Two-Dimensional Values and Information Sharing in Auctions*

Xu Tan[†]

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Abstract

Incentives to share private information ahead of auctions are explored in a setting with two-dimensional valuations: there are both common and private components to bidders valuations and private information is held on both dimensions. This setting fits a lot of applications where bidders care about both their private preference and some common future value, such as the housing market. We show that full revelation of the common-value signals is the (unique) sequential equilibrium, and such revelation is also consistent with the seller's interest. Thus, the argument that sharing information is strictly dominated in pure common-value auctions is not robust to a slight perturbation. Moreover, the auctions only involve pure private values after the revelation of common-value signals, such that the doubts on nonexistence of equilibrium and efficiency loss by previous studies on auctions with such two-dimensional valuations disappear.

Keywords: information sharing, auctions, two-dimensional values

JEL Classification Codes: D44, D82

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[†]Department of Economics, Stanford University, Stanford, California 94305-6072 USA. Emails: xutan@stanford.edu.

1 Introduction

In many market settings, there is both a private and a common component to valuations. For example, in the housing market: potential buyers have private values for living in the house, and also care about the future price (common value) of the house. Generally, in markets with clear private values such as wine and gallery auctions, the common component appears as long as the item will be resold in the future with the same price for all agents; also in markets with clear common values such as oil drilling and sale of timber-harvesting contracts, the private component appears as long as agents have different technologies and costs. Thus, the combination of common and private components fits in many applications, like treasury auctions as well.

In these auctions, agents observe two private signals of the valuation: one is their private component and the other is correlated to the common component. In this paper, we explore the incentives to share these private information prior to the auction. The communication of information is a critical part even in a competitive world. Competing firms can often commit to share relevant information with their competitors. Moreover, in housing market agents share and discuss their predictions about the future with each other. In the literature, Kovenock, Morath and Munster (2010) considered the information sharing in all-pay auctions, and they showed that in pure common-value auctions, sharing information is strictly dominated. The reason is: when agents have complete information about the common value, the expected return from auctions is reduced to zero. In other words, agents lose their information rent by revealing signals.

However, we show that the result on no information sharing is not robust to a slight perturbation of valuations. In fact, full revelation of the common-value signal is the unique sequential equilibrium in auctions with both private and common components in valuations. Intuitively, revealing the smallest common-value signal publicly reduces other agents' bids. With the private component, agents now are strictly better off if all others reduce their bids. Following the standard argument in information revelations literature, given the smallest common-value signal is revealed, revelation of the second smallest common-value signal also reduces the bids by the same logic. Eventually, it leads to a full revelation of the common-value signals.

Despite the bidders' incentives to share information, the seller also prefers to have the information shared among bidders. Since complete information reduces bidders' information rent and increases the expected winning price. For instance, without complete information about the common value, bidders would bid less aggressively due to the winner's curse.

1.1 Related Literature

This paper is related to two strings of literature: the study on valuations with both common and private components, and the study on information sharing in auctions.

When moving away from pure valuations, we lose efficiency as discussed by Pesendorfer and Swinkels (2000) and existence of equilibrium as shown in Jackson (2009). Pesendorfer and Swinkels (2000) showed the asymptotic efficiency could be achieved in a large society while without knowing whether the equilibrium exists. Goeree and Offerman (2003) solved the equilibrium given a specific (linear) form of the common component, while further confirmed the inefficiency in the two-dimensional valuation setting. Jackson (2009) showed with a simple example that equilibria can fail to exist in second-price auction with two-dimensional valuations. In our model, agents fully reveal the common-value signals prior to the auction such that the auction become a pure private-value auction. Thus, here the equilibrium can be easily solved by the standard auction theory and there is no inefficiency since the winner is always the agent with the highest private component.

Milgrom and Weber (1982) considered the incentive of the seller to share information about the private information of the values. Kovenock, Morath and Munster (2010) considered the information sharing in all-pay auctions, and they showed that in pure common-value auctions, sharing information is strictly dominated.

2 Model

A group of agents, $N = \{1, ..., n\}$, bid over a single indivisible item in an auction.

