an outstanding debt obligation? Rational investors will assess probability distribution of the country's future income shocks based on the its action today, which is a signal of the unobserved fundamental. In particular, it is only natural that repayment is positively associated and default is negatively associated with a good fundamental shock. Since a good fundamental means more favorable income shocks in the future, investors will offer a cheaper insurance premium for a repay-er than a defaulter. This provides an incentive for the country to repay its debt. A similar argument works when information is imperfect in many periods.

There is another interesting related empirical observable. The figure below shows that many emerging market economies simultaneously hold large amounts of external debt and large amounts of reserves. The trend is becoming increasingly pronounced in recent years. Given that the interest rate on borrowing is much higher than that on reserves, this phenomenon has raised interesting discussions regarding the cost and benefits of reserve accumulation. The phenomenon is also a reflection on the international scale of a long standing puzzle in domestic macroeconomics: why do consumers hold high levels of credit card debt while they also have high levels of savings at the same time (the

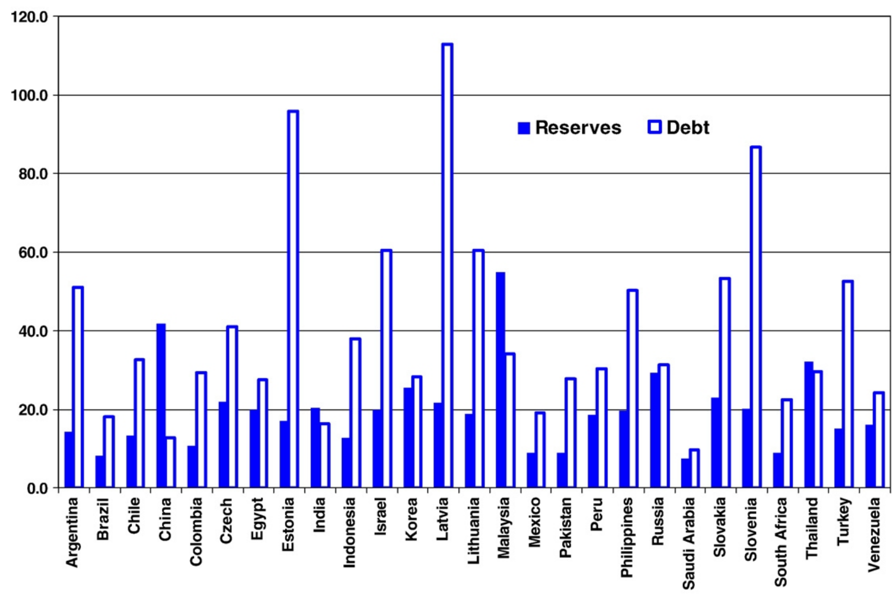

FIGURE 1. Reserve holding and external debt of emerging market economies in 2006 (as % of GDP). Source: Alfaro and Kanczuk (2009).

so-called "credit card debt puzzle"). A common explanation is that savings has more liquidity than borrowing, and thus savings acts as an insurance mechanism against unexpected shocks.

The model in this paper proposes another explanation, without using liquidity. In an incomplete financial market where instruments are not indexed to the private fundamental shocks, the difference in interest rates is not an arbitrage opportunity, but simply a reflection of the fact that debt will be repaid only when fundamental is in good state. Debt acts like an Arrow security that transfers wealth from the good state of tomorrow to today. The consumer also wants to save to transfer wealth from today to the bad state of tomorrow.

The paper as a whole gives an interesting implication. Unlike typical markets for consumption goods, the market for international financial products might not always benefit from more information. Improving the verifiability of private information on the side of the debtors might harm their ability to borrow. In particular, if information is perfect then we are back to the Bulow and Rogoff (1989) benchmark, and the debt market thus collapses. This leads to a policy implication to the international financial market: more verification and indexation might not increase the country's ability to share its risks with the international financial market as the "original sin" literature (see related literature below) have usually suggested. "Ignorance can be a bliss".

Related Literature. This paper is related to a few branches of the macroeconomic literature. In international macroeconomics, it is linked to the large collection of papers on sovereign debt following Bulow and Rogoff (1989). Among them the closest models to ours are the imperfect information models of Cole and Kehoe (1998), Sandleris (2008), and Catão et al. (2009). [1] The first two papers however require the country to have another relationship, exogenous from borrowing and lending, either with workers as in Cole and Kehoe (1998), or with foreign direct investors as in Sandleris (2008). Catão et al. (2009) requires collateral to sustain borrowing. Furthermore the last two models are only three period models, while ours has infinitely many periods. Our paper also relates to a moral hazard model of Atkeson (1991), and incomplete market models of Eaton and Gersovitz (1981), Grossman and Van Huyck (1988), Cole et al. (1995). However, these papers sustain debt by the ability of investors to exclude defaulters from payment-in-advance contracts, and hence do not address the puzzle posed by Bulow and Rogoff (1989) for competitive financial markets. Thomas (1992) and Grossman and Han (1999) address the Bulow Rogoff puzzle by assuming that there is an exogenous asymmetry between borrowing and saving: debt is more state-contingent than saving. Our model does not need to make this assumption. Finally, these models in general cannot predict simultaneous borrowing and saving.

In closed economies where borrowers are individuals instead of countries, Chatterjee et al. (2008) has a signaling mechanism similar to ours. However their paper has finite horizon, and more importantly, assumes that the defaulter is excluded from savings and insurance in the default period, and hence their paper does not address the challenge of debt sustainability posed by Bulow and Rogoff (1989).

