Avoiding deforestation efficiently and fairly: a mechanism design perspective

Very preliminary version - please do not quote

Charles Figuières and Estelle Midler

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Abstract

The international community recently agreed on a cost-effective mechanism called REDD+ to reduce deforestation in tropical countries. However the mechanism would probably fail to induce an optimal reduction of deforestation. The aim of this article is to propose an alternative class of mechanisms for negative externalities that is both efficient and satifies some fairness properties. It implements the Pareto optimum as a Nash Subgame Perfect Equilibrium. It is also individually rational and could lead to envy free allocations.

1 Introduction

Deforestation in tropical countries accounts for up to 20% of global emmissions of CO2. It is the second most important source of Greenhouse Gas Emissions in the world and the first one in developing countries. It is also the leading cause of loss of global biodiversity. A new scheme called REDD, for Reduction of Emissions from Deforestation and Degradation of forests, has been agreed on at the 16th COP of the UNFCCC to reward countries with low deforestation rates. The principle is to compensate developing countries that reduce their deforestation with financial incentives. However, there is still no consensus on the way such financial incentives should be calculated and allocated. The REDD transfers would be allocated per unit of real reduction of deforestation level compared to a reference level, called the baseline (see for instance Parker et al, 2008). In spirit, this is a cost-effectiveness approach of the problem: how to effectively reach an exogenous limitation of deforestation? Not surpringly then, the mechanism (or the class of mechanisms embedded in the REDD program) has no reason to induce a Pareto optimal reduction of deforestation (see Figuières et al, 2010).

In this paper, we propose to attack the question from a different angle. We let the goal be Pareto optimality, supplemented by additional requirements of fairness and acceptability that seem relevant for an international externality problem llike deforestation, and we engineer a proposal to achieve it.

The paper is organised as follows. Section 2 presents a simple North-South Deforestation model. Section 3 introduces a class of incentive mechanisms - call it REDD* - directly inspired from the compensation mechanism (see Danziger & Schnytzer, 1991, Varian, 1994), and analyses its efficiency first under complete information and then under some form of bounded rationality. Equity issues are discussed in section 4. Section 5 concludes.

2 A north-south deforestation framework

Consider m countries in the developing South with a high endowment of tropical forests. Deforestation provides land and capital for development. Let $d_i \in \left[0, \bar{d}_i\right[$, be the amount of hectare deforested by the country i, where $\bar{d}_i$ is the total remaining forest area in country i. Each country is endowed with an exogenous wealth y^i. Country i's preferences are defined over the pairs (d_i, y^i), and represented by an additively separable utility function:

$$U^i(d_i, y^i) = u_i\left(d_i\right) + y^i,\ i = 1, ..., m.$$

The functions $u_i\left(.\right)$ are increasing and concave, $u_i'' \leq 0 \leq u_i'$.

As regards deforestation there is a country-specific limit d_i^{bau}, beyond which nature cannot be turned into arable lands within the time-scale captured by our static model; or put differently, for geographical, bio-physical or economic reasons the marginal product is zero beyond those thresholds, $u_i'\left(d_i\right) = 0,\ \forall\ d_i \geq d_i^{bau}$. Therefore, on a non cooperative basis, southern countries push deforestation up to that threshold d_i^{bau}.

The north is a block that will be treated as a single country. It is also endowed with an exogenous wealth y^n and it is interested in the aggregate preservation of tropical forest, $D = \sum_i d_i$, because it reduces carbon emissions. Its preferences are captured by a utility function:

$$U^n(D, y^n) = u_n(D) + y^n,$$

which is decreasing and concave with respect to the first argument, $u_n' \leq 0$, $u_n'' \leq 0$.

This model is simple, yet it accounts for the asymmetric nature of the deforestation problem: at the business-as-usual, the South deforests too much, for it fails to take into account of the negative externality it generates. Pareto optimal deforestation levels, denoted $(d_1^*, ..., d_m^*)$, on the contrary, would equalize the marginal benefit for the south with the marginal cost for the the North, and would solve the following equations (technical details are given in Appendix A):

$$u_i' = -u_n' \ , \quad i = 1, ..., m. \tag{1}$$

Pareto optimality calls for lower deforestation, because of its external negative effect. But avoided deforestation represents an opportunity cost for southern countries.

