Obfuscating to Persuade[*]

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Abstract

A decision maker faces a choice between a risky new policy and a risk-free status quo, and consults an expert adviser who knows how risky the new policy can be and how to best mitigate the risk. The adviser wants to persuade the decision maker to adopt the new policy through his report. This paper shows that strategic communication is characterized by *obfuscation*: the adviser gives a vague report to signal risklessness, even though it causes the decision maker to misinterpret how to execute the policy. The less detailed the report is, the less risky the new policy sounds. In contrast to the past finding, the adviser's message becomes more vague as the parties' interests becomes more congruent: vagueness signals congruence.

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1 Introduction

A decision maker often consults an expert adviser about a risky new policy or project. A large literature has studied various aspects of this important economic situation as signaling games. This paper presents a novel feature of strategic communication: an adviser *obfuscates* policy information to *persuade* a decision maker to adopt a risky policy. The main insight is that the adviser's desire to persuade the decision maker makes him to obfuscate policy information to signal that the policy risk is low. This paper develops the idea by shedding light on the nature of language.

In a variety of situations, an expert adviser knows both how risky a new policy can be and how to best mitigate the risk. For instance, a financial adviser knows how good the economy is and which company's stocks to buy, a legislative committee knows how uncertain a new infrastructure investment plan is and which area makes the investment most effective, and a cookbook author knows how difficult a particular dish is and what procedures make the dish most delicious. When the decision maker has a risk-free alternative such as a status quo, it is only when he thinks that the risk is low relative to his valuation of the new policy that he takes a chance on attaining a higher outcome. In an environment in which the adviser values a new policy more than the decision maker does, the adviser tends to claim that the risk magnitude is low even when it is not. This paper is concerned about understanding how such a credibility issue influences the way the adviser elaborates on a policy parameter.

The incentive issue arises in the model because the adviser puts a higher value on adoption of a new policy than the decision maker does. For instance, a financial adviser gets a bonus from his employer if his client buys financial commodities. A committee member receives a commission from construction industry if an investment plan is accepted by the legislature. A cookbook author attains an intrinsic satisfaction if his reader tries his recipe. This difference in policy valuation is the only conflict of the parties' interests in the model. Importantly, we do not assume that the adviser has a bias in terms of how to execute the new policy. Since both parties want to mitigate the policy risk as much as they can, the adviser prefers to tell the decision maker the policy parameter precisely if the policy is executed. What prevents precise information transmission is the adviser's motive to convey information about risk magnitude. To increase the attraction of the new policy he claims that the risk magnitude is low. Since such a claim is not credible, he needs to show some evidence to the decision maker. This is manifested in his act of obfuscation.

Obfuscation comes with cost to both parties. If the adviser speaks vaguely about the policy parameter, then the decision maker is likely to misinterpret his true intention. For instance, a recommendation to buy "a company in the future energies industry" can be interpreted to mean a solar energy company or a geothermal energy company. A statement that investment is most required at "areas of a severe traffic jam" can be understood to indicate Brentwood or Richmond Heights. A recipe instruction to deep-fry shrimp "thoroughly in medium high heat" can be followed as to fry it in 320 degrees for two minutes or 350 degrees for three minutes. The informational loss from misinterpretation is less significant when the risk magnitude is lower. The lower the risk magnitude is, the more interpretive loss the adviser can afford. This creates the adviser to use a vague speech as a costly signal of risklessness.

Obfuscation for a signaling purpose is observed extensively. For instance, a financial adviser may only mention that future energies industry is good, when he is asked, at the time of economic boom, which company's stocks he recommends. A committee may just report that any area of a severe traffic jam is a profitable site for an infrastructural investment, if he is certain about the general profitability of investment in that city. A cookbook offers a terser recipe of grilled salmon than that of fish and chips, because it is less hazardous in terms of a cooking outcome to grill salmon over coals than to deep-fry fish in hot oil.

This paper relates to several strands of literature. Since the seminal paper by Crawford and Sobel [4], the various aspects of strategic communication have been analyzed.

This paper adds a new finding to the literature: a sender sacrifices a part of information transmission to signal another part of information. The model appears to be a costless signaling game with two dimensional private information. Yet, the way communication is described makes it rather different from typical cheap talk models.

An interesting feature of this paper is that costly signaling emerges when cheap talk is possible. In the literature of signaling games, there existed a methodological divide in that a signal is either costly or costless. One adopted either approach in a model as a plausible assumption to address a motivating example. But often times the boundary was quite fuzzy. Recently, Kartik et al. [10] and Kartik [9] studied two important questions: when signal is costly or costless, and how signaling cost affects economic analysis. In this paper, a sender has an option to send whether a costless signal or a costly signal, and chooses the latter over the former. All signals are costless to send but a vague signal incurs *interpretive cost* to both parties. The result suggests that further exploration on this topic may be possible.

The idea that a speaker's intention does not automatically translate to a listener's interpretation has been recently picked up by several authors. Dewatripont and Tirole [8] argue that communication is a team play by a speaker and a listener, both of whom need to incur effort costs for successful communication. Other authors assume that a speaker is able to manipulate the degree of vagueness by dint of language. Blume et al. [3] and Blume and Board [2] point out the possibility that a vague speech can facilitate communication in cheap talk environments. Dewan and Myatt [7] pay attention to issues of leadership in coordination environments and show that a certain type of leaders may obfuscate their message to attract an audience. This paper advances this line of research.

In terms of strategic environment, this paper is close in spirit to Che et al. [5]. As this paper emulates their title, it shares the setting with Che et al. [5] in which the expert adviser tries to persuade the decision maker to follow his advice instead of taking a status quo. The two papers differ in the other respects. They show that the adviser panders to better-looking projects for the sake of persuasion, whereas this paper shows that the adviser obfuscates policy information of less risky policies for persuasion.

The rest of the paper proceeds as follows. Section 2 describes the signaling game of interest. The model is set up in Section 2.1 and the interpretation of an important part of the model is provided in Section 2.2. Section 3 presents a prediction of the game. After a classification of equilibria, benchmark results are discussed in Section 3.1 and the most important class of equilibria are characterized in Section 3.2. Then a comparative statics result follows in Section 3.3. Section 4 discusses the robustness of the analysis and the last section concludes this paper. All proofs are found in Appendix.