2.1 Preference

Agent i has a utility for the item which is described by

$$u_i(v_i, q) = \lambda v_i + (1 - \lambda)q$$

where $v_i \in [v_L, v_H]$ is a private component, $q \in [q_L, q_H]$ is a common component and $\lambda \in (0, 1)$ is a weighting factor such that the valuations have both common and private components. When $\lambda = 0$, the setting is a standard common-value auction, and when $\lambda = 1$, the setting is a standard private-value auction. Moreover, we could have heterogenous weighting factors, and the remark following the main theorem shows it won't affect the results.

Agents' private components, $\{v_i\}$, are drawn independently from the interval $[v_L, v_H]$ according to the same probability distribution F. We assume that F has a continuous and strictly positive density f. Given the world is discrete, we also show that our main result is easily generalized to discrete valuations. See the discussion after the main theorem.

Agent i doesn't observe the true common component, q, instead observes an realization of the random signal $s_i \in [s_L, s_H]$. All signals, $\{s_i\}$, are drawn independently from the interval $[s_L, s_H]$ according to the same probability distribution G. We assume that G has a continuous and strictly positive density g. Then the common component is fully determined by all signals[1]

$$q = q(s_1, ..., s_n)$$

where q is strictly increasing and continuous in each s_i. For instance, the true common component is the average of the signal $q = (s_1 + ... + s_n)/n$[2]. The common-value signals are independent of the private values.

All of the setting are common knowledge expect for the private information (v_i, s_i). All agents are risk neutral.

We remark that this two-dimensional valuation is different from the affiliated signals discussed in Milgrom and Weber (1982). The affiliation among signals is general, however each bidder observes only one private signal such that the bidding strategy is a one-to-one mapping. When there are two private signals for each bidder, the bidding strategy could be more complicated: for instance it is hard to define the monotonicity of a bidding strategy.

2.2 The Game and Equilibrium

Prior to the auction, agents could meet and communicate signals in the form of verifiable revelation of signals[3]. In particular, agents can choose to reveal the common-value signal or not, but they cannot "lie". This matches a variety of applications where the signal of the future value (common component) is in the form of verifiable facts. For instance in the housing market, there are several reports and summary from government and some experts which could indicate the future market. Moreover, in many applications it is hard to reveal the private component with evidence since it is more about individual taste and personal preference. We consider the revelation of both signals in Section **??**, and before that the revelation is only about common component.

More formally, a revelation strategy for an agent $i \in N$ is a function:

$$r_i(s_i) \in \triangle\{s_i, \emptyset\}$$

We use $r(s) = (r_1(s_1), ..., r_n(s_n))$ to represent the full revelation strategy.

[1]The remarks after the main theorem shows the result holds even if we only know the distribution of the common component after observing all signals.

[2]Goeree and Offerman (2003) studied auctions where the common component has this specific relationship with all signals.

[3]We discuss the robustness of this verifiable revelation assumption and cheap-talk communication in Section **??**.

In a first stage, agents with observed private signals simultaneously choose their revealing strategies. Any revealed signals are then public, and so observed by all agents. In a second stage, agents bid, $b_i(v_i, s_i, r(s))$, conditional on all available information in an auction with specific rules to be define later. For instance, the following section we consider standard sealed-bid second-price auction such that the agent with the highest bid wins and pays the price equaling the second highest bid. Ties are broken randomly. All auctions with Vickrey payoffs and English auctions have the same results.

An *equilibrium* is a list of strategies, r_i in the first stage and $b_i(v_i, s_i, r(s))$ in the second stage for each $i \in N$, that form a sequential equilibrium, where both the revealing and bidding strategies could be mixed. Moreover, we require the bidding strategy is undominated which is explained below.

There are some refinements we consider to rule out some sorts of degenerate equilibrium. It is well-known that for the second-price auction there always exists an asymmetric equilibrium. For example, let bidder 1 unconditionally bid X, where $X = v_H + q_H$, and have all other bidders bid 0. This is an equilibrium regardless of the weighting factor λ. In order to eliminate this, let us consider two different refinements: (i) symmetric equilibrium and (ii) equilibrium that only uses strategies that are in the closure of the set of undominated strategies. Here, the refinement to undominated strategies is more general than a refinement to symmetric equilibrium, while symmetric equilibria are the main ones studied in the literature. Thus we include results concerning both.

