Some New Results on Impulse Balance in First-Price Auctions

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Outline

1. Background
2. Regret-based models of behaviour in auctions
3. Details on Impulse Balance Equilibrium
4. New Results in the Symmetric Case
5. Deep Thoughts on Behavioural Solution Concepts
6. The Model Has Some Bite: Results in Asymmetric Auctions

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The puzzle of aggressive bidding

- This talk is about the puzzle of modeling how subjects formulate bids in first-price auctions in the laboratory.
- The setting:
  - $N \geq 2$ bidders.
  - One indivisible object for sale.
  - Each bidder i has a private value v_i for the object.
  - Values are private information, but drawn from commonly-known distributions.
  - Bidders simultaneously submit bids; highest bid wins, and winning bidder earns the difference between his private value and his bid.

The puzzle of aggressive bidding

Stylised fact 1

In the case where values are distributed uniformly over a common interval, bidding is substantially more aggressive than the risk-neutral Nash equilibrium.

- The earnings consequences of this are often substantial.
- For example, in Turocy, Watson, and Battalio (*Experimental Economics* 2007), we conducted 60 periods with 3 bidders each.
- We found that on average, bidders earned around \$12 - but would have earned around \$20 if they unilaterally had followed the risk-neutral (expected earnings) maximising bidding strategy.

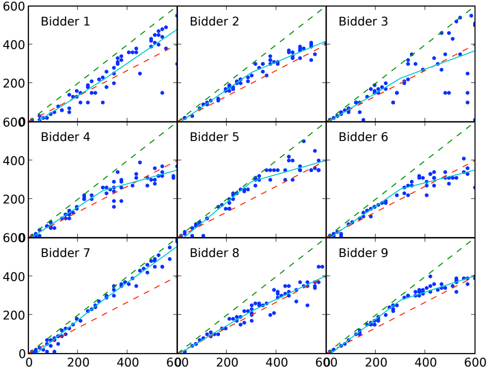
The puzzle of aggressive bidding

Stylised fact 2

- In the case where values are distributed uniformly over a common interval, most bidders' bid functions are close to linear.
- Where bidding is not approximately linear, it is usually because the bid function become concave (flatter) for private values close to the top of the interval of values.

- The literature makes a bigger deal out of the first point and tends to minimise the second one.
- However, in TWB (2007) and a followup paper Turocy and Watson (2011, under review at *SEJ*), we find some bidders exhibit noticeable concavity/flatness.

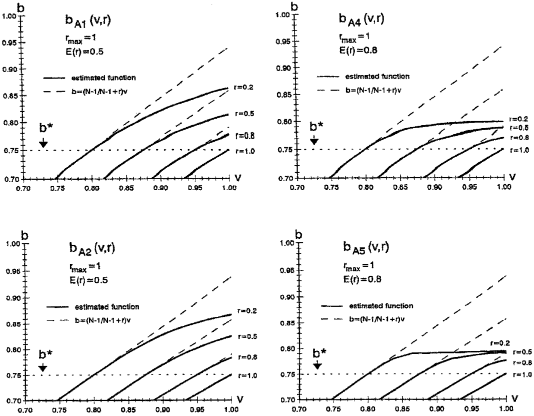
The puzzle of aggressive bidding



Risk attitudes as an explanation

- The risk-neutral Nash equilibrium of the auction predicts linear bidding, but at a less aggressive rate.
- If bidders are assumed to have CRRA utility functions over per-auction earnings, then equilibrium bid functions are (Cox, Smith, Walker 1988; van Boening, Rassenti, Smith 1994).
    - Linear for all values up to some threshold value $v^\star$, which depends on the distribution of CRRA parameters;
    - Generally concave above $v^\star$, with the degree of concavity depending on how risk averse the bidder in question is compared to the population of bidders as a whole.

