From Boston to Shanghai to Deferred Acceptance: Theory and Experiments on A Family of School Choice Mechanisms *

Yan Chen Onur Kesten

March 31, 2011

Abstract

We characterize a family of proposal-refusal school choice mechanisms, including the Boston, Shanghai, and Deferred Acceptance (DA) mechanisms as special cases. We find that moving from one extreme member to the other results in systematic changes in both the incentive properties and nested Nash equilibria. In the laboratory, we find that participants are most likely to reveal their preferences truthfully under the DA mechanism, followed by the Shanghai and then Boston mechanisms. Furthermore, while DA is significantly more stable than Shanghai or Boston, the efficiency comparison varies across environments. In our 4-school environment, DA is weakly more efficient than Boston. However, in our 6-school environment, Boston achieves significantly higher efficiency than Shanghai, which outperforms DA.

Keywords: school choice, Boston mechanism, Shanghai mechanism, deferred acceptance , experiment

JEL Classification Numbers: C78, C92, D82

*We would like to thank Dirk Bergeman, Fuhito Kojima, Erin Krupka, Paul Resnick, Al Roth, Rahul Sami and seminar participants at Columbia, Florida State, Michigan, Microsoft Research, UECE Lisbon Meetings (2010) for helpful discussions and comments, Malvika Deshmukh, Ming Jiang, Robert Ketcham, Tracy Xiao Liu and Kai Ou for excellent research assistance. The financial support from the National Science Foundation through grants no. SES-0720943 and SES-0962492 is gratefully acknowledged. Chen: School of Information, University of Michigan, 105 South State Street, Ann Arbor, MI 48109-2112. Email: yanchen@umich.edu. Kesten: Tepper School of Business, Carnegie Mellon University, PA 15213. Email: okesten@andrew.cmu.edu.

1 Introduction

School choice has been one of the most important and widely-debated education policies in the past two decades (Hoxby 2003), with game theory playing a major role in the adoption of school choice mechanisms. Some school districts have reexamined their school choice mechanisms after game theoretic analysis (Abdulkadiroğlu and Sönmez 2003, Ergin and Sönmez 2006) and experimental evidence (Chen and Sönmez 2006) indicated that one of the most popular school choice mechanisms, the Boston mechanism, is vulnerable to strategic manipulation and thus might not result in efficient allocations. Following intensive policy discussions, in 2003, New York City public schools decided to replace its allocation mechanism with a version of the student-proposing deferred acceptance (DA) mechanism (Gale and Shapley 1962, Abdulkadiroğlu, Pathak and Roth 2005b). Similarly, in 2005, the Boston Public School Committee voted to replace the existing Boston school choice mechanism with the deferred acceptance mechanism (Abdulkadiroğlu, Pathak, Roth and Sönmez 2005a).

Despite the concern regarding potential manipulation, some recent literature on school choice has provided a more optimistic view of the Boston mechanism. Pathak and Sönmez (September 2008) and Özek (2008) propose justifications as to why some school districts using the Boston mechanism showed reluctance to transition to DA, and highlight some of the virtues of the Boston mechanism. In a similar vein, Abdulkadiroğlu, Che and Yasuda (2011) and Miralles (2009) emphasize possible *ex ante* welfare advantages of the Boston mechanism compared to DA. In a recent experimental study Featherstone and Niederle (2008) show that from an *ex ante* point of view, truthtelling can be an equilibrium under the Boston mechanism. Furthermore, the Boston mechanism can stochastically dominate DA in terms of efficiency.

In this paper, we strive to reconcile these seemingly conflicting points of view regarding the Boston mechanism and its comparison to DA. Specifically, our goal in this study is to better assess the efficiency-incentive trade-offs one faces when transitioning from the Boston mechanism to a mechanism such as the DA. A mechanism that we believe could provide key insights to this issue is one pioneered in Shanghai for high school admissions,[1] and later adopted by a number of provinces in Chinese college admissions. In the latter context, it is called the parallel mechanism.

The Chinese college admissions mechanisms are centralized matching processes via standardized tests, with each province implementing an independent matching process. These matching mechanisms fall into three classes: sequential, parallel, and partial parallel. The sequential mechanism is a priority matching mechanism similar to the Boston mechanism, but executed sequentially across categories in decreasing prestige and desirability. In the sequential mechanism, each college belongs to a category. Within each category, the Boston mechanism is used. When assignments in the first category are finalized, the assignment process in the second category starts, and so on. A problem with the sequential mechanism is the ability to game the system (Nie 2007).

To alleviate the student incentive to manipulate their preferences under the sequential mechanism, the parallel mechanism was developed. In the parallel mechanism, students can place several "parallel" colleges in each subcategory. For example, a student's first choice can contain four colleges, A, B, C and D, in

[1] This mechanism was adopted in Shanghai for high school admissions as early as 2003, `http://edu.sina.com.cn/l/2003-05-15/42912.html`, retrieved on February 2, 2011.

decreasing desirability. Allocation within the parallel colleges is temporary until all four colleges have considered the student's application. Thus, this mechanism lies between the Boston mechanism, where every step is final, and DA, where every step is temporary until all seats are filled. This method has yielded some improved allocation outcomes. For example, in 2008, when students in Beijing were allowed to fill in only one college for their first choice, but four parallel colleges as their second choice.[2] only 30 students with high exam scores (above 600) did not get into their second choice colleges vs. 200 in the previous year's sequential mechanism.[3] To our knowledge, the parallel mechanism has never been theoretically studied or empirically tested in the laboratory. In this paper, we call the entire class of parallel mechanisms the *Chinese parallel mechanisms*, and the simplest member of this class the *Shanghai mechanism*.

To study the performance of the different mechanisms more formally, we first conduct a theoretical analysis. We then conduct a series of experiments to compare the manipulability, stability and efficiency of each. In our theoretical analysis, we first present a family of *proposal-refusal* mechanisms in which each member is characterized by some positive integer $e \in \mathbb{N}_+$ of rounds through which the proposal and refusal process continues before assignments are made permanent. More precisely, the mechanism works as follows: During rounds 1 through e, students apply to schools in order of reported preference from the most preferred to the least, and schools tentatively admit applicants up to their capacity in order of priority going from the highest to the lowest. At the end of round e students tentatively held at a school are permanently accepted into that school. The remaining students participate in a new proposal and refusal process from round $e + 1$ through round $2e$. The process continues in this fashion until no student remains unassigned.

It is quite easy to see that as e increases, we go from the familiar Boston mechanism ($e = 1$) to the Chinese parallel mechanisms ($e \in [2, \infty)$) which include the Shanghai mechanism ($e = 2$), and from those to DA ($e = \infty$). In this framework, we find that, as one moves from one extreme member of this family to the other, the experienced trade-offs are in terms of efficiency, stability and strategic immunity. Within this family, Boston is the only Pareto efficient mechanism given truthtelling, and DA is the only mechanism that is strategy-proof and stable. We also show that members of this family can be ranked according to their immunity against strategic action. Any given member is more manipulable than a member with a higher e number (Proposition 1). On the welfare side, a more subtle comparison result emerges. For a given problem, the number of students receiving their reported first choices diminishes with an increasing e (Proposition 2). As far as stability or Pareto efficiency is concerned, the ranking is ambiguous (Proposition 3).

Since the theoretical efficiency ranking in this family of mechanisms assumes truthtelling, which is a dominant strategy only under DA, it is important to assess the behavioral response to members of this family. On a broader level, as the theoretical ranking for stability or Pareto efficiency is ambiguous, empirical rankings of aggregate performance measures for these mechanisms in different controlled laboratory settings are informative for policymakers in reforming school choice or college admissions.

For these reasons, we evaluate three members of this family in two environments in the laboratory. In both environments, we find that the proportion of truthtelling follows the order of DA > Shanghai > Boston,

[2]This type of mechanism is called the "partial parallel" college admissions mechanism, as it is a hybrid between the sequential and the parallel system.

[3]Source: Beijing Educational Testing Bureau statistics reported on China News Web `http://edu.sina.com.cn/gaokao/2009-06-07/1129203534.shtml`, retrieved on June 8, 2009.

while the proportion of District School Bias follows the reverse order. Consistent with theory, DA achieves a significantly higher proportion of stable outcomes than either Shanghai or Boston. However, the efficiency comparison is sensitive to the environment. In our 4-school environment, DA is weakly more efficient than Boston, while the Shanghai mechanism is not significantly different from either. In comparison, in our 6-school environment, the Boston mechanism achieves significantly higher efficiency than the Shanghai mechanism, which in turn is significantly more efficient than DA.

Our findings on the manipulability and stability of the Boston mechanism compared to DA are consistent with earlier experimental work on school choice (Chen and Sönmez 2006, Pais and Pintér 2008, Calsamiglia, Haeringer and Klijn 2010). However, we differ from previous research in that we present the first experimental evidence for the performance of the Shanghai mechanism. Furthermore, compared to the one-shot implementation of previous experiments on school choice except Featherstone and Niederle (2008),[4] our experimental design with repeated random re-matching enables us to compare the performance of the mechanisms with experienced participants. In doing so, we find that learning separates the performance of the mechanisms both in terms of efficiency and stability. Lastly, our theoretical characterization of the complete set of Nash equilibria provides a benchmark for analyzing the experimental data, which reveals that stable Nash equilibrium outcomes are significantly more likely to arise than the unstable ones even when the latter Pareto dominates the former. To our knowledge, this is the first experimental evidence on equilibrium selection in school choice mechanisms.

The rest of this paper is organized as follows. Section 2 formally introduces the school choice problem and the family of mechanisms. Section 3 presents the theoretical results. Section 4 describes the experimental design. Section 5 presents the hypotheses. Section 6 summarizes the results of the experiments. Section 7 concludes.

2 School choice problem and the three mechanisms

A school choice problem (Abdulkadiroğlu and Sönmez 2003) consists of a number of students each of whom is to be assigned a seat at one of a number of schools. Further, each school has a maximum capacity, and the total number of seats in the schools is no less than the number of students. We denote the set of students by $I \equiv \{i_1, i_2, \ldots, i_n\}$. A generic element in I is denoted by i. Likewise, we denote the set of schools by $S \equiv \{s_1, s_2, \ldots, s_m\}$. A generic element in S is denoted by s. Each school has a number of available seats. Let q_s be the number of available seats at school s, or the **quota** of s. For each school, there is a strict priority order of all students, and each student has strict preferences over all schools. The priority orders are determined according to state or local laws as well as the criteria of school districts. We denote the priority order for school s by $\succ_s$, and the preferences of student i by P_i. Let R_i denote the at-least-as-good-as relation associated with P_i.

A **school choice problem**, or simply a problem, is a pair $((\succ_s)_{s\in S}, (P_i)_{i\in I})$ consisting of a collection of priority orders and a preference profile. For a given problem, at a matching, each student is placed at

[4]Featherstone and Niederle (2008) investigate the performance of the Boston and DA mechanisms under incomplete information, whereas we study the family of mechanisms under complete information. While their experiment is implemented under a random re-matching protocol, they do not explicitly analyze the effects of learning.

one school and the number of students placed to a particular school does not exceed the number of available seats at that school. A matching is **Pareto efficient** if there is no other matching which makes all students at least as well off and at least one student better off.

A closely related problem to the school choice problem is the *college admissions problem* (Gale and Shapley 1962). In the college admissions problem, schools have preferences over students whereas in a school choice problem, schools are merely objects to be consumed. A key concept in college admissions is stability. A matching is **stable** if there is no student-school pair (i, s) such that student i prefers school s to the school he is assigned to, and school s prefers student i to at least one student who is assigned to it. The natural counterpart of stability in our context is defined by Balinski and Sönmez (1999). Specifically, for a given matching μ, the priority of student i for school s is violated if i would rather be placed at s and a student j is assigned to s who has lower priority for s than student i. A matching is **stable** if no student's priority for any school is violated.

A **school choice mechanism**, or simply a mechanism, selects a matching for each problem. A mechanism is Pareto efficient if it always selects Pareto efficient matchings. A mechanism is stable if it always selects stable matchings. A mechanism is **strategy-proof** if no student can ever gain by misrepresenting her preferences. More precisely, there does not exist a problem $((\succ_s)_{s\in S}, (P_i)_{i\in I})$, a student i, and a preference P_i' such that student i prefers the school she is placed at when she reports P_i' to the school she is placed at when she reports P_i.

We now describe the three mechanisms that are central to our study. The first two are the familiar Boston and DA mechanisms, while the third one is a stylized version of the Shanghai mechanism.

2.1 Boston Mechanism (BOS)

Our first mechanism is the most common school choice mechanism observed in practice. Its outcome can be calculated via the following algorithm for a given problem:

- An application to the first choice school is sent for each applicant.

