

THE CONDORCET JUR(IES) THEOREM*

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Abstract

Two issues can be decided in a single general election by a single committee or in two special elections by disjoint committees. Similarly, two defendants can be tried together in a joint trial or tried separately in severed trials. The multiplicity of issues or defendants introduces new strategic considerations. As in the standard Condorcet Jury Theorem, we study environments with common values and incomplete information as the number of voters goes to infinity. The joint trial by a single committee can aggregate information if and only if the severed trials by separate committees can aggregate information. Specifically, suppose that either for the joint trial or for the severed trials there exists a sequence of equilibria that implements the optimal outcome with probability approaching one. Then a sequence of equilibria with similar asymptotic efficiency exists for the other format. Thus, the advocacy of a particular format cannot hinge on pure information aggregation with many signals. A counterpart of the sufficient statistical assumption for a single issue suffices for information aggregation in the severed trials, therefore suffices in the joint trial as well. The equivalence of asymptotic efficiency across formats is maintained even when three or more issues are divided into arbitrary partitions, or when abstention is allowed.

1 Introduction

Many group decisions involve two or more related issues. The issues are decided jointly by a single committee of voters or separately by different committees. We compare these two institutions for environments with incomplete information and common values. For example, suppose two defendants are indicted for related crimes. The court can either try both defendants in a joint trial, or try the defendants separately in severed trials. Which format is more likely to result in correct verdicts? Or, suppose a legislature is deciding whether to pass two related policy measures. These measures can both be decided in a single committee, or be separately decided in different committees. Which committee structure is more likely to pass the right combination of measures?

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Beginning with the observation of Condorcet, a large literature on the Jury Theorem studies when large juries can aggregate information to reach the optimal outcome. These insights for elections with common values are limited to settings with a single issue or defendant. Ahn and Oliveros (2010) study simultaneous voting over multiples issues, but only for cases with pure private values. This paper takes first steps in developing our theoretical knowledge of joint versus severed trials or committees in environments with common values. In particular, we execute the first comparison of simultaneous versus separate resolution of issues under common values.

Our main result is an equivalence: one format can aggregate information if and only if the other format can also aggregate information. This equivalence contrasts with the private values case. With private values, the simultaneous elections of two issues can prevent a Condorcet winner from being implemented (Ahn and Oliveros 2010, Example 1). This inefficiency is related to the wedge between the unconditional resolution of the second trial and the conditional resolution of the second trial when a voter is pivotal for the first trial. A straightforward solution is to close this wedge by having separate elections for the two issues. In fact, it is straightforward to show that, in the private values environment, with separate and disjoint committees, there is always a limit equilibrium that implements the Condorcet winner. However, this paper proves that this divergence between the joint and severed elections disappears in the common values case. This leaves an interesting divergence between the two extreme assumptions, one that is left for future work.

To outline the rest of the paper, Section 2 introduces the voting model and the two election formats. Section 3 presents examples that illustrate the different strategic considerations in the joint trial and the severed trials. As with a single issue, conditioning on being pivotal for the outcome is essential to equilibrium behavior. As observed by Austen-Smith and Banks (1996), being pivotal often eliminates the strategic incentive to vote sincerely. But deciding multiples issues introduces additional complications. The implications of being pivotal depend on whether the issues are joined or severed. In some situations, voting sincerely based on one's private signal is efficient and an equilibrium for the joint trial, but fails to be an equilibrium for the severed trials. In other situations, sincere voting is efficient and an equilibrium for the severed trials, but not for the joint trial.

The examples illustrate the strategic subtlety of voting over multiple issues. Nonetheless, Section 4 establishes an equivalence between weak information aggregation in a joint trial and in severed trials. Specifically, suppose that there exists a sequence of equilibria for one format such that the probability of correct verdicts goes to one as the number of voters goes to infinity. Then there exists an analogous asymptotically efficient sequence of equilibria for the other format.

Section 5 exploits this equivalence to provide conditions on the primitives that suffice for information aggregation. Section 6 considers two extensions of the equivalence result. First, we show that with three or more issues, the asymptotic efficiency of any division of the issues implies the asymptotic efficiency of all divisions of the issues. Second, we show that equivalence between joint and severed trials is maintained when voters are allowed to abstain.

2 Model

There is a set $X = \{1, 2\}$ of two up-down issues that need to be decided, e.g. the passage of two referendums or the guiltiness of two defendants. The set of possible outcomes is the power set of bundles: $\mathcal{X} = \{\{1, 2\}, \{1\}, \{2\}, \emptyset\}$. In the referendums setting, each bundle corresponds to the set of initiatives that are approved. In the jury setting, each bundle outcome corresponds to the set of defendants that are found guilty.

Let Ω denote a finite set of states of the world. The prior probability of state ω is denoted $P(\omega)$. The common utility for all voters for outcome A when the state of the world is $\omega \in \Omega$ is denoted $U(A|\omega)$. We assume that there is a unique best outcome $A_\omega \in \mathcal{X}$ that maximizes $U(A|\omega)$ for each state of the world. For any set $A \in \mathcal{X}$, we let $A^1 = A \cap \{1\}$ and $A^2 = A \cap \{2\}$ denote the projections onto the first and second issues.

The finite set S is a set of possible signals. The conditional probability of signal $s \in S$ given ω is denoted $F(s|\omega)$. Given the state of the world ω , each voter receives a conditionally independent signal from the distribution $F(\cdot|\omega)$. The conditional product probability of the signal profile $\mathbf{s} = (s_1, \dots, s_I)$ is denoted $F(\mathbf{s}|\omega)$.