3 The compensation mechanism to curb deforestation

3.1 The mechanism

There is a class of mechanisms, generically referred to as "the compensation mechanism", that rests on the following logic: agents involved in an economic environment with externalities solve the social dilemma by mean of cross-subsidies (in case of positive externalities) or cross-taxes (in case of negative externalities) whose magnitude they decide by themselves. The classic reference is Varian (1994), but crucial predecessors are Guttman (1978, 1985 and 1987) and Danziger and Schnytzer (1991). These solutions implement first best allocations as subgame perfect Nash equilibria.

That kind of solution cannot be applied as it is in our context of transnational negative externalities, because it would involve the developed North taxing the developing South! But a trick can be found to retain the spirit of the mechanism, while turning taxes into subsidies. The description of what we call REDD* is as follows. The North can now decide to subsidize developing countries who are willing to reduce their deforestation throught a two-stage mechanism:

1. In the first stage, the announcement stage, countries choose subvention/tax levels simultaneously. Developing country i chooses a tax level $t_i^s \in [0, \bar{t}]$ and the North chooses a vector of subsidies $(t_1^n, ..., t_m^n)$, where $t_i^n \in [0, t_i^s]$ is the subsidy level offered to developing country i [1]. The regulator collects those rates and implements the transfers as follows: the North pays $T^n = \sum_i T_i^n$, with

$$T_i^n = \begin{cases} t_i^s(d_i^b - d_i) & if \quad d_i < d_i^b \\ 0 & otherwise \end{cases}$$

and each country i receives:

$$S_i = \begin{cases} t_i^n(d_i^b - d_i) - \varepsilon_i(t_i^n - t_i^s)^2 & if \quad d_i < d_i^b \\ 0 & otherwise \end{cases}$$

.

2. In the choice stage, each southern country i $(i = 1, ..., m)$ determines its level of deforestation d_i. Transfers are then implemented by the regulator.

So, under the mechanism, incomes become:

$$y^i = y_0^i + t_i^n(d_i^b - d_i) - \varepsilon_i(t_i^n - t_i^s)^2 \ ,$$

and:

$$y^n = y_0^n - \sum_i t_i^s(d_i^b - d_i) \ .$$

We can study the mechanism properties within a complete information structure. The model is solved, as usual, by backward induction.

In the last decision period, developing countries choose their optimal deforestation level d_i^* which maximizes their utility under the mechanism, knowing t_i^n and t_i^s The first order condition for an interior optimal deforestation is:

$$\begin{aligned} \frac{\partial U^i}{\partial d_i} &= u_i'(.) - t_i^n = 0 \\ &\Longleftrightarrow u_i'(.) = t_i^n \end{aligned} \tag{2}$$

[1] As a result, if the North chooses $t_i^n > t_i^s$, tranfers are not implemented.

Assuming that $u_i'(.)$ is invertible, d_i^*will be a function of t_i^n, which we can write:

$$d_i^* = d_i^*(t_i^n).$$

Applying the implicit function theorem to (2), we can deduce that the larger the subsidy rate, the lower the deforestation:

$$d_i^{*\prime}(t_i^n) = \frac{1}{u_i''} \leq 0.$$

In the first period, countries choose the tax levels. In the South, the program $\max_{t_i^s} U^i$ implies the following first order condition for an interior optimal decision:

$$\begin{aligned} &\Rightarrow \quad 2\varepsilon_i(t_i^n - t_i^s) = 0 \\ &\iff \quad t_i^s = t_i^n \end{aligned} \tag{3}$$

In the North:

$$\begin{aligned} \underset{t_i^n}{max}\ U^n \quad &\Rightarrow \quad u_n' \frac{\partial d_i^*}{\partial t_i^n} + \frac{\partial y^n}{\partial d_i^*}\frac{\partial d_i^*}{\partial t_i^n} = 0 \\ &\iff \quad u_n' \frac{\partial d_i^*}{\partial t_i^n} + t_i^s \frac{\partial d_i^*}{\partial t_i^n} = 0 \\ &\iff \quad -u_n' = t_i^s \end{aligned} \tag{4}$$

Then, since we have (2), (3) and (4):

$$t_i^s = -u_n' = t_i^n = u_i' \ . \tag{5}$$

This last equation characterizes all the subgame perfect interior nash equilibria. Since the Pareto Optimum requires $-u_n' = u_i'$, it can be reached through the mechanism. However there could be multiple Nash equilibria. If it did, countries could face a coordination problem and therefore may not reach the optimum.