  Each school accepts the students with the highest priority for that school up to its capacity. These students and their assignments are removed from the system. The remaining applications for each respective school are rejected.

- The rejected applications are sent to the student's second choice.

  If a school still has vacant seats, then it accepts students with the highest priority up to its capacity and rejects the remaining applications.

  ......

- The algorithm terminates when there are no students or schools left.

Note that the allocation is final in each round. More formally, the algorithm can be described as follow:

Round 1: For each school s, consider only those students admissable to it who have listed it as their first acceptable choice. Up to q_s students among them with the highest priority for school s (all students if fewer than q_s) are placed to school s.

Round k, $k \geq 2$: Consider the remaining students. For each school s with q_s^k available seats, consider only those students admissable to it who have listed it as their k-th acceptable choice. Those q_s^k students among them with the highest priority for school s (all students if fewer than q_s^k) are placed to school s.

The algorithm terminates when there are no students or schools left. Any student who is not placed to any school acceptable to her remains unassigned. An important critique of the Boston mechanism is that it gives students strong incentives to misrepresent their preferences. Because a student who has high priority for a school may lose her advantage for that school if she does not list it as her first choice, students are forced to act strategically to game the system.

2.2 Deferred Acceptance Mechanism (DA)

A second matching mechanism is the student-optimal stable mechanism (Gale and Shapley 1962), which finds the stable (*stable* in our context) allocation that is most favorable to each student for any given two-sided matching problem (*school choice problem* in our context). Its outcome can be calculated via the following *deferred acceptance algorithm* for a given problem:

- An application to the first ranked school is sent for each student.
- Throughout the allocation process, a school can hold no more applications than its capacity.

  If a school receives more applications than its capacity, then it temporarily retains the students with the highest priority up to its capacity and rejects the remaining students.
- Whenever an applicant is rejected at a school, his application is sent to the next highest-ranked school.
- Whenever a school receives new applications, these applications are considered together with the retained applications for that school. Among the retained and new applications, the ones with the highest priority up to its capacity are held temporarily.
- The allocation is finalized when no more applications can be rejected.

  Each participant is assigned a slot at the school that holds his or her application at the end of the process.

Note that allocation is temporary in each round until the last round. More formally, the algorithm can be described as follow:

Round 1: Each student applies to his favorite acceptable school. For each school s, up to q_s applicants admissable to it who have the highest priority for school s are tentatively placed to school s. The remaining applicants are rejected.

Round k, $k \geq 2$: Each student rejected from a school at step $k-1$ applies to his next favorite acceptable school. For each school s, up to q_s students admissable to it who have the highest priority for school s among the new applicants, and those tentatively on hold from an earlier step, are tentatively placed at school s. The remaining applicants are rejected.

The algorithm terminates when each student is either tentatively placed to a school, or has been rejected by every school acceptable to him. DA has several desirable theoretical properties, most notably incentives and stability. Under DA, it is a dominant strategy for students to state true preferences (Roth 1982, Dubins and Freedman 1981). Another desirable property of DA is that it is stable. Although it is not Pareto efficient, it is the most efficient among the stable school choice mechanisms.

In practice, DA has been the leading mechanism for school choice reforms. For example, DA has been adopted by New York City and Boston public school systems, which had suffered from congestion and incentive problems from their previous mechanisms, respectively.

2.3 Shanghai Mechanism (SH)

As mentioned, the Shanghai mechanism was first implemented as a high school admissions mechanism in Shanghai. In 2008, variants of the mechanism were implemented in nine provinces as the parallel college admissions mechanisms to replace the sequential mechanisms, which corresponds to the Boston mechanism with categories.[5] In this study, we use a stylized version of the (Shanghai) parallel mechanism, adapted for the school choice context.

- An application to the first ranked school is sent for each student.
- Throughout the allocation process, a school can hold no more applications than its capacity.

  If a school receives more applications than its capacity, it retains the students with the highest priority up to its capacity and rejects the remaining students.
- Whenever a student is rejected at a school, his or her application is sent to the next highest-ranked school.
- Whenever a school receives new applications, these applications are considered together with the retained applications for that school. Among the retained and new applications, the ones with the highest priority up to its capacity are retained.
- The allocation is finalized every e steps. That is, in steps e, $2e$ and $3e$ etc., each participant is assigned a school that holds his or her application in that step. These students and their assignments are removed from the system.
- The allocation process ends when no more applications can be rejected.

In the next section, we offer a formal definition of the parallel mechanisms and characterize the theoretical properties of this family of matching mechanisms.

[5] The admission process is executed sequentially by category. Within each category, universities are again divided into subcategories. Here is a typical 4-category division: Category 0 includes art institutes, military academies, normal universities attached to the Ministry of Education, and Hong Kong universities. Category 1 consists of four-year universities and colleges, which are in turn divided into subcategories, usually by the prestige of the schools. Category 2 consists of arts junior colleges (professional schools), while Category 3 consists of other junior colleges (professional schools).

3 Theoretical Analysis

In this section, we characterize the theoretical properties of the family of proposal-refusal mechanisms. Given (strict) student preferences, (strict) school priorities, and fixed school quotas, consider the following general *proposal-refusal algorithm* that indexes each member of the family by a *permanency-execution period* e:

Round $\mathbf{r} = \mathbf{1}$:

∘ Each student applies to her first choice. Each school s then considers its applicants. Those students with the highest s-priority are tentatively assigned to school s based on the quota of school s. The rest are rejected.

∘ If $r \equiv 0\ (mod\ e)$, then all tentative assignments are final. Reduce each school's quota by the number of students currently tentatively assigned to it.

$$\vdots$$

In general,

Round $\mathbf{r} \geq \mathbf{2}$:

∘ Each rejected student applies to her next choice school. Each school s considers its applicants in this round together with those students who have been tentatively assigned to it in an earlier round (if any). Those students with the highest s-priority are tentatively assigned to school s based on the quota of school s. The rest are rejected.

∘ If $r \equiv 0\ (mod\ e)$, then all tentative assignments are final. Reduce each school's quota by the number of students assigned to it at this round.

The algorithm terminates when all students have been assigned to a school.

Remark 1 *For a given problem $\xi = ((\succ_s)_{s\in S}, (P_i)_{i\in I})$, the outcome of the above proposal-refusal algorithm (e) coincides with*

(i) the Boston mechanism when $e = 1$;

(ii) the Shanghai mechanism when $e = 2$

(iii) the Chinese parallel mechanism when $2 \leq e < \infty$

(iv) the DA mechanism when $e = \infty$.

We can now formalize our first observation about the proposal-refusal family of mechanisms.

Lemma 1 *Within the class of mechanisms spanned by the proposal-refusal algorithm, i.e., $e \in \{1, 2, \ldots, \infty\}$,*

(i) there is exactly one member that is Pareto efficient. This is the Boston mechanism;

(ii) there is exactly one member that is strategy-proof. This is the DA mechanism.

(ii) there is exactly one member that is stable. This is the DA mechanism.

Note that part (i) of Lemma 1 refers to Pareto efficiency with respect to reported preferences. Since the Boston mechanism is not strategy-proof, we expect that efficiency with respect to reported and true preferences will be different. Furthermore, recall that, following the convention in the matching literature, our notion of Pareto efficiency is with respect to ordinal rather than cardinal preferences.

3.1 Manipulation strategies for proposal-refusal mechanisms

As Lemma 1 shows, a proposal-refusal (e) mechanism is manipulable if $e < \infty$. Hence students need to make hard judgments to determine their optimal strategies. Since tentative assignments are made permanent at the end of round e, ultimately what matters for a student is to be tentatively held at a particular school at the end of round e, $2e$, $3e$ etc. The best strategy is the one that ensures that the student is held at her target school at the end of round e. In general, listing a school high on a preference list does not necessarily increase her chances of being assigned to that school. This is because an early application to a school may trigger a rejection chain that may cause the early applicant to be rejected from that school before the end of the first e rounds (see Example 1a).

On the other hand, if a student is rejected from a school at round e, then she has already lost her priority for schools she has not yet applied to (to students who did so in the first e rounds). Hence a good strategy needs to optimally trade off the advantage from listing a school high in the preference list with the disadvantage of being rejected from that school at round e. More concretely, we next discuss two ways a proposal-refusal ($e < \infty$) mechanism may be manipulated.

Example 1a. (*within-round manipulation*): We first illustrate a *within-round manipulation*. Consider a school choice problem with four students and four schools, each with one seat. Fixed priority orders and possible student preferences are as follows.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$
i_3	i_2	i_3	$\vdots$
i_1	i_3	i_4	$\vdots$
i_2	$\vdots$	$\vdots$	$\vdots$

P_{i_1}	P'_{i_1}	P_{i_2}	P_{i_3}	P'_{i_3}	P_{i_4}
s_1	s_3	s_1	s_2	s_1	s_3
s_3	s_1	s_2	s_1	$\vdots$	s_1
s_4	s_4	$\vdots$	s_4		$\vdots$

The following two tables illustrate the steps of the proposal-refusal mechanism ($e = 3$) applied to the problem (P_{i_1}, P_{-i_1}). A student tentatively placed to a school at a particular step is indicated with a box.

Round 1	s_1 ($q_{s_1}^{r=1} = 1$)	s_2 ($q_{s_2}^{r=1} = 1$)	s_3 ($q_{s_3}^{r=1} = 1$)	s_4 ($q_{s_4}^{r=1} = 1$)
Step 1	$\boxed{i_1}$, i_2	$\boxed{i_3}$	$\boxed{i_4}$	
Step 2	$\boxed{i_1}$	$\boxed{i_2}$, i_3	$\boxed{i_4}$	
Step 3	$\boxed{i_3}$, i_1	$\boxed{i_2}$	$\boxed{i_4}$	

Round 2	s_1 ($q_{s_1}^{r=2} = 0$)	s_2 ($q_{s_2}^{r=2} = 0$)	s_3 ($q_{s_3}^{r=2} = 0$)	s_4 ($q_{s_4}^{r=2} = 1$)
Step 4			i_1	
Step 5	$\vdots$			$\boxed{i_1}$
Step 6	$\vdots$	$\vdots$		$\boxed{i_1}$

In the above rounds, at the problem (P_{i_1}, P_{-i_1}), student i_1 applies to school s_1 in the first step but has her placement rescinded at the end of the third step. Her final assignment is school s_4, which is determined in the second round. Now consider the following rounds of the mechanism when student i_1 reports P'_{i_1}.

Round 1	s_1 $(q_{s_1}^{r=1} = 1)$	s_2 $(q_{s_2}^{r=1} = 1)$	s_3 $(q_{s_3}^{r=1} = 1)$	s_4 $(q_{s_4}^{r=1} = 1)$
Step 1	$\boxed{i_2}$	$\boxed{i_3}$	$\boxed{i_4}, i_1$	
Step 2	$\boxed{i_1}, i_2$	$\boxed{i_3}$	$\boxed{i_4}$	
Step 3	$\boxed{i_1}$	$\boxed{i_2}, i_3$	$\boxed{i_4}$	
Round 2	s_1 $(q_{s_1}^{r=2} = 0)$	s_2 $(q_{s_2}^{r=2} = 0)$	s_3 $(q_{s_3}^{r=2} = 0)$	s_4 $(q_{s_4}^{r=2} = 1)$
Step 4	i_3			
Step 5	$\vdots$			$\boxed{i_3}$
Step 6	$\vdots$	$\vdots$		$\boxed{i_3}$

In this case, at the problem (P'_{i_1}, P_{-i_1}), student i_1 can secure her place at school s_1 by delaying her application to this school to the second step. Note that, at the problem (P_{i_1}, P_{-i_1}) the application of student i_1 in step 1 initiates a rejection chain which eventually causes her to be rejected from this school. On the other hand, this adverse effect is eliminated when she deliberately wastes her first choice at a school (namely, school s_3) where she does not have any chance of being admitted. We refer to such a manipulation as a *within-round manipulation*. In fact, this sort of a strategy is reminiscent of the sniping behavior observed in eBay auctions (Ockenfels and Roth 2002).

In Example 1a, note that the outcome of the proposal-refusal mechanism ($e = 3$) for the problem (P'_{i_1}, P_{-i_1}) involves a situation of justified envy for student i_3 for school s_1 since she has higher priority at this school than student i_1. This happens simply because student i_3 gets 'too late' to apply to school s_1 and loses her priority to student i_1. We next illustrate a second kind of manipulation, whereby student i_3 misrepresents her preference to improve her final allocation.