2 The Model

2.1 Setup

There are two players: a female decision maker (DM) and a male expert adviser (A), where gender is assumed to ease the notation of personal pronouns. The decision maker is deciding whether to adopt a new policy ($p = 1$) or to keep a status quo ($p = 0$).

The new policy involves two kinds of risk. The one is purely informational and it increases quadratically with a distance between a policy execution a and a policy parameter x. The decision maker sets $a \in \mathbb{R}$ to reduce this risk given her information about $x \in \mathbb{R}$. The other is a non-informational risk and it is captured by a publicly known parameter $c \in (0, 1)$. Both risks are magnified by a common parameter $z \in [\underline{z}, 1] \equiv Z$, where $\underline{z} \in (0, 1)$, so that the total risk of the new policy is expressed as $z\left[(a - x)^2 + c\right]$. A pair (x, z) is the adviser's private information.

Utilities of player $i \in \{DM, A\}$ are expressed as:

$$u_i(p, a, x, z) = \begin{cases} v_i - z\left[(a - x)^2 + c\right] & \text{if } p = 1; \\ 0 & \text{if } p = 0, \end{cases}$$

where $v_i \in [0, 1]$ is i's valuation of the new policy. Since we are interested in situations in which the adviser is an advocate of the new policy, we assume that v_A is always higher than v_{DM} and fix $v_A = 1$. We let $v_{DM} = v$ for brevity. A scalar v is the decision maker's private information.

The initial common priors about (x, z, v) are assumed as follows. x has a flat (improper) prior on $\mathbb{R}$, which implies that the decision maker initially has no information about x and faces an infinite degree of informational risk. z is distributed according to an arbitrary distribution F with density f that has a full support on Z. v is uniformly distributed on [0,1], which captures the adviser's lack of information about v.[1]

The heart of this paper lies in a description of communication between the adviser and the decision maker. I briefly describe it here and discuss the underlying assumptions in Section 2.2. The adviser sends a costless signal (cheap talk) to the decision maker. The signal consists of an intention (sent message) $m_s \in \mathbb{R}$ and a degree of vagueness $b \in \mathbb{R}_+$. Listening to this signal, the decision maker identifies the vagueness level b and forms an interpretation (received message) m_r, which is a random variable drawn from $G(\cdot; m_s, b)$, the distribution with mean m_s and variance b; the density is $g(\cdot; m_s, b)$. Our assumption is that the decision maker cannot directly observe the adviser's intention m_s but can gain an unbiased estimator m_r of it from what she heard.

Timing of plays is as follows. First, Nature reveals (x, z) to the adviser and v to the decision maker. Second, the adviser sends a signal (m_s, b). Third, the decision maker observes b and m_r, a random draw from $G(\cdot; m_s, b)$. Last, the decision maker takes an action (p, a).

2.2 Interpretation of Setup

The adviser's signal is a pair (m_s, b). He can choose to talk about m_s in specific terms and set $b = 0$ so that the decision maker gets a "correct" interpretation $m_r = m_s$. Alternatively, he can choose to talk in vague terms and set $b > 0$ so that she gets an "incorrect" interpretation $m_r \neq m_s$ except in measure zero events.

[1]The uniform distribution assumption is more than necessary. We will discuss this in Section 4.

The reason why there can be a gap between the adviser's intention and the decision maker's interpretation is due to the nature of our language: language can be specific or vague depending on how it is expressed. One can identify an object or an issue precisely by carefully choosing words with enough information and without unnecessary information. Or one can obfuscate them by supplying information either in short or in excess, or by using rhetoric, so that a listener has to make interpretative efforts to understand what is heard.

Although we and the players in this game are apt at this art of language, this paper does not model it more than G is explained. In our description of communication we made four underlying assumptions: (i) the adviser can precisely manipulate the degree of vagueness; (ii) the decision maker can precisely discern a vagueness level of the adviser's signal; (iii) the decision maker's interpretation is an unbiased estimator of the adviser's intention; and (iv) the decision maker's interpretations are distributed with variance which corresponds to a vagueness level chosen by the adviser.

Note that players in the game are capable of identifying each $x \in \mathbb{R}$ and transmitting the information precisely through communication. No assumption is made for bounded rationality in this paper. What makes misinterpretation possible is the existence of vague language as a communication device, which I take as given. As Lipman [11] attempts, it is interesting to challenge a philosophical question why vague language exists, but my motivation in this paper is to study how it is used in strategic communication.

3 Equilibrium Analysis

We pay attention to perfect Bayesian equilibria. Let us introduce some notations to classify a set of equilibria. Let $\mu(x,z)$ and $\beta(x,z)$ be the adviser's pure strategies for intention and vagueness, respectively. We say that an equilibrium is *separating* if β is one-to-one, and *pooling* if $|\beta(T)| = 1$.

We focus our analysis on a class of equilibria in which the adviser is truthful in intention ($\mu(x,z) = x$) and vagueness is orthogonal to policy parameter ($\beta(x,z) = \beta(z)$).[2] Naturally we are most interested in this class because the parties have no conflict of interests about how to use an unbiased estimator $\theta(m_r, b)$ of x. They both agree that if the decision maker has no more information about x than $\theta(m_r, b)$, then she should set $a = \theta(m_r, b)$ in executing a new policy. With truthful intention, vagueness plays a role to convey information about risk magnitude. We elaborate on this in the subsequent subsections.

3.1 Benchmark Results

To see most clearly the roles of our model assumptions, we consider the two benchmark cases in which only one assumption is modified. In both benchmark cases, the most informative equilibrium is pooling without obfuscation ($\beta = 0$).

3.1.1 Absence of non-informational risk

Proposition 1 shows that if the policy risk is just informational, then risk magnitude is irrelevant since the risk is totally eliminated through communication. Here there is no need to signal risk magnitude.

Proposition 1. *When $c = 0$, an equilibrium exists in which the adviser sends a signal $(m_s, b) = (x, 0)$ and the decision maker takes an action $(p, a) = (1, m_r)$.*

In the absence of non-information risk, the adviser can totally eliminate the risk of a new policy by speaking precisely ($b = 0$) and revealing the realization ($m_r = m_s = x$) so that the decision maker always executes a new policy without incurring any risk.