In explaining the coexistence of borrowing and saving, a related paper to ours is Alfaro and Kanczuk (2009): they argue that countries need both debt and savings to smooth incomes in an incomplete financial markets. However, debt repayment is not sustainable in Alfaro and Kanczuk (2009) without invoking an exogenous cost of default. In general, most models that explain the simultaneous-saving-and-borrowing

[1] Readers who are interested in non-informational explanations for sovereign debt repayment can see Mitchener and Weidenmier (2010) or Panizza et al. (2009) for a survey of trade sanctions and military sanctions against defaulting countries. In a different direction, some recent works (Kletzer and Wright (2000), Kehoe and Perri (2002), Hellwig and Lorenzoni (2009)) show that debt could exist between agents that equally lack commitment.

puzzle unfortunately cannot simultaneously explain the uncollaralized-debt-repayment puzzle.

There are two companion working papers. Phan (2012b) studies a similar environment with asymmetric information, but the country can send a costless signal to the foreign investors about its hidden states, and the investors find the optimal dynamic contract that gives the country incentives to tell the truth and to fully repay. Phan (2012a) applies private information in a competitive unsecured debt market to explaining the credit card debt puzzle.

On the theoretical front, this paper uses adverse selection to explains unsecured borrowing, and hence benefits from the theoretical foundations laid down by the literature on competitive insurance markets with adverse selection (Rothschild and Stiglitz (1976); Wilson (1977); Eichenbaum and Peled (1987)).

Finally, on the policy front, the paper relates to Borensztein and Mauro (2004) and Griffith-Jones and Sharma (2005), which propose an initiative led by an international organization (like the International Monetary Fund or the World Bank) to verify GDP data of developing countries, in order to create a GDP-indexed bond market. This verification program should be handled with care, however, since information can cause the debt market to break down as we have discussed earlier.

Plan of paper. Section 2 lays out the basic framework. Then the benchmark case of full information in section 3 recalls the Bulow and Rogoff result that debt is not sustainable. Section 4 shows that debt is sustainable if there is a hidden shock at $t = 1$. Section 5 extends the model to show that borrowing and saving happen at the same time in equilibrium. Section 6 generalizes the main result of debt sustainability for cases where there are hidden shocks in more than one periods. Finally section 7 discusses further extensions and concludes.

2. Environment

The dynamic game consists of a sovereign country[2] and a continuum of foreign investors.

The sovereign country. Consider an infinite horizon endowment economy with a single non-storable consumption good at each date $t \in \{0, 1, 2, \dots\}$. In each period, the endowment is a random variable y_t, whose realizations are publicly observable to the world. The country also experiences a shock δ_t to its discount factor of future utility.

[2]The game can be easily reformulated so that the single agent is replaced by a continuum of idiosyncratic agents. My working paper Phan (2012a) takes this approach.

Only the country observes the realization of this preference shock. Let $z_t = y_t \times \delta_t$ be the vector of period t shocks. Assume these vectors $\{z_t\}_{t=0}^{\infty}$ follow a first order Markov process, whose transitional probabilities are denoted by $\pi_t(z_t|z_{t-1}) = \pi_t(y_t, \delta_t|y_{t-1}, \delta_{t-1})$. The marginal transitional probability of y_t is denoted by $\pi_t(y_t|y_{t-1}, \delta_{t-1})$. For simplicity, assume in each period there are only two possible states for the discount factor shock, $\delta_t^h = 1$ or $\delta_t^l = \delta^l < 1$, and there are finitely many possible states for the endowment, $\{y_t^{(1)}, \ldots, y_t^{(s)}\}$, where $0 < y_t^{(1)} < \cdots < y_t^{(s)}$. Here we see how the preference shocks δ_ts affect the probability distributions of future observable shocks y_ts. We interpret this as a country with a more patient government is more likely to have better future incomes, since it is less prone to impulsive economic policies. More details on this serial correlation will come in section 4.

A shock history z^t is a sequence of realizations $\{z_0, z_1, \ldots, z_t\}$. A partial history z_t^{t+j} is a sequence of shock realizations between period t and period $t+j \geq t$, $\{z_t, z_{t+1}, \ldots, z_{t+j}\}$. A history node z^s is said to follow another node z^t, if there is a sequence of realizations $z_{t+1}, \ldots, z_s$ so that $z^s = (z^t, z_{t+1}, \ldots, z_s)$. In notation: $z^s \succeq z^t$. Similarly we define the ordering $y^s \succeq y^t$

The unconditional probability of z^t is denoted by $\pi(z^t)$. We assume $\pi(z^t) > 0$ for all $z^t \in \mathcal{Z}$. Let $\mathcal{Z}^t$ be the set of all possible endowment histories from time zero to time t, and let $\mathcal{Z}$ denote the set of all possible endowment histories (hence $\mathcal{Z} = \cup_{t\geq 0}\mathcal{Z}^t$). We define y^t, y_t^{t+j}, $\mathcal{Y}^t$, $\mathcal{Y}$, δ^t, δ_t^{t+j}, Δ^t and Δ in a similar way.

There is one representative agent in the country. Her preference over a consumption sequence $\{c(z^t)\}_{z^t\in\mathcal{Z}}$ is represented by the lifetime expected utility

$$U(\{c(z^t)\}_{z^t\in\mathcal{Z}}) = \sum_{z^t\in\mathcal{Z}} \beta^t \pi(z^t)\delta(z^t)u(c(z^t)),$$

where the aggregate discount factor shock for each history node z^t is $\delta(z^t) \equiv \delta_1\delta_2\cdots\delta_t$. Similarly define $\delta(z_t^{t+j}) \equiv \delta_t\delta_{t+1}\cdots\delta_{t+j}$. Assume that the period utility function $u(\cdot)$ is strictly increasing, strictly concave and twice continuously differentiable, and satisfies the standard Inada conditions.