We consider two voting games. The first is a joint election on both issues, where a single committee of I voters decides both issues using q -majority rule. The set of possible ballots for each voter is $\mathcal{X} = \{\{1, 2\}, \{1\}, \{2\}, \emptyset\}$, where a ballot A is interpreted as voting for every issue in A and voting against every issue in the complement of A . If more than qI of the voters vote up on an issue, that issue passes. The final outcome is the set of issues that are supported by more than q fraction of the voters. Formally, the aggregation rule $\mathcal{F}(A_1, \dots, A_I)$ is defined by

$$\mathcal{F}(A_1, \dots, A_I) = \{x : \#\{i : x \in A_i\} \geq qI\}.$$

A strategy for juror i is a function $\sigma_i : S \rightarrow \Delta\mathcal{X}$ that assigns a distribution over ballots $\sigma_i(s)$ to each signal s . When $\sigma_i(s)$ is a degenerate point mass on the ballot A , we slightly abuse notation and write $\sigma_i(s) = A$. A profile $(\sigma_1, \dots, \sigma_I)$ of strategies is *symmetric* if $\sigma_i = \sigma_j$ for all i, j . When referring to symmetric strategy profiles, we drop the subscript. The common expected utility for the strategy profile $\boldsymbol{\sigma}(\mathbf{s}) = \sigma_1(s_1), \dots, \sigma_I(s_I)$ is

$$EU(\boldsymbol{\sigma}) = \int_{\Omega} \int_S \int_{\mathcal{X}^I} U(\mathcal{F}(A_1, \dots, A_I) | \omega) d\boldsymbol{\sigma}(\mathbf{s}) dF(\mathbf{s}|\omega) dP(\omega).$$

In the jury setting, this corresponds to a single trial for both defendants, where qI of the jurors must find each defendant guilty to reach a guilty verdict for that defendant. We refer to this game as a *joint trial*. We will study symmetric Nash equilibria of the joint trial.¹ We consider the limit of symmetric equilibria as the number of voters goes to infinity and let the subscript denote the size of the electorate rather than a specific individual voter. We are interested in whether a sequence

¹All symmetry assumptions are for expositional convenience. Suitable analogs of the results hold for possibly asymmetric strategies.

of strategies (σ_I) will enact the optimal outcome A_ω in large elections. In particular, we say that the *probability of error goes to zero* if, for every ω ,

$$\int_S \int_{\mathcal{X}^I} U(\mathcal{F}(A_1, \dots, A_I) | \omega) d\sigma_I(\mathbf{s}_I) dF(\mathbf{s}_I | \omega) \rightarrow U(A_\omega | \omega),$$

as I goes to infinity. The probability of error goes to zero if and only if the probability of the optimal outcome A_ω goes to one for every state of the world.

In the second game, a total of $2I$ voters are divided into two disjoint committees of I voters that decide each issue separately using q -majority rule.² Let the first $1, \dots, I$ voters constitute the first committee and the last $I + 1, \dots, 2I$ voters constitute the second committee. The voters in the first committee can either vote up or down on the first issue $\mathcal{X}_1 = \{\{1\}, \emptyset\}$ and the voters in the second committee can vote up or down on the second issue $\mathcal{X}_2 = \{\{2\}, \emptyset\}$. The outcome of the first committee is

$$\mathcal{F}^1(A_1, \dots, A_I) = \begin{cases} \{1\} & \text{if } \#\{i : A_i = \{1\}\} \geq qI \\ \emptyset & \text{otherwise} \end{cases}.$$

The outcome of the second committee $\mathcal{F}^2(A_{I+1}, \dots, A_{2I})$ is defined analogously. The outcome of the game is $\mathcal{F}^1(A_1, \dots, A_I) \cup \mathcal{F}^2(A_{I+1}, \dots, A_{2I})$. A strategy for a member i of the first committee is a function $\sigma_i^1 : S \rightarrow \{\{1\}, \emptyset\}$ and for a member j of the second committee is a function $\sigma_j^2 : S \rightarrow \{\{2\}, \emptyset\}$. A profile of strategies is *semi-symmetric* if $\sigma_i = \sigma_{i'}$ for all voters i, i' in the first committee and $\sigma_j = \sigma_{j'}$ for all voters j, j' in the second committee. The common expected utility for the strategy profile $(\boldsymbol{\sigma}^1(\mathbf{s}), \boldsymbol{\sigma}^2(\mathbf{s})) = (\sigma_1^1(s_1), \dots, \sigma_I^1(s_I), \sigma_{I+1}^2(s_{I+1}), \dots, \sigma_{2I}^2(s_{2I}))$ is

$$EU(\boldsymbol{\sigma}^1, \boldsymbol{\sigma}^2) = \int_\Omega \int_S \int_{[\mathcal{X}^1]^I \times [\mathcal{X}^2]^I} U(\mathcal{F}^1(A_1^1, \dots, A_I^1) \cup \mathcal{F}^2(A_{I+1}^2, \dots, A_{2I}^2) | \omega) d\boldsymbol{\sigma}(\mathbf{s}) dF(\mathbf{s} | \omega) dP(\omega).$$

In the jury setting, this corresponds to having a separate trial for each defendant. We refer to this game as *severed trials*. We will study semi-symmetric Nash equilibria of the severed trials. For a sequence of semi-symmetric strategies (σ_I^1, σ_I^2) , we say the *probability of error goes to zero* if, for all ω :

$$\int_S \int_{[\mathcal{X}^1]^I \times [\mathcal{X}^2]^I} U(\mathcal{F}^1(A_1^1, \dots, A_I^1) \cup \mathcal{F}^2(A_{I+1}^2, \dots, A_{2I}^2) | \omega) d\boldsymbol{\sigma}(\mathbf{s}) dF(\mathbf{s} | \omega) \rightarrow U(A_\omega | \omega),$$

as I goes to infinity. As in the joint trial, the probability of error goes to zero if and only if the the probability of both trials reflecting the optimal outcome A_ω goes to one.

²For a fixed integer I , there are more voters in the split juries game than in the unified jury game. However, since our results consider information aggregation in large elections, this formality is irrelevant.

3 Examples

The following examples illustrate some of the strategic subtleties in deciding multiple issues with common values. In all the examples, sincere voting is informative and asymptotically efficient. However, sincere voting is incentive compatible in either the joint trial or the severed trials, but not incentive compatible in the other format.