[2] As is the case in costless signaling games, there exist a class of bubbling equilibria, in which no information about x is transmitted through the adviser's signal. Then, irrespective of information about z, the decision maker faces an infinite degree of informational risk in executing a new policy and therefore chooses a status quo ($p = 0$).

3.1.2 Certainty of risk magnitude

Proposition 2 shows that if the adviser's private information is just a policy parameter, then the adviser prefers to speak precisely and the informational risk is totally eliminated through communication. Here there is no information to signal.

Proposition 2. *When z is publicly known, an equilibrium exists in which the adviser sends a signal $(m_s, b) = (x, 0)$ and the decision maker takes an action $(p, a) = (1, m_r)$ if $v \geq zc$ and $p = 0$ otherwise.*

If there is no uncertainty about risk magnitude, then it is in the adviser's best interest to speak clearly ($b = 0$) and reveal the realization ($m_r = m_s = x$). If the decision maker turns out to value the new policy relatively high to non-informational risk, then she will execute it without incurring any informational risk.

Here we can see that the lower z is, the more likely the decision maker executes a new policy. Hence we can interpret the inverse of z as the degree of *congruence* between the parties' interests.[3]

3.2 Separating Equilibrium with Obfuscation

Recall that we focus on equilibria in which intention is truthful ($\mu(x, z) = x$) and vagueness is orthogonal to policy parameter ($\beta(x, z) = \beta(z)$). In what follows we first assume it and later confirm that this is compatible with the adviser's rationality. Let $\pi(b, v)$ and $\alpha(m_r, b)$ be the decision maker's pure strategies for policy selection and policy execution, respectively, and $\gamma(z; b)$ be the decision maker's posterior belief. Then we can express an equilibrium as a tuple $(\beta, \pi, \alpha, \gamma)$ such that the following conditions are satisfied.[4]

[3]This definition of congruence is different from the standard one in strategic communication literature, which is the inverse of the distance of the parties' bliss points in policy space.

[4]We dropped the dependence of α on v and of π on m_r since they are irrelevant. This will be apparent when we later characterize α and π.

(i) The adviser is rational in that for each (x, z),

$$\beta(z) \in \arg\max_{b\in\mathbb{R}_+} \int_0^1 \pi(b,v) \int_{-\infty}^{\infty} \left(1-z\left[\left(\alpha(m_r,b)-x\right)^2+c\right]\right) g(m_r;x,b)\, dm_r\, dv. \quad (1)$$

(ii) The decision maker is rational in that for each (m_r, b, v),

$$\alpha(m_r,b) \in \arg\max_{a\in\mathbb{R}} \int_{\underline{z}}^1 \int_{-\infty}^{\infty} \left(v - z\left[(a-x)^2 + c\right]\right) g(x;m_r,b)\, dx\, \gamma(z;b)\, dz, \quad (2)$$

$$\pi(b,v) = \begin{cases} 1 & \text{if } E_{x,z}\left[u_{DM}\left(1, \alpha(m_r,b), x, z\right)\right] \geq 0; \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

(iii) The decision maker derives her belief by Bayes' rule such that if $b \in \beta(Z)$ then

$$\gamma(z;b) = \begin{cases} \frac{f(z)}{\int_{\{\beta^{-1}(b)\}} f(z)\, dz} & \text{if } z \in \beta^{-1}(b); \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

Now we characterize an equilibrium. First, we can see here that quadratic utilities assumption makes algebra easy. For the decision maker, the second integral on the right-hand side of (2) simplifies to

$$\int_{\mathbb{R}} \left(v - z\left[(a-x)^2 + c\right]\right) g(x;m_r,b)\, dx = v - z\left[(a-m_r)^2 + b + c\right].$$

It then follows that $\alpha(m_r, b) = m_r$ and (3) is rewritten as

$$\pi(b,v) = 1 \text{ iff } v \geq \int_{\underline{z}}^1 z(b+c)\gamma(z;b)\, dz. \quad (5)$$

Now, for the adviser, the second integral on the right-hand side of (1) simplifies to

$$\int_{\mathbb{R}} \left(1 - z\left[(m_r - x)^2 + c\right]\right) g(m_r;x,b)\, dm_r = 1 - z(b+c), \quad (6)$$

and we can rewrite (1) as

$$\beta(z) \in \arg\max_{b\in\mathbb{R}_+} \int_0^1 \pi(b,v)\left[1 - z(b+c)\right] dv. \quad (7)$$

We can confirm in (6) that the adviser chooses to be truthful in intention ($\mu(x,z) = x$) and in (7) that vagueness does not convey information about policy parameter ($\beta(x,z) = \beta(z)$).

We make several remarks here. First, vagueness b comes into the adviser's expected utility, while it does not appear in his ex post utility as the signal is just cheap talk. Second, vagueness plays different roles for the two players. For the decision maker, it is a variance of x as she is uncertain about the policy parameter if the adviser speaks vaguely. For the adviser, it is a variance of m_r as he is uncertain about how the decision maker interprets his vague speech. Third, the expected informational cost zb is manipulable for the adviser through vagueness b. I call it "interpretive cost" as it is the cost of information loss due to misinterpretation following a vague signal. So the vague signal is costly. We will later see that the adviser uses this cost of vagueness to signal the level of non-informational risk zc.

Next, we let $\hat{q}(b) = \int_0^1 \pi(b,v)\,dv$ denote the adviser's expected probability that the decision maker chooses a new policy after she listened to his signal with vagueness b. Then $q(z) = \hat{q}(\beta(z))$ is his expected policy selection probability when he emulates the equilibrium behavior of type z. Using this notation, we can write the adviser's incentive compatibility conditions as follows: for all $z, z' \in Z$,

$$q(z)\left[1 - z\left(\beta(z) + c\right)\right] \geq q(z')\left[1 - z\left(\beta(z') + c\right)\right]. \tag{IC}$$

Furthermore, we let $U_A(b,z) = \hat{q}(b)\left(1 - z(b+c)\right)$ denote type z's expected utility when he chooses b. Then $U_A^*(z) = U_A\left(\beta(z), z\right)$ is type z's expected utility in equilibrium. Lemma 1 states the behavior of U_A^*.

Lemma 1. *U_A^* is positive and continuous on Z and strictly decreasing in z.*

It says that the adviser of any risk magnitude can expect the decision maker to execute the new policy with a positive probability and that his expected utility is higher if he has lower risk magnitude. Given this result, Lemma 2 states the behavior of β and q.