Competitive foreign investors. The agent can share her income risks with a continuum $I = [0, 1]$ of infinitely-lived, risk-neutral foreign investors. Investors have access to a safe storage technology with a deterministic rate of return $R > 1$. They discount the future at rate $\frac{1}{R}$.

For convenience, assume all investors have "deep pockets", i.e. their endowment is sufficiently large so that they are never financially constrained in a transaction with the agent.

Each investor's objective is to maximize his expected life-time profit. If his profit in each period is $\gamma_t(z^t)$ (which can be negative or positive), then his expected life-time profit is

$$U^i(\{\gamma(z^t)\}_{z^t\in\mathcal{Z}}) = \sum_{z^t\in\mathcal{Z}} \frac{1}{R^t}\pi(z^t)\gamma(z^t).$$

The investors can fully observe the country's endowment shocks y_ts, but cannot observe the preference shocks δ_ts. Thus this is an economic environment with asymmetric information.

Market structure. The country wishes to obtain insurance against stochastic endowment fluctuations and smooth consumption inter-temporally by borrowing/saving with risk neutral foreign investors. We will characterize risk-sharing contracts that competitive investors offer to a country that cannot commit to honor these contracts. After the realization of shocks, but before consumption takes place, the country's representative agent is free to leave the current contract and start a new contract with another investor. We take the extreme position in Bulow and Rogoff (1989) that breaking a contract is painless, and that there is no legal international framework to extract any of the country's endowment in case of default. On the other hand, *an investor is fully committed to honor his contracts.* Hence this is an environment with one-sided commitment.

For each endowment history node y^t, a y_{t+1}-contingent short-term contract $\{d^i(y^t), b^i(y^{t+1})\}$ with an investor i specifies a payment of $d^i(y^t)$ from the investor to the country, and a promised payment of $b^i(y^{t+1})$ in period $t+1$ at node y^{t+1}. Note that since δ_{t+1}s are not observable, we do not have δ_{t+1}-contingent contracts.

A long-term contract with investor i that begins at y^t is simply a sequence of short-term contracts. Denote a long term contract by

$$B^i(y^t) = \{d^i(y^{t+j}), b^i(y^{t+j+1})\}_{y^{t+j}\succeq y^t}.$$

The continuation payoff of the country at node z^t under this contract is given by

$$U(B^i(y^t)|y^t,\delta^t) = \sum_{s\geq t}\beta^{s-t}\sum_{z_t^s}\pi(z_t^s|y_t,\delta_t)\delta(z_t^s)u(c(y_t^s)) \tag{1}$$

where each $c(y_t^s)$ comes from the following budget equation

$$c(y_t^s) = y_s - b^i(y^s) + d^i(y^s).$$

For compactness, throughout the paper we assume that $b(y^t)\in[b_{t,\min}, b_{t,\max}]$, where the bounds are deterministic, finite, and sufficiently generous that they do not bind in equilibrium. Furthermore, we rule out Ponzi schemes by simply assuming that the upper bounds satisfy [3]

$$\lim_{t\to\infty} \frac{b_{t,\max}}{R^t} \leq 0. \tag{2}$$

The country starts out with no asset, $b(y^0) \equiv 0$.

We now formulate a risk-sharing game with *one-sided commitment, perfect competition and asymmetric information.*

Timing of events in the game.

(1) At the begging of each date t, nature draws the country's new endowment shock y_t and preference shock δ_t . Everybody observes y_t, but only the country sees δ_t.
(2) If the country is currently in a contract that specifies an obligation to repay a positive amount of debt, it decides whether to default on the contract (repay nothing), or to stay (and repay fully). Investors then update their beliefs based on this default decision.
    - Notation: $x_t = 1$ if the country defaults, and $x_t = 0$ if the country repays. When there is no debt obligation, let $x_t \equiv 0$ by convention.
(3) Each investor $j \in I$ offers a new long-term contract $\{d^j(y^{t+j}), b^j(y^{t+j+1})\}_{y^{t+j} \succeq y^t}$ to the country.
(4) If the country has defaulted, then it decides which new long-term contract to choose. If the country has not defaulted, then it decides whether to stay with the same contract, or choose a new contract.

Public history and private history. A *public* history encodes all the information that investors has before they offer new contracts at t. Formally, it is simply a collection of past observable shocks, aggregate distributions of past contract offers, and past defaults:

$$\hat{H}^t \equiv y^t \times \{B_1, \ldots, B_t\} \times \{x_1, \ldots, x_t\}.$$

A *private* history encodes all the information that the country uses to decide whether to repay in period t. Formally:

$$H^t \equiv \delta^t \times y^t \times \{B_1, \ldots, B_t\} \times \{x_1, \ldots, x_{t-1}\}.$$

Notice there are two differences between H^t and $\hat{H}^t$. First, H^t includes δ^t while $\hat{H}^t$ does not. This reflects the fact that hidden shocks are observe only to the country.

[3] In an environment of symmetric information, Hellwig and Lorenzoni (2009) shows that the borrower can indefinitely roll over her debt in a Ponzi scheme. Our paper shows that debt is possible without resorting to Ponzi schemes.

Second, H^t does not include x_t (as the country uses H^t to decide x_t), while $\hat{H}^t$ does (as investors update their beliefs based on observation of x_t).