With a single issue, being pivotal for the outcome provides additional information regarding the state of the world. This conditioning often precludes sincere or informative voting from being an equilibrium (Austen-Smith and Banks 1996). Since incentive compatibility is maintained in one format but not the other, the strategic reasoning in the following examples is necessarily distinct from the standard story. In the first example, sincere voting is not incentive compatible in the joint trial because each voter can deviate on both issues simultaneously, while this deviation is precluded by severing the trials. In the second example, sincere voting is not incentive compatible because each voter conditions the resolution of the second issue on being pivotal for the first, while this conditioning is irrelevant in the severed trials. In the third example, the joint trial allows voters to coordinate across issues, while this coordination is not possible in the severed trials.

In the first example, two defendants are accused of the same crime and exactly one is guilty. The sincere strategy profile is efficient and incentive compatible in the severed trials. However, it is not an equilibrium in the joint trial. This is because the space of deviations is larger in the joint trial: when she is pivotal for either defendant, any juror is better off finding both defendants innocent, an option that is not available to her in the severed trials.

Example 1 (Too many actions in joint trial). Let $q = \frac{1}{2}$. Suppose $\Omega = \{\{1\}, \{2\}\}$ and $P(\omega) = \frac{1}{2}$ for every ω .

$$U(A|\omega) = \begin{cases} 1 & \text{if } A = \omega \\ \frac{2}{3} & \text{if } A = \emptyset, \{1, 2\} \\ 0 & \text{if } A = \omega^c \end{cases}$$

Exactly one of the defendants is guilty. Jurors are risk averse in the number of correct verdicts: the marginal utility for deciding at least one of the verdict correctly (which can be guaranteed by finding both innocent) rather than none of them correctly, $U(\emptyset|\omega) - U(\omega^c|\omega) = \frac{2}{3}$, is greater than the marginal utility for deciding both rather than only one of the verdicts correctly, $U(\omega|\omega) - U(\emptyset|\omega) = \frac{1}{3}$.

Suppose $S = \Omega$ and

$$F(s|\omega) = \begin{cases} \frac{3}{5} & \text{if } s = \omega \\ \frac{2}{5} & \text{if } s \neq \omega \end{cases}$$

First consider the severed trials. The sincere strategy profile $\sigma^1(s) = s \cap \{1\}$ and $\sigma^2(s) = s \cap \{2\}$ aggregates information and is also incentive compatible in large juries. To see this, consider a juror in the first trial who assumes she is pivotal for the first defendant. By information aggregation, this juror can be made arbitrarily confident that the second defendant will be convicted if guilty

and acquitted if innocent. When she is pivotal for the first trial, the other jurors' signals cancel each other, so her posterior is based on her private signal, namely that the probability of the state $\omega = \{1\}$ is $\frac{3}{5}$. Therefore, conditional on being pivotal, her expected utility after seeing the signal $s = \{1\}$ of convicting the first defendant can be made arbitrarily close to

$$\frac{3}{5}U(\{1\}|\{1\}) + \frac{2}{5}U(\{1, 2\}|\{2\}) = \frac{3}{5} \times 1 + \frac{2}{5} \times \frac{2}{3}.$$

On the other hand, her expected utility after seeing the signal $\{1\}$ of acquitting can be made arbitrarily close to

$$\frac{3}{5}U(\emptyset|\{1\}) + \frac{2}{5}U(\{2\}|\{2\}) = \frac{3}{5} \times \frac{2}{3} + \frac{2}{5} \times \frac{2}{3}.$$

The second quantity is strictly less than the first. So for sufficiently large jury sizes, this juror's best response is to vote for conviction when she sees a guilty signal. The other cases can be argued symmetrically.

Now consider the joint trial. The sincere strategy $\sigma(s) = s$ aggregates information, but is not incentive compatible. To see this, suppose juror i observes signal $s_i = \{1\}$. Suppose she is pivotal for some issue. Then half of the other voters submitted the ballot $\{1\}$ and the other half submitted the ballot $\{2\}$, so she is pivotal for both issues. Moreover, the other voters' signals cancel themselves and her posterior based on her private signal $\{1\}$ is that the probability $\omega = (1, 0)$ is $\frac{3}{5}$. Then voting to convict the first defendant alone, i.e. submitting the ballot $\{1\}$, provides an expected utility of $\frac{3}{5}U(\{1\}|\{1\}) + \frac{2}{5}U(\{1\}|\{2\}) = \frac{3}{5}$. On the other hand, voting to acquit both defendants, i.e. submitting the ballot \emptyset , provides a strictly greater (sure) expected utility of $\frac{2}{3}$. So the suggested strategy is not incentive compatible. Interestingly, if the space of admissible ballots had been restricted to include only $\{1\}$ or $\{2\}$, then sincerity would be incentive compatible by arguments from the literature on strategic information aggregation with binary outcomes.

In the next example, sincerity is again efficient and incentive compatible for severed trials, but again not an equilibrium in the joint trial. The reasoning is different, however. Here, in the joint trial, being pivotal on the first issue provides information that the other voters are making an error on the second issue.