Example 1b. *(across-round manipulation)*: Now consider the problem $(P'_{i_1}, P_{i_2}, P'_{i_3}, P_{i_4})$ that results when student i_3 submits school s_1 as her first choice (as opposed to her second). The following table illustrates the resulting proposal-refusal mechanism (e=3) outcome.

Round 1	s_1 $(q_{s_1}^{r=1} = 1)$	s_2 $(q_{s_2}^{r=1} = 1)$	s_3 $(q_{s_3}^{r=1} = 1)$	s_4 $(q_{s_4}^{r=1} = 1)$
Step 1	$\boxed{i_3}, i_2$		$\boxed{i_4}, i_1$	
Step 2	$\boxed{i_3}, i_1$	$\boxed{i_2}$	$\boxed{i_4}$	
Step 3	$\boxed{i_3}$	$\boxed{i_2}$	$\boxed{i_4}$	$\boxed{i_1}$

By submitting P'_{i_3}, student i_3 improves the ranking of school s_1 and secures a seat at the end of round 1 as opposed to round 2. We refer to such a manipulation as an *across-round manipulation*. This manipulation strategy has been widely observed under the Boston mechanism and diagnosed as one of the main factors behind the gaming issues that arose under the Boston mechanism.

We next provide an incentive-based ranking of the family of proposal-refusal mechanisms. However, we first impose a 'richness' assumption on the school choice problems that we study. Let $((\succ_s)_{s\in S}, (P_i)_i \in I)$ be a school choice problem. Let s_i^k denote the school that student i ranks k-th. We define school s_i^k as *unachievable* for student i if

1. there exists a set $I_1^i \subset I$ with $|I_1^i| \geq q_{s_i^1}$ such that each $j_1 \in I_1^i$ ranks s_i^1 first and has higher s_i^1-priority than i;

2. there exists a set $I_2^i \subset I \backslash I_1^i$ with $|I_2^i| \geq q_{s_i^2}$ such that each $j_2 \in I_2^i$ ranks s_i^2 second or higher and has higher s_i^2-priority than i;

$$\vdots$$

k there exists a set $I_k^i \subset I \backslash \{I_1^i \cup I_2^i \cup \cdots \cup I_{k-1}^i\}$ with $|I_k^i| \geq q_{s_i^k}$ such that each $j_k \in I_k^i$ ranks s_i^k k-th or higher and has higher s_i^k-priority than i.

Presence of Unachievable Schools (PUS^f): *A school choice problem satisfies the PUS^f assumption if there exist at least f unachievable schools for any student.*

The PUS assumption gives each student the possibility of making 'dead' choices, thus increasing their options to manipulate their preferences. For example, by listing an unachievable school, a student might be able to avoid a rejection chain. This assumption is primarily used to compare the properties of the Shanghai mechanism with those of the DA. Note that the unachievable set of each student could presumably be different. It is straightforward to observe that, given a problem satisfying the PUS^f assumption, under a proposal-refusal mechanism (e) with $e \leq f$, no student will be assigned to one of her unachievable schools. In practice, most students can easily identify a school within the district for which they have a very low chance of being accepted due to the popularity of the school, short quota, etc.[6] Even though it may seem counterintuitive, we will see that students may find it in their best interest to use unachievable schools in their preference rankings under certain members of the proposal-refusal family. The proof of the next proposition makes it clear that the seemingly irrational behavior of choosing an unattainable school could be a very useful strategy. We first define manipulability following Pathak and Sönmez (2011). That is, mechanism ϕ is said to be *manipulable by student* j at problem $(\succ, P)$ if there exists P_j' such that $\phi_j(\succ, P_j', P_{-j})$ P_j $\phi_j(\succ, P)$. Thus, mechanism ϕ is said to be *manipulable* if there exists some student j and some problem $(\succ, P)$ at which ϕ is manipulable by student j.

Lemma 2 *Assume PUS. If the outcome of a proposal-refusal mechanism $\varphi(e)$ is unstable for a problem $((\succ_s)_{s\in S}, (P_i)_{i\in I})$, then an envious student can successfully manipulate at this problem.*

Proposition 1 (Manipulability) *If the Shanghai mechanism is manipulable by a student at some problem, then the Boston mechanism is also manipulable by a student at the same problem. However, the converse is*

[6] Moreover, in practice, a student can list the same school multiple times in her preference list, even though it will have no effect on her chance of being admitted to the particular school.

not true. In general, if a proposal-refusal mechanism (e) with $e > 1$ is manipulable by a student at some problem, then any proposal-refusal mechanism (e') with $e' < e$ is also manipulable at the same problem, provided that it satisfies PUS $^{e'-1}$. However, the converse is not true.

Proposition 1 consists of two parts. The first part pertains to the manipulability comparison between the Boston and Shanghai mechanisms and does not require any assumption on the specific problem. However, because of the more sophisticated strategic environment that emerges when $e > 2$, the second part of Proposition 1 relies on the PUS assumption. In order to give insight into this result, we next outline an example demonstrating the idea behind the Boston vs. Shanghai comparison.

Example 2a. (The Boston mechanism is manipulable whenever the Shanghai mechanism is) Consider the following example with the given priority structure and the profile of preferences. Schools s_1 and s_2 each have a quota of one, while school s_3 has a quota of two.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$
i_4	i_1	$\vdots$
i_1	i_3	
i_2	i_4	
i_3	i_2	

P_{i_1}	P'_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P'_{i_4}
s_1	s_2	s_1	s_2	s_2	s_1
s_2	$\vdots$	s_3	s_3	s_1	$\vdots$
s_3	$\vdots$	s_2	s_1	s_3	$\vdots$

The following two tables illustrate the steps of the Shanghai mechanism applied to the problem $(\succ, P)$. A student tentatively placed at a school at a particular step is outlined in a box.

Round 1	s_1 $(q_{s_1}^{r=1} = 1)$	s_2 $(q_{s_2}^{r=1} = 1)$	s_3 $(q_{s_3}^{r=1} = 2)$
Step 1	$\boxed{i_1}, i_2$	$\boxed{i_3}, i_4$	
Step 2	$\boxed{i_4}, i_1$	$\boxed{i_3}$	$\boxed{i_2}$
Round 2	s_1 $(q_{s_1}^{r=2} = 0)$	s_2 $(q_{s_2}^{r=2} = 0)$	s_3 $(q_{s_3}^{r=2} = 1)$
Step 3		i_1	
Step 4	$\vdots$		$\boxed{i_1}$

In the above tables, observe that student i_1 ends up at her last choice at problem $(\succ, P)$. Now consider the following two tables that illustrate the steps of the Shanghai mechanism when student i_1 reports P'_{i_1}, as opposed to P_{i_1}.

Round 1	s_1 $(q_{s_1}^{r=1} = 1)$	s_2 $(q_{s_2}^{r=1} = 1)$	s_3 $(q_{s_3}^{r=1} = 2)$
Step 1	$\boxed{i_2}$	$\boxed{i_1}, i_3, i_4$	
Step 2	$\boxed{i_4}, i_2$	$\boxed{i_1}$	$\boxed{i_3}$
Round 2	s_1 $(q_{s_1}^{r=2} = 0)$	s_2 $(q_{s_2}^{r=2} = 0)$	s_3 $(q_{s_3}^{r=2} = 1)$
Step 3			$\boxed{i_2}$

In this case, student i_1 ends up at school s_2. Thus, the Shanghai mechanism is manipulable by student i_1 at problem $(\succ, P)$. Next, let us apply the Boston mechanism to problem $(\succ, P)$. The specifications are illustrated in the following tables.

Round 1	s_1 $(q^{r=1}_{s_1} = 1)$	s_2 $(q^{r=1}_{s_2} = 1)$	s_3 $(q^{r=1}_{s_3} = 2)$
Step 1	$\boxed{i_1}$, i_2	$\boxed{i_3}$, i_4	
Round 2	s_1 $(q^{r=2}_{s_1} = 0)$	s_2 $(q^{r=2}_{s_2} = 0)$	s_3 $(q^{r=2}_{s_3} = 2)$
Step 2	i_4		$\boxed{i_2}$
Round 3	s_1 $(q^{r=2}_{s_1} = 0)$	s_2 $(q^{r=2}_{s_2} = 0)$	s_3 $(q^{r=2}_{s_3} = 1)$
Step 3			$\boxed{i_4}$

Observe that student i_1 ends up at her first choice at problem $(\succ, P)$, and thus cannot gain by a misreport, but student i_4 ends up at her last choice at problem $(\succ, P)$. Next consider the following tables that illustrate the steps of the Boston mechanism when student i_4 reports P'_{i_4}, as opposed to P_{i_4}.

Round 1	s_1 $(q^{r=1}_{s_1} = 1)$	s_2 $(q^{r=1}_{s_2} = 1)$	s_3 $(q^{r=1}_{s_3} = 2)$
Step 1	$\boxed{i_4}$, i_1, i_2	$\boxed{i_3}$	
Round 2	s_1 $(q^{r=2}_{s_1} = 0)$	s_2 $(q^{r=2}_{s_2} = 0)$	s_3 $(q^{r=2}_{s_3} = 2)$
Step 2		i_1	$\boxed{i_2}$
Round 3	s_1 $(q^{r=2}_{s_1} = 0)$	s_2 $(q^{r=2}_{s_2} = 0)$	s_3 $(q^{r=2}_{s_3} = 1)$
Step 3			$\boxed{i_1}$

Now student i_4 ends up at school s_1. Thus, the Boston mechanism is also manipulable at problem $(\succ, P)$.

Example 2b. (Shanghai mechanism need not be manipulable when the Boston mechanism is) Consider the following example with the given priority structure and the profile of preferences. Each school, s_1, s_2, and s_3, has a quota of one.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$
i_1	i_2	$\vdots$
i_2	i_3	
$\vdots$	$\vdots$	

P_{i_1}	P_{i_2}	P'_{i_2}	P_{i_3}
s_1	s_1	s_2	s_2
$\vdots$	s_2	$\vdots$	s_3
	s_3		$\vdots$

Clearly, at problem $(\succ, P)$ under the Boston mechanism, student i_2 can successfully obtain a seat at s_2 by submitting P'_{i_2} as opposed P_{i_2} which places her at s_3. Note, however, that under the Shanghai mechanism no student can ever gain by lying at problem $(\succ, P)$.

The next result enables us to see the efficiency-incentive tension within the class of mechanisms. A common method for evaluating mechanism efficiency is to assess the number of students assigned to their (reported) first choice. For example, in evaluating the outcome of the Boston mechanism, Cookson Jr. (1994) reports that 75% of all students entering the Cambridge public school system at the K-8 levels gained admission to the school of their first choice. Similarly, the analysis of the Boston and NYC school district data by Abdulkadiroğlu, Pathak, Roth and Sönmez (2006) and Abdulkadiroğlu, Pathak and Roth (2009) also report the number of first choices of students.

To assess efficiency, we rank the mechanisms within the family based on the number of first choices they assign. The Boston mechanism turns out to be the most generous in terms of first choice assignments, whereas the DA is the least. Hence, the next result suggests that, within the family of proposal-refusal mechanisms, the decrease in the possibility of manipulation with increasing e value comes at the cost of a diminishing number of first choice assignments.

Proposition 2 (First Choice Accommodation) *Given any school choice problem, the proposal-refusal* (e) *mechanism assigns no fewer students to their (reported) first choices than the proposal-refusal* (e') *mechanism with* $e' > e$.

Note that one needs to be cautious when interpreting Proposition 2. Since all members of the family with the exception of DA violate strategy-proofness, student preference submission strategies may also vary across mechanisms.

Proposition 3 (Stability and Pareto Efficiency) *Consider any school choice problem.*
(i) If the outcome of the proposal-refusal (e) *mechanism with* $e > 1$ *is unstable, then the outcome of any proposal-refusal mechanism* (e') *with* $e' < e$ *is not necessarily unstable.*
(ii) If the outcome of the proposal-refusal (e) *mechanism with* $e > 1$ *is Pareto efficient, then the outcome of any proposal-refusal mechanism* (e') *with* $e' < e$ *is not necessarily Pareto efficient.*

Proposition 3 indicates that there is no problem-wise systematic ranking in terms of stability or Pareto efficiency within the Chinese parallel mechanisms.

3.2 Stability and the Nash Equilibria of the Induced Preference Revelation Games

An important implication of Proposition 1 is that the set of Nash equilibrium strategies corresponding to the preference revelation games associated with members of the proposal-refusal family have a nested structure.[7] Hence, for any given problem, DA has the largest set of equilibrium strategies within the entire family, whereas Boston has the smallest. Consequently, coordination issues may become more serious for the Chinese parallel mechanisms as e increases. Interestingly, this finding suggests that DA mechanism would be subject to a more difficult coordination problem than Boston if not for its strategy-proof property.