System of beliefs. Not being able to observe δ^t, each investor forms his own beliefs over these hidden shocks. A system of beliefs μ is a specification of a probability distribution for each public history $\hat{H}^t$, i.e. $\mu(\delta^t|\hat{H}^t) \geq 0$ for all $\delta^t \in \Delta^t$, and $\sum_{\delta^t \in \Delta^t} \mu(\delta^t|\hat{H}^t) = 1$. To conserve on notation, we also let $\mu(\cdot|\hat{H}^t)$ denote the probability distribution of δ_t (instead of the whole history $\delta^t = \{\delta_0, \ldots, \delta_t\}$) given $\hat{H}^t$, where $\mu(\delta^h|H^t) \geq 0$ and $\mu(\delta^l|\hat{H}^t) \geq 0$, and $\mu(\delta^h|\hat{H}^t) + \mu(\delta^l|\hat{H}^t) = 1$. Based on μ, an investor can form a probability distribution of future endowment shocks y_{t+j}, $j \geq 0$. Using the same notation μ, let us define

$$\mu(y_{t+1}|\hat{H}^t) \equiv \mu(\delta^h|\hat{H}^t)\pi_t(y_{t+1}|\delta^h, y_t) + \mu(\delta^l|\hat{H}^t)\pi_t(y_{t+1}|\delta^l, y_t).$$

Equilibrium. The equilibrium concept we use is perfect Bayesian Nash equilibrium. [4]

A perfect Bayesian Nash equilibrium is a collection of history-contingent contract offers $B^{i*}(\hat{H}^t)$ for each investor $i \in I$, default strategy $x^*(H^t)$, and a belief system μ^* such that:

- At each public history $\hat{H}^t$, taking as given the country's default strategy σ^x and other investor's strategies, each investor chooses a long-term contract offer $B^i(y^t)$ that maximize his expected profit

$$(3) \qquad U^i(B^i(y^t)|y^t, \mu^*, x^*) = E\left\{\sum_{j\geq 0} \frac{1}{R^t}\left(-d_{t+j} + \frac{1}{R}b_{t+j+1}(1-\delta_{t+j+1})\right)|y^t, \mu, x^*\right\}.$$

- At each private history H^t, the country chooses whether to quit a current contract and which contract to move to, in order to maximize its expected utility.
- The belief system μ^* is derived from the strategy profile σ through Bayes rule whenever possible. That is for any public history $\hat{H}^t$ that is reached with a positive probability under the strategy profile σ (i.e. $\Pr(\hat{H}^t|\sigma) > 0$), we must have

$$\mu^*(\delta^t|\hat{H}^t) = \frac{\Pr(\delta^t|\sigma)}{\Pr(\hat{H}^t|\sigma)}, \ \forall \omega^t \in \Omega.$$

This paper focuses only on symmetric strategies, i.e. strategies that specify the same action for all investors at any public history $\hat{h}^t$.

The next section revisits Bulow and Rogoff (1989)'s controversial debt impossibility result.

[4] Some papers, including Chari and Kehoe (1990, 1993); CHARI et al. (1998), prefer to use the phrase sustainable equilibrium instead.

3. Full Information Benchmark

In this section suppose $\delta_t = 1$ for all $t \geq 0$, so there is no private shock on the discount factor, and $z^t y^t$. The country always discounts future utility at a constant rate β. As a consequence there is no private information, and hence no Bayesian updating of investor beliefs. The concept of perfect Bayesian Nash equilibrium simply reduces to the concept of sub-game perfect Nash equilibrium.

Non-existence of debt. The following is the classic result from Bulow and Rogoff (1989). It states that sovereign debt cannot be sustainable.

Theorem. *Any contract in which debt is positive, i.e.* $b_t > 0$ *for some period* $t \geq 0$*, always generates a negative expected profit for an investor. Thus in equilibrium, the country cannot borrow, i.e.* $b_t^* \leq 0$ *on the equilibrium path.*

Proof. Appendix. □

Equivalence result. The following results will be useful for section 4.

Proposition 1. *The equilibrium consumption profile* $C^* = \{c_t^*(y^t)\}_{y^t \in \mathcal{Y}}$ *is the unique solution to the following consumption smoothing problem with no borrowing:*

$$\max_{\{b_t(y^t)\}_{y^t \in \mathcal{Y}}} \sum_{y^t \in \mathcal{Y}} \beta^t \pi(y^t) u(c_t(y^t)) \tag{4}$$

subject to budget constraint

$$c_t(y^t) = y_t - b_t(y^t) + \frac{1}{R} \sum_{y_{t+1}} \pi(y_{t+1}|y_t) b_{t+1}(y^{t+1})$$

and no borrowing constraint

$$b_t(y^t) \leq 0, \ \forall y^t \in \mathcal{Y}.$$

Proof. Appendix. □

Corollary 1. [Recursive reformulation] *The equilibrium payoff for the country at* $t = 0$ *is* $V_0(0, y_0)$*, where* $V_t(b_t, y_t), t \geq 0$ *solves the following recursive consumption insurance problem with no borrowing:*

$$V_t(b, y_t) = \max_{b'(y_{t+1})} u\left(y_t - b + \frac{1}{R} E_t[b'(y_{t+1})|y_t]\right) + \beta E_t[V_{t+1}(b'(y_{t+1}), y_{t+1})|y_t] \tag{5}$$

$$\text{subject to } b'(y_{t+1}^{(j)}) \leq 0, \ \forall y_{t+1}^{(j)} \in \{y_{t+1}^{(1)}, \ldots, y_{t+1}^{(s)}\}.$$

Proof. Appendix. ☐

In the next section we will show that debt repayment is sustainable when there is one period with private information.