In some cases, we also consider the ex-ante stable notion of equilibrium: in the equilibrium, agents decide whether to reveal the signals or not before observing them, and when they observe the signals they don't have incentives to deviate from the predetermined strategy. While without this notion, there would more types of equilibria, for instance if a common-component signal is not revealed, other agents believe that the actual signal is the highest one which in turn gives agents with lower signals incentives to reveal the signal. This notion refine the sequential equilibria, such that now when a signal is not revealed, agents' belief is the same as the ex ante one.

3 Second-Price Auctions

Let's start with two bidders ($n = 2$) in a standard second-price auction , which highlights the incentives and effects of sharing information.

3.1 Pure Common-Value Auctions

As a benchmark, we first state the results on information sharing in auctions with pure common values where $\lambda = 0$.

There are two bidders: if agent 1 reveal the signal, agent 2's undominated strategy is bidding the true common value in second-price auction which gives agent 1 an expected payoff of zero regardless of agent 2's revealing strategy. Thus, agents revealing the signals have a zero expected payoff since they lose the information rent.

LEMMA **1** *In second-price auction with two bidders and pure common values, sharing common-value signals are weakly dominated.*

The proof is obvious and thus is omitted.

Intuitively, revealing signals would reduce the information rents such that the expected return from the auction is reduced to zero. Kovenock, Florian and Munster (2010) provided similar results in all-pay auctions. However, we will show shortly that this argument is not robust to a slight perturbation in agents' valuations.

3.2 Equilibrium

Let us say that an equilibrium is *fully-revealing equivalent* if the outcome is the same as in an equilibrium in a game where all agents reveal their common-value signals.

Consider a equilibrium σ which is not fully-revealing equivalent. For each agent i, we define $D_i \subset [s_L, s_H]$ to be the set of common-value signals that agent i is hiding. In σ, we can find some i such that D_i contains more than one signals. Let $\underline{s}_i$ denoted the the infimum of the signals in D_i, $\underline{s}_i = \inf D_i$. The following lemma describes the characters of such σ.

LEMMA **2** *Consider a equilibrium σ which is not fully-revealing equivalent:*

- *D_i is an interval which is either $(\underline{s}_i, s_H]$ or $[\underline{s}_i, s_H]$;*
- *In other words, $r_i(s) = \emptyset$ for all $s > \underline{s}_i$ such that agent i hides all signals above $\underline{s}_i$;*
- *The bidding strategy is $b_i(v_i, s_i, r_{-i}(s_{-i})) = \lambda v_i + (1-\lambda) q(s_i, s^r_{-i})$ where the inferred signal $s^r_j = \underline{s}_j$ if $r_j(s_j) = \emptyset$, otherwise $s^r_j = r_j(s_j)$.*

Proof of Lemma 2: In second-price auction, agents bid the true value if they observe it. Given agent i's information, $b_i(v_i, s_i, r_{-i}(s_{-i})) = \lambda v_i + (1-\lambda) q(s_i, s^r_{-i})$ is the lowest possible value for agent i. Thus, agent i would not want to lower the bid. On the other hand, if there is some agent i with a bid higher than the lowest possible value, revealing signals very close to the bottom signal $\underline{s}$ reduces agent i's bid and increases other agents' expected payoff. In all, bidding the lowest possible value is the only reasonable strategy in a equilibrium σ.

Given all agents bid the lowest possible value given their information, agent i prefers not to reveal signals above $\underline{s}_i$. Since if not revealing the signal $s_i > \underline{s}_i$, other agents bid as if they knew the signal for agent i is $\underline{s}_i$; if revealing the signal, other agents bid as the signal for agent i is s_i which increases other agents' bid and reduces agent i's expected payoff. ∎

Before formally prove the main theorem, let consider the discrete case to gain some intuition. If some agent i observes the smallest common-value signal, revealing it publicly reduces other agents' expectations of the common component and thus reduces bids. So agent i now has strict incentives to reveal the signal. Given the smallest signal is revealed for sure, agents observing the second smallest signal now have incentives to reveal it by the same logic. So on and so forth, we have full revelation of the common-value signals.

In the continuous case, the lemma tells us when some agent i hides more than one signal, other agents need to bid as if they knew the signal for agent i is the smallest signal getting hidden in order to make sure agent i doesn't have the incentive to reveal the smallest hidden signal. However, bidding the lowest possible value given the information is too conservative to be an equilibrium such that the equilibrium σ which is not fully-revealing equivalent doesn't exist.