Example 2 (Informational linkage in joint trial). Let $q = \frac{1}{2}$ and suppose $\Omega = \{\{1, 2\}, \{1\}, \{2\}, \emptyset\}$. It is better to mismatch both issues with the state of the world than to match just a single issue. Formally, let

$$U(A|\omega) = \begin{cases} 1 & \text{if } A = \omega \\ \frac{1}{2} & \text{if } A^1 \neq \omega^1 \text{ and } A^2 \neq \omega^2 \\ 0 & \text{otherwise} \end{cases}$$

Suppose $S = \Omega \times \{+, \circ\}$. Recall our notation for issue-by-issue projections of a set A : $A^1 = A \cap \{1\}$ and $A^2 = A \cap \{2\}$. We can therefore denote a signal by three components: $s = (s^1, s^2, s^3)$, with

the first two components being the projections onto the two issues. The signal's last component, + or \circ , denotes its accuracy. The $(\omega^1, \omega^2, +)$ signal guarantees that the state of the world is ω . Formally, fix $\delta > 0$ and let

$$F(s|\omega) = \begin{cases} \delta & \text{if } s = (\omega, +) \\ \frac{2}{3} - \delta & \text{if } s = (\omega, \circ) \\ \frac{1}{6} & \text{if } s^1 = \omega^1, s^2 \neq \omega^2, s^3 = \circ \\ \frac{1}{6} & \text{if } s^1 \neq \omega^1, s^2 \neq \omega^2, s^3 = \circ \\ 0 & \text{otherwise} \end{cases}$$

In words, there is a $\frac{2}{3}$ probability of getting a signal that matches the state of the world on both issues, a $\frac{1}{6}$ probability of getting a signal that matches issue 1 but mismatches issue 2, and a $\frac{1}{6}$ probability of getting a signal that mismatches both issues. Moreover, conditional on getting the aligned signal, there is a probability, captured by δ , that this signal is perfectly informative.

Now consider the sincere strategy profile $(\sigma^{1*}, \sigma^{2*})$ of the split jury environment defined by

$$\begin{aligned} \sigma^{1*}(s) &= s^1 \\ \sigma^{2*}(s) &= s^2 \end{aligned}$$

This is an equilibrium for sufficiently large I in the severed trials. To see this, first observe that the strategy profile aggregates information. Consider a voter in the first jury who observes $s = (\{1, 2\}, +)$. Then, she is almost sure that issue 2 will pass. So, her optimal vote is to vote up on issue 1. Now consider a voter in the first jury who receives the signal $(\{1, 2\}, \circ)$. Because of information aggregation, she can be sure that the second jury will match issue 2 with the state of the world. When she is pivotal in the first jury, she knows that half the other voters in the first jury observed a signal with $s^1 = \{1\}$, i.e. $s = \{1, 2\}$ or $s = \{1\}$, and the other half observed a signal with $s^1 = \emptyset$, i.e. $s = \{2\}$ or $s = \emptyset$. Given that the probability that $s^1 = \omega^1$ is the same in all states of the world, the other voters' signals regarding the correct resolution of issue 1, i.e. regarding ω^1 , cancel themselves. So, this pivotal voter should follow her own signal regarding issue 1, since if she is correct she will very likely get utility 1 and if she is wrong she will very likely get utility 0 (because issue 2 will be correctly passed). A similar analysis for the other cases demonstrates that $(\sigma^{1*}, \sigma^{2*})$ is an equilibrium in a large electorate.

However, consider the sincere strategy profile $\sigma(s_i) = (s)$ for the joint trial. This strategy is not an equilibrium. Suppose a voter i receives signal $s_i = (\{1, 2\}, +)$. She is perfectly informed that the state of the world is $\{1, 2\}$. Unconditionally, she can be confident that issue 2 will pass. But whenever she is pivotal on issue 1, half of the residual voters voted down on issue 1 because they received the signal \emptyset . These voters will also vote down on issue 2. So, when she is pivotal on issue 1, this voter knows that the voters are making an error on issue 2. Given the structure of payoffs, this voter should therefore vote against issue 1, yielding utility $\frac{1}{2}$ rather than 0.

This voter, after receiving signal $(\{1, 2\}, +)$, knows the state of the world perfectly, so her strategic considerations are unrelated to whether she properly updates her posterior regarding the underlying state. However, simultaneously deciding two issues introduces an additional strategic concern. When she is pivotal for issue 1, she is sure that issue 2 will fail and mismatch the state of the world. This strategic linkage across issues is broken by severing the issues into separate elections.

In the last example, sincere voting is efficient and incentive compatible in the joint trial, but fails to be incentive compatible in the severed trials. In this environment, the two issues are substitutes: it is best to pass one issue or the other, but it is very bad to pass both issues together. The joint trial provides a way for voters to coordinate their votes, but this coordination is broken when the trials are severed.

Example 3 (No coordination in severed trials). Suppose $\Omega = \{\{1\}, \{2\}\}$ and $S = \{\{1\}, \{2\}\}$. Let

$$U(A|\omega) = \begin{cases} 1 & \text{if } A = \omega \\ \frac{3}{4} & \text{if } A^1 \neq \omega^1 \text{ and } A^2 \neq \omega^2 \\ \frac{1}{2} & \text{if } A = \emptyset \\ 0 & \text{if } A = \{1, 2\} \end{cases}$$

Suppose

$$F(s|\omega) = \begin{cases} \frac{3}{5} & \text{if } s = \omega \\ \frac{2}{5} & \text{if } s \neq \omega \end{cases}$$

For example, suppose that a state that faces excess traffic can either build a new highway or a new high speed railway. One of the options is better than the other. Conditional on the state of the world, the best outcome is to build the better option, but even the inferior option is better than doing nothing at all. However, the worst possible outcome would be spending the money to build both the highway and the railway.

The sincere strategy profile $\sigma^*(s) = s$ is an equilibrium of a single election, and takes the probability of an error to zero. To see that it is an equilibrium, consider a voter who sees the signal $\{1\}$. If she is pivotal for either issue, then she is pivotal for both issues. Then her expected utility for submitting the ballot $\{1\}$ is

$$\frac{3}{5}U(\{1\}|\{1\}) + \frac{2}{5}U(\{2\}|\{2\}) = \frac{3}{5} \times 1 + \frac{2}{5} \times \frac{3}{4}.$$

Her expected utility for submitting the ballot $\{2\}$ is

$$\frac{3}{5}U(\{2\}|\{1\}) + \frac{2}{5}U(\{2\}|0, 1) = \frac{3}{5} \times \frac{3}{4} + \frac{2}{5} \times 1.$$

This is strictly worse than submitting $\{1\}$. Finally, when she is pivotal, her (sure) expected utility for submitting the ballot $\{1, 2\}$ is 0, and her (sure) expected utility for submitting the ballot \emptyset is $\frac{1}{2}$. These are both worse as well. So sincere voting is incentive compatible in the single election.