4. Private Information in One Period

This section assumes there is private information only in period $t = 1$. There is no private shock in $t = 0$ ($\delta_0 = 1$) and in $t \geq 2$ ($\delta_t = 1$). But in $t = 1$, δ_1 is a random variable that takes value $\delta^h = 1$ with probability $\pi^h \in (0, 1)$, and takes value $\delta^l < 1$ with probability $\pi^l = 1 - \pi^h$.

Assumption: The probability distribution $\pi_2(y_2|y_1^{(k)}, \delta^h)$ strictly first order stochastic dominates $\pi_2(y_2|y_1^{(k)}, \delta_1^l)$ for any realization $y_1^{(k)} \in \{y_1^{(1)}, \ldots, y_1^{(s)}\}$.

Hence the private preference shock is positively correlated with future endowment shocks.

For simplicity, assume $y_0 = y_0^{(1)}$ and $y_1 = y_1^{(1)}$ with probability one, so there is no uncertainty at $t = 0$. Results for the case y_0 and y_1 being random variables will be provided in the appendix. To conserve on notation, we write $\pi^h(y_2)$ instead of $\pi_2(y_2|y_1, \delta_1^h)$, and $\pi^l(y_2)$ instead of $\pi_2(y_2|y_1, \delta_1^l)$. Similarly, we write $E_1^h[\cdot]$ instead of $E[\cdot|y_1, \delta_1^h]$, and $E_1^l[\cdot]$ instead of $E[\cdot|y_1, \delta_1^l]$. For convenience, we will call a country that has experienced a low discount factor shock δ^l a low type, and a country that has experienced a high shock δ^h a high type.

We now solve for an equilibrium using backward induction.

4.1. Periods $t \geq 2$.

Information is symmetric in periods $t \geq 2$, hence Bulow and Rogoff's arguments give us the following result:

Proposition 2. *Sovereign debt cannot be sustainable in periods after information has becomes symmetric. In other words, any contract in which debt b_t is positive in some period $t > 1$ always generates a negative expected profit for an investor.*

Proof. The is a direct corollary of Bulow and Rogoff's result. See appendix for details. ☐

Corollary 2. *The continuation payoff for the country at $t = 2$ is summarized by $V_2(b_2, y_2)$.*

4.2. **Period** $t = 1$**.** The country begins period $t = 1$ with endowment y_1, hidden preference shock δ_1, and inherits a level of debt b from period $t = 0$. The country also inherits a "credit rating" in the form of investor's belief $\mu(\delta^h|\hat{H}^1)$ on how likely the country is the high type.

We will find the optimal contracts, taking investor's belief as given. Then when we solve back to $t = 0$, we will verify that beliefs are consistent with Bayes law on the equilibrium path.

Competitive insurance with $\mu(h|\hat{H}^1) = \mu^h \equiv 1$. Suppose investors belief that the country is the high type with probability one. Given this belief, competitive investors will offer long-term contracts in which the short-term contract $\{d^{*h}(b), b^{*h}(b, y_2)\}$ between period $t = 1$ and $t = 2$ solves the following utility maximization problem for the country:

$$
\begin{aligned}
V_1^h(b, \mu^h) \quad \equiv \quad & \max_{d', b'(y_2)} u\left(y_1 - b + d'\right) + \beta E^h[V_2(b'(y_2), y_2)]. \\
& \text{subject to} \\
& d' = \frac{1}{R} E^h[b'(y_2)] \\
& b'(y_2^{(j)}) \leq 0, \ \forall y_2^{(j)} \in \{y_{t+1}^{(1)}, \ldots, y_{t+1}^{(s)}\}
\end{aligned} \tag{6}
$$

$V_1^h(b, \mu^h)$ is the continuation payoff for the high type if it repays b at $t = 1$ and gets the credit rating associated with μ^h. The last no borrowing constraint comes from proposition 2, and the future continuation payoff function V_2 comes from corollary 2.

Lemma 1. *For each b and y_1, $V_1^h(b, \mu^g)$ is well-defined, and the maximization problem (6) has a unique solution $\{d^{*h}(b), b^{*h}(b, y_2)\}$. Furthermore, the value function is strictly decreasing in b: $\frac{\partial}{\partial b} V_1^h(b, \mu^g) < 0$.*

Proof. Existence is guaranteed by continuity of the maximand and compactness of the domain. Uniqueness comes from strict concavity of u and concavity of V_2^{nb}. By the envelope theorem

$$
\frac{\partial}{\partial b} V_1^h(b, \mu^h) = -u'(y_1 - b + d^{*h}(b)) < 0.
$$

□

Competitive insurance with $\mu(h|\hat{H}^1) = \mu^l \equiv 0$. Suppose investors belief that the country is the good type with probability zero, and thus the country is the bad type with probability one. Given this belief, investors will offer long-term contracts in which the short-term contract $\{d^{*l}, b^{*l}(y_2)\}$ between period $t = 1$ and $t = 2$ solves

$$V_1^l(0,\mu^l) \quad = \quad \max_{d', b'(y_2)} u\left(y_1 - 0 + d'\right) + \beta E^l[V_2(b'(y_2), y_2)]. \tag{7}$$

$$\text{subject to}$$

$$d' = \frac{1}{R} E^l[b'(y_2)]$$

$$b'(y_2^{(j)}) \le 0, \ \forall y_2^{(j)} \in \{y_2^{(1)}, \ldots, y_2^{(s)}\}.$$

$V_1^l(0,\mu^l)$ is the continuation payoff for the low type for defaulting at $t = 1$ and gets the credit rating associated with μ^l. Like before, the last no borrowing constraint comes from proposition 2, and the future continuation payoff function V_2 comes from corollary 2.