THEOREM 1 *All symmetric sequential equilibria or equilibria in undominated strategies are fully-revealing equivalent.*

Proof of Theorem 1: Suppose there exists a equilibrium σ which is not fully-revealing equivalent such that we can find some i such that D_i contains more than one signals. We need to show that the bidding strategy in Lemma 2 is not an equilibrium strategy.

Given some revelation of signals in the first stage where agent i reveals no signal, we consider agent j's problem where $j \neq i$. All other agents' bidding strategy is described in the lemma such that $b_l(v_l, s_l, r_{-l}(s_{-l})) = \lambda v_l + (1-\lambda)q(s_l, s^r_{-l})$ where $l \neq j$. Let β_j represent the highest bid among all other agents, $\beta_j = \max_{l \neq j} b_l(v_l, s_l, r_{-l}(s_{-l}))$. The expected payoff for agent j with a bid b_j is

$$\int_{s_{-j}} \int_{\underline{\beta}}^{b_j} (\lambda v_j + (1-\lambda)q(s_j, s_{-j}) - \beta_j) h(\beta_j) g(s_{-j}) d\beta_j ds_{-j}$$

where $h(\beta_j)$ is the density of $h(\beta_j)$. The first order condition implies

$$\int_{s_{-j}} (\lambda v_j + (1-\lambda)q(s_j, s_{-j}) - b_j) h(b_j) g(s_{-j}) ds_{-j} = 0$$

However, the first order condition is strictly positive if agent j uses the bidding strategy in the lemma. Because if $b_j = \lambda v_j + (1-\lambda)q(s_j, s^r_{-j})$, the condition becomes

$$\int_{s_{-j}} (1-\lambda)(q(s_j, s_{-j}) - q(s_j, s^r_{-j}))h(b_j)g(s_{-j})ds_{-j} > 0$$

which is positive since at least for agent i, D_i contains some higher signals while $s^r_i = \underline{s}_i$ always.

Thus, such equilibrium σ doesn't exist and all sequential equilibria are fully-revealing equivalent. ▮

We remark that the results hold in a more general and broader setting:

- Heterogeneous preference: if agents put different weight factors, $\lambda_i \in (0,1)$, on the private and common components, the results of the theorem still hold. Since as long as other bidders care the common-value component, revealing the smallest common-value signal could reduce their bids and thus increase the expected payoff of oneself.

- Noisy signals: another case is the common-value component is not fully determined by all signals, then we consider the expected common-value component since all agents are risk-neutral. Revealing small common-value signals reduces the expectation of the common component such that it reduces the bids from other bidders. Thus, full revelation is still the only possible equilibrium.

- Multiple signals: it is possible that agents get more than one signal of the common-value component, $(s_i^{(1)}, ..., s_i^{(m)})$. Such as in the housing market, there are reports from the government and from different experts and researchers. The intuition still works such as in the discrete case, agents have incentives to reveal the smallest common-value signals, then the second smallest, and so on and so forth until the full revelation.

- General preference: the utility of agents could depend on the private and common components in a more complex way than the linear case: $u(v_i, q)$. As long as it is strictly increasing in both components, agents' incentive of revealing small common-value signal is still there.

Results in the pure common-value setting are not robust to a slight perturbation in agents' valuations. Since with pure common value, agents observing the smallest common-value signal is indifferent between revealing the signal or not since the expected payoff is zero anyway. With a (tiny) private component, agents now are strictly better off by revealing the smallest common-value signal since this reduces all other agents' bids. And given the smallest common-value signal is revealed, revelation of the second smallest common-value signal also reduces the bids by the same logic. Eventually, it leads to a full revelation of the common-value signals.

It is efficient to give the item to the agent with the highest private component since the common component is the same for all agents. However in auctions without information sharing, there is a positive expected efficiency loss[4]. Because it is quite possible that the winner of the auction observing a high enough common-value signal but not having the highest private component. While if we allow information sharing, agents fully reveal their common-value signal in the first stage such that in the second stage, the auction only involves private information of the private values and thus winner is the agent with the highest private component in equilibrium. Then there is no efficiency loss with information sharing.