Lemma 2. *β and q are weakly decreasing in z.*

It says that the lower the risk magnitude is, the more vague the signal tends to be and the more likely a risky new policy tends to be selected.

With no restriction on the decision maker's beliefs off equilibrium ($b \notin \beta(Z)$), there exist every class of equilibria: pooling, partially pooling, and separating. To obtain a sharper prediction, we introduce a restriction on off-equilibrium beliefs. We adopt the belief refinement concept of universal divinity proposed by Banks and Sobel [1], which requires that a sender's deviation to off-equilibrium signals is believed to be taken by the sender types who are most likely to deviate in the sense that they prefer to deviate for a bigger set of a receiver's best responses.[5]

Given any off-equilibrium signal $b \notin \beta(Z)$, let

$$\xi(z; b) = \frac{q(z)\left[1 - z\left(\beta(z) + c\right)\right]}{1 - z(b + c)}.$$

be the expected policy selection probability which makes the adviser indifferent between sending his equilibrium signal and deviating to the off-equilibrium vagueness b. The universal divinity criterion requires $\gamma(z'; b) > 0$ iff $z' \in \arg\min_{z \in Z} \xi(z; b)$. Intuitively, the universal divinity criterion dictates that the decision maker, on observing an off-equilibrium vagueness of the adviser, believes that the signal was sent by the adviser of type z who is "most likely" to deviate in that he has the lowest threshold of the expected policy selection probability which makes him deviate from his equilibrium behavior: given the belief on the decision maker's strategies fixed, whenever some type wanted to deviate, this most likely type would also want to deviate. Our main result, summarized in Theorem 1, follows from this belief refinement criterion.

Theorem 1. *A universal divine equilibrium uniquely exists and is separating.*

[5] Note that the D2 criterion in Cho and Kreps [6] yields the identical result. In contrast, the intuitive criterion has no bite here. Also note that universal divinity is stronger than is required for our results. A strong form of divinity suffices.

It says that the adviser obfuscates information of policy parameter to signal risklessness of the policy and consequently persuades some, if not all, decision makers to execute the new policy. The lower the risk magnitude is, the more interpretive cost the adviser can afford, and the more vague his signal becomes. In turn, the more likely the new policy is executed by the decision maker. In equilibrium a decision maker can identify the risk magnitude of a new policy and conduct a cost-benefit analysis given her valuation of the new policy to decide between the new policy and a status quo. The cost of signaling for the adviser–and for the decision maker–is the loss of information clarity on policy parameter through the decision maker's misinterpretation of the adviser's intention. The important point is that this interpretive cost is endogenously created by the adviser.

Earlier we interpreted the inverse of z as the degree of congruence between the parties' interests. The above theorem states that vagueness signals congruence. This positive relationship contrasts to the finding of Blume and Board [2]. They show that intentional vagueness decreases with the degree of congruence. It is fruitless to make simple comparisons as the strategic environments are rather different in two papers, but it is interesting to see how vagueness is used for different purposes.

3.3 Comparative Statics

In the unique separating equilibrium, β satisfies

$$\beta(z) = c(z^{-\frac{1}{2}} - 1), \tag{8}$$

which is pictured in Figure 1 for particular parameters.

As is seen in (8), $\beta(z;c)$ is decreasing in c for all z and $\beta(z;c)$ converges to zero uniformly as c goes to 0. If we recall Proposition 1 in the first benchmark case, we can conclude that a separating equilibrium is continuous in $c \in [0, 1)$ if it is viewed as a function of the parameter.

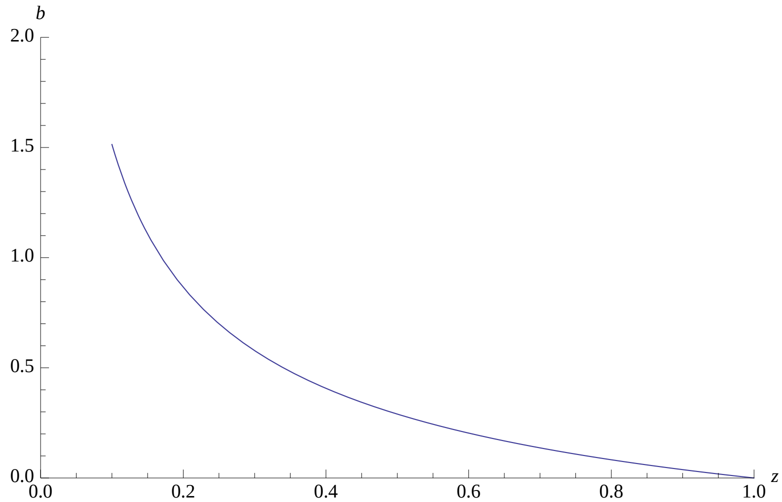

Figure 1: β when $c = 0.7$ and $\underline{z} = 0.1$

Note that the lower bound of risk magnitude ($\underline{z}$) only expands the domain of β and does not affect its shape. Also, it is clear that our result does not depend on the shape of risk distribution F.

4 Discussion

All of our results hold true when we take the space of policy parameter to be a circle of large circumference instead of the real line. The important assumption is that the space does not have a boundary and not that the state space is unbounded.

The uniform distribution of the decision maker's valuation is only necessary for a closed-form expression of equilibrium. Appendix C shows that all the results including comparative statics remain unchanged if we assume that twice continuously differentiable distribution with full support on $[0, 1]$.

The model assumed that the adviser is uncertain about the decision maker's valuation. I believe that this is plausible in most of consulting situations. To simply illustrate the possibility that vagueness may be used as a signal of congruence, we would not need this assumption of receiver uncertainty. The assumption allowed us to use the differential approach and obtain a maximum prediction power. It is the unique observation that the adviser obfuscates to persuade and vagueness increases with the degree of congruence.

To highlight the use of vague language, we specified a particular setup of communication. Naturally, questions arise as to what would happen in alternative communication environments. How robust is the result to the timing of communication? What if the decision maker is allowed to speak? To analyze these questions, we need to find what the optimal communication mechanisms are from each player's perspective

5 Conclusion

We found that the adviser obfuscates to persuade the decision maker. In the environment in which a signal is just cheap talk, the adviser chooses to obfuscate the information about policy parameter to make the decision maker's interpretation unpredictable to himself. Then the vague signal becomes costly because the decision maker will misinterpret the adviser's intention and execute the policy "incorrectly." This interpretive cost is affordable if the adviser knows that the risk magnitude is low. In this way a vague report can be utilized to signal risklessness.