Lemma 2. *For each y_1, $V_1^l(0,\mu^l)$ is well-defined, and the maximization problem (7) has a unique solution $\{d^{*l}, b^{*l}(y_2)\}$.*

Proof. Similar to proof of lemma 1. □

Endogenous debt limit. Define

$$V_1^h(0,\mu^l) \equiv u(y_1 - 0 + d^{*l}) + \beta E^h[V_2(b^{*l}(y_2), y_2)].$$

This is the payoff for the high type for defaulting and thus receiving the "low contract" $\{d^{*l}, b^{*l}(y_2)\}$ associated with low belief μ^l. Similarly define

$$V_1^l(b,\mu^h) \equiv u(y_1 - b + d^{*h}(b)) + \beta E^l[V_2(b^{*h}(b, y_2), y_2)].$$

This is the payoff for the low type for repaying b and thus receiving the "high contract" $\{d^{*h}(b), b^{*h}(b, y_2)\}$ associated with high belief μ^h.

Proposition 3.

(1) There exists a unique $\bar{b} > 0$ such that given beliefs μ^h, μ^l, the high type is indifferent between repaying or defaulting on $\bar{b}$:

$$V_1^h(\bar{b},\mu^h) = V_1^h(0,\mu^l). \tag{8}$$

(a) There exists a unique $\underline{b} > 0$ such that given beliefs μ^h, μ^l, the low type is indifferent between repaying or defaulting on $\underline{b}$:

$$V_1^l(\underline{b},\mu^h) = V_1^l(0,\mu^l).$$

Proof. Part 1. First we will show that $V_1^h(0,\mu^h) > V_1^h(0,\mu^l)$.

By definition of $V_1^h(0,\mu^h)$ as the value of the maximization problem (6), it follows that

$$\begin{aligned} & V_1^h(0,\mu^h) \\ (9) \qquad & \geq u(y_1 - 0 + \frac{1}{R}E^h[b^{*l}(y_2)]) + \beta E^h[V_2(b^{*l}(y_2), y_2)]. \end{aligned}$$

Since $b^{*l}(y_2)$ is the solution to problem (7), b^{*l} must be counter-cyclical, i.e. $b^{*l}(y_2^{(j)}) < b^{*l}(y_2^{(k)})$ for any $j > k$ (otherwise b^{*l} provides no insurance value against y_2 shock). So $b^{*l}(\cdot)$ is a strictly decreasing function in y_2. Since $\pi^h(y_2)$ first order stochastic dominates $\pi^l(y_2)$, it follows that

$$(10) \qquad E^h[b^{*l}(y_2)] < E^l[b^{*l}(y_2)] = d^{*l}.$$

Combining (9) and (10):

$$\begin{array}{lll} & V_1^h(0,y_1) & \\ \geq & u(y_1 - 0 + \frac{1}{R}E^h[b^{*l}(y_2)]) & +\beta E^h[V_2(b^{*l}(y_2), y_2)] \\ > & u(y_1 - 0 + d^{*l}(y_1)) & +\beta E^h[V_2(b^{*l}(y_2), y_2)] \\ = & V_1^h(0,\mu^l). & \end{array}$$

Second, since $V_1^h(0,\mu^h)$ is continuous and strictly decreasing in b, there must exists a unique $\overline{b} > 0$ such that (8) holds. It is immediate then that the high type strictly prefers to repay any $b < \overline{b}$, and strictly prefers to default on any $b > \overline{b}$.

Part 2 is similar. Its proof is provided in the appendix. □

Lemma 3. *Define the difference in future continuation payoffs between the "low contract" and the "high contract" associated with $\overline{b}$ by*

$$\Delta_2(\overline{b}, y_2) \equiv V_2(b^{*h}(\overline{b}, y_2), y_2) - V_2(b^{*l}(y_2), y_2).$$

If the low discount shock $\delta^l = \delta$ is sufficiently low so that

$$(11) \qquad \delta E^l[\Delta_2(\overline{b}, y_2)] < E^h[\Delta_2(\overline{b}, y_2)],$$

then given beliefs μ^h and μ^l, the low type strictly prefers to default on $\overline{b}$:

$$(12) \qquad V_1^l(\overline{b}, \mu^h) < V_1^l(0, \mu^l).$$

Proof. The differences between the sides of (12) is

$$\begin{aligned}
& V_1^l(\bar{b},\mu^h) - V_1^l(0,\mu^l) \\
= & \quad u(y_1 - \bar{b} + d^{*h}(\bar{b})) \quad + \beta\delta E^l[V_2(b^{*h}(\bar{b},y_2),y_2)] \\
& \quad -u(y_1 + d^{*l}) \quad - \beta\delta E^l[V_2(b^{*l}(\bar{b},y_2),y_2)] \\
= & \quad [u(y_1 - \bar{b} + d^{*h}(\bar{b})) - u(y_1 + d^{*l})] \quad + \beta\delta E^l[\Delta_2(\bar{b},y_2)].
\end{aligned}$$

Combining this with assumption (11) yields:

$$\begin{aligned}
& V_1^l(\bar{b},\mu^h) - V_1^l(0,\mu^l) \\
< & \quad [u(y_1 - \bar{b} + d^{*h}(\bar{b})) - u(y_1 + d^{*l})] \quad + E^h[\Delta_2(\bar{b},y_2)].
\end{aligned}$$

But

$$\begin{aligned}
& [u(y_1 - \bar{b} + d^{*h}(\bar{b})) - u(y_1 + d^{*l})] \quad + E^h[\Delta_2(\bar{b},y_2)] \\
= & \quad V_1^h(\bar{b},\mu^h) - V_1^h(0,\mu^l) \\
= & \quad 0.
\end{aligned}$$

Hence

$$V_1^l(\bar{b},\mu^h) - V_1^l(0,\mu^l) < 0.$$

$\square$

Corollary 3. $\bar{b} > \underline{b}$.