Corollary **1** *There is no efficiency loss in auctions with information sharing.*

Not only the social planner support the sharing of information, the seller also prefer agents to share their information. In order to compare the seller's payoff, we need to know the equilibrium in both cases such that we consider the following setting in the example where we learn the equilibrium from Goeree and Offerman (2003). The next example shows that the expected payoffs of agents decrease with information sharing because they lose their information rent and thus the expected payoff of the seller increases.

Example **1** *The expected payoff of seller is higher if agents share their information.*

Suppose there are two agents in the society ($n = 2$), the densities f and g are logconcave[5], and the common value is the average of the two signals: $q = (s_1 + s_2)/2$. Let $c_i = v_i + s_i/2$ be the surplus of each agent. From Goeree and Offerman (2003), without information sharing the symmetric bidding strategy (for agent 1) is a increasing function of the surplus

$$b(c_1) = c_1 + E(s_2/2|c_2 = c_1)$$

With information sharing, the auction is a standard private-value second-price auction such that the bid equals the true value.

Given the private components (v_1, v_2). With information sharing, the expected payoff of agent 1 is

$$\pi^*(v_1, v_2) = max(v_1 - v_2, 0)$$

Without information sharing, the expected payoff of agent 1 is

$$\pi^{**}(v_1, v_2) = E[(v_1 - v_2) + (s_1 - s_2)/2|(v_1 - v_2) + (s_1 - s_2)/2 > 0)]$$

[4]See Pesendorfer and Swinkels (2000) and Goeree and Offerman (2003).

[5]This is enough to ensure the conditional expectations of the private-value and common-value are non-decreasing with the surplus. See Goeree and Offerman (2003) for details.

When $v_1 \leq v_2$, $\pi^{**} \geq \pi^* = 0$. When $v_1 < v_2$, $\pi^{**} = \pi^* + E[(s_1 - s_2)/2|(v_1 - v_2) + (s_1 - s_2)/2 > 0)]$. Since s_1 and s_2 are identically distributed, the density of $s_1 - s_2$ is symmetric with zero mean such that $E[(s_1 - s_2)/2|(v_1 - v_2) + (s_1 - s_2)/2 > 0)] \geq 0$. So $\pi^{**} \geq \pi^*$.

The expected payoffs of agents decrease with information sharing, and then the expected payoff of the seller must increase.

Intuitively, we consider the pure common-value auctions: if all signals are common knowledge, all agents submit the same bid which is the true common value in the equilibrium. Thus, the expected payoff is zero for all agents. However, if the signals are private information, all agents have a positive expected payoff expect the agents with the smallest signal. So with full information sharing agents' expected payoff reduces and the seller's expected payoff increases.

4 Revelation of Common-Value Signals

In this section, we consider two situations that generalize the previous simple case: (i) multiple bidders in second-price auctions; and (ii) other types of auctions such as all-pay auctions. We show that the same intuition still roughly works in these general settings.

4.1 Multiple Bidders

Again, we start with the benchmark case where there are only pure common values.

When there are more than two bidders, if agent i reveals the signal while all other agents don't, the bidding strategy is not clear since no strategy is dominated at this time. Thus we cannot use the same refinement as the two-bidder case to claim no one prefers to revealing signals. However, we will show shortly that it is still true that agents revealing signals have a zero expected return such that revealing signals is not a wise choice. Thus, agents prefer not to reveal the signal if it is done ex-ante, such that before observing the signals.

LEMMA **3** *In every symmetric equilibrium with pure common values and second-price auction, agents revealing signals have zero expected payoff. Thus agents prefer not to reveal signals ex-ante.*

Proof of Lemma 3: We start with the equilibrium in the second stage: if all common-value signals are revealed, in equilibrium all agents bid the true common value q and their expected payoffs are zero; if all but one common-value signal (i.e. s_1) are revealed, in equilibrium agent 1 bids the true common value and the rest agents bid x where x is at most the smallest possible common value, and agent 1 has a positive expected payoff while all others' expected payoff is zero.