We obtained two insights of strategic communication from the model. First, one part of information can be transmitted as a result of a compromise on another part of information. We saw that the adviser can transmit the information about risk magnitude by intentionally giving up information clarity on the policy parameter. Second, vagueness can be used as a signal of congruence. We observed that the adviser sends a more vague report if he knows that the risk magnitude is lower and more likely the decision maker wants to adopt the new policy.

A Proofs

A.1 Proof of Lemma 1

First, observe that for all $z \in Z$ there exists $\bar{v} \in (0,1)$ such that for all $v \in [\bar{v}, 1]$, $\pi(0, v) = 1$, since for any belief $\gamma(\cdot; 0)$, $\int_{\underline{z}}^{1} zc\,\gamma(z; 0)dz \leq c < 1$. Since the adviser can always choose $b = 0$, it must be that for any $z \in Z$, $q(z) > 0$ and $U_A^*(z) > 0$. Then, note that if $z < z'$ then $U_A^*(z) \geq U_A(\beta(z'), z) > U_A^*(z')$. Last, suppose that there exist $\hat{z} \in Z$ and $\epsilon > 0$ such that $U_A^*(\hat{z}-) - U_A^*(\hat{z}+) = \epsilon$. From the uniform continuity of $U_A(b, z)$ in z, there exists $\delta > 0$ such that if $|z - z'| < \delta$ then $|U_A(\beta(z), z) - U_A(\beta(z), z')| < \epsilon$. But, for $z < \hat{z} < z'$, $U_A(\beta(z), z) = U_A^*(z) > U_A^*(\hat{z}) > U_A^*(z') \geq U_A(\beta(z), z')$ and hence $U_A(\beta(z), z) - U_A(\beta(z), z') > \epsilon$, a contradiction.

A.2 Proof of Lemma 2

Let $z < z'$. From (IC) for z and z',

$$q(z)\left[1 - z\left(\beta(z) + c\right)\right] \geq q(z')\left[1 - z\left(\beta(z') + c\right)\right]$$
$$q(z')\left[1 - z'\left(\beta(z') + c\right)\right] \geq q(z)\left[1 - z'\left(\beta(z) + c\right)\right]$$

Since $\forall z \in Z, U_A^*(z) > 0$, we can arrange the above inequalities as:

$$\frac{1 - z\left(\beta(z') + c\right)}{1 - z\left(\beta(z) + c\right)} \leq \frac{q(z)}{q(z')} \leq \frac{1 - z'\left(\beta(z') + c\right)}{1 - z'\left(\beta(z) + c\right)} \tag{9}$$

Define $h(\tilde{z}) = \frac{1-\tilde{z}(\beta(z')+c)}{1-\tilde{z}(\beta(z)+c)}$ and suppose $\beta(z) < \beta(z')$. Since

$$h'(\tilde{z}) = \frac{\beta(z) - \beta(z')}{\left[1 - \tilde{z}\left(\beta(z) + c\right)\right]^2} < 0,$$

it follows that $h(z) > h(z')$, which contradicts the inequalities in (9). Therefore $\beta(z) \geq \beta(z')$. Then, from the left inequality in (9), we obtain $\frac{q(z)}{q(z')} \geq h(z) \geq 1$ and so $q(z) \geq q(z')$.

A.3 Proof of Theorem 1

I prove it in steps.

Step 1. *Look at the universal divinity requirement for off-equilibrium beliefs.*

Lemma 3 summarizes the universal divinity requirement for the adviser's belief $\gamma(z;b)$ at the decision maker's off-equilibrium signal $b \notin \beta(Z)$.

Lemma 3. *In an universal divine equilibrium, γ must satisfy:*

(i) *if $b \in [0, \beta(1))$, then $\gamma(z;b) > 0$ iff $z = 1$;*

(ii) *if $b \in (\beta(\underline{z}), \infty)$, then $\gamma(z;b) > 0$ iff $z = \underline{z}$; and*

(iii) *if $b \in (\beta(\tilde{z}+), \beta(\tilde{z}-))$ for $\tilde{z} \in Z$, then $\gamma(z;b) > 0$ iff $z = \tilde{z}$,*
where for $\tilde{z} = \underline{z}$ or 1, we let $\beta(\underline{z}-) = \beta(\underline{z})$ and $\beta(1+) = \beta(1)$.

Proof. Let $z < z'$ and $b \notin \beta(Z)$. Define $\phi(b) = \frac{1-z(b+c)}{1-z'(b+c)}$ and observe that

$$\phi'(b) = \frac{z' - z}{[1 - z'(b+c)]^2} > 0$$

Consider the following cases.

(1) When $b < \beta(z') < \beta(z)$, we have

$$\begin{aligned}
\frac{\xi(z;b)}{\xi(z';b)} &= \frac{q(z)}{q(z')} \cdot \frac{1 - z\left(\beta(z) + c\right)}{1 - z'\left(\beta(z') + c\right)} \cdot \frac{1 - z'\left(b + c\right)}{1 - z\left(b + c\right)} \\
&\geq \frac{1 - z\left(\beta(z') + c\right)}{1 - z\left(\beta(z) + c\right)} \cdot \frac{1 - z\left(\beta(z) + c\right)}{1 - z'\left(\beta(z') + c\right)} \cdot \frac{1 - z'\left(b + c\right)}{1 - z\left(b + c\right)} && (\because (9)) \\
&= \frac{\phi\left(\beta(z')\right)}{\phi(b)} > 1.
\end{aligned}$$

(2) When $b < \beta(z') = \beta(z)$, we have $q(z) = q(z')$ and

$$\begin{aligned}
\frac{\xi(z;b)}{\xi(z';b)} &= \frac{q(z)}{q(z')} \cdot \frac{1 - z\left(\beta(z) + c\right)}{1 - z'\left(\beta(z') + c\right)} \cdot \frac{1 - z'\left(b + c\right)}{1 - z\left(b + c\right)} \\
&= \frac{\phi\left(\beta(z')\right)}{\phi(b)} > 1.
\end{aligned}$$