For the rest of the paper we will impose the assumption in lemma 3.

Assumption. Low discount factor show δ is sufficient low so that (11) holds.

Competitive insurance with $\mu(h|\hat{H}^1) = \mu \in (0,1)$. This is the classic competitive insurance problem with adverse selection (Rothschild and Stiglitz (1976); Wilson (1977); Eichenbaum and Peled (1987)).

Investor's offers will provide a menu of short-term contracts $\{d'^h, b'^h(y_2)\}, \{d'^l, b'^l(y_2)\}$ between $t = 1$ and $t = 2$ that solves

$$(13) \qquad \max_{\{d'^h, b'^h(y_2)\},\{d'^l, b'^l(y_2)\}} \mu \cdot \left\{ u(y_1 - b + d'^h) + \beta E^h[V_2^{nb}(b'^h(y_2), y_2)].\right\} \\ + (1-\mu) \cdot \left\{ u(y_1 - b + d'^l) + \beta\delta E^l[V_2^{nb}(b'^l(y_2), y_2)]\right\}$$

subject to no borrowing constraint

$$b'^h(y_2^{(j)}), b'^l(y_2^{(j)}) \leq 0, \ \forall y_2^{(j)} \in \{y_{t+1}^{(1)}, \dots, y_{t+1}^{(s)}\}$$

and break-even condition

$$r \cdot (\mu d'^h + (1-\mu) d'^l) \leq \mu \frac{1}{R} E^h[b'^h(y_2)] + (1-\mu)\frac{1}{R} E^l[b'^l(y_2)], \tag{14}$$

and subject to two additional incentive compatibility constraints

$$\begin{aligned} & u(y_1 - b + d'^h) + \beta E^h[V_2^{nb}(b'^h(y_2), y_2)] \\ \geq & u(y_1 - b + d'^l) + \beta\delta E^l[V_2^{nb}(b'^l(y_2), y_2)], \end{aligned} \tag{15}$$

$$\begin{aligned} & u(y_1 - b + d'^l) + \beta\delta E^l[V_2^{nb}(b'^l(y_2), y_2)] \\ \geq & u(y_1 - b + d'^h) + \beta E^l[V_2^{nb}(b'^h(y_2), y_2)]. \end{aligned} \tag{16}$$

The appendix shows the necessary and sufficient condition for the existence of a pure strategy Nash equilibrium for this insurance game, based on Eichenbaum and Peled (1987). It also shows that when this condition fails, a mixed strategy equilibrium always exists, based on Rosenthal and Weiss (1984).

4.3. **Period $t = 0$ and separating equilibrium.** We are now ready to establish the major theorem of the paper:

Theorem 1.

(1) The largest possible debt level in any equilibrium is $\bar{b}$.
   (a) Suppose there is sufficient need for borrowing at $t = 0$ so that the endogenous debt limit is binding; formally:

$$u'\left(y_0 + \frac{1}{R}\pi^h \bar{b}\right) \geq \beta R \frac{\partial}{\partial b} V_1^h(\bar{b}, \mu^h), \tag{17}$$

   then there is a separating equilibrium[5] in which debt in $t = 1$ is positive and equal to the debt limit $\bar{b}$. The system of beliefs specifies:

$$\mu(h|\hat{H}^1) = \begin{cases} 1 & \text{if } x_1 = 0, b_1 \geq \underline{b} \\ \pi^h & \text{if } x_1 = 0, b_1 < \underline{b} \\ 0 & \text{if } x_1 = 1. \end{cases}$$

Proof. Appendix. Basically (17) guarantees that the country wants to borrow a sufficient amount at $t = 0$ that the borrowing constraints $\bar{b}$ will always bind. At this debt

[5]An equilibrium is separating if on the equilibrium path, the high type repays debt and the low type defaults on debt in period $t = 1$.

limit, the high type wants to repay and the low types wants to default. Hence the separating beliefs are consistent with Bayesian updating. □

5. Simultaneous Borrowing and Savings

This section will extend the model in section 4 to show that borrowing and saving occur at the same time in equilibrium.

Suppose in period $t = 0$, beside a financial contract with investors, the country can save at the risk free rate R as well. So the timing of events is the following: in period $t = 0$, investors offer contracts to the country, who then chooses how much to save at the risk free rate. Everybody observes how much the country saves. The events in periods $t \geq 1$ are the same as before.

If information is perfect as in section 3, then there is no reason for the country to save at the safe rate: investors offer the country access to a set of fully state contingent contracts, which dominates safe storage in smoothing consumption volatility for the country. However, if information is imperfect as in section 4, then the situation is very different. The following result shows that borrowing and saving will occur at the same time on the equilibrium path:

Theorem 2. *Suppose information is imperfect in period $t = 1$ as in section 4. Then there is a range of parameters:*

$$\beta R \frac{\partial}{\partial b} V_1^h(\bar{b}, \mu^h) \leq u'(y_0 + \frac{\pi^h}{R}\bar{b}) < \beta R \frac{\partial}{\partial b} V_1^l(0, \mu^l).$$

so that in an equilibrium, the country simultaneously borrows from investors and saves at the safe rate R.