Concave portions of CRRA bid functions



Weaknesses of risk aversion

- Risk aversion is not a particularly satisfying explanation for the behaviour we observe.
- The CRRA model assumes bidders are risk averse over income in each auction period, and the implied risk aversion parameter estimates make them very risk averse over that income.
- But CRRA not only seems implausible, it has done poorly in organising behaviour in related environments:
  - The Turocy, Watson, Battalio (2007) paper was over 60 periods, and subjects knew this. (Other papers have used large numbers of repetitions are observed the same aggressive bidding as well.)
  - Isaac and Walker (2000) show that bidders who are implicitly "risk averse" because they bid aggressively in auctions make choices which imply risk-seeking behaviour in the Becker-DeGroot-Marschak procedure, and vice-versa.
  - Turocy and Watson (2011) manipulate the way payoffs are calculated in the first-price auction, and do not observe the comparative statics predicted by CRRA.
  - ... and many more.

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Episode IV: A new hope

- There have been a recent series of papers investigating **ex-post regret** as explaining aggressive bidding (Englebrecht-Wiggans and Katok *ET* 2007; Filiz-Ozbay and Ozbay *AER* 2007; Ockenfels and Selten *GEB* 2005).
- When the outcome of an auction is realised, there are basically three possibilities for a bidder:
  1. The bidder won the auction. Since this implies he strictly had the highest bid (except for the rare case of a tie), he experiences **winner regret**: ex-post he has learned he could have bid less aggressively, still won the auction, and earned more.
  2. The bidder lost the auction, but the winning bid was less than the bidder's value. Here, the bidder experiences **loser regret**: ex-post he had learned he has missed out on a potential chance to win the auction profitably, had he bid more aggressively.
  3. The bidder lost the auction, but the winning bid was more than the bidder's value. Here, the bidder experiences no regrets; there is nothing he could have done to have made more money in this auction, the way things played out.

Regret: A new hope

- If we suppose that loser regret is more salient than winner regret, then we would expect that bidders would bid more aggressively, so as to avoid loser regret.
- Filiz-Ozbay and Ozbay (2007) manipulate the feedback bidders receive to emphasize one outcome or the other, and find the comparative statics of bids is in the expected direction, with the condition emphasizing loser regret resulting in more aggressive bidding.
  - Others have observed feedback effects which are consistent with this result.
- Turocy and Watson (2011) manipulate the way outcomes are described and payoffs are calculated, and also find bidding to be significantly less aggressive in a treatment in which loser regret seems a priori likely to be less salient.

Impulse Balance Equilibrium

- Ockenfels and Selten's (2005) approach to regret is based in the spirit of learning models (such as Selten's earlier learning direction theory).
  - When a bidder experiences winner regret, OS say he will experience a **downward impulse**, i.e., he will be inclined to bid less in future.
  - When a bidder experiences loser regret, OS say he will experience an **upward impulse**, i.e., he will be inclined to be more in future.
- Learning direction theory predicts only directions; impulse balance augments this by assigning magnitudes to the impulses.
- Magnitudes are operationalised as being proportional to the ex-post maximum surplus the bidder could have received.
- An **impulse balance equilibrium** obtains when the expected upward impulse equals the expected downward impulse.
  - Note that this use of "equilibrium" is more like its use in physical systems; it does not imply any sort of optimisation or best response.

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Notation

- There are N bidders; bidders are generically indexed by i.
- Each bidder i has a private value v_i. Bidder i's distributed on an interval $[\bar{v}_i, \underline{v}_i]$ with distribution function F_i; bidder values are realised independently.
- Each bidder adopts a bid function $b_i(v_i)$.
- A key random variable for bidder i is the distribution of the largest bid submitted by the other bidders, $w_i = \max_{j \neq i} b_i(v_i)$. Given fixed set of bid functions $b_j(v_j)$ for bidders $j \neq i$, denote the distribution of w_i by $M_i(w_i)$.