(3) When $\beta(z') < \beta(z) < b$, we have

$$\begin{aligned}\frac{\xi(z;b)}{\xi(z';b)} &= \frac{q(z)}{q(z')} \cdot \frac{1 - z\left(\beta(z) + c\right)}{1 - z'\left(\beta(z') + c\right)} \cdot \frac{1 - z'\left(b + c\right)}{1 - z\left(b + c\right)} \\ &\leq \frac{1 - z'\left(\beta(z') + c\right)}{1 - z'\left(\beta(z) + c\right)} \cdot \frac{1 - z\left(\beta(z) + c\right)}{1 - z'\left(\beta(z') + c\right)} \cdot \frac{1 - z'\left(b + c\right)}{1 - z\left(b + c\right)} \qquad (\because\ (9)) \\ &= \frac{\phi\left(\beta(z)\right)}{\phi(b)} < 1.\end{aligned}$$

(4) When $\beta(z') = \beta(z) < b$, we have $q(z) = q(z')$ and

$$\begin{aligned}\frac{\xi(z;b)}{\xi(z';b)} &= \frac{q(z)}{q(z')} \cdot \frac{1 - z\left(\beta(z) + c\right)}{1 - z'\left(\beta(z') + c\right)} \cdot \frac{1 - z'\left(b + c\right)}{1 - z\left(b + c\right)} \\ &= \frac{\phi\left(\beta(z)\right)}{\phi(b)} < 1.\end{aligned}$$

Since $\gamma(z';b) > 0$ iff $z' \in \arg\min_{z \in Z} \xi(z;b)$, the result (i) follows from (1) and (2), and (ii) from (3) and (4). To see (iii), note that $\beta(z) \geq \beta(\tilde{z}-)$ for all $z < \tilde{z}$ and $\beta(\tilde{z}+) \geq \beta(z)$ for all $z > \tilde{z}$, which implies that (iii) follows from the similar arguments to the ones for (i) and (ii). $\square$

Step 2. *Look at the behavior of β.*

Since β is monotone, if it has discontinuities then they are jump discontinuities and the points of discontinuities are countable. If β has a jump discontinuity at z then either $\beta(z) = \beta(z-)$ or $\beta(z) = \beta(z+)$, since otherwise $\beta(z) \in (\beta(z+), \beta(z-))$ and the decision maker identifies type z on signal $\beta(z)$, but then Lemma 3 (iii) implies that type z deviates to any $b \in (\beta(z+), \beta(z))$. Let J be the set of discontinuities of β and let B be the set of off-equilibrium vagueness levels.[6] From the above argument,

$$B = [0, \beta(1)) \bigcup (\beta(\underline{z}), \infty) \bigcup \left\{ \bigcup_{z \in J} (\beta(z+), \beta(z-)) \right\}.$$

[6]Note that J can be dense in Z.

Next, let us define, for each $\hat{z} \in Z$,

$$T(\hat{z}) = \{z \in Z : \beta(z) = \beta(\hat{z})\},$$
$$T^{-}(\hat{z}) = \{z \in Z : \beta(z) = \beta(\hat{z}-)\}, \text{ and}$$
$$T^{+}(\hat{z}) = \{z \in Z : \beta(z) = \beta(\hat{z}+)\}.$$

We say that β is *separating at* $\hat{z}$ if $|T(\hat{z})| = 1$ and *pooling at* $\hat{z}$ otherwise.[7] Also, β is *left-separating at* $\hat{z}$ if $|T^{-}(\hat{z})| = 1$ and *left-pooling at* $\hat{z}$ otherwise; β is *right-separating at* $\hat{z}$ if $|T^{+}(\hat{z})| = 1$ and *right-pooling at* $\hat{z}$ otherwise. Finally, define $z^m(\hat{z}) = \int_{T(\hat{z})} z f(z) dz$ and note that if $b = \beta(\hat{z})$ for some $\hat{z} \in Z$ then $\hat{q}(b) = 1 - z^m(\hat{z})(b + c)$.

Step 3. *Prove that β is separating everywhere.*

First, I claim that β is separating at $\underline{z}$. Otherwise $z^m(\hat{\underline{z}}) > \underline{z}$, which implies that type $\underline{z}$ deviates to any $b \in (\beta(\underline{z}), \beta(\underline{z}) + \epsilon)$ for some $\epsilon > 0$.

Now, by way of contradiction, suppose that $J \neq \emptyset$. Let z' be an arbitrary point in J. If β is right-pooling at z', then type z' deviates to any $b \in (\beta(z'), \beta(z') + \epsilon)$ for some $\epsilon > 0$, since $z^m(z') > z'$. If β is left-pooling at z', then either $\inf T(z') = \underline{z}$ or $\inf T(z') = z'' \in J$, since otherwise U_A^* cannot be continuous at $\inf T(z')$. In the former, type $\underline{z}$ deviates to any $b \in (\beta(z'), \beta(\underline{z}))$. In the latter, type z'' deviates to any $b \in (\beta(z'), \beta(z') + \epsilon)$ for some $\epsilon > 0$, since $z^m(z') > z''$. It then follows that β is both left-separating and right-separating at z'. But this implies that $U_A(\beta(z'-), z') < U_A(\beta(z'+), z')$ and contradicts the continuity of U_A^*. We thus conclude that $J = \emptyset$ and β is continuous everywhere.

Recall that β is separating at $\underline{z}$. If β is pooling somewhere, then β must change from separating to pooling at some point, say z', at which U_A^* cannot be continuous since $z^m(z') > z'$. Therefore, β is separating everywhere.

Step 4. *Find a unique separating equilibrium from the necessary condition.*

[7] Note that if $|T(\hat{z})| > 1$ then the monotonicity of β implies that $T(\hat{z})$ is an interval.