However, the associated strategy profile $\sigma^1(s) = [\sigma^*(s)]^1$ and $\sigma^2(s) = [\sigma^*(s)]^2$ is not an equilibrium if the issues are decided separately by disjoint committees. To see this, consider a voter i in the first committee and observes the signal $s_i = \{1\}$. When she is pivotal, the other voters' signals cancel each other and her posterior probability is simply based on her private signal. So, the posterior probability of $\omega = \{1\}$ is $\frac{3}{5}$. With probability approaching one, if $\omega = \{1\}$, then the second committee playing strategy σ^2 will vote against the railway, while if $\omega = \{2\}$ it will support the railway. So, for sufficiently large I , her expected utility for voting for the highway can be made arbitrarily close to

$$\frac{3}{5}U(\{1\}|\{1\}) + \frac{2}{5}U(\{1, 2\}|\{2\}) = \frac{3}{5}.$$

Her expected utility for voting against the highway can be made arbitrarily close to

$$\frac{3}{5}U(\{0\}|\{1\}) + \frac{2}{5}U(\{2\}|\{2\}) = \frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times 1 = \frac{7}{10}.$$

So, for a sufficiently large I , her best response to this strategy profile is to vote down on issue 1. Hence, the associated strategy (σ^1, σ^2) in the split juries game is not an equilibrium.

4 Asymptotic equivalence

The examples in Section 3 suggest that the strategic considerations are different in joint and severed trials. Nonetheless, we demonstrate that if there exists an efficient sequence of equilibria in either format, then there exists an efficient sequence of equilibria in the other. Therefore, an argument for the superiority of either format cannot hinge on information aggregation with many voters, but must appeal to other considerations.

Proposition 1. *There exists a sequence of symmetric equilibria (σ_I^*) in the joint trial such such that the probability of error goes to zero if and only if there exists a sequence of semi-symmetric equilibria $(\sigma_I^{1*}, \sigma_I^{2*})$ in the severed trials such that the probability of error goes to zero .*

Proposition 1 is a corollary of the following two lemmata. The first adapts an insight of McLennan (1998) for common value elections: if any strategy profile aggregates information, then there exists a Nash equilibrium that aggregates information.

Lemma 1 (McLennan 1998). *The following are true:*

- (i) *If there exists a sequence of symmetric strategies (σ_I) in the joint trial such that the probability of error goes to zero, then there exists a sequence of symmetric equilibria (σ_I^*) such that the probability of error goes to zero.*

- (ii) *If there exists a sequence of semi-symmetric strategies (σ_I^1, σ_I^2) in the severed trials such that the probability of error goes to zero, then there exists a sequence of semi-symmetric equilibria $(\sigma_I^{1*}, \sigma_I^{2*})$ such that the probability of error goes to zero.*

Proof. We will prove the first claim, the proof of the second claim is nearly identical. Consider a fixed I . A straightforward adaptation of the proof of Theorem 2 of McLennan (1998) demonstrates that if $\bar{\sigma}_I$ maximizes the common expected utility of the agents among all symmetric strategy profiles, then it is a symmetric equilibrium. The common expected utility $EU(\sigma_1, \dots, \sigma_I)$ is a continuous function on the compact space of symmetric strategy profiles, so the maximizing $\bar{\sigma}_I$ exists and is an equilibrium.

Now suppose some sequence (σ_I) take the error probability to zero, i.e. the common expected utility goes to one. Then the sequence of equilibria $(\bar{\sigma}_I)$ must also take the error probability to zero. If it did not, then there would be a state of the world ω where the probability of an error is strictly positive for arbitrarily large juries. Then the common expected utility of $(\bar{\sigma}_I)$ strictly less than the common expected utility achieved by (σ_I) , which would contradict its optimality over all symmetric strategy profiles. \square

The standard application of McLennan’s observation is to argue for efficiency *within* a fixed voting institution: for example, McLennan (1998) shows that if sincere voting aggregates information, then there exists some equilibrium that also aggregates information. In contrast, we use McLennan’s observation to argue *across* institutions: we show that information aggregation under one mechanism implies information aggregation under *another* mechanism. In particular, if (σ_I) aggregates information in a joint trial, then the corresponding strategies in the separated trials where each juror in trial x plays the marginal distribution of $\sigma(s)$ for issue x also aggregates information. Conversely, if (σ_I^1, σ_I^2) achieves full efficiency in the separated trials, then the strategies in the joint trial defined by the product distribution of σ_I^1 and σ_I^2 also achieve full efficiency.

Lemma 2. *There exists a sequence (σ_I) of symmetric strategies for the joint trial that takes the probability of error to zero if and only if there exists a sequence (σ_I^1, σ_I^2) of semi-symmetric strategies for severed trials that takes the probability of error to zero.*

Proof. We first prove the “only if” direction. Suppose there exists a sequence (σ_I) of symmetric strategies for the joint trial that takes the probability of error to zero. Now consider the following semi-symmetric strategies for the split trials:

$$\begin{aligned} [\sigma_I^1(s)](\{1\}) &= [\sigma_I(s)](\{1, 2\}) + [\sigma_I(s)](\{1\}) \\ [\sigma_I^2(s)](\{2\}) &= [\sigma_I(s)](\{1, 2\}) + [\sigma_I(s)](\{2\}) \end{aligned}$$

Without loss of generality, consider a state ω where the optimal outcome is $A_\omega = \{1, 2\}$. The vote count on issue 1 in the joint trial when voters use strategy σ_I follows a binomial distribution of I

draws with a success probability equal to the probability of including issue 1 in the ballot:

$$\sum_{s \in S} F(s|\omega) ([\sigma_I(s)](\{1, 2\}) + [\sigma_I(s)](\{1\})).$$

By construction, this is exactly the distribution of the vote count in the first trial when voters use strategy σ_I^1 . By assumption, the probability that the vote count on the first issue is greater than or equal to qI goes to one in the joint trial, so the probability that vote count in the first of the severed trials is greater than or equal to qI also goes to one. Similarly, the probability the vote count on the second issue is greater than or equal to qI also goes to one.