Expected impulse

- Fix a value v and a bid b. A bidder will experience an upward impulse in the case when:
  1. He is outbid: $w > b$;
  2. He could have won the auction profitably: $v > w$.
- Remembering that w is a random variable from the perspective of the bidder, his expected upward impulse can be written

$$U(b; v) = \int_b^v (v - w) dM(w).$$

- Fix a value v and a bid b. A bidder will experience a downward impulse in any case in which he wins the auction at a price strictly above the second-highest bid, $b > w$.
- Remembering that w is a random variable from the perspective of the bidder, his expected downward impulse is

$$D(b; v) = \int_0^b (b - w) dM(w)$$

Impulse Balance Equilibrium

- A (weighted) impulse balance equilibrium occurs when

$$\begin{aligned} U(b;v) &= \lambda D(b;v) \\ \int_b^v (v-w)dM(w) &= \lambda \int_0^b (b-w)dM(w) \end{aligned}$$

- The scalar constant λ allows upward and downward impulse to be measured on different scales, i.e., allows surplus lost in the winner regret condition to have a different effect on average than surplus lost in the loser regret condition.
- When $\lambda < 1$, upward impulse is more salient; measured in terms of earnings, earnings lost by being outbid result in a larger impulse than the same amount of earnings lost by winning the auction at a bid strictly higher than the second-higest bid.

Impulse Balance Equilibrium: The Linear Case

- Ockenfels and Selten compute a closed-form solution for the impulse balance equilibrium in a special case:
  - The symmetric first-price auction with uniform values
  - Assuming bidders adopt linear bid functions, $b(v) = \alpha v$
  - And all bidders have a common impulse weight λ.
- The impulse balance condition is assumed to hold in expectation over all values:

$$\int_{\underline{v}}^{\bar{v}} \int_{b}^{v} (v - w) dM(w) dF(v) = \lambda \int_{\underline{v}}^{\bar{v}} \int_{0}^{b} (b - w) dM(w) dF(v)$$

- The impulse balance equilibrium slope is given by

$$\alpha = 1 + \frac{\lambda}{N(N-1)} - \sqrt{\left(1 + \frac{\lambda}{N(N-1)}\right)^2 - 1}.$$

- Based on their experiments and experiments previously reported by Isaac and Walker (1985), they estimate $\lambda \approx 0.34$.
  - Surplus lost by being outbid has roughly three times the weight of surplus lost by leaving money on the table when winning.

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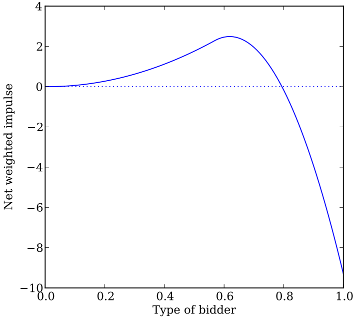
Impulse Balance Equilibrium: The Linear Case

- Why did Ockenfels and Selten restrict attention to the linear case?
  1. To a first approximation, bidding in experiments is roughly linear.
  2. Based on previous work by Selten and coauthors (using the "strategy method," for example), there is evidence that subjects do adjust their bids holistically.
  3. Impulse balance equilibrium is in the spirit of bounded rationality models, so it is appropriate to consider bid functions which have a limited number of free parameters.
- But also
  1. Restricting attention to the linear case allows them to exhibit have a closed-form solution, which is easier to explain and more palatable in theory outlets.

Impulse Balance Equilibrium: The Linear Case

- The condition used by OS states that expected net impulse is zero, where the expectation is taken across all possible realizations of v.
  - Just because something is zero on average, does not imply that it is close to zero most of the time, or that the deviations from zero are not systematic.
  - In fact, there is a systematic pattern in how expected impulse, conditional on v, deviates from zero:



Impulse Balance Equilibrium: The Linear Case

- Bidders with high realizations of v experience a net downward expected impulse which is systematic, and large in magnitude.
- These bidders are very likely to have the highest realized value, and therefore have a high probability of winning the auction.
  - They very rarely experience upward impulse.
  - They very often experience downward impulse, in some cases in large magnitudes.
- In experiments, these are the bidders who have the strongest financial incentives.
- We therefore ask: what happens if we suppose bidders are a bit more sophisticated, and do not adopt only linear bidding functions?