Two important remarks, about the differences between this mechanism (REDD*), the one proposed by Varian and the REDD+ one, are in order:

- Within the Varian's mechanism, payoffs are a linear function of the amount of negative externality produced. Therein, payoffs are a linear function of $(d_i^b - d_i)$. Thereby it rewards the deforestation effort of the South as desired by the international community rather than taxing the net deforestation level.

- Within the REDD+ mechanism, each tropical country willing to reduce its deforestation level below its reference level would receive a tranfert $t(d_i^b - d_i)$ with t being the exogenous carbon price on the market. Our mechanism differs because subsidy rates are determined endogenously so they equal the marginal cost of deforestation for the North.

3.2 About the information structure

The solution concept used above to describe non cooperative decisions is indicative of the information structure underwhich the mechanism is most relevant: the "regulator", whatever it may be, does not have any information about countries' preferences but countries themselves know a great deal more. In the terminology of game theory, there is complete information and common knowledge.

It could be argued that players do not always know perfectly each other preferences. For now, we have considered the case where each country knows the other ones' preferences. If this is not the case, and if the mechanism is repeated over time, they could proceed by tatonnement to find t_i^s and t_i^n. This kind of informational structure could correspond to an international repeated negotiation.

Consider two periods, the period t and the period $t+1$.

Then, at each period, country i in the South and the North can ajust their subsidy level as follows:

$$\begin{cases} t_{i,t+1}^s = t_{i,t}^n \ , \\ t_{i,t+1}^n = t_{i,t}^n - \gamma \left[U_1^n(D_t, y_t^n) + t_{i,t}^s U_2^n(D_t, y_t^n) \right] \ , \end{cases}$$

with $\gamma > 0$ a parameter.

The South will match its level of transfer at $t+1$ with the one from the North at t. The north will adjust its chosen level of transfer, if it sees that their is a marginal gain (respectively loss) from increasing D_t, then it will decrease (resp. increase) t_i^n proportionally.

Proposition 1 *Assume countries behave myopically as defined by the above adjustemnt process and every country does not know the other countries' preferences, then if the mechanism is repeated over time, it converges asymptotically to a Pareto Optimum.*

Proof. The previous system can be written as a matrix equation:

$$\begin{bmatrix} t^s_{i,t+1} \\ t^n_{i,t+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\gamma U_2^n & 1 \end{bmatrix} \begin{bmatrix} t^s_{i,t} \\ t^n_{i,t} \end{bmatrix} + \begin{bmatrix} 0 \\ -\gamma U_1^n \end{bmatrix} \tag{6}$$

For simplicity matters we will set:

$$\begin{aligned} t_{i,t+1} &= \begin{bmatrix} t^s_{i,t+1} \\ ti,^n_{t+1} \end{bmatrix}, \ A = \begin{bmatrix} 0 & 1 \\ -\gamma U_2^n & 1 \end{bmatrix}, \\ t_{i,t} &= \begin{bmatrix} t^s_{i,t} \\ t^n_{i,t} \end{bmatrix}, \text{ and } b = \begin{bmatrix} 0 \\ -\gamma U_1^n \end{bmatrix}. \end{aligned}$$

(6) became:

$$t_{i,t+1} = At_{i,t} + b \tag{7}$$

As we can see, if the optimum is reached at t, at $t+1$ we will have $t^n_{i,t+1} = t^n_{i,t}$. Therefore Pareto Optima are the stationnary states of the dynamics. We can infer the stability of the stationnary states by studying A.

A's eigenvalues λ_1 and λ_2 solve $P(\lambda) = \lambda - \lambda^2 + \gamma U_2^n = 0$. If $\gamma < \frac{U_2^n}{4}$, the shape of P is presented in figure 1.