If $m \geq 2$ signals are not revealed, without loss of generality, assume agent i $(1 \leq i \leq m)$ hides signals and the rest agents reveal signals. Let y_i be the highest signal observed by agent j, such that $1 \leq j \leq m$ and $j \neq i$. In equilibrium, agents revealing signals bid x (x is at most the smallest possible common value) and other agents bid as if there is only the first m agents such that for agent i with $1 \leq i \leq m$

$$b_i(s_i, s_{m+1}, ..., s_n) = E(q|s_i, y_i = s_i, s_{m+1}, ..., s_n)$$

which follows from the standard auction theory. We need to check bidding zero is indeed the best response for agents revealing signals. Otherwise, if agent j $(m+1 \leq j \leq n)$ submits a positive bid, the payoff changes only when this is the highest bid and agent j needs to pay a price

$$p = E(q|s_{(1)}, s_{(2)} = s_{(1)}, s_{m+1}, ..., s_n)$$

where $s_{(1)}$ is the highest hidden signal and $s_{(2)} = s_{(1)}$ means the second highest signal equals the highest signal. However, the expect common value should be conditional on $s_{(2)} < s_{(1)}$ such that the common value is lower than the price. Agent j loses money by winning such that bidding zero is the best choice.

Then we need to show agents revealing signals cannot have positive expected payoff in all equilibria. Suppose there is one equilibrium where agent j $(m+1 \leq j \leq n)$ uses a mixed strategy and has a positive expected payoff. Still agent j needs to pay a price

$$p' > E(q|s_{(1)}, s_{(2)} < s_{(1)}, s_{m+1}, ..., s_n)$$

Agent j loses money by winning such that bidding zero is the best choice.

Thus, in the second stage, all agents revealing signals always have zero expected payoffs while agents hiding signals sometimes have positive payoffs. So in equilibrium, agents hides the common-value signals. ∎

Then we move on to the case where the valuations have both common-value and private-value components. Let say agents have *independent belief* if the belief on each agent doesn't depend on the behaviors of the rest agents.

If we loose the independent belief assumption such that one deviation leads to changes in believes for more agents, none-revealing is possible to appear in sequential equilibrium.

EXAMPLE **2** *Sequential equilibrium with none-revealing of common-value signals.*

Suppose there are four agents in the society ($n = 4$), the densities f and g are uniform on [0,1][6], and the common value is the average of the four signals: $q = (s_1 + s_2)/4$. Let

[6] Again, this is enough to ensure the conditional expectations of the private-value and common-value are non-decreasing with the surplus. See Goeree and Offerman (2003) for details.

$c_i = v_i + s_i/2$ be the surplus of each agent and y_i be the highest surplus among all agents except i. From Goeree and Offerman (2003), without information sharing the symmetric bidding strategy (for agent i) is a increasing function of the surplus

$$b(c_i) = c_i + E(s/4|c = c_i) + 2 * E(s/4|c < c_i)$$

Thus $b(c_i) < c_i + 1/4 + 2 * 1/8$.

Suppose in the equilibrium, all 4 agents won't reveal the common-value signals. If some agent (say agent 1) observes a signal of 0 and reveal this signal, all other agents believes that they are in fact in a fully revealing equilibrium such that agents only hides the highest signal. Then all other agents bid the common value under their believes $b'(c_i) = c_i + 0 + 2 * 1/4$ which is higher than the original bid. Thus, revealing the lowest signal can increase other agents' such that no one wants to deviate. None-revealing is indeed a sequential equilibrium.

Let's say the bidding strategy in equilibrium is *monotone*[7] if $b(v, s)$ (or $b(v)$ if s is revealed) is strictly increasing in v, continuous in v, and differentiable in v at all but finitely many points.

Theorem 2 *All symmetric monotone sequential equilibria with independent belief are fully-revealing equivalent.*

Proof of Theorem 2: We show that fully-hiding is not a equilibrium and it is easy to generate the proof to show that partially-hiding is also not a equilibrium.

If in the equilibrium σ all agents hide the common-value signal, the bidding strategy (following Proposition 1 in Pesendorfer and Swinkels (2000)) has the form

$$b(v_i, s_i) = E(v_i + q|d = b(v_i, s_i), s_i)$$

where d is the highest bid among all other agents except i.

If agent 1 observes a lowest common-value signal and reveals the signal, this deviation won't affect the believes on behaviors of all other agents by the independence. Now the bidding strategy of agents i ($i \geq 2$) is

$$b'(v_i, s_i) = E(v_i + q|d = b(v_i, s_i), s_i, s_L)$$

where d is the highest bid among all other agents except i. Since $b'(v_i, s_i) < b(v_i, s_i)$, agent i has strict incentive to reveal the signal.