A Nonexistence Result

- Following the standard way (strategic) equilibrium is defined, we first suppose that the entire bid function $b(\cdot)$ is specified such that the zero net impulse condition holds for all values v.
- This leads to a disappointing result:

Theorem

*Consider a symmetric first-price auction with N bidders. There **does not exist** an impulse balance equilibrium in which the zero net impulse condition is required to hold for all values v.*

- The intuition for the result can be built up rather quickly.

A Nonexistence Result

Definition

Fix a distribution $M(w)$ of the maximum order statistics of other bidders' bids. Given a value v, a bid $b(v)$ is an **impulse reply** to M if it solves the zero net impulse condition for v.

- The terminology is intended to remind that this is the analog of a "best reply" in strategic equilibria.
- However, note that this is not a maximiser of some objective function, but rather simply the zero to an equation.

Impulse Responses are Monotonic

Theorem

Fix a distribution $M(w)$. Then, $b(v)$ is a nondecreasing function of v; that is, impulse responses are monotonic.

- The proof of the result is done by contradiction.
- It is based on the following observations:
  - The upward impulse function is nondecreasing in v and nonincreasing in b.
  - The downward impulse function is nondecreasing in b and is independent of v.

Impulse Balance Equilibria would be "Flat at the Top"

Theorem

In order for $b(v)$ to be an impulse balance equilibrium, there must be some $\varepsilon > 0$ such that $b(v)$ is constant on the interval $(\bar{v} - \varepsilon, \bar{v}]$.

- Suppose instead $b(v)$ is strictly increasing on all intervals for sufficiently small ε.
- Because we are considering a symmetric impulse balance equilibrium, and because $b(v)$ must be monotonic everywhere, it must be that a bidder of type $\bar{v}$ wins with probability one.
- Therefore, this bidder receives no upward impulses.
- Therefore, this bidder must never receive a downward impulse.
- Which is impossible, so we have a contradiction.

Impulse Balance Equilibria Just Don't Exist

Theorem

Consider a symmetric first-price auction with N bidders. There ***does not exist*** *an impulse balance equilibrium in which the zero net impulse condition is required to hold for all values v.*

- Suppose $b(v)$ is an impulse balance equilibrium.
- There exists some $\varepsilon > 0$ such that $b(v)$ is constant, $b(v) \equiv \beta$ on the interval $(\bar{v} - \varepsilon, \bar{v}]$.
- Pick any two values $v_1 \neq v_2$ on that interval.
- The expected net impulse impulse for type v_1 is

$$\frac{1}{2} P(tie)(v_1 - \beta) - \lambda \int_0^\beta (\beta - w)\, dM(w) = 0.$$

Similarly, the expected net impulse for type v_2 is

$$\frac{1}{2} P(tie)(v_2 - \beta) - \lambda \int_0^\beta (\beta - w)\, dM(w) = 0.$$

- Since the probability of a tie is positive, this cannot be.

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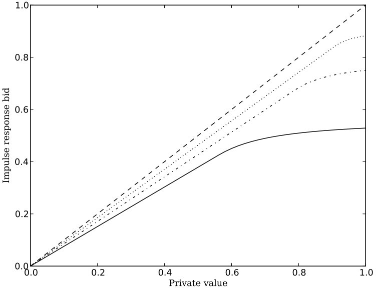
But this is being unfair

- Ordinarily, if a concept is shown to define an empty set, we think of it as being not a particularly interesting concept.
- However, in some ways, this theorem is being unfair on impulse balance.
- As a model of boundedly-rational decision-making, we would not expect a bidder to be formulating a bid more or less independently for each possible value, which is what we have demanded here.
- ***Food for thought:*** The way we ought to be defining and operationalising solution concepts based on behavioural principles, and the expectations we have on them in terms of the domain on which they are well-defined, is different from classical "fully rational" game theory.