Corollary 1 (Nested Nash Equilibria) *Assume PUS*$^{e-1}$. *Given any school choice problem, any Nash equilibrium of the preference revelation game under the proposal-refusal* (e) *mechanism with* $1 \leq e < \infty$, *is also a Nash equilibrium of the preference revelation game of the proposal-refusal* (e') *mechanism with* $e' > e$.

[7] A similar observation is made by Haeringer and Klijn (2008) for the revelation games under the Boston mechanism when the number of school choices a student can make (in her preference list) is limited by a quota.

We next study the properties of the Nash equilibrium outcomes induced by the preference revelation games under the proposal-refusal mechanisms. In this analysis, we see that the main theorem of Ergin and Sönmez (2006) continues to hold for exactly one more member of the proposal-refusal family. This is the Shanghai mechanism.

Proposition 4 (Nash Equilibria and Stability) *Let P_I be the list of true student preferences. Consider the preference revelation game induced by the proposal-refusal (e) mechanism.*
(i) If $e \in \{1,2\}$, then the set of Nash equilibrium outcomes of this game is equal to the set of stable matchings under the true preferences P_I.
(ii) If $e \notin \{1,2\}$, there exist Nash equilibrium outcomes of this game which are unstable.
(iii) If $e \notin \{1,2,\infty\}$, there exist stable outcomes of this mechanism which correspond to non-equilibrium strategy-profiles.

Note that Proposition 4 does not assume PUS. However, if this assumption is imposed, Lemma 2 implies that any Nash equilibrium outcome of any proposal-refusal mechanism but DA is stable.

An immediate corollary of Proposition 4 is that the Boston mechanism, the Shanghai mechanism and DA are the only members of the family for which a stable matching can arise only as a Nash equilibrium outcome.[8]

Corollary 2 *Let P_I be the list of true student preferences. Let μ be a stable matching (under the true preferences) given by the proposal-refusal (e) mechanism for a preference profile Q. If $e \in \{1,2,\infty\}$, then Q is a Nash equilibrium profile.*

The next corollary suggests that the Shanghai mechanism provided greater incentive for truthtelling over the Boston mechanism at no efficiency cost.[9]

Corollary 3 (Boston vs. Shanghai) *Consider any school choice problem (possibly violating the PUS assumption).*
(i) If a student can successfully manipulate the Shanghai mechanism, then she can also successfully manipulate the Boston mechanism. However, the converse is not true.
(ii) The set of Nash equilibrium outcomes of the preference revelation game induced by the Boston mechanism is equal to the set of Nash equilibrium outcomes of the preference revelation game induced by the Shanghai mechanism. This is the stable set under students' true preferences.

Parts (i) and (ii) follow from Propositions 1 and 4, respectively. Based on Corollary 3, we expect that the proportion of truthtelling is higher under the Shanghai mechanism than the Boston mechanism, while

[8]Note that this result was shown for the Boston and Shanghai mechanisms in Proposition 4. It is also true for DA since it is strategy-proof. That is any outcome of DA (stable or not) is an equilibrium outcome.

[9]We do not give a proof of this result. It is however straightforward to check that when $e = 2$ with slight modification, the proof of Proposition 1 still holds without any need for the PUS assumption. The only modification needed is for the preference lists used in the proof in which all first $e-1$ choices of a student contain an unachievable school and her e-th choice is some target school s. Simply replacing any such preference lists in the proof with a preference list in which the corresponding student lists school s first allows us to use the same proof.

the efficiency under the Shanghai mechanism is not lower than under the Boston mechanism. We will test this prediction, along with other theoretical predictions in our experiment.

Overall, our theoretical analysis indicates that the incentive and welfare properties vary systematically within the family of proposal-refusal mechanisms. In particular, the Shanghai mechanism is not as manipulable as the Boston mechanism, while DA is strategy-proof. In terms of first choice accommodation as an empirical measure of efficiency, the Boston mechanism assigns the highest number of reported first choices, followed by Shanghai, which in turn, is followed by DA. We now compare the mechanisms in an experimental setting.

4 Experimental Design

We compare the performance of the Boston, Shanghai and the DA mechanisms, with particular attention to the questions of truthful preference revelation, Nash equilibrium outcomes, efficiency and stability comparisons. The environments are designed to capture the key aspects of the school choice problem and to simulate the complexity inherent in potential applications of the mechanisms.

Under complete information, we design our experiment based on the theoretical characterization of the family of proposal-refusal mechanisms in Section 3. We implement a 3(mechanisms) $\times$ 2(environments) factorial design to evaluate the performance of the three mechanisms {BOS, SH, DA} in two different environments, a simple 4-school environment and a more complex 6-school environment.

4.1 The 4-School Environment

The first environment, which we call the **4-school environment**, has four students, $i \in \{1, 2, 3, 4\}$, and four schools, $s \in \{a, b, c, d\}$. Each school has one slot, which is allocated to one participant. We choose the parameters of this environment to satisfy several criteria: (1) no one lives in the district of her top or bottom choices; (2) the first choice accommodation index, i.e., the proportion of first choices an environment can accommodate, is 1/2; (3) the average efficiency under truth-telling is 81% for DA, reflecting the tension between stability and efficiency; and (4) there is a small number of Nash equilibrium outcomes, which reduces the complexity of the games.

The payoffs for each student are presented in Table 1. The square brackets, **[ ]**, indicate the resident of each school district, who has higher priority in that school than other applicants. Payoffs range from 16 points for a first-choice school to 5 points for a last-choice school. Each student resides in her second-choice school.

For each session in the 4-school environment, there are 12 participants of four different types. Participants are randomly assigned types at the beginning of each session. At the beginning of each period, they are randomly re-matched into groups of four, each of which contains one of each of the four different types. Four schools are available for each group. In each period, each participant ranks the schools. After all participants have submitted their rankings, the server allocates the schools in each group and informs each person of his school allocation and respective payoff. The experiment consists of 20 periods to facilitate learning. Furthermore, we change the priority queue every five periods to investigate whether participant

Table 1: Payoff Table for the 4-School Environment

	a	b	c	d
Payoff to Type 1	[**11**]	7	5	16
Payoff to Type 2	5	[**11**]	7	16
Payoff to Type 3	7	16	[**11**]	5
Payoff to Type 4	5	16	7	[**11**]

strategies are conditional on their priority. More specifically, the priority queue among the non-residents in each five-period block is outlined in Table 2.

Table 2: Priority Queue for the 4-School Environment

	Type 1	Type 2	Type 3	Type 4
Block 1: periods 1-5	1	2	3	4
Block 2: periods 6-10	2	3	4	1
Block 3: periods 11-15	3	4	1	2
Block 4: periods 16-20	4	1	2	3

It follows from Proposition 4 that the Nash equilibrium outcomes of Boston and Shanghai mechanisms are the same but the equilibrium strategy profiles leading to those outcomes may differ.

For each of the 4 different queues, we compute the Nash equilibrium outcomes under the Boston and Shanghai mechanisms (which are the same) as well as under DA. For all four blocks, Boston and Shanghai each have a unique Nash equilibrium outcome, where each student is assigned to her district school. This college/student-optimal matching, $\mu^{C/S}$, is Pareto inefficient, with an aggregate payoff of 44:

$$\mu^{C/S} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ a & b & c & d \end{pmatrix}$$

For all four blocks, the matching $\mu^{C/S}$ is also a Nash equilibrium outcome under DA. However, DA has exactly one more Nash equilibrium outcome for all four cases, which is the following Pareto efficient matching μ^*, with an aggregate payoff of 54:

$$\mu^* = \begin{pmatrix} 1 & 2 & 3 & 4 \\ a & d & c & b \end{pmatrix}.$$

The Nash equilibrium profile that sustains outcome μ^* is the following (asterisks are arbitrary): This is an equilibrium profile regardless of the priority order.[10] Note that, in this equilibrium profile, types 1 and 3 misrepresent their first choices by reporting their district school as their first choices, while types 2 and 4 report their true top choices.[11]

[10]This is a Nash equilibrium because, for example, if student 1 (or 3) submits a profile where she lists school d (resp. b) as her first choice, then she may kick out student 2 (resp. 4) in the first step but 2 (resp. 4) would then apply to b (resp. d) and kick out 4 (resp. 2) who would in turn apply to d (resp. b) and kick out 1 (resp. 3). Hence student 1 (or 3), even though she may have higher priority than 2 (resp. 4), she cannot secure a seat at b (resp. d) under DA.

[11]Note that types 1 and 3's manipulation benefits types 2 and 4, thus it does not violate truthtelling as a weakly dominant strategy,

P_1	P_2	P_3	P_4
a	d	c	b
$*$	b	$*$	d
$*$	$*$	$*$	$*$
$*$	$*$	$*$	$*$

We now analyze participant incentives to reveal their true preferences in this environment. We observe that, in blocks 1 and 3, while truth-telling is a Nash equilibrium strategy under the Shanghai mechanism, it is not a Nash equilibrium under Boston. Furthermore, under truth-telling, Shanghai and DA yield the same Pareto inefficient outcome. Recall that Proposition 1 implies that, if truth-telling is a Nash equilibrium under Boston, then it is also a Nash equilibrium under the Shanghai mechanism, but the converse is not true. Blocks 1 and 3 are examples of the latter.

Table 3: Truthtelling and Nash Equilibrium Outcomes in the 4-School Environment

	Truthful Preference Revelation			Nash Equilibrium Outcomes		
	BOS	SH	DA	BOS	SH	DA
Block 1	not NE	NE	dominant strategy			
Block 2	not NE	not NE	dominant strategy	$\mu^{C/S}$	$\mu^{C/S}$	$\{\mu^{C/S}, \mu^*\}$
Block 3	not NE	NE	dominant strategy			
Block 4	not NE	not NE	dominant strategy			

In comparison, for blocks 2 and 4, truth-telling is not a Nash equilibrium strategy under either Shanghai or Boston. Under truth-telling, Boston, Shanghai and DA each yield different outcomes. While the outcome under SH is Pareto efficient, those under DA is not. Table 3 summarizes our analysis on truthtelling and Nash equilibrium outcomes.

4.2 The 6-School Environment

The 4-school environment is designed to compare the mechanisms in a simple context. We now test the mechanisms in a more complex environment where student preferences are generated by school proximity and quality.

In this **6-school environment**, each group consists of six students, $i \in \{1, 2, \ldots, 6\}$, and six schools $s \in \{a, b, \ldots, f\}$. Each school has one slot. Following Chen and Sönmez (2006), each student's ranking of the schools is generated by a utility function, which depends on school quality, school proximity and a random factor. There are two types of students: for notation purposes, odd labelled students are gifted in sciences while even labelled students are gifted in the arts. Schools *a* and *b* are higher quality schools, while *c*-*f* are lower quality schools. School *a* is stronger in the arts and *b* is stronger in sciences: *a* is a first tier school in the arts and second tier in sciences, while *b* is a second tier school in the arts and first tier

since type 1 (resp. 3) is indifferent between truthtelling and lying. If type 1 (resp. 3) reverts to truthtelling, she will then cause a rejection chain which gives everyone their district school, including herself. Therefore, she is not better off by deviating from the efficient but unstable Nash equilibrium strategy.

in sciences; *c*-*f* are each third tier in both arts and sciences. The utility function of each student has three components:

$$u^i(s) = u^i_p(s) + u^i_q(s) + u^i_r(s), \tag{1}$$

where the first component, $u^i_p(s)$, represents the proximity utility for student i for school s. We designate this as 10 if student i lives within the walk zone of School s and 0 otherwise. The second component, $u^i_q(s)$, represents the quality utility for student i at school s. For odd labelled students, $u^i_q(a) = 20$, $u^i_q(b) = 40$, and $u^i_q(s) = 10$ for $s = c - f$. For even labelled students, $u^i_q(a) = 40$, $u^i_q(b) = 20$, and $u^i_q(s) = 10$ for $s = c - f$. Finally, the third component, $u^i_r(s)$, represents a random utility (uniform in the range 0-40) which includes diversity in tastes. Based on this utility function, we randomly generate 20 environments. We choose an environment which again satisfies several criteria: (1) no one lives within the district of her top or bottom choices; (2) the first choice accommodation index is 1/3, a more competitive scenario than the 4-school environment; and (3) the average efficiency under truth-telling is 84% for DA, again, reflecting the tension between stability and efficiency.

We use Equation (1) to generate payoffs. We then normalize the payoffs such that the payoff from the first to last choice schools spans $\{16, 13, 11, 9, 7, 5\}$, the same payoff range as in the 4-school environment. The normalized payoff table is reported in Table 4.