Thus, such equilibrium σ doesn't exist and all sequential equilibria are fully-revealing equivalent. ∎

With multiple bidders, it is still true that agent observing a low common-value signal wants to reveal the signal and reduce the bids from other agents. Thus, the only reasonable equilibrium is fully-revealing equivalent.

[7]This corresponds to assumption 2 in Pesendorfer and Swinkels (2000).

4.2 Other Auction Mechanisms

There are many other forms of auctions and mechanisms where we are interested in knowing the incentives of sharing information, such as first-price auctions and all-pay auctions. For instance, competing firms can sometimes commit to share information with their competitors. All-pay auctions are often used as a benchmark to represent such competitions among firms, such as their investments in R&D. We proceed in this section by showing that all the intuition of sharing information of common-value signals could be generalized to most auctions.

Let $n = 2$. For any auction mechanism (p, x) such that $p_i(b_1, b_2)$ is the probability that agent i is the winner and $x_i(b_1, b_2)$ is the money agent i needs to pay to the seller. Moreover, we assume this auction is:

- Symmetric: $p_1(b_1, b_2) = p_2(b_2, b_1)$ and $x_1(b_1, b_2) = x_2(b_2, b_1)$;
- Monotonic: $p_1(b_1 > b_2) > p_2(b_1 > b_2) = 0$ such that agent with a lower bid has zero chance of winning;
- Continuous: $x_i(b_1, b_2)$ is continuous in b.

Lemma 4 *In every sequential equilibrium in any symmetric, monotonic and continuous auction with pure common value, revealing common-value signal is dominated.*

Proof of Lemma 4: Consider the agent who reveals the common-value signal, we claim that such agent must use mixed strategy and claim that the expect payoff of such agent must be zero. ∎

However, with private component, agents now have incentives to reveal the low common-value signals in order to lower the other agent's bid.

Formally, we consider the auction mechanism with monotone equilibrium such that

(A1) $b(v, q) < b(v, q')$ when $q < q'$, such that the bidding strategies are higher when both private values are higher;

(A2) $b(v, \xi) < b(v, \zeta)$ where ξ and ζ are distribution of the common value and ξ is first-order stochastic dominated by ζ.

Theorem 3 *Fully revelation of common-value signals is a sequential equilibrium under condition (A1); it is the unique sequential equilibrium if condition (A2) is true.*

Proof of Theorem 3: ∎

Then we show that Assumption 2 is satisfied by other auctions beside second-price auction.

Example 3 *(A2) is satisfied in all-pay auctions.*

In all pay auctions (contests of R&D), there is no seller, thus we only need to discuss the social welfare.

Example 4 *The social welfare might decrease with information sharing.*

Suppose there are two firms in the industry ($n = 2$) and they compete in an all-pay auction. The densities f and g are logconcave, and the common value is the average of the two signals: $q = (s_1 + s_2)/2$. Let $c_i = v_i + s_i/2$ be the surplus of each agent. Without information sharing the symmetric bidding strategy (for agent 1) is a increasing function of the surplus

$$b(c_1) = \int_{c_L}^{c_1} (x + E(s_2/2|c_2 = x)dF_c(x)$$

With information sharing, the auction is a standard private-value all-pay auction such that the bidding strategy in equilibrium is

$$\beta(v_1) = \int_{v_L}^{v_1} ydF(y)$$

The expected payoff of firm 1 is the same as in Example 1 such that

$$\pi^{**}(c_1) \geq \pi^*(v_1)$$

Since in contests, the social welfare is the sum of the payoff of both firms, which is higher without information sharing.

Thus, information sharing sometimes benefits the society such as in first-price and second-price auctions, but not always the case such as in all-pay auctions.

5 Extensions

[[Extensions to be added]]

Now we consider three extensions and applications we can use the model to explore: (i) revelation of both common-value and private-value signals; (ii) agents' and seller's incentive of inquiring information (signals s); (iii) cheap-talk communication and noise in revealing signals.

5.1 Revelation of Both Private and Common Components

Without the restriction of independent revelation, agents now might find it not worth revealing the lowest common-value signal if other agents believe that such revelation is correlated with certain type of private value and bid even more aggressive.

Example 5 *None signal is revealed in a correlated equilibrium*

5.2 Incentive of Inquiring Information

5.3 Cheap talk and Noise in Revealing Signals

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