Predictions of impulse replies

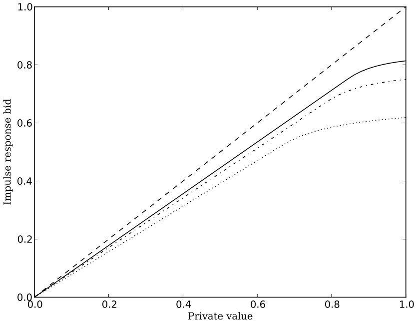
- Nevertheless, the work so far allows us to investigate what impulse replies look like when other bidders are assumed to bid according to some linear bid function $b(x) = \alpha x$.
- It is possible, with modest calculations, to write down equations characterising the impulse reply functions.
- Only in the special case of $N = 2$ is it possible to solve the equation explicitly for b.
- Nevertheless, we can plot impulse replies for selected parameter choices of λ and N.
- In the slides that follow, we plot impulse replies against the linear-restricted solution found by OS.

Impulse replies varying N



- Fixing $\lambda = 0.34$; $N = 2$ is solid line; $N = 4$ is dot-dash line; $N = 9$ is dotted line.
- Responses become more aggressive as N increases.
- For small N, a concave region occurs for large realisations of the private value.
- The reply has a non-differentiable point when v equals the maximum bid being submitted by the other bidders.

Impulse replies varying λ



- Fixing $N = 4$, let $\lambda = 0.17$ (solid line), $\lambda = 0.34$ (dot-dash line), $\lambda = 1.0$ (dotted line)
- As λ gets smaller, replies are more aggressive (as expected).
- Impulse replies exhibit concave regions for large realisations of the private value.

Impulse replies versus CRRA

- There is a qualitative similarity between these impulse reply functions and the CRRA equilibrium functions shown earlier.
- The intuitive reasons for the concave shapes have some relation, but are not identical:
  - CRRA equilibrium functions are concave near the top because the bidders ***bid*** is large enough that his bid beats some other bidder's bid with probability one.
  - Impulse replies are concave because the bidder's ***value*** is large enough that he will experience upward impulse with probability one.
- Perhaps the impulse reply model is simpler and more plausible, and therefore preferable.

A richer impulse balance model

- The technical reason for the failure of impulse balance equilibrium to exist when bids can be set independently type-by-type is that the bidder with the maximum possible value will never experience upward impulse. Everything unravels from there.
  - Another way to think about it: Take some initial monotonic bid function b^0. Let b^1 be the impulse reply to b^0, b^2 the impulse reply to b^1, and so on. Loosely, the sequence $\{b^k\}$ will be such that it converges downward to the constant function at zero. However, the limiting function does not satisfy the impulse balance condition.
  - So, it's an open set problem or discontinuity problem, whichever you prefer.
- But this is a knife-edge result. It comes because there exists some $w < \bar{v}$ such that $M(w) = 1$. If we simply make it so that $M(w) < 1$ for all $\bar{v}$ (or at least values sufficiently close to $\bar{v}$, then the unraveling does not occur.

A way forward

- We can instead think of computing impulse balance equilibria with a richer set of "types":
  - Some bidders choose a linear bid function according to the zero expected impulse condition.
  - Some bidders choose a piecewise linear bid function, with one, two, or more knots, with zero expected impulse at the knots.
  - Some bidders choose a type-by-type impulse reply.
- Possibly even one might consider adding
  - Some bidders dogmatically choose a linear bid function using some other rule-of-thumb heuristic - a kind of "level-0" bidder.
- This removes the well-definedness problem entirely, and qualitatively matches the variety of behaviours we observe in the data.
- Objective (for work actively in progress now): If one tries to estimate a more sophisticated model like this, do we still get the same estimates for λ that OS got looking only a linear bidding?