To prove the “if” direction, suppose there exists a sequence (σ_I^1, σ_I^2) of semi-symmetric strategies for severed trials that takes the probability of error to zero. Consider the following symmetric strategies for the unified trial:

$$[\sigma_I(s)](A) = [\sigma_I^1(s)](A \cap \{1\}) \times [\sigma_I^2(s)](A \cap \{2\}).^3$$

Without loss of generality, consider a state ω where the optimal outcome is $A_\omega = \{1, 2\}$. Then the conditional probability that both issues will pass in the severed trials goes to one. The vote count in the first of the severed trials follows a binomial distribution defined by I draws with a success probability of

$$\sum_{s \in S} F(s|\omega) [\sigma_I^1(s)](\{1\}).$$

The vote count on the first issue in the joint trial follows a binomial distribution with success probability of

$$\begin{aligned} & \sum_{s \in S} F(s|\omega) \{[\sigma_I(s)](\{1, 2\}) + [\sigma_I(s)](\{1\})\} \\ &= \sum_{s \in S} F(s|\omega) \{[\sigma_I^1(s)](\{1\}) \times [\sigma_I^2(s)](\{2\}) + [\sigma_I^1(s)](\{1\}) \cdot [\sigma_I^2(s)](\emptyset)\} \\ &= \sum_{s \in S} F(s|\omega) \{[\sigma_I^1(s)](\{1\}) \times ([\sigma_I^2(s)](\{2\}) + [\sigma_I^2(s)](\emptyset))\} \\ &= \sum_{s \in S} F(s|\omega) [\sigma_I^1(s)](\{1\}). \end{aligned}$$

So, the probability that the vote count on the first issue in the joint trial will be greater than or equal to qI hereditarily goes to one. Similarly, the probability the second issue passes in the joint trial also goes to one. \square

We should what mention Proposition 1 leaves open. It only maintains the equivalence of the existence of an asymptotically efficient sequence of equilibria across formats. There could exist an additional inefficient sequence of equilibria in one format, but with no analogous sequence in the

³It is straightforward to verify that this is a well-defined mixed strategy.

other format, leaving miscoordination as a potential disadvantage of one format. There is also no comparison of welfare with finite voters.⁴ Finally, in cases where information fails to aggregate, Proposition 1 provides no guidance regarding which environment is superior.

5 Sufficient conditions for information aggregation

A useful application of Proposition 1 is generating sufficient conditions for information aggregation in the joint trial. Sufficient conditions from the single-issue case can be suitably adapted for the separated trials, since the separate trial for the first or the second defendant is a single-issue trial. Proposition 1 proves that such conditions are also sufficient for information aggregation in the joint trial.

Definition 1. Let $\Omega_x = \{\omega \in \Omega : x \in A_\omega\}$. The signal technology $F(\cdot|\cdot)$ is **informative for issue x** if there exist a pair of disjoint subsets $S_x, T_x \subset S$ such that

$$\begin{aligned} F(S_x|\omega) &> F(T_x|\omega) \text{ for all } \omega \in \Omega_x \\ F(S_x|\omega) &< F(T_x|\omega) \text{ for all } \omega \notin \Omega_x. \end{aligned}$$

The technology is **informative** if it is informative for both issues.

Austen-Smith and Banks (1996) assumed an analogous environment with a single issue and Wit (1998) showed that this assumption guarantees information aggregation in equilibrium. Specifically, in these papers there are two states of the world (“innocent” and “guilty”) and two corresponding signals, with the probability of a correct signal greater than one half in either state. Definition 1 adapts this assumption to our environment, with potentially many states and many signals, by imposing it across states and across signals.⁵

Proposition 2. *Suppose $q \in (0, 1)$. If the signal technology $F(\cdot|\cdot)$ is informative, then there exists a sequence of semi-symmetric strategies in the severed trials that takes the probability of error to zero.*

Proof. Without loss of generality, assume $q \geq \frac{1}{2}$. Consider the first trial in the split juries environment. We will describe a strategy by a function $\sigma : S \rightarrow [0, 1]$ that takes each signal s into a probability $\sigma(s)$ of submitting the ballot $\{1\}$ and a probability $1 - \sigma(s)$ of submitting the ballot \emptyset .

⁴A difficulty in comparing finite trials is that splitting the trials would halve the number of signals for each trial (or would require twice as many jurors). While this is irrelevant for our asymptotic treatment, the effects of jury sizes must be explicitly controlled in the finite case, and has different small-sample statistical implications.

⁵Definition 1 is technically more general than some other assumptions in the literature. In particular, our definition of an informative technology does not require: first, any global monotone likelihood ratio property, or second, any assumptions about the strength of the signal in relation to the preference of the voters. Duggan and Martinelli (2001) use both previous assumptions (see assumptions A3 and A4 in page 266) but allowing for continuous signals. Feddersen and Pesendorfer (1997) use the first assumption (see assumption 4 in page 1033) but allowing for some private component in voter’s preferences. In the standard two signal–two state model, our assumption is equivalent to Austen-Smith and Banks (1996) and Wit (1998).