In a separating equilibrium, (5) implies that $q(z) = 1 - z(\beta(z) + c)$. The incentive compatibility condition for $z \in Z$ is

$$z \in \arg\max_{\tilde{z} \in Z} [1 - \tilde{z}(\beta(\tilde{z}) + c)] [1 - z(\beta(\tilde{z}) + c)].$$

As a monotonic function, β is differentiable almost everywhere and hence on a dense subset of Z. The necessary first order condition at points of differentiability yields

$$\beta'(z) = -\frac{\beta(z) + c}{2z}. \tag{10}$$

This is a linear ordinary differential equation of order 1 with variable coefficients and is solvable as

$$\beta(z) = kz^{-\frac{1}{2}} - c, \tag{11}$$

where $k \in \mathbb{R}$ must be unique for β to be continuous.[8] Since a continuous mapping is determined by its values on a dense subset of its domain, (11) describes β everywhere and hence equilibrium uniqueness is obtained. The boundary condition is given by $\beta(1) = 0$, since otherwise a profitable deviation is possible for $z = 1$. Substituting this condition into (11) yields $k = c$, and a separating equilibrium is uniquely described by

$$\beta(z) = c(z^{-\frac{1}{2}} - 1).$$

Step 5. *Show that the necessary condition is sufficient.*

We shall show that the strict incentive compatibility condition is satisfied for any $z \in Z$.[9] Define $V(z, \tilde{z}, b) = [1 - \tilde{z}(b + c)][1 - z(b + c)]$. We need to show that for any $z \in Z$,

$$\{\beta(z)\} = \arg\max_{b \in \beta(Z)} V(z, \beta^{-1}(b), b). \tag{12}$$

Note that (10) can be written as

$$\beta'(z) = -\frac{V_2(z, z, \beta(z))}{V_3(z, z, \beta(z))}. \tag{13}$$

[8] See Appendix B for the derivation.

[9] The technique found here and in Appendix C is due to Mailath [12].

Also, if $1 - z(b+c) > 0$, then for any $\tilde{z} \in Z$,

$$V_2(z, \tilde{z}, b) = -(b+c)\left[1 - z(b+c)\right] < 0. \tag{14}$$

Furthermore, for any $z, \tilde{z} \in Z$,

$$\frac{\partial}{\partial z}\left[\frac{V_3(z, \tilde{z}, \beta(\tilde{z}))}{V_2(z, \tilde{z}, \beta(\tilde{z}))}\right] = \frac{1 - \tilde{z}(\beta(\tilde{z}) + c)}{(\beta(\tilde{z}) + c)^2\left[1 - z(\beta(\tilde{z}) + c)\right]^2} > 0. \tag{15}$$

By way of contradiction, suppose that there exists $z \in Z$ such that (12) is not true. Let b' be a maximizer of V such that $b' \neq \beta(z)$. Note that $V_2(z, \tilde{z}, b') \neq 0$ for any $\tilde{z} \in Z$, since $1 - z(b' + c) > 0$ by Lemma 1.

(i) Suppose that $\beta^{-1}(b') = z' \in \mathrm{int}Z$. By the first order condition for z in (12),

$$V_2(z, z', b') \left.\frac{d\beta^{-1}}{db}\right|_{b=\beta(z')} + V_3(z, z', b') = 0.$$

From (13),

$$V_2(z', z', b') \left.\frac{d\beta^{-1}}{db}\right|_{b=\beta(z')} + V_3(z', z', b') = 0.$$

Combining these equations yields

$$\frac{V_3(z, z', b')}{V_2(z, z', b')} = \frac{V_3(z', z', b')}{V_2(z', z', b')},$$

which contradicts (15).

(ii) Suppose that $\beta^{-1}(b') = \underline{z}$. By the first order condition for z in (12),

$$V_2(z, \underline{z}, b') \left.\frac{d\beta^{-1}}{db}\right|_{b=\beta(\underline{z})} + V_3(z, \underline{z}, b') \geq 0.$$

Then, by (13),

$$V_2(z, \underline{z}, b')\left[-\frac{V_3(\underline{z}, \underline{z}, b')}{V_2(\underline{z}, \underline{z}, b')} + \frac{V_3(z, \underline{z}, b')}{V_2(z, \underline{z}, b')}\right] \geq 0.$$

But (14) and (15) imply the opposite strict inequality, a contradiction.

(iii) Suppose that $\beta^{-1}(b') = 1$. By the first order condition for z,

$$V_2(z, 1, b') \left.\frac{d\beta^{-1}}{db}\right|_{b=\beta(1)} + V_3(z, 1, b') \leq 0.$$

Then, by (13),

$$V_2(z, 1, b')\left[-\frac{V_3(1, 1, b')}{V_2(1, 1, b')} + \frac{V_3(z, 1, b')}{V_2(z, 1, b')}\right] \leq 0.$$

But (14) and (15) imply the opposite strict inequality, a contradiction.

B Differential Equation

We want to solve the following ordinary differential equation,

$$\beta'(z) = -\frac{\beta(z) + c}{2z}.$$

Rewrite it as

$$\beta'(z) + \frac{1}{2z}\beta(z) = -\frac{c}{2z}.$$

Multiply the integrating factor $e^{\int_{\underline{z}}^{z} \frac{1}{2y}\,dy}$ throughout to obtain

$$\beta'(z)e^{\int_{\underline{z}}^{z} \frac{1}{2y}\,dy} + \frac{1}{2z}\beta(z)e^{\int_{\underline{z}}^{z} \frac{1}{2y}\,dy} = -\frac{c}{2z}e^{\int_{\underline{z}}^{z} \frac{1}{2y}\,dy},$$

which simplifies due to the product rule to

$$\frac{d}{dz}\left[\beta(z)e^{\int_{\underline{z}}^{z} \frac{1}{2y}\,dy}\right] = -\frac{c}{2z}e^{\int_{\underline{z}}^{z} \frac{1}{2y}\,dy}.$$

Integrate both sides over $[\underline{z}, z]$ to obtain

$$\beta(z)e^{\int_{\underline{z}}^{z} \frac{1}{2y}\,dy} - \beta(\underline{z}) = -\int_{\underline{z}}^{z} \frac{c}{2w}e^{\int_{\underline{z}}^{w} \frac{1}{2y}\,dy}\,dw + l,$$

where l is the constant of integration. If we note that $e^{\int_{\underline{z}}^{z} \frac{1}{2y}\,dy} = e^{\frac{1}{2}\ln(\frac{z}{\underline{z}})} = (\frac{z}{\underline{z}})^{\frac{1}{2}}$, then easy algebra shows

$$\beta(z) = kz^{-\frac{1}{2}} - c,$$

where $k = \underline{z}^{\frac{1}{2}}(c + l + \beta(\underline{z}))$.