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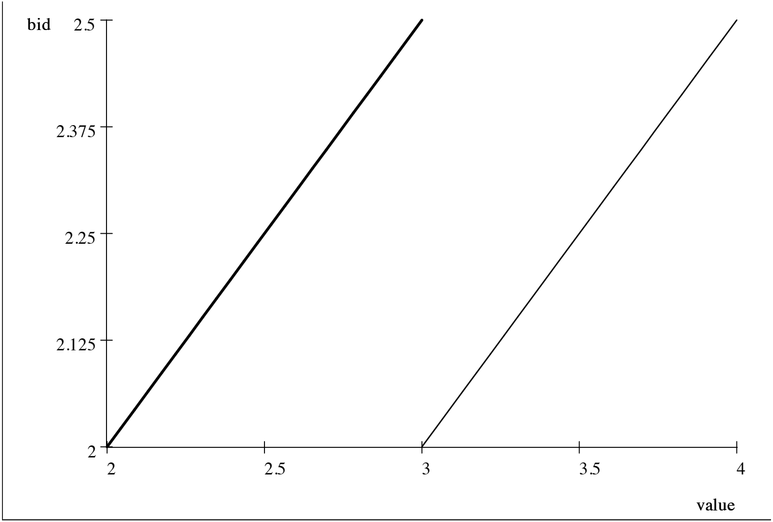
A second application: An asymmetric auction

- Recall that one of the reasons for distrusting CRRA as an "explanation" for aggressive bidding is that it does a poor job of predicting behaviour across institutions we might consider to be "similar."
- The impulse balance results so far appear to be promising, but there is no result so far which offers a qualitative prediction which is sharply different than best-response with some suitably chosen utility function.
- We also have the problem that there is a free parameter λ in the impulse balance model, which we can choose based on the data. While the general shape predictions of impulse balance are roughly independent of λ, phenomena like very aggressive bidding do require λ to be in particular ranges.
- To make impulse balance a more compelling explanation, we need some sense that it offers predictions which are robust to different designs.

An interesting asymmetric auction

- Kaplan and Zamir (2008) have considered the case of two-bidder first price private-values auctions, which are asymmetric insofar as the two bidders' values are drawn from different distributions (and these different distributions are common knowledge).
- They provide a characterisation that shows necessary and sufficient conditions on the distributions of values, and the minimum bid in the auction, so that the resulting equilibrium bid functions are linear.
- These form a one-dimensional set of auction parameters, which can be indexed by the amount of asymmetry in the ranges of values.
- It includes both the standard symmetric case with minimum bid equals zero as a special case.
- All the equilibria involve bid functions with slope equal to one-half.

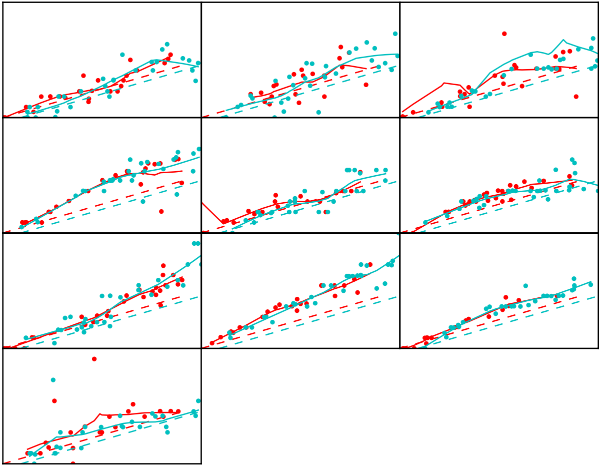
An interesting asymmetric auction



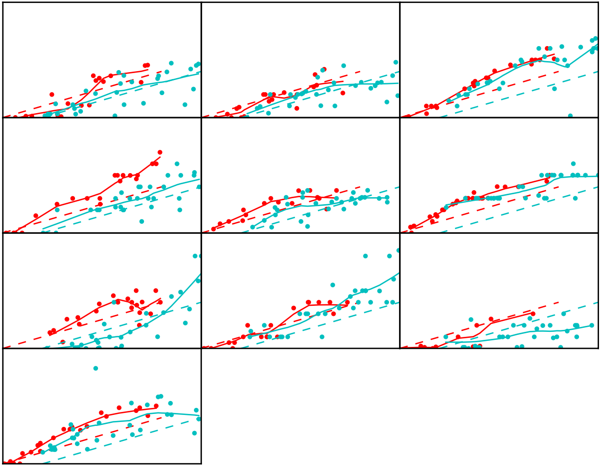
An interesting asymmetric auction

- Kaplan and Turocy (in preparation, presented last July in a CBESS seminar) conducted experiments at the FEELE lab at Exeter where we tested the effects of changing the amount of asymmetry.
- We had four treatments: small asymmetry, medium asymmetry, large asymmetry, and extremely large asymmetry (in which the strong bidder always had a value strictly higher than the weak bidder).
- We did a within-subjects design; each subject alternated between strong and weak roles in each period.
- The next four frames show bid scatterplots from each of those treatments, in that order.