Let $\delta = \frac{1-q}{2}$, which is strictly positive because $q \neq 1$. We study the following strategy:

$$\sigma(s) = \begin{cases} q + \delta & \text{if } s \in S_1 \\ q - \delta & \text{if } s \in T_1 \\ q & \text{if } s \notin S_1 \cup T_1. \end{cases}$$

Let

$$X_i = \begin{cases} 1 & \text{if } \sigma(s_i) = \{1\} \\ 0 & \text{otherwise} \end{cases}$$

be the indicator function on the event voter i (using strategy σ) supported issue 1. Without loss of generality, assume that the state of the world is $\omega \in \Omega_1$, i.e. the optimal outcome in state ω includes passing issue 1; the following statements are conditioning on ω . Issue 1 passes if and only if

$$\frac{\sum_{i=1}^I X_i}{I} \geq q.$$

The expectation of X_i , conditional on ω is:

$$E[X_i] = q(1 - F(S_1|\omega) - F(T_1|\omega)) + (q + \delta)F(S_1|\omega) + (q - \delta)F(T_1|\omega).$$

Since $F(S_1|\omega) > F(T_1|\omega)$, the inequality $E[X_i] > q$ holds. Let $\varepsilon = \frac{E[X_i] - q}{2} > 0$. By the weak law of large numbers, the probability

$$\Pr\left(\frac{\sum_{i=1}^I X_i}{I} \geq q\right) \geq \Pr\left(\frac{\sum_{i=1}^I X_i}{I} \geq E[X_i] - \varepsilon\right) \rightarrow 1.$$

So the probability that issue 1 will pass, in any state $\omega \in \Omega_1$ where passing issue 1 is part of the optimal outcome, goes to one.

The nearly identical argument establishes that if $\omega \notin \Omega_1$, then the probability of passing issue 1 goes to zero. \square

Corollary 1. *Suppose $q \in (0, 1)$. If the signal technology $F(\cdot|\cdot)$ is informative, then there exist a sequence of semi-symmetric equilibria in the severed trials and a sequence of symmetric equilibria in the joint trial that take the probability of error to zero.*

We note that asymptotic efficiency is implied only when the decision rule is not unanimous, i.e. when $q \neq 0, 1$. The restriction to non-unanimous rules reiterates findings from the single-issue case (Feddersen and Pesendorfer 1998).

6 Extensions

In this section, we consider the following two extensions. First, suppose there are three or more issues. Then there are myriad ways to divide the issues into committees. If any of these divisions yields an asymptotically efficient sequence of equilibria, than any of these divisions will yield a corresponding efficient sequence as well. Second, suppose voters can abstain from participating in a particular election. The equivalence of efficiency in joint and severed trials is robust to allowing for abstention.

6.1 More than two issues

Now suppose the space of issues X is arbitrary. There are multiple ways to divide the issues into separate committees. Let $\Pi = \{X^1, X^2, \dots, X^N\}$ be a partition of X . Each committee $n = 1, \dots, N$ decides all of the issues in its associated set of issues X^n . Committee n must decide what subset $A^n \in \mathcal{X}^n = 2^{X^n}$ should be passed. A strategy for a voter in committee n is a map $\sigma^n : S \rightarrow \mathcal{X}^n$ that takes signals into ballots over the relevant issues.

Proposition 3. *Suppose there exists a partition Π of X such that there exists a sequence $(\sigma_I^1, \dots, \sigma_I^N)$ of semi-symmetric equilibria such that the error probability goes to zero. Then for any partition $\Pi' = \{Y^1, \dots, Y^M\}$ of X , there exists a sequence of semi-symmetric equilibria that take the error probability to zero.*

Proof. The proposition is proven using two lemmas that are analogous to Lemmas 1 and 2. First, McLennan’s reasoning regarding common interest games is maintained in the setting with many issues.

Lemma 3. *If there exists a sequence of strategies $(\sigma_I^1, \dots, \sigma_I^N)$ for the Π -severed trials that takes the error probability to zero, then there exists a sequence of semi-symmetric equilibria $(\sigma_I^{1*}, \dots, \sigma_I^{N*})$ for the Π -severed trials that takes the probability of error to zero.*

Let $\Pi^* = \{\{x\} : x \in X\}$ denote the partition of singletons. The second lemma

Lemma 4. *There exists a sequence of semi-symmetric strategies $(\sigma_I^1, \dots, \sigma_I^N)$ for the Π -severed trials such that the probability of error goes to zero if and only if there exists sequence of semi-symmetric strategies $(\sigma_I^1, \dots, \sigma_I^{|X|})$ for the Π^* -severed trials such that the probability of error goes to zero.*

Proof. The proof is nearly identical to the argument for Lemma 2. To prove the “only if” direction, suppose $(\sigma_I^1, \dots, \sigma_I^N)$ takes the probability of error to zero in the Π -severed trials. Then the following semi-symmetric strategy:

$$[\sigma_I^x(s)](\{x\}) = \sum_{A: x \in A} [\sigma_I^k(s)](A),$$

where $x \in X^k$, takes the probability of error to zero in the Π^* -severed trials.

To prove the “if” direction, suppose $(\sigma_I^1, \dots, \sigma_I^{|X|})$ take the probability of error to zero in the Π^* -severed trials. Then the following semi-symmetric strategy:

$$\left[\sigma^k(s) \right] (A) = \prod_{x \in A} \sigma^x(A \cap \{x\})$$

takes the probability of error to zero in the Π -severed trials. \square

To argue the Proposition, suppose there exists a partition Π and a sequence of semi-symmetric equilibria $(\sigma_I^1, \dots, \sigma_I^N)$. By the first part of Lemma 4, there exists a sequence of semi-symmetric strategies in the Π^* -severed trials that takes the probability of error to zero. By the second part of Lemma 4, there exists a sequence of semi-symmetric strategies in the Π' -severed trials that takes the probability of error to zero. Finally, Lemma 3 provides a sequence of semi-symmetric equilibria in the Π' -severed trials that takes the probability of error to zero. \square

6.2 Abstention

We finally compare the joint and severed trials when voters are allowed to abstain. We should stress that we consider the joint trial where voters can abstain from one issue or defendant while voting on the other, i.e. that voters can “roll-off.” As in the single-issue case, the “swing voter’s curse” can induce strategic abstention (Feddersen and Pesendorfer 1996). But in addition, there will be new strategic considerations regarding abstention due to the interdependence of the issues. For example, equilibrium in the joint trial may involve roll-off. However, McLennan’s observation affords a comparison of formats without explicit computation of equilibria.