Table 4: Payoff Table for the 6-School Environment

	a	b	c	d	e	f
Payoff to Type 1	[**9**]	16	11	13	7	5
Payoff to Type 2	16	[**11**]	5	13	9	7
Payoff to Type 3	9	16	[**7**]	11	5	13
Payoff to Type 4	16	7	9	[**13**]	5	11
Payoff to Type 5	16	13	11	7	[**9**]	5
Payoff to Type 6	16	13	11	5	7	[**9**]

For each session in the 6-school environment, we include 18 participants of six different types. Participants are randomly assigned types at the beginning of each session. The experiment consists of 30 periods, with random re-matching into three groups of six in each period. Again, we change the priority queue every five periods. Specifically, the priority queue for non-district applicants in each five-period block is presented in Table 5.

Compared with the 4-school environment, the 6-school environment has a much larger set of Nash equilibrium outcomes. By Proposition 4, the Nash equilibrium outcomes of Boston and Shanghai are the same, but the equilibrium strategy profiles leading to those outcomes may differ. Furthermore, there are more equilibrium strategy profiles under Shanghai than under Boston. We examine the 6 different priority queues and compute the Nash equilibrium outcomes under Boston and Shanghai, which are the same. The list of Nash equilibrium outcomes for each block is included in Appendix B.

Lastly, we present the efficiency analysis for the 6-school environment. The allocations that maximizes the sum of payoffs are the following ones, each leading to an aggregate payoff of 78.

Table 5: Priority Queue for the 6-School Environment

	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6
Block 1: periods 1-5	1	2	3	4	5	6
Block 2: periods 6-10	2	3	4	5	6	1
Block 3: periods 11-15	3	4	5	6	1	2
Block 4: periods 16-20	4	5	6	1	2	3
Block 5: periods 21-25	5	6	1	2	3	4
Block 6: periods 26-30	6	1	2	3	4	5

Table 6: Features of Experimental Sessions

Treatment	Mechanism	Environment	# Subjects $\times$ # sessions	Total # of subjects
BOS_4	Boston	4-school	12×4	48
SH_4	Shanghai	4-school	12×4	48
DA_4	Deferred Acceptance	4-school	12×4	48
BOS_6	Boston	6-school	18×4	72
SH_6	Shanghai	6-school	18×4	72
DA_6	Deferred Acceptance	6-school	18×4	72

$\mu_1^* = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & d & f & a & e & c \end{pmatrix}$ or $\mu_2^* = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & a & f & d & e & c \end{pmatrix}$.

In comparison, the No Choice benchmark, where each student is assigned to her district school, generates an aggregate payoff of 58, which generates an efficiency of 74%.

4.3 Experimental Procedures

In each experimental session, each participant is randomly assigned an ID number and is seated in front of a terminal in the laboratory. The experimenter then reads the instructions aloud. Subjects have the opportunity to ask questions, which are answered in public. Subjects are then given 10 minutes to read the instructions at their own pace and to finish the review questions. After everyone finishes the review questions, the experimenter distributes the answers and goes over the answers in public. Afterwards, participants go through 20 (respectively 30) periods of a school choice experiment in the 4-school (respectively 6-school) environment. At the end of the experiment, each participant fills out a demographics and strategy survey on the computer. Each participant is paid in private at the end of the experiment. The experiment is programmed in z-Tree (Fischbacher 2007).

Table 6 summarizes the features of the experimental sessions. For each mechanism in each environment, we conduct four independent sessions between May 2009 and July 2010 at the Behavioral Economics and Cognition Experimental Lab at the University of Michigan. The subjects are students from the University of Michigan. No one participates in more than one session. This gives us a total of 24 independent sessions and 360 subjects. Each 4-school session consists of 20 periods. These sessions last approximately 60 minutes. In comparison, each 6-school session consists of 30 periods. These sessions last approximately 90 minutes.

The first 20-30 minutes in each session are used for instructions. The conversion rate is \$1 = 20 points for all treatments. Each subject also receives a participation fee of \$5, and up to \$3.5 for answering the Review Questions correctly. The average earning (including participation fee) is \$19.23 for the 4-school treatments, and \$25.58 for the 6-school treatments. Experimental instructions are included in Appendix C. Data are available from the authors upon request.

5 Hypotheses

Our first set of hypotheses concern truthful preference revelation. For those participants who do not reveal their preferences truthfully, we investigate patterns of manipulation, especially district school bias, which has been documented in previous experimental studies. Corollary 3 suggests that the Shanghai mechanism is less manipulable than the Boston.

HYPOTHESIS 1 (Truthtelling: BOS vs. SH) *There will be a higher proportion of truthtelling under SH than BOS.*

In the 4-player environment, in cases 1 and 3 (periods 1-5, 11-15) truthtelling is a Nash equilibrium under SH but not under BOS, whereas in cases 2 and 4 (periods 6-10, 16-20) truth-telling is not a Nash equilibrium under either mechanism. Thus, when we analyze preference revelation by lottery (blocks of 5 periods), we expect that the gap between SH and BOS will be larger in cases 1 and 3.

In particular, we expect that there will be less manipulation of first choices under SH than under BOS. Furthermore, in the 4-player environment, we expect the gap to be larger in cases 1 and 3.

HYPOTHESIS 2 (Truthful First Choice: BOS vs. SH) *A higher proportion of reported first choices will be true first choices under the Shanghai mechanism than under the Boston mechanism.*

Furthermore, under DA, truthtelling is a weakly dominant strategy. This leads to our next hypothesis.

HYPOTHESIS 3 (Truthtelling: DA) *Under DA, participants will be more likely to reveal their preferences truthfully than under either BOS or SH.*

In the 4-player environment under DA, truthful preference revelation leads to an inefficient Nash equilibrium outcome which coincides with the No Choice outcome, i.e., each student stays in his or her district school. In this environment, compared to the dominant strategy equilibrium, there is a set of unstable Nash equilibrium profiles leading to a Pareto efficient outcome. Each of these unstable Nash equilibrium profiles is characterized by players 1 and 3 listing their district school as their top choice while players 2 and 4 reveal their preferences truthfully. If players can coordinate on this Pareto superior outcome rather than playing a weakly dominant strategy, we should observe district school bias in players 1 and 3. Alternatively, we expect that players 2 and 4 are more likely to tell the truth than players 1 and 3.

As district school bias is the main form of manipulation under the Boston mechanism in Chen and Sonmez (2006), we expect this type of manipulation to be present in our BOS treatment as well. We also expect that there will be less district school bias under the Shanghai mechanism, especially in the 4-player environment.

HYPOTHESIS 4 (District School Bias) *There will be less district school bias in reported preferences under the Shanghai vs. Boston mechanisms. Furthermore, in the 4-player environment under DA, players 1 and 3 are more likely to exhibit district school bias than players 2 and 4.*

The second set of hypotheses examine Nash equilibrium outcomes. Generically, there are multiple Nash equilibria in the proposal-refusal family of mechanisms, among which the student-optimal Nash equilibrium Pareto dominates the rest (Ergin and Sönmez 2006). We conjecture that Pareto dominance might provide a useful criterion for equilibrium selection in this context.

HYPOTHESIS 5 (Equilibrium Selection: SH and BOS) *Under BOS and SH, the student-optimal Nash equilibrium outcome arises more frequently than the other Nash equilibria.*

Under DA, although there are multiple Nash equilibria, truthtelling is a weakly dominant strategy. The Pareto criterion predicts that the Pareto optimal unstable Nash equilibrium should be selected. However, experimental results from secure implementation suggest that the dominant strategy equilibrium, when coinciding with Nash, is more likely to be chosen (Cason, Saijo, Sjöström and Yamato 2006). This predicts that the student-optimal Nash equilibrium outcome is more likely to arise.

HYPOTHESIS 6 (Equilibrium Selection: DA) *Under DA, the student-optimal Nash equilibrium outcome will arise more frequently than the other Nash equilibria..*

At the aggregate level, we examine three performance measures, first choice accommodation, efficiency and stability.

HYPOTHESIS 7 (First Choice Accommodation) *The proportion of participants receiving their reported top choices will be the highest under the Boston mechanism, followed by the Shanghai mechanism, and then the DA mechanism.*

Since DA has an additional Pareto optimal Nash equilibrium compared to Shanghai and Boston, we expect that it will generate higher efficiency than either of the other mechanisms in the 4-school environment, i.e., DA $>$ BOS and DA $>$ SH.

For stability, Lemma 1 and Proposition 3 suggest that, while DA is stable, we cannot systematically rank the Boston and the Shanghai mechanisms.

HYPOTHESIS 8 (Stability) *There will be more stable outcomes achieved under DA than under either Boston or Shanghai.*

6 Experimental Results

Several questions are important in evaluating this family of mechanisms. First, to what extent do individuals report their preferences truthfully? Second, which equilibrium will be selected when there are multiple equilibrium outcomes? At the outcome level, we want to compare first choice accommodation, efficiency and stability of the three mechanisms. Finally, are the results sensitive to changes in the environments?

In presenting the results, we introduce some shorthand notations. Let $x > y$ denote that a measure under mechanism x is greater than the corresponding measure under mechanism y at the 5% significance level or less. Let $x \geq y$ denote that a measure under mechanism x is greater than the corresponding measure under mechanism y, but the comparison is not statistically significant at the 5% level.

6.1 Individual Behavior

We first examine the extent to which individuals reveal their preferences truthfully, and the pattern of any preference manipulation under each of the three mechanisms.

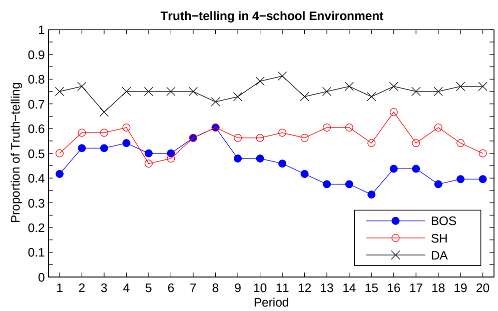

Figure 1: Proportion of Truth-Telling in the 4-school Environment

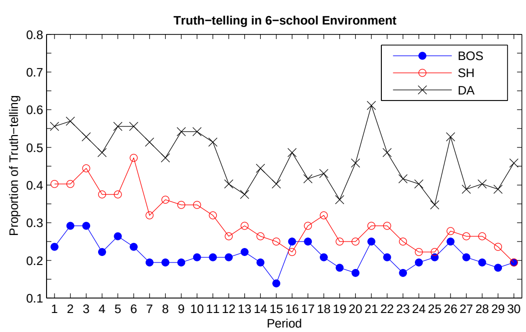

Figure 2: Proportion of Truth-Telling in the 6-School Environments

Figures 1 and 2 present proportion of truthtelling in the 4- and 6-school environments under each mechanism, respectively. Note that, under the Boston and Shanghai mechanisms, truthful preference revelation requires that the entire reported ranking is identical to a participant's true preference ranking.[12] Under DA, however, truthful preference revelation requires that the reported ranking is identical to the true preference ranking from the first choice up to the participant's district school. The remaining rankings, from the district school to the last choice, are irrelevant under DA. While DA has robustly higher proportion of truthtelling than the other two mechanisms, SH also outperforms BOS. Further, under each mechanism, the proportion

[12]The only exception is when a participant's district school is her top choice. In this case, truthful preference revelation entails stating the top choice. However, by design, this case never arises in our experiment, as no one's district school is her first choice.

of truthtelling is higher under the 4-school than under the 6-school environment, especially under DA, which indicates that it is easier to figure out the dominant strategy in a simpler environment.

Table 7: Proportions of Truthful Preference Revelation and Misrepresentations

	Truthful Preference Revelation			District School Bias		
First Period	Proportion	H_a	p-value	Proportion	H_a	p-value
BOS_4	0.417	BOS < SH:	$p = 0.270$	0.542	BOS > SH:	$p = 0.270$
SH_4	0.500	SH < DA:	$p = 0.020$	0.458	SH > DA:	$p = 0.008$
DA_4	0.750	BOS < DA:	$p = 0.002$	0.208	BOS > DA:	$p = 0.001$
BOS_6	0.236	BOS < SH:	$p = 0.024$	0.611	BOS > SH:	$p = 0.002$
SH_6	0.403	SH < DA:	$p = 0.047$	0.361	SH > DA:	$p = 0.048$
DA_6	0.556	BOS < DA:	$p = 0.000$	0.222	BOS > DA:	$p = 0.000$
All Periods						
BOS_4	0.456	BOS < SH:	$p = 0.014$	0.478	BOS > SH:	$p = 0.014$
SH_4	0.563	SH < DA:	$p = 0.014$	0.310	SH > DA:	$p = 0.014$
DA_4	0.751	BOS < DA:	$p = 0.014$	0.107	BOS > DA:	$p = 0.014$
BOS_6	0.214	BOS < SH:	$p = 0.086$	0.593	BOS > SH:	$p = 0.086$
SH_6	0.303	SH < DA:	$p = 0.014$	0.459	SH > DA:	$p = 0.014$
DA_6	0.468	BOS < DA:	$p = 0.014$	0.144	BOS > DA:	$p = 0.014$

Notes:
1. First period: p-values computed from 1-sided proportion of t-tests, treating each individual as one observation.
2. All periods: p-values computed from 1-sided permutation tests, treating each session as one observation.

