The replacement principle and the egalitarian rule
Extended Abstract

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Abstract

We study two widely applicable resource allocation problems in which agents cannot or should not be treated symmetrically. In the first problem, shares of jobs with predetermined processing times have to be assigned to workers who may not be qualified to perform every job. The second problem concerns a stylized networked market in which a commodity is to be transferred from a set of sellers to a set of buyers; a transfer between a seller and a buyer is possible only when they are connected via the network. For both problems, we rule out monetary compensations and assume that agents have single-peaked preferences over their assignments: workers have ideal workloads and traders have ideal trade volumes, below and beyond which their welfare is decreasing.

For the first problem, Bochet, İlkılıç, and Moulin [1] introduce an assignment mechanism they call the egalitarian rule. They characterize it on the basis of Pareto-efficiency, strategy-proofness, and an equity condition. For the second problem, Bochet, İlkılıç, Moulin, and Sethuraman [2] propose and characterize another assignment mechanism along similar lines. Here, we study the implications of the "replacement principle," as was formulated by Thomson [4], and provide alternative characterizations of the assignment mechanisms of [1] and [2].

Keywords: Bilateral trade; Fairness; Envy-freeness; Welfare-domination under preference-replacement; Networks; Single-peaked preferences

1 Introduction

Consider the assignment of shares of jobs with predetermined processing times. In general, only some employees are qualified to perform a given job. Employees have preferences over their total workloads and we rule out monetary compensations. The idea is that, once a person is hired, she is subject to typically varying workloads while her salary is constant.

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Our goal is to find "desirable" workloads for each worker. To do so, we identify a set of appealing fairness and solidarity criteria, or axioms, and study their implications on the possible workload assignments.

We will refer to the problem of finding these assignments as the *Job Assignment Problem*. We will focus on a, mathematically, more general version of this problem: a market for a divisible good under a fixed prices. Here, a buyer can only buy from a subset of sellers and has preferences over her total consumption assignment. Similarly, a seller can only sell to a subset of buyers and has preferences over her total transfer assignment, her net sales. Since the price is fixed, we rule out competitive price equilibria and need to specify the transfer assignments for each agent. We call this the *Transfer Assignment Problem*.

For both problems, we assume that agents have single-peaked preferences over their assignments: workers have ideal workloads and traders have ideal trade volumes, below and beyond which their welfare is decreasing. In many application this assumption is natural: it is implied by the convexity of preferences over an underlying consumption space or by the convexity of production sets. For instance, in the Transfer Assignment Problem, single-peakedness follows, for buyers, if they have strictly convex preferences over the space of bundles of the good being traded and a composite commodity (money). Here, the restriction of these preferences to the budget lines are single-peaked.

In this paper we will evaluate **rules** mapping each case of one of the above problems to "desirable" **allocations** specifying each agents' assignment. This type of analysis was initiated by Sprumont [3] for the Job Assignment Problem *when there is a single job* to be assigned. He specified that a rule should satisfy *Pareto-efficiency*, *strategy-proofness*, and either *anonymity* or *no-envy*. Sprumont proved that there is a unique rule satisfying these criteria, the "uniform rule."

Bochet, İlkılıç, Moulin (2010) (henceforth BIM) consider the full-fledged Job Assignment Problem and propose the egalitarian rule as a solution. They provide a characterization of this rule parallel to Sprumont's characterization of the uniform rule. Bochet, İlkılıç, Moulin, and Sethuraman (BIMS) (henceforth BIMS) initiated the axiomatic analysis of the problem we called the Transfer Assignment Problem. BIMS proposed a rule (which they also call egalitarian) and characterize it on the basis of *Pareto-efficiency*, *strategy-proofness*, a constrained version of *equal-treatment of equals* and *voluntary participation*.

This paper provides a unified approach to the Job Assignment and Transfer Assignment problems. Unlike BIM and BIMS, we are primarily concerned with notions of fairness and solidarity when agents cannot be treated symmetrically.

Our main contribution is the formulation of an operationally useful solidarity requirement. We study the implications of the "replacement principle," as formulated by Thomson [4]. This solidarity requirement specifies that a change in one agent's preferences affects all other agents in the same direction, welfare-wise. We posit a weak version of the property which is compatible with *Pareto-efficiency* and various distributional requirements.

The conclusions reached here confirm the importance of the egalitarian rules proposed by BIM and BIMS. Our main result is an alternative characterization of BIMS' egalitarian

rule. We prove that the egalitarian rule is the only rule satisfying the following requirements:

- *One-sided preference-replacemet*: This is the weak version of the replacement principle already mentioned.
- *Constrained no-envy*: The allocation recommended is such that if an agent envies another, then there is no other feasible allocation improving upon her assignment while leaving the assignments of all other agents, except for the envied agent, unchanged.
- *Pareto-efficiecy*: The allocation recommended is such that there is no other feasible allocation in which some agent can be made better off while not making any other agent worse off.
- *Voluntary participation*: the allocation recommended is such that each agent finds her assignment at least as good as receiving no assignment at all.
- *Replication-invariance*: Suppose that we are given two "identical" instances of the Transfer Assignment Problem, the only difference being that there is no agent in common between the two instances. Now suppose that we bring both problems together into a joint problem. *Replication-invariance* requires that the allocation recommended for the joint problem be consistent with that recommended for the two original problems: the assignment recommended for each agent in the joint problem is the same as that recommended for her in the original problem (that she was a part of).

References

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