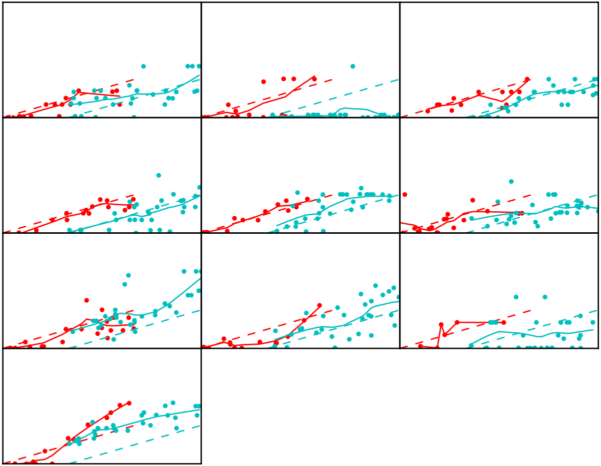
Scatterplots: Small asymmetry



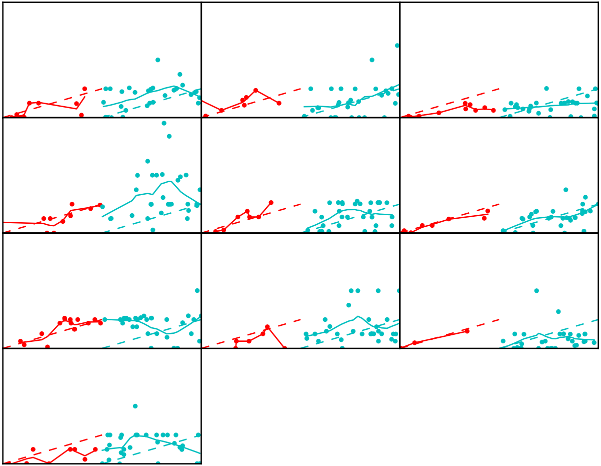
Scatterplots: Medium asymmetry



Scatterplots: Large asymmetry



Scatterplots: Extremely large asymmetry



What the....?

- We observe that, in general, bidders in the weak role bid roughly as we are accustomed to seeing in symmetric auctions; a bit more aggressive than the equilibrium, but with a clear monotonic increase as a function of their realised private value.
- The strong bidders, however, do not follow this pattern. When asymmetry is significant, the response of the strong bidders to variation in their private value is much weaker, with some bidders bidding almost independently of their value.
- In particular, when the asymmetry is sufficiently large that there is no overlap in the intervals of values, the strong bidders bid strictly above the minimum bid, which is incompatible with maximising expected utility over money.

Impulse reply to the rescue!

Impulse reply explains these facts!

- When there is no overlap, the strong bidder is always in a situation in which he will receive upward impulse if he loses.
- Therefore
  1. The impulse reply involves bidding more than the minimum bid, even if the strong bidder has his smallest possible value.
  2. The impulse reply is much flatter than the best reply.
- Furthermore, the magnitudes of the bids we observed the strong types submitting are consistent with $\lambda \approx 0.33$.

Where next?

- Carrying out some more detailed calculations in the symmetric case allowing for multiple bidder types, and seeing whether the estimate of $\lambda \approx \frac{1}{3}$ quoted by OS holds up.
- Estimating λ more carefully in the Kaplan-Turocy asymmetric auctions.
- Looking for other auction variants where we might be able to show that impulse reply ideas have equal power to organise observations?