Table 7 presents the proportion of truthful preference revelations, as well as the proportion of district school bias, a prevalent form of misrepresentation, for each treatment. As most of the previous school choice experiments are implemented as one-shot games, we first report truth-telling and district school bias in the first period (upper panel), where p-values are computed from one-sided proportion of t-tests, treating each individual as an independent observation. We then report the same information averaged over all periods (lower panel), where p-values are computed from one-sided permutation tests, treating each session as an independent observation.

Result 1 (Truth-telling) : *In both the 4-school and 6-school environments, over all periods, the proportion of truthful preference revelation under DA is significantly higher than that under Boston or Shanghai. The proportion of truthful preference revelation under Shanghai is significantly (weakly) higher than that under Boston in the 4-school (6-school) environment.*

By Result 1, we reject the null in favor of Hypothesis 1 that the Shanghai mechanism is less manipulable than Boston at the 5% (10%) level in the 4-school (6-school) environment for all periods. Furthermore, we reject the null in favor of Hypothesis 3 that DA is less manipulable than either Shanghai or Boston. The result is similar for inexperienced participants (first period). While the ranking of truthtelling between BOS and DA is consistent with Chen and Sönmez (2006), manipulability of the Shanghai mechanism is reported for the first time. Even though truthtelling is not a dominant strategy under the Shanghai mechanism, the extent of manipulation is significantly less than under Boston. We also observe the environment effect, i.e., the proportion of truthful preference revelation is significantly higher in the simple 4-school environment than that under the 6-school environment ($p = 0.014$, one-sided permutation test for each pairwise comparison).

A main critique of the Boston mechanism is centered around the fact that the mechanism is putting a lot of pressure on how to choose the first school to list. The Shanghai mechanism alleviates this pressure. We now examine the likelihood that participants reveal their first choices truthfully under each mechanism.

Result 2 (Truth-telling: First Choice) : *In the 4-school (6-school) environment, the proportion of truthful first choices is 77% (55%) under DA, 62% (46%) under SH, and 49% (37%) under BOS, resulting in statistically significant ranking of DA > SH > BOS for truthful first choices for both environments.*

SUPPORT: Using each session as an independent observation, pairwise comparisons of the proportion of truthful first choices each yields $p = 0.014$ (one-sided permutation tests) for the 4-school environment. For the 6-school environment, using the same tests, we obtain DA > BOS ($p = 0.014$), DA > SH ($p = 0.029$), and SH > BOS ($p = 0.043$). ∎

By Result 2, we reject the null in favor of Hypothesis 2 that the Shanghai mechanism generates higher proportion of truthful first choices than Boston. Thus, the truthful preference revelation ranking of the mechanisms is stronger in first choices. Regardless of the environment, participants are more likely to submit true top choices under the Shanghai mechanism than under Boston.

While we do not observe 100% truthtelling under DA, it outperforms both the Shanghai and the Boston mechanisms in truthtelling. Thus, we reject the null in favor of Hypothesis 3. Furthermore, we observe that the proportion of truthtelling in DA is significantly higher in the 4-school environment than in the 6-school environment ($p = 0.014$, one-sided permutation test), probably because of the relative simplicity of the environment.

We now analyze truthtelling by block in the 4-school treatments. Since truth-telling is a Nash equilibrium under SH but not under BOS in blocks 1 and 3, while it is not a NE under either mechanism in blocks 2 and 4. We thus expect the gap between the two mechanisms to be larger in blocks 1 and 3. In block 1, the gap between SH and BOS is 5% and weakly significant ($p = 0.086$, one-sided permutation test). In block 2, the gap is reduced to 3% and not significant ($p = 0.343$). Blocks 3 and 4 each have an average gap of 19% and 16% respectively ($p = 0.014$). Thus, block level analysis is largely consistent with our theoretical prediction, and confirms our intuition from Figure 1.

Of the preference manipulations reported in previous experimental studies of the Boston mechanism, the most prevalent one is District School Bias, where a participant puts her district school into a higher position than that in the true preference order.

Result 3 (District School Bias) : *Averaged over all periods, the proportion of participants who exhibits District School Bias follows the ranking of $BOS_4 > SH_4 > DA_4$, and $BOS_6 \geq SH_6 > DA_6$.*

SUPPORT: See columns under "District School Bias" in the lower panel of Table 7. ∎

By Result 3, we reject the null in favor of Hypothesis 4. Our regression analysis indicates that participants are significantly more likely to submit their true top choices if they have a better position in the lottery under BOS or SH. In comparison, such lottery position effect is absent under DA. This indicates that they understand the incentives in each mechanism.

The proceeding analysis of individual behavior has implications for Nash equilibrium outcomes. Recall that our school choice games of complete information each have multiple Nash equilibrium outcomes. Thus, from both the theoretical and practical implementation perspectives, it is important to investigate which equilibrium outcomes are more likely to arise.

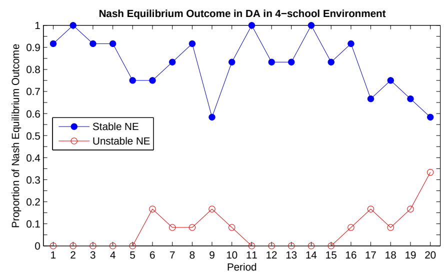

Figure 3: Proportion of Stable and Unstable Nash Equilibrium Outcomes under DA in the 4-School Environment

Recall that in our 4-school environment, the Pareto inefficient Nash equilibrium outcome, $\mu^{C/S}$, is the unique Nash equilibrium outcome under BOS and SH, while there are two Nash equilibrium outcomes under DA, $\mu^{C/S}$ and μ^*, where the latter Pareto dominates the former. We find that, on average, 60.4%, 67.5% and 82.5% of the outcomes are the Pareto inefficient Nash equilibrium outcome $\mu^{C/S}$ under BOS, SH and DA respectively. The proportion of this Nash equilibrium outcome follows BOS $\leq$ SH $<$ DA, where the p-values from pairwise permutation tests are presented in Table 8. More interestingly, we examine which of the two equilibrium outcomes arises more frequently under DA. Figure 3 reports the proportion of the stable and unstable equilibrium outcomes over time under DA in the 4-school environment, while Table 3 reports session level information for each mechanism and pairwise comparisons between mechanisms and outcomes.

Table 8: Proportion of Nash Equilibrium Outcomes in the 4-School Environment

	BOS ($\mu^{C/S}$)	SH ($\mu^{C/S}$)	DA ($\mu^{C/S}$)	DA (μ^*)	H_a	p-value
Session 1	0.683	0.717	0.950	0.017	BOS $\neq$ SH	0.143
Session 2	0.600	0.700	0.717	0.133	BOS $<$ DA	0.014
Session 3	0.600	0.633	0.800	0.017	SH $<$ DA	0.029
Session 4	0.533	0.650	0.833	0.117	DA(μ^*) $<$ DA($\mu^{C/S}$)	0.063

Result 4 (Nash Equilibrium Outcomes: 4-school) : *Under DA, the proportion of the inefficient but stable Nash equilibrium outcome (82.5%) is weakly higher than that of the efficient but unstable Nash equilibrium outcome (8.9%).*

SUPPORT: The last column in Table 8 presents p-values of permutation tests comparing the proportion of equilibrium outcomes under different mechanisms. The null of equal proportion against the H_a of DA(μ^*) $<$ DA($\mu^{C/S}$) yields $p = 0.063$ (paired permutation test, one-sided). ∎

By Result 4, we reject the null in favor of Hypothesis 6. We conjecture that the stable Nash equilibrium outcome ($\mu^{C/S}$) is observed more often despite being Pareto dominated by μ^*, because the former requires truthful preference revelation, the weakly dominant strategy adopted by about 75% of the participants under DA, while the latter requires coordinated manipulation of top choices by players 1 and 3. However, we do note an increase of the unstable but efficient Nash equilibrium outcome, μ^*, in the last block in Figure 3, indicating that players 1 and 3 learn to coordinate their manipulation towards the end of the game. This increase will have direct implications for the efficiency comparisons in Result 7.

In comparison to the 4-school environment, the 6-school environment generates many more Nash equilibrium outcomes. Because of this multitude of Nash equilibria, without strategy-proofness, on average, 10% and 16% the outcomes are Nash equilibrium outcomes under Boston and Shanghai, respectively. In contrast, 79% of the outcomes under DA are Nash equilibrium outcomes. The proportion of this NE outcome follows DA $>$ BOS ($p = 0.014$), DA $>$ SH ($p = 0.014$), and SH $\geq$ BOS ($p = 0.086$). If we break down the Nash equilibrium outcomes under DA into stable and unstable equilibria, we again observe that the stable outcomes arise much more frequently (Figure 4).

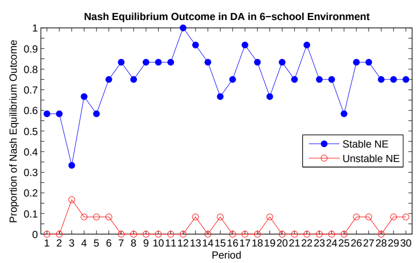

Figure 4: Proportion of Stable and Unstable Nash Equilibrium Outcomes under DA in the 6-School Environment

Table 9: Proportions of Nash Equilibrium Outcomes in the 6-School Environment

	BOS	SH	DA	DA(Stable)	DA(Unstable)	H_a	p-value
Session 1	0.089	0.144	0.822	0.811	0.011	BOS $\neq$ SH	0.171
Session 2	0.156	0.189	0.778	0.778	0.000	BOS $<$ DA	0.014
Session 3	0.022	0.100	0.844	0.789	0.056	SH $<$ DA	0.014
Session 4	0.133	0.211	0.711	0.644	0.067	DA(unstable) $<$ DA(stable)	0.000

Result 5 (Nash Equilibrium Outcomes: 6-school) : *Under DA, the proportion of the stable Nash equilibrium outcomes (75.5%) is significantly higher than that of the unstable Nash equilibrium outcomes (3.3%).*

SUPPORT: The last column in Table 9 presents p-values of permutation tests comparing the proportion of equilibrium outcomes under different mechanisms. The null of equal proportion against the H_a of

DA(unstable) $<$ DA(stable) yields $p < 0.001$ (paired permutation test, one-sided). ∎

By Result 5, we again reject the null in favor of Hypothesis 6. .

6.2 Aggregate Performance

Having presented the individual behavior and equilibrium outcomes, we proceed to evaluate the aggregate performance of the mechanisms using three measures, i.e., the proportion of participants receiving their reported and true first choices, the efficiency achieved, and the stability under each mechanism.

In the education literature, the performance of a school choice mechanism is often evaluated through the proportion of students who get their reported top choices. We compare the proportion of participants receiving their reported top choices, as well as the proportion who actually receive their true top choices.

Table 10: First Choice Accommodation: Reported versus True First Choices

	Proportion Receiving Reported First Choice					Proportion Receiving True First Choice				
4-school	BOS	SH	DA	H_a	p-value	BOS	SH	DA	H_a	p-value
Session 1	0.596	0.450	0.138	BOS $>$ SH	0.014	0.088	0.096	0.017	BOS $\neq$ SH	0.257
Session 2	0.617	0.488	0.271	BOS $>$ DA	0.014	0.113	0.100	0.121	BOS $\neq$ DA	0.114
Session 3	0.583	0.463	0.192	SH $>$ DA	0.014	0.121	0.108	0.071	SH $\neq$ DA	0.257
Session 4	0.608	0.325	0.183			0.138	0.096	0.075		
6-school	BOS	SH	DA	H_a	p-value	BOS	SH	DA	H_a	p-value
Session 1	0.611	0.422	0.196	BOS $>$ SH	0.014	0.191	0.133	0.109	BOS $\neq$ SH	0.143
Session 2	0.587	0.402	0.270	BOS $>$ DA	0.014	0.181	0.143	0.111	BOS $\neq$ DA	0.029
Session 3	0.572	0.461	0.231	SH $>$ DA	0.014	0.220	0.207	0.085	SH $\neq$ DA	0.029
Session 4	0.606	0.435	0.241			0.178	0.159	0.120		

Table 10 reports the proportion of participants receiving their reported and true first choices in each session in each treatment. The alternative hypotheses for reported first choices are based on Proposition 2. However, since neither the Boston nor the Shanghai mechanism is strategy proof, we are agnostic about the ranking of the proportion receiving their true top choices. Thus, the alternative hypotheses comparing mechanisms accommodating true first choices are inequalities. The results are summarized below.