So $(\lambda_1, \lambda_2) \in]-1, 1[^2$ and consequently $\underset{t\to\infty}{lim}\, y_t = 0$.

The repeated mechanism is therefore converging to a stationnary state which is the optimum. We have: $t_i^s = t_i^n = t_j^n = t$ with $(i, j) = (1, ..., m)^2$.

So, finally:

$$\begin{aligned} t &= t - \gamma(U_1^n + tU_2^n) \\ &= \frac{-U_1^n}{U_2^n} \end{aligned} \tag{8}$$

■

We get the same t as before, when countries were supposed to have common knowledge. Therefore the efficiency of the mechanism does not necessarily disappears when countries does not have all the information on each other's preferences. The proposed mechanism implements the optimum under less restrictive conditions than one may think at first sight. This property remains whatever the ε_i and the d_i^b chosen, allowing us to choose baselines which satisfy some fairness properties. Various types of baselines are discussed in the next section.

4 Baselines and equity

An important topic of the international debate about financing avoided deforestation in the South is the definition of the baselines. Several possibilities are under consideration. They could be based only on historical levels of deforestation but it would promote countries that had bad past behavior. It could also take into account countries' development paths. In that case, countries that have not cleared a lot of their forest until now would be favored. For more details on possible baseline definitions see Bush et al (2009). What is more likely to happen is a mix solution where both the present effort made and the past behavior are taken into account.

In addition, there exists an academic literature that addresses the question of equity from a more general perspective and that already gives a substantial and well organized bulk of knowledge (see Maniquet [**?**] for a survey, and Fleurbaey [**?**]). We will first borrow important notions from this literature and then get back to the concerns expressed by those currently involved in the design of REDD.

Definition 1 *A Pareto optimal allocation* $\left(d_1^*, ..., d_m^*, y^{1*}, ..., y^{m*}, y^{n*}\right)$ *is individually rational (IR) if:*

$$\begin{aligned} u_i\left(d_i^*\right) + y^{i*} &\geq u_i\left(d_i^{bau}\right) + y_0^i \ , \qquad i = 1, ..., m, \\ u_n\left(\sum\nolimits_i d_i^*\right) + y^{n*} &\geq u_n\left(\sum\nolimits_i d_i^{bau}\right) + y_0^n \ . \end{aligned}$$

Proposition 2 *Assume that the sum of baselines is not larger than the sum of business-as-usual levels, i.e.* $\sum_i d_i^b \leq \sum_i d_i^{bau}$. *Then the REDD* mechanism implements a Pareto optimal allocation which is individually rational.*

Proof. We already know that Pareto optimality obtains under the REDD* mechanism. But individual rationality must be ascertained.

For southern countries, note that after the mechanim is introduced, each could unilaterally secure the level of utility it enjoyed under the business-as-usual scenario. It suffices to set $t_i^s = 0$. Then, because $t_i^n \in [0, t_i^s]$, necessarily $t_i^n = 0$ and $d_i^*\left(t_i^n\right) = d_i^*\left(0\right) = d_i^{bau}$ while $y^{i*} = y_0^i$. If countries' equilibrium tax rates are not zero, $t_i^{s*} \neq 0$, then it must be the case that $u_i\left(d_i^*\right) + y^{i*} \geq u_i\left(d_i^{bau}\right) + y_0^i$, $i = 1, ..., m$. Note that this inequality does not depend on the profile of baselines $(d_1^b, ..., d_m^b)$.

As for the North, because $u_n(.)$ is concave

$$u_n\left(D^b\right) \leq u_n\left(D^*\right) + u_n'\left(D^*\right)\left(D^b - D^*\right).$$

But, since at a Pareto optimal allocation $u_n'(D^*) = -t_i^s = -t_i^n = -t^*$, the above inequality reads as:

$$u_n\left(D^b\right) \leq u_n\left(D^*\right) - t^* \sum_i \left(d_i^b - d_i^*\right).$$

When the baselines are set at the business-as-usual levels, this inequality can be re-written:

$$t^* \sum_i \left(d_i^{bau} - d_i^*\right) \leq u_n\left(D^*\right) - u_n\left(D^{bau}\right) = WTP. \tag{9}$$

It means that, at the implemented allocation, what the north is required to pay (the left hand-side) is less than what it would accept to pay (the right hand-side) to move to the optimum, so individual rationality obtains. Would the same inequality prevail with different baselines?