Proof. Appendix. Basically the optimal contract and saving at $t = 0$ solve the following problem:

$$\begin{aligned} \max_{a,d,b} u & \left(y_0 + d - \frac{1}{R} a \right) \\ & + \pi^h \beta V_1^h(b - a, \mu^h) \\ & + \pi^l \beta \delta V_1^l(-a, \mu^l) \\ \text{subject to } d & \leq \frac{\pi^h}{R} b \\ b & \leq \bar{b} \\ a & \geq 0. \end{aligned}$$

□

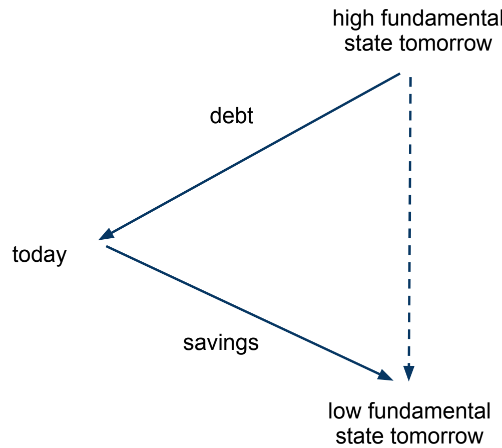

FIGURE 2. Intra-temporal transfer in incomplete markets is achieved by combination of borrowing and savings.

The intuition is illustrated in the following diagram:

The country borrows and then repays in high state in order to transfer wealth from the high state in period $t = 1$ to period $t = 0$ (the top arrow in the diagram). It also saves at safe rate R to transfer wealth from $t = 0$ to the state in period $t = 1$ (the bottom arrow). Thus by borrowing and saving at the same time at $t = 0$ and then default in the low state in $t = 1$, she can transfer some wealth from the high state to the low state, and hence reduce the volatility of her consumption in $t = 1$. (This is a form of "ex-post state-contingency", as the agent's financial wealth is different in different states not by the face value of the contracts, but by different actions in different states in the period the contracts mature.)

6. Private Information in Many Periods

This subsection establishes debt sustainability in more general scenarios of information asymmetry.

Let $\mathcal{P}$ be the set of periods in which there are hidden shocks: $\mathcal{P} \equiv \{t : \operatorname{Var}_{t-1}(\delta_t) > 0\}$. Let $T \equiv \sup \mathcal{P}$. Throughout this section we assume that $\mathcal{P} \neq \emptyset$, so T is well-defined (and it could be infinity). Examples:

(1) $\mathcal{P} = \{0\}$, $T = 1$. This is the case studied in the previous subsection.
(2) $\mathcal{P} = \{t\}$, $T = t$. This is when y_t is private and y_{t+1} is public, then $\{y_{t+j}\}_{j \geq 2}$ is either all public or all private information.

(3) $\mathcal{P} = \{0, n, 2n, \dots\}$ for some integer $n > 1$, $T = \infty$. This illustrates a scenario in which the income of the country experiences hidden shocks in every n periods (say, an election or a new economy policy occurs every n years).

This section will be explicit about endowment growth. Let $y_t^{(i)} = g^t y_0^{(i)}$, where $g > 0$ is a deterministic growth rate. Let ω_ts be independently and identically distributed with discrete weights π^h, π^l over $\{\delta^h, \delta^l\}$. De-trended shocks $\overline{y}_t = \frac{y_t}{g^t}$ follow a stationary Markov process with transition probability $\pi(\overline{y}'|\overline{y}, \omega)$.

Theorem 3. *There exists $\overline{g} > 0$ such that whenever $g \geq \overline{g}$, there is an equilibrium in which repayments of positive debt obligations occur in high endowment states on the equilibrium path from $t = 0$ to $t = T$.*

Proof. Appendix. Intuitively, if T is finite then the mechanism in section 4 that sustains debt: the agent has an incentive to repay debt in period T to signal that $\omega_T = \omega^h$, and thus gets a cheaper insurance premium against the y_{T+1} shock. Hence some amount of debt repayment can be sustainable in period T. Interestingly, this gives the agent an incentive to repay debt in periods $t < T$. The reason is simple: if the country defaults in $t < T$ then it will lose the ability to borrow in period $T - 1$. Thus in every period $t < T$ the agent will weigh the gain and loss from defaulting.

The proof by contradiction for $T = \infty$ is slightly more delicate, and will be provided in the appendix. □

7. Conclusion

This paper shows the role of asymmetric information in the sustainability of unsecured debt. If information is perfect then debt is not possible. If information is imperfect in every period, and simultaneous borrowing and saving is disallowed, then debt is also not possible. If information imperfection is somewhere in between (as formalized in theorem 3) then debt is sustainable. Alternatively, if information is imperfect in every period, but the agent can borrow and save at the same time then debt is also sustainable. Finally, the paper shows that in a financial market with only non-contingent borrowing and saving instruments, the agent would like to borrow and save at the same time, and default in bad income states, to achieve partial risk sharing.

The paper draws interesting implications. In the context of a competitive unsecured debt and insurance market with free entry, more information can make the default option more attractive as the insurance market for a defaulter is improved, and thus more information can shrink the debt market, and if there is sufficient average growth in the borrower's income, then more information can reduce her welfare. This has a policy implication: if the IMF or the World Bank is to implement a GDP data verification program in developing countries, in order to facilitate the creation of a GDP-indexed bond market, then they should take into account the adverse role of information. If GDP data remains verified after a country defaults, then the country can enter GDP-contingent insurance contracts, which are precisely the contracts that make the punishment for default impotent, as pointed out by Bulow and Rogoff (1989). On the other hand, if the GDP is verified only if the country has not defaulted, then debt becomes more sustainable, and the country's welfare is improved. Furthermore, facilitating a country's ability to save (via reserve accumulation or sovereign wealth funds) can increase its ability to issue sovereign bonds.

Appendix

Will be available online on the author's website: `www.toanphan.org` .

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