Result 6 (First Choice Accommodation) : *In both the 4-school and 6-school environments, the following ranking of the proportion of participants receiving their reported first choices is significant: BOS $>$ SH $>$ DA. However, when looking at the proportion receiving their true first choices, the Boston and Shanghai mechanisms are not significantly different from each other, and both significantly outperform DA in the 6-school environment.*

SUPPORT: Treating each session as an observation, p-values from the corresponding permutation tests are reported in Table 10. ∎

By Result 6, we reject the null in favor of Hypothesis 7 for reported first choices. Looking at accommodation of true first choices, however, we find that using reported top choices to evaluate mechanism performance is not a good measure when the incentives properties under each mechanism are different. In the 4-school environment, the three mechanisms are not significantly different from each other, while in the 6-school environment, Boston and Shanghai are not significantly different from each other while both outperform DA.

We next investigate efficiency comparisons of the mechanisms in each environment. Following the convention in the experimental economics literature, we define a normalized efficiency measure as

$$\text{Normalized Efficiency} = \frac{\text{Actual sum of payoffs - No Choice sum of payoffs}}{\text{Maximum sum of payoffs - No Choice sum of payoffs}}, \tag{2}$$

where the *No Choice* benchmark assigns each student to her district school without any choices, which is the traditional public school assignment mechanism. Thus, this definition measures the efficiency gain above the No Choice benchmark. The No Choice sum of payoffs are 44 and 58 for the 4-school and 6-school environments, respectively. Likewise, the maximum sum of payoffs are 54 and 78 for the 4-school and 6-school environments, respectively. Because of the normalization, this measure is not bounded below by zero. When the submitted preferences are such that some participants are assigned to a school worse than her district school, the normalized efficiency can become negative.

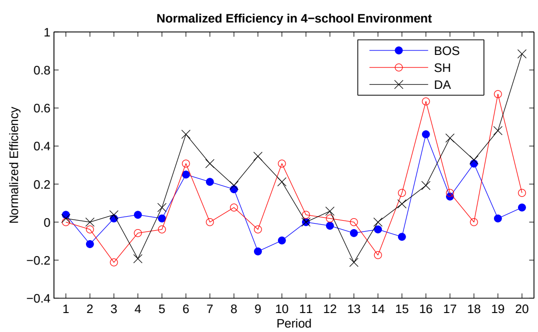

Figure 5: Normalized Efficiency in the 4-School Environment

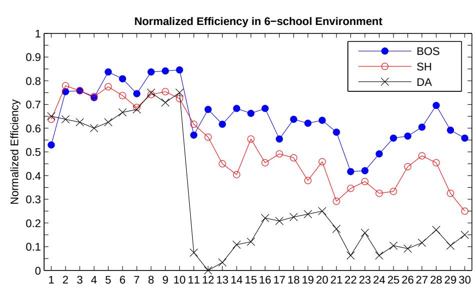

Figure 6: Normalized Efficiency in the 6-School Environment

Figures 5 and 6 present the normalized efficiency under each mechanism in the 4-school and 6-school environment respectively. Session level normalized efficiency for the first and last blocks, as well as the average efficiency over all periods are reported in Table 11.

Table 11: Normalized Efficiency: First Block, Last Block and All Periods

	First Block (periods 1-5)			Last Block			All Periods		
4-school	BOS	SH	DA	BOS	SH	DA	BOS	SH	DA
Session 1	0.000	-0.013	0.000	0.053	0.100	0.073	-0.018	0.068	0.017
Session 2	0.033	-0.020	-0.007	0.100	0.200	0.240	0.048	0.060	0.148
Session 3	0.013	-0.033	0.007	0.047	0.227	0.160	0.038	0.030	0.038
Session 4	-0.047	-0.053	-0.020	0.147	0.033	0.333	0.035	0.012	0.120
6-school	BOS	SH	DA	BOS	SH	DA	BOS	SH	DA
Session 1	0.760	0.780	0.600	0.470	0.267	0.033	0.636	0.488	0.285
Session 2	0.703	0.707	0.613	0.560	0.463	0.097	0.608	0.478	0.303
Session 3	0.677	0.820	0.680	0.773	0.470	0.020	0.748	0.634	0.284
Session 4	0.747	0.640	0.617	0.610	0.360	0.357	0.610	0.506	0.376

Result 7 (Efficiency) : *While DA is more efficient in the 4-school environment, Boston is more efficient than Shanghai, which in turn is more efficient than DA in the 6-school environment. Specifically,*

- *First period: None of the pairwise efficiency comparisons in the 4-school or 6-school environment is significant.*
- *First block:* $DA_4 > SH_4$; $BOS_6 > DA_6$, $SH_6 > DA_6$.
- *Last block:* $DA_4 \geq BOS_4$; $BOS_6 > SH_6 > DA_6$.
- *All periods:* $BOS_6 > SH_6 > DA_6$, *while none of the pairwise efficiency comparisons in the 4-school environment is significant.*

SUPPORT: Using one-sided permutation tests with each session as an observation (except first period), we find that

- First period: None of the pairwise efficiency comparisons in the 4-school or 6-school environment is significant at the 10% level when we treat each group of 4 or 6 as an observation.
- First block: $\text{DA}_4 > \text{SH}_4$ ($p = 0.043$); $\text{BOS}_6 > \text{DA}_6$ ($p = 0.029$), $\text{SH}_6 > \text{DA}_6$ ($p = 0.029$).
- Last block: $\text{DA}_4 \geq \text{BOS}_4$ ($p = 0.057$); $\text{BOS}_6 > \text{SH}_6$ ($p = 0.029$); $\text{BOS}_6 > \text{DA}_6$ ($p = 0.014$); $\text{SH}_6 > \text{DA}_6$ ($p = 0.029$).
- All periods: $\text{BOS}_6 > \text{SH}_6$ ($p = 0.043$); $\text{BOS}_6 > \text{DA}_6$ ($p = 0.014$); $\text{SH}_6 > \text{DA}_6$ ($p = 0.014$). while none of the pairwise efficiency comparisons in the 4-school environment is significant. ■

While Result 7 is consistent with part (ii) of Proposition 3 that there is no systematic efficiency ranking within the class of the Chinese parallel mechanisms, it contributes to our understanding of the empirical performance of the school choice mechanisms and the empirical study of them in several ways. First, efficiency comparison is environment sensitive. While not a single mechanism is more efficient in both environments, the Shanghai mechanism is never the worst. Its performance tends to be sandwiched between Boston and

DA. Second, while first-period pairwise efficiency comparison is not significant in either environment, separation of performance occurs with learning so that the last block ranking is significant. The first period results are consistent with Calsamiglia, Haeringer and Klijn (2011). It points to the importance of allowing subjects to learn in school choice experiments. Lastly, our finding that DA is weakly more efficient than Boston in the last block is driven by the rise of the unstable but efficient Nash equilibrium outcome observed in Figure 3.

To evaluate the stability achieved under each mechanism, we look at the proportion of stable allocations among all allocations in a block or averaged over all periods in a session. Again, all statistics presented are session averages.

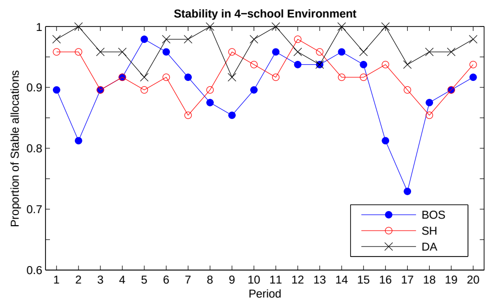

Figure 7: Proportion of stable Allocations in the 4-school Environment

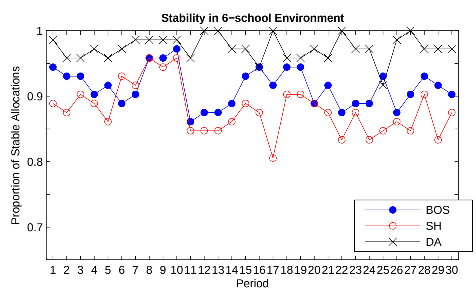

Figure 8: Proportion of stable Allocations in the 6-school Environment

Figures 7 and 8 present the proportion of stable allocations under each mechanism in the 4-school and 6-school environment respectively.

Result 8 (Stability) : *Stability comparisons are as follows:*

- *First period: while* $DA_6 > SH_6$*, none of the other pairwise stability comparison is significant.*
- *First block:* $DA_4 > BOS_4$*;* $DA_6 > BOS_6 > SH_6$*.*
- *Last block:* $DA_4 \geq SH_4 > BOS_4$*;* $DA_6 > BOS_6 > SH_6$*.*
- *All periods:* $DA_4 > SH_4 > BOS_4$*;* $DA_6 > BOS_6 > SH_6$*.*

Table 12: Stability: First Block, Last Block and All Periods

	First Block (periods 1-5)			Last Block			All Periods		
4-school	BOS	SH	DA	BOS	SH	DA	BOS	SH	DA
Session 1	0.933	0.967	1.000	0.850	0.883	0.983	0.904	0.938	0.992
Session 2	0.883	0.933	0.900	0.833	0.917	0.983	0.904	0.925	0.950
Session 3	0.933	0.933	0.983	0.850	0.900	0.900	0.900	0.908	0.946
Session 4	0.850	0.867	0.967	0.850	0.917	1.000	0.883	0.908	0.983
6-school	BOS	SH	DA	BOS	SH	DA	BOS	SH	DA
Session 1	0.944	0.867	0.967	0.878	0.867	0.989	0.920	0.880	0.976
Session 2	0.933	0.867	0.944	0.933	0.867	0.978	0.907	0.880	0.963
Session 3	0.922	0.878	1.000	0.911	0.833	0.989	0.917	0.867	0.991
Session 4	0.900	0.922	0.956	0.900	0.889	0.967	0.909	0.891	0.965

SUPPORT: Using one-sided permutation tests with each session as an observation (except first period), we find that

- First period: $\text{DA}_4 \geq \text{BOS}_4$ ($p = 0.103$), $\text{DA}_4 \geq \text{SH}_4$ ($p = 0.500$), $\text{SH}_4 \neq \text{BOS}_4$ ($p = 0.435$), $\text{DA}_6 \geq \text{BOS}_6$ ($p = 0.183$), $\text{DA}_6 > \text{SH}_6$ ($p = 0.017$), $\text{SH}_6 \neq \text{BOS}_6$;($p = 0.364$).
- First block: $\text{DA}_4 > \text{BOS}_4$ ($p = 0.043$); $\text{DA}_6 > \text{BOS}_6$ ($p = 0.029$), $\text{DA}_6 > \text{SH}_6$ ($p = 0.014$), $\text{BOS}_6 > \text{SH}_6$ ($p = 0.043$).
- Last block: $\text{DA}_4 \geq \text{SH}_4$ ($p = 0.057$), $\text{DA}_4 > \text{BOS}_4$ ($p = 0.014$), $\text{SH}_4 > \text{BOS}_4$ ($p = 0.029$); $\text{DA}_6 > \text{BOS}_6$ ($p = 0.014$), $\text{DA}_6 > \text{SH}_6$ ($p = 0.014$), $\text{BOS}_6 > \text{SH}_6$ ($p = 0.029$).
- All periods: $\text{DA}_4 > \text{SH}_4$ ($p = 0.014$), $\text{DA}_4 > \text{BOS}_4$ ($p = 0.014$), $\text{SH}_4 > \text{BOS}_4$ ($p = 0.014$); $\text{DA}_6 > \text{BOS}_6$ ($p = 0.014$), $\text{DA}_6 > \text{SH}_6$ ($p = 0.014$), $\text{BOS}_6 > \text{SH}_6$ ($p = 0.014$).

∎

By Result 8, we reject the null in favor of Hypothesis 8. Thus, consistent with Lemma 3, DA achieves significantly higher proportion of stable allocations than either the Boston or Shanghai mechanisms in both environments. The ranking between DA and Boston in stability is consistent with Calsamiglia et al. (2010). Even though the theoretical stability ranking of the Boston and Shanghai mechanisms is not clear, in our experiment, we find that the Shanghai mechanism achieves significantly higher proportion of stable outcomes in the 4-school environment, while the reversed ranking holds in the 6-school environment.