When $D^b < D^{bau}$, from (9) we can deduce:

$$t^* \sum_i \left(d_i^b - d_i^*\right) < t^* \sum_i \left(d_i^{bau} - d_i^*\right) \leq u_n\left(D^*\right) - u_n\left(D^{bau}\right),$$

and individual rationality obtains again.

When $D^b > D^{bau}$:

$$t^* \sum_i \left(d_i^b - d_i^*\right) > t^* \sum_i \left(d_i^{bau} - d_i^*\right),$$

and it is no longer guaranted that the WTP exceeds the transfer. ■

Remark 2 *The above proposition identifies a sufficient condition to impose on baselines in order to ensure individual rationality. It does not necessarily mean that if baselines are larger than the business-as-usual levels, IR is violated. But, clearly, being too lax on baselines has the effect of increasing the volume of transfers, at the risk of transgressing individual rationality of the north.*

Another criterion for equity is the no-envy test. We will limit the use of this notion to southern countries. An allocation $\left(d_1^*, ..., d_m^*, y^{1*}, ..., y^{m*}, y^{n*}\right)$ has no-envy in the South if no southern country would prefer the deforestation - income pair of another southern country. More precisely:

Definition 3 *There is no envy (NE) in the South if there exists no pair of developing countries i and j such that:*

$$u_i\left(d_j^*\right) + y^{j*} > u_i\left(d_i^*\right) + y^{i*}.$$

The above notion could be criticized in our context, because it does not question the domain over which it is reasonable to use the absence of envy as a guide for equity. This test could be modified in order to discard from the domain of justice the exogenous endowment of incomes, y_0^i, for trying to redress a feeling of envy based on such a exogenous variable could hurt the intuition. A modified and weaker axiom, referred to as NE', would then just discard the possibility that:

$$u_i\left(d_j^*\right) + t^*\left(d_j^b - d_j^*\right) > u_i\left(d_i^*\right) + t^*\left(d_i^b - d_i^*\right)$$

Proposition 3 *Assume that the same baselines are offered to any southern countries. Then the REDD* mechanism implements a Pareto optimal allocation and satisfies NE'.*

Proof. The modified no-envy test in the South requires that:

$$u_i\left(d_j^*\right) + t^*\left(d^b - d_j^*\right) \leq u_i\left(d_i^*\right) + t^*\left(d^b - d_i^*\right) \ ,$$

$$\iff \quad u_i\left(d_j^*\right) \leq u_i\left(d_i^*\right) + t^*\left(d_j^* - d_i^*\right) \ .$$

Now, because $u_i'\left(d_i{}^*\right) = t^*$:

$$u_i\left(d_j^*\right) \leq u_i\left(d_i^*\right) + u_i'\left(d_i{}^*\right)\left(d_j^* - d_i^*\right),$$

an equality that is verified because the functions $u_i\left(.\right)$ are concave. ■

Results so far indicate that both individual rationality and (some form of) no-envy are compatible. This can be achieved for instance by setting the same baseline d^b to each country and in such a way that their sum is not larger than $\sum_i d_i^{bau}$. For instance $d^b = \overline{d}^{bau} = \frac{\sum_i d_i^{bau}}{m}$ would do the job.

Getting back to propositions currently discussed at the UN, there is a concern that, based on observed current deforestation behaviors, some countries are more deserving than other and should be rewarded; on the contrary some countries bear more responsibility about the environmental problem, and should be penalized. A possible measure of "environmental merit" could be the gap between the total possible deforestation and the BAU deforestation, $\bar{d}_i - d_i^{bau}$, that is, the contribution on a voluntary basis to pristine nature. However such a measure would attribute the same merit to countries with the same gap but with large differences in potential contributions, because some countries have much larger $\bar{d}_i$ than others. This objection is overcome if the merit is measured in relative terms, with the ratio:

$$M_i = \frac{\bar{d}_i - d_i^{bau}}{\bar{d}_i}.$$

Let us note $\overline{M}_i$ the average relative merit and define $\Delta M_i = M_i - \overline{M}_i$. From the point of view of their contributions to the environment, countries can be partionned into two subsets, those who are much deserving ($\Delta M_i > 0$) and those who are less ($\Delta M_i \leq 0$). Let us also impose the following requirement on transfers:

Definition 4 *A transfer scheme recognizes environmental merit (EM) if, other things being equal, "deserving" countries are rewarded whereas "undeserving" countries are penalized.*

We are now in position to make the following proposal. Baselines could be set as follows:

$$d_i^{b*} = \alpha \Delta M_i \sum_{h=1}^{m} \left(\bar{d}_h - d_h^{bau} \right) + (1-\alpha)\, d_i^{bau} \ , \qquad \alpha \in [0,1].$$

Proposition 4 *Let the baselines be given by d_i^{b*}. Then the REDD* mechanism satisfies PO, IR and EM.*

Proof. By construction, if the baselines d_i^{b*} are chosen, the mechanism recognizes environmental merit. Besides we already know that the mechanism implements pareto optimal allocations. If $d_i^b = d_i^{b*}$, we have:

$$\begin{aligned} \sum_i d_i^b &= \alpha \sum_h \bar{d}_h - d_h^{bau} \sum_i \Delta M_i + (1-\alpha) \sum_i d_i^{bau} \\ &= (1-\alpha) \sum_i d_i^{bau} \leq \sum_i d_i^{bau}. \end{aligned}$$

And by proposition 2, the mechanism is individually rational. ■

In summary, the proposed mechanism allows us to choose some fairness properties without loosing efficiency.

5 Conclusion

In this article we propose a class of incentive mechanisms, REDD*, to curb deforestation efficiently in tropical countries. It is derived fromt the Compensation Mechanism (Varian, 1994) and adapted to the context of international negative externalities where no tax can be imposed on the polluter. This class of mechanisms implement pareto optimal allocations and it is individually rational. Besides, it can satisfy some fairness properties such as a form of no-envy or reward the environmental merit of countries.

Even if the aim of the paper is to study the problem of deforestation in tropical countries, this mechanism could clearly be applyed to other environmental issues such as pollution or climate change.

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Appendix

A Pareto optimal allocations

Pareto optimal allocations can be found as a solution to the program:

$$\max_{\{d_i\}_{i=1}^m,\{y^i\}_{i=1}^m,y^n} u_n\left(\sum\nolimits_i d_i\right)+y^n$$
$$\text{s.t.}\ \begin{cases} u_i\left(d_i\right)+y^i \geq \bar{U^i}\ , & i=1,...,m, \\ y^n+\sum\nolimits_i y^i=\Omega\ . \end{cases}$$

The Lagrangian for this problem is:

$$\mathcal{L}=u_n\left(\sum\nolimits_i d_i\right)+y^n+\sum\nolimits_{i=1}^m \sigma_i\left[u_i\left(d_i\right)+y^i-\bar{U^i}\right]+\lambda\left(y^n+\sum\nolimits_i y^i-\Omega\right)$$

The necessary conditions for optimality read as:

$$\frac{\partial\mathcal{L}}{\partial\ d_i}=u_n'+\sigma_i u_i'=0\ , \quad i=1,...,m, \tag{10}$$

$$\frac{\partial\mathcal{L}}{\partial\ y^i}=\lambda+\sigma_i\ =0, \quad i=1,...,m, \tag{11}$$

$$\frac{\partial\mathcal{L}}{\partial\ \sigma_i}=u_i\left(d_i\right)+y^i-\bar{U^i}=0\ , \quad i=1,...,m, \tag{12}$$

$$\frac{\partial\mathcal{L}}{\partial\ \lambda}=y^n+\sum\nolimits_i y^i-\Omega=0\ , \tag{13}$$

$$\frac{\partial\mathcal{L}}{\partial\ y^n}=1+\lambda=0. \tag{14}$$

>From (11) and (14):

$$\sigma_i\ =1, \quad i=1,...,m.$$

Using this information in (10), one can deduce:

$$u_i'=-u_n'\ ,$$

Figure 1: Eigen Values polynome

as indicated in the text by expression (1).