The space of pure actions in the joint trial is now $\mathcal{D} = \{(Y, N) \in \mathcal{X} \times \mathcal{X} : Y \cap N = \emptyset\}$. The interpretation is that the first set D^Y is the set of issues that the voter supports (or votes “Yes” for), the second set D^N is the set of issues that the voter rejects (or votes “No” against), and the complement $X \setminus [D^Y \cup D^N]$ is the set of issues from which the voter abstains.

Recall that $\mathcal{X}_1 = \{\emptyset, \{1\}\}$. In the severed trials, the pure actions in the first trial is $\mathcal{D}_1 = \{(Y, N) \in \mathcal{X}_1 \times \mathcal{X}_1 : Y \cap N = \emptyset\}$. Define \mathcal{D}_2 as the set of pure actions for the second trial analogously.⁶

The aggregation rule depends on the number of voters that submit a Yes or No ballot, rather than abstaining. In the joint trial, the final outcome is:

$$\mathcal{F}(A_1, \dots, A_I) = \left\{ x \in X : \frac{\#\{i : x \in A_i^Y\}}{\#\{i : x \in A_i^Y\} + \#\{i : x \in A_i^N\}} \geq q \right\}.$$

In the severed trials, the outcome of the first trial is:

$$\mathcal{F}^1(A_1, \dots, A_I) = \begin{cases} \{1\} & \text{if } \frac{\#\{i : A_i^Y = \{1\}\}}{\#\{i : A_i^Y = \{1\}\} + \#\{i : A_i^N = \{1\}\}} \geq q \\ \emptyset & \text{otherwise} \end{cases}.$$

⁶The set notation is used here to maintain consistency with the basic model.

$\mathcal{F}^2(A_{I+1}, \dots, A_{2I})$ is defined analogously. The outcome of the severed trials is $\mathcal{F}^1(A_1, \dots, A_I) \cup \mathcal{F}^2(A_{I+1}, \dots, A_{2I})$.

The implied Bayesian games can be defined verbatim using the definitions in Section 2, using the suitably modified notation that was just introduced. As in the case without abstention, the existence of an efficient sequence of equilibria in one format guarantees an analogous sequence in the other format.

Proposition 4. *There exists a sequence of symmetric equilibria (σ_I^*) in the joint trial with abstention such that the probability of error goes to zero if and only if there exists a sequence of semi-symmetric equilibria $(\sigma_I^{1*}, \sigma_I^{2*})$ in the severed trials with abstention such that the probability of error goes to zero.*

The proof of Proposition 4 is essentially similar to the proof of Proposition 1. We begin by recording without proof the counterpart of McLennan’s insight.

Lemma 5. *The following are true:*

- (i) *If there exists a sequence of symmetric strategies (σ_I) in the severed trials with abstention such that the probability of error goes to zero, then there exists a sequence of symmetric equilibria (σ_I^*) such that the probability of error goes to zero.*
- (ii) *If there exists a sequence of semi-symmetric strategies (σ_I^1, σ_I^2) in the joint trial with abstention such that the probability of error goes to zero, then there exists a sequence of semi-symmetric equilibria $(\sigma_I^{1*}, \sigma_I^{2*})$ such that the probability of error goes to zero.*

Next, we observe that an efficient strategy in one format can be translated into an efficient strategy in the other.

Lemma 6. *There exists a sequence of symmetric strategies (σ_I) in the joint trial with abstention such that the probability of error goes to zero if and only if there exists a sequence of semi-symmetric strategies (σ_I^1, σ_I^2) in the severed trials with abstention such that the probability of error goes to zero.*

Proof. To prove the “only if” direction, suppose (σ_I) takes the probability of error to zero in the joint trial. Define

$$\begin{aligned} [\sigma^1(s)](\{1\}, \emptyset) &= [\sigma_I(s)](\{1, 2\}, \emptyset) + [\sigma_I(s)](\{1\}, \emptyset) + [\sigma_I(s)](\{1\}, \{2\}) \\ [\sigma^1(s)](\emptyset, \{1\}) &= [\sigma_I(s)](\emptyset, \{1\}) + [\sigma_I(s)](\emptyset, \{1, 2\}) + [\sigma_I(s)](\{2\}, \{1\}) \\ [\sigma^1(s)](\emptyset, \emptyset) &= [\sigma_I(s)](\emptyset, \emptyset) + [\sigma_I(s)](\emptyset, \{2\}) + [\sigma_I(s)](\{2\}, \emptyset). \end{aligned}$$

In words, the probability of voting up, voting down, or abstaining in the first trial is equal to the probability of voting up, voting down, or abstaining on the first issue in the joint trial. Define $\sigma^2(s)$ analogously. Then if σ_I takes the probability of error to zero in the joint trial, then (σ^1, σ^2) also takes the probability of error to zero in the severed trials.

To prove the “if” direction, suppose (σ_I^1, σ_I^2) takes the probability of error to zero in the severed trials. Define

$$[\sigma_I(s)](D^Y, D^N) = \sigma^1(s)[D^Y \cap \{1\}, D^N \cap \{1\}] \cdot \sigma^2(s)[D^Y \cap \{2\}, D^N \cap \{2\}].$$

Then (σ_I) hereditarily takes the probability of error to zero in the joint trial. □

Finally, we note that an informative signal technology, as defined in Definition 1, is sufficient for information aggregation and asymptotic efficiency when abstention is allowed. To see this, observe that any strategy in the joint trial or severed trials without abstention can be replicated in the joint trial or severed trials with abstention, namely by not having any voter abstain. Therefore, again using McLennan’s insight, an equilibrium that delivers at least as much common expected utility must exist in the game with abstention.

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