C General Distribution

In this appendix I show that all the qualitative results remain unchanged if we assume a more general distribution of v.

Assume that distribution $\Psi(v)$ is twice continuously differentiable with full support on $[0,1]$. We then have $q(z) = 1 - \Psi\left[z\left(\beta(z)+c\right)\right]$ and the incentive compatibility condition for $z \in Z$ is

$$z \in \arg\max_{\tilde{z}\in Z} \left[1-\Psi\left[\tilde{z}\left(\beta(\tilde{z})+c\right)\right]\right]\left[1-z\left(\beta(\tilde{z})+c\right)\right].$$

The necessary first order condition at points of differentiability yields

$$\beta'(z) = \frac{-\left(\beta(z)+c\right)\left[1-z(\beta(z)+c)\right]\Psi'\left[z(\beta(z)+c)\right]}{z\left[1-z(\beta(z)+c)\right]\Psi'\left[z(\beta(z)+c)\right]+z\left[1-\Psi\left[z(\beta(z)+c)\right]\right]}. \tag{16}$$

Let $\eta(z,\beta)$ denote the right hand side of (16) with $\beta(z)$ being replaced by β. We need to find a unique solution $\tilde{\beta}$ to the terminal value problem

$$\beta' = \eta(z,\beta),\ \beta(1) = 0.$$

Define $D = \left\{(z,\beta) : \underline{z} \le z \le 1,\ 0 \le \beta < \frac{1}{z} - c\right\}$. Since $\eta \in C(D)$ and $(1,0) \in D$, there exists a local solution $\tilde{\beta}$ on $(\tilde{z},1]$ for some $\tilde{z} \in [\underline{z},1)$. Since $\eta(z,\beta)$ is Lipschitz continuous in β (as $\frac{\partial\eta}{\partial\beta} \in C(D)$), the uniqueness of solutions is guaranteed. It remains to show that the local solution $\tilde{\beta}$ is continuable to $[\underline{z},1]$, which requires Lemma 4.[10]

Lemma 4. *Let $\tilde{D}$ be a nonempty connected set in the (z,β) domain and let η be a bounded and continuous function on $\tilde{D}$. Suppose $\tilde{\beta}$ is a solution of $\beta' = \eta(z,\beta)$ on the interval $(\tilde{z},1]$. Then (i) the left-hand limit of $\tilde{\beta}$ at $\tilde{z}$, $\tilde{\beta}(\tilde{z}+)$ exists, and (ii) if $(\tilde{z},\tilde{\beta}(\tilde{z}+)) \in \tilde{D}$ then the solution $\tilde{\beta}$ is continuable to the left past the point $z = \tilde{z}$.*

If we can find $\tilde{D}$ such that $(z,\tilde{\beta}(z+)) \in \tilde{D}$ for any $z \in (\underline{z},1)$, then we can conclude that $\tilde{\beta}$ is continuable to $[\underline{z},1]$. Let $D_\epsilon = \left\{(z,\beta) : \underline{z} \le z \le 1,\ 0 \le \beta \le \frac{1}{z} - c - \epsilon\right\} \subseteq D$ for some $\epsilon \in (0,1-c)$. For any $(z,\beta) \in D_\epsilon$, $z(\beta+c) < 1 - z\epsilon$ and $\Psi\left[z(\beta+c)\right] < \Psi(1-z\epsilon)$. Also $\Psi'(z)$ is bounded away from 0. Thus $\eta(z,\beta)$ is bounded on D_ϵ. I claim that $(z,\tilde{\beta}(z+)) \in D_\epsilon$ for any $z \in (\underline{z},1)$. To see this, note that for any $(z,\beta(z)) \in D_\epsilon$,

$$|\beta'(z)| = \frac{\beta(z)+c}{z} \cdot \left[1 + \frac{1-\Psi[z(\beta(z)+c)]}{[1-z(\beta(z)+c)]\Psi'[z(\beta(z)+c)]}\right]^{-1} < \frac{1}{z^2}.$$

[10] This is Theorem 3.1 in Miller and Michel [13].

Then we observe that for any $z' \in (\underline{z}, 1)$,

$$\begin{aligned}
\tilde{\beta}(z'+) &= \tilde{\beta}(1) - \lim_{z \downarrow z'} \int_z^1 \tilde{\beta}'(z)\, dz \\
&< 1 - c - \epsilon - \int_{z'}^1 \left(-\frac{1}{z^2}\right) dz \\
&< 1 - c - \epsilon - \int_{z'}^1 \frac{d}{dz}\left(\frac{1}{z} - c\right) dz \\
&= \frac{1}{z'} - c - \epsilon.
\end{aligned}$$

For sufficiency of the first order condition, redefine

$$V(z, \tilde{z}, b) = [1 - \Psi[\tilde{z}(b+c)]]\,[1 - z(b+c)]\,.$$

If $1 - z(b+c) > 0$, then for any $\tilde{z} \in Z$,

$$V_2(z, \tilde{z}, b) = -(b+c)\Psi'[\tilde{z}(b+c)]\,[1 - z(b+c)] < 0.$$

Also, for any $z, \tilde{z} \in Z$,

$$\frac{\partial}{\partial z}\left[\frac{V_3(z, \tilde{z}, \beta(\tilde{z}))}{V_2(z, \tilde{z}, \beta(\tilde{z}))}\right] = \frac{1 - \Psi[\tilde{z}(\beta(\tilde{z}) + c)]}{[V_2(z, \tilde{z}, \beta(\tilde{z})]^2} > 0.$$

Then the same arguments in Step 5 go through.

It is apparent from (16) that β is strictly decreasing. To see comparative statics results, note that $\beta(z) < \frac{\tau(c)}{z} - c$ for any $z \in [\underline{z}, 1]$, where $\tau(c) = 1 - (1 - \Psi(c))(1 - zc)$, since we have $\beta(1) = 0$ and β must satisfy

$$1 - z(\beta(z) + c) > [1 - \Psi[z(\beta(z) + c)]]\,[1 - z(\beta(z) + c)] > (1 - \Psi(c))(1 - zc).$$

As c goes to 0, the term $\frac{\tau(c)}{z} - c$ vanishes everywhere and hence $\beta(z; c)$ converges to zero uniformly.

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