7 Conclusions

School choice and college admissions have profound implications for the education and labor market outcomes of the students involved in these processes world wide. The actual mechanisms used for the matching of students to schools or colleges differ in their strategic, welfare and stability properties. In this paper, we synthesize well known matching mechanisms for school choice and college admissions, and characterize them as a family of proposal-refusal mechanisms, which includes the Boston mechanism, the Shanghai mechanism, and the Gale-Shapley Deferred Acceptance mechanism as special cases. A key insight is that the Shanghai mechanism used for both high school admissions in Shanghai and for college admissions in many provinces in China bridges the well studied Boston and DA mechanisms. The Boston and DA mechanisms are the two extreme members of this family while variants of the Chinese parallel mechanisms constitute the intermediate members of this family.

Our theoretical analysis indicates a systematic change in the incentive properties of this family of mechanisms as one moves from one extreme member towards the other. The Nash equilibrium strategies corresponding to the induced preference revelation games associated with members of the proposal-refusal family are nested. The Boston and the Shanghai mechanisms are the only members of the family for which the set of Nash equilibrium outcomes of the preference revelation game is equal to the set of stable matchings under students' true preferences. This finding suggests that the Shanghai mechanism could remedy the incentive problems observed under the Boston mechanism at no welfare cost.

To test the theoretical predictions and to search for behavioral regularities where theory is silent, we conduct laboratory experiments in two environments differentiated by their complexity. We find that the proportion of truth-telling follows the order of DA $>$ Shanghai $>$ Boston, while the proportion of District School Bias follows the reverse order. While the manipulability ranking of DA and Boston is consistent with both theory and prior experimental findings, the manipulability of the Shanghai mechanism is reported for the first time. As predicted by theory, it is sandwiched in between the Boston and the DA mechanisms. While theory is silent about equilibrium selection, we find that stable Nash equilibrium outcomes are more likely to arise than unstable ones. On the stability front, consistent with theory, DA achieves significantly higher proportion of stable outcomes than either Shanghai or Boston in both environments. However, the efficiency comparison is sensitive to the environment. In our 4-school environment, DA is weakly more efficient than Boston, while the Shanghai mechanism is not significantly different from either. In comparison, in our 6-school environment, Boston achieves significantly higher efficiency than Shanghai, which, in turn, outperforms DA.

Compared to previous research on school choice mechanisms, this is the first theoretical and experimental investigation of the Shanghai mechanism, and more generally, the Chinese parallel mechanisms. The analysis yields valuable insights which enable us to treat this class of mechanisms as a family, and systematically study their behavioral properties. More importantly, our results have policy implications for school choice and college admissions. As the Shanghai mechanism is less manipulable than the Boston mechanism, and its achieved efficiency is robustly sandwiched in between the two extremes whose efficiency vary with the environment, it might be a less radical replacement for the Boston mechanism compared to DA.

Like school choice in the United States, college admissions reform is among the most intensively discussed public policies in recent years in China. Each year more than 10 million high school seniors compete for approximately 6 million spots at various colleges and universities. To our knowledge, this annual event is the largest centralized matching process in the world. Since 2008, variants of the parallel mechanism have been implemented in various provinces to replace the sequential mechanism, i.e., Boston with categories, to address the latter's incentive problems. However, the choice of the number of parallel options (e) seems arbitrary. Our study provides the first theoretical analysis and experimental data on the effects of the number of parallel options on the incentives and aggregate performance of these mechanisms.

In the future, we hope to extend this stream of research to evaluate this family of mechanisms in the incomplete information setting, and in the college admissions setting.

Appendix A: Proofs

Proof of Lemma 1: (First statement) It is easy to see that the Boston mechanism is Pareto efficient. Now consider the following problem with four students and four schools each with one seat. Priority orders and student preferences are as follows.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$
i_4	i_2	$\vdots$	$\vdots$
i_2	i_3		
i_1	i_4		
i_3	i_1		

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}
s_1	s_1	s_2	s_2
s_4	s_2	s_3	s_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$

The outcome of the proposal-refusal mechanism (e) for all $e \geq 2$ is the Pareto inefficient matching $\mu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ s_4 & s_2 & s_3 & s_1 \end{pmatrix}$.

(Second statement) Consider the following problem. Let $I = \{i_1, i_2, \ldots, i_{\kappa+1}\}$ and $S = \{s_1, s_2, \ldots, s_{\kappa+1}\}$ where each school has a quota of one. Suppose student i_1 ranks school s_1 first. Suppose also that each $i_k \in I$ with $k \geq 2$ ranks school s_{k-1} first and school s_k second. Suppose further that each $i_k \in I$ has the highest priority for school s_k. Let us apply the proposal-refusal (e) mechanism with $e < \kappa$ to any such problem. Consider student i_{e+1}. It is easy to see that she applies to school s_{e+1} in round $e+1$ of the algorithm when a lower student is already permanently assigned to this school at the end of round e. Hence her final assignment is necessarily worse than school s_{e+1}.Then the outcome of the proposal-refusal (e) mechanism for this problem is clearly unstable. Note however that student i_{e+1} can secure a seat at school s_{e+1} when she submits an alternative preference list in which she ranks it first. ∎

Proof of Lemma 2: Clearly, $e \neq \kappa$. Suppose there exists $i, j \in I$ and $s \in S$ such that $\varphi_j(e)(P) \equiv s$ $P_i \varphi_i(e)(P)$ and $i \succ_s j$.Let P'_i be the preference in which all first $e-1$ choices of student i contain an unachievable school for i and that her e-th choice is school s. By comparing the first step of $\varphi(e)$ for problem P with its first step for problem (P'_i, P_{-i}), it is easy to see that $\varphi(e)(P'_i, P_{-i}) \equiv s$. ∎

Proof of Proposition 1: (First part of the proof). Fix a school choice problem $((\succ_s)_{s \in S}, (P_i)_i \in I)$. We will suppress the priorities and denote the problem by $P = (P_i)_{i \in I}$. Let $\varphi(e)$ and $\varphi(e')$ denote the two proposal-refusal mechanisms with $e' < e$. Suppose there exists $i \in I$ with P'_i such that $\varphi_i(e)(P'_i, P_{-i})$ P_i $\varphi_i(e)(P)$.Suppose by contradiction that no student can manipulate $\varphi(e')$ at problem P.

Claim 1: for all $j \in I$ $\varphi_j(e')(P)$ R_j $\varphi_j(e)(P)$. *Proof:* Suppose that at problem (P) student j applies to school s at some round of some step t. Let P'_j be the preference in which all first $e-1$ choices of student j contain an unachievable school for j and that her e-th choice is school $\varphi_j(e)(P)$. We first show that $\varphi(e)(P'_j, P_{-j}) \equiv \varphi_j(e)(P)$. To see this, note that when we compare the rounds of the first step of $\varphi(e)$ for problems (P) and (P'_j, P_{-j}), the only difference is that for the latter problem student i does not make any applications until the last round of the first step. We consider two cases. Case (1): at (P'_j, P_{-j}) student applies to school $\varphi_j(e)(P)$ in some step $t > 1$: In this case school $\varphi_j(e)(P)$ does not fill all its quota at the end of the first step. Then at the first step of $\varphi(e)$ applied to (P), there are no more students applying to school $\varphi_j(e)(P)$ than at the first step of $\varphi(e)$ applied to (P'_j, P_{-j}). Thus the claim is true. Case (2): at (P'_j, P_{-j}) student j applies to school $\varphi_j(e)(P)$ in step $t = 1$: Since student j does not apply to any school before the last round, the number of applications to each school at P is no greater than the number of applications at (P'_j, P_{-j}). Then student j cannot be rejected from school $\varphi_j(e)(P'_j, P_{-j})$ at (P'_j, P_{-j}) either.

Now we turn to the proof of Claim 1. We show that under $\varphi(e')$ at problem P, student j can guarantee a seat at school $\varphi_j(e)(R)$ by submitting P^*_j (as opposed to P_j) in which all her first $e' - 1$ choices contain an unachievable school for j and that her e'-th choice is $\varphi_j(e)(P)$. We claim that $\varphi(e')(P^*_j, P_{-j}) = \varphi_j(e)(P)$. Note that the first $e' - 1$ rounds of of $\varphi(e)$ for (P''_j, P_{-i}) are identical to the first $e' - 1$ rounds of $\varphi(e')$ for (P^*_j, P_{-j}). Then similarly to the above Case (2) student j cannot be rejected from school s at the e'-th round of $\varphi(e')$ when applied to (P^*_j, P_{-j}). ∎

Claim 2: $\varphi_i(e')(P)\ P_i\ \varphi_i(e)(P)$. ***Proof:*** This can be shown similarly to the proof of Claim 1. Since $\varphi_i(e)(P'_i, P_{-i}) \equiv s\ P_i\ \varphi_i(e)(P)$, we must have $\varphi_i(e)(P^{\tilde{}}_i, P_{-i}) = s$ where $P^{\tilde{}}_i$ is preference in which all first $e - 1$ choices of student i contain an unachievable school for i and that her e-th choice is s. We show that under $\varphi(e')$ at problem P, student i can guarantee a seat at school s by submitting P^*_i (as opposed to P_i) in which all her first $e' - 1$ choices contain an unachievable school and that her e'-th choice is s. One can then see that $\varphi(e')(P^*_i, P_{-i}) = s$ since the first $e' - 1$ rounds of $\varphi(e)$ for $(P^{\tilde{}}_i, P_{-i})$ are identical to the first $e' - 1$ rounds of $\varphi(e')$ for (P^*_i, P_{-i}). Then student i cannot be rejected from school s at the e'-th round of $\varphi(e')$ when applied to (P^*_i, P_{-i}). ∎

Claim 1 and 2 together imply that allocation $\varphi(e')(P)$ Pareto dominates allocation $\varphi(e)(P)$, i.e., no student is worse off, and there is at least one student (namely i) who is better off at $\varphi(e')(P)$ compared to $\varphi(e)(P)$. Let $\varphi_i(e')(P) = a \neq \varphi_i(e)(P)$. Then there is $j \in I\backslash\{i\}$ with $j \in \varphi(e)(P)(a)\backslash\varphi(e')(P)(a)$ who is also better off at $\varphi(e')(P)$. Suppose $\varphi_j(e')(P) = b \neq a = \varphi_j(e)(P)$. Then there is $k \in I\backslash\{i,j\}$ with $j \in \varphi(e)(P)(b)\backslash\varphi(e')(P)(b)$ who is also better off at $\varphi(e')(P)$. Iterating this argument, we obtain a set of students $N_1 = \{i^1_1, i^1_2, \ldots, i^1_m\}$ with $m \geq 2$ such that $\varphi_{i_t}(e')(P) = \varphi_{i_{t+1}}(e)(P)$ for all $i_t \in N_1$ [where $m + 1 \equiv 1$]. Now considering the $\varphi(e)$ algorithm applied to problem P, this is possible only if each $i \in N_1$ applies to the corresponding school $\varphi_i(e)(P)$ within the same step of the procedure since the assignments are being made permanent at the end of each step. Let $i^1_{k^*} \in N_1$ be the *last* student among those in the set N_1 to apply to her permanent assignment at problem P. Suppose this application takes place at some round $r > 1$ of the $\varphi(e)$ procedure. By the choice of student $i^1_{k^*}$, student $i^1_{k^*-1} \in N_1$ must be rejected from $\varphi_{i^1_{k^*}}(e)(P)$ at an earlier round than r. This means that the quota of school $\varphi_{i^1_{k^*}}(e)(P)$ is full at the beginning of round r of the $\varphi(e)$ algorithm applied to problem P. Then there is $j_1 \in I\backslash N_1$ with $j_1 \succ_{\varphi_{i^1_{k^*}}(e)(P)} i^1_{k^*-1}$ who is rejected from school $\varphi_{i_{k^*}}(e)(P)$ when student i_{k^*} is permanently admitted at round r. This also implies that student j_1 applies to school $\varphi_{j_1}(e)(P)$ at some round $r' > r$.

Now consider the following two possibilities: if $\varphi_{i_{k^*-1}}(e')(P)\ P_{j_1}\ \varphi_{j_1}(e')(P)$, then student j_1 is envious of student $i^1_{k^*-1}$ at $\varphi(e')(P)$ leading to a contradiction by Lemma 2. If $\varphi_{j_1}(e')(P)\ R_{j_1}\ \varphi_{i_{k^*-1}}(e')(P)$, then student j_1 is also better off at $\varphi(e')(P)$ compared to $\varphi(e)(P)$. Repeating the argument in the previous paragraph, we obtain a set $N_2 = \{i^2_1, i^2_2, \ldots, i^2_{m'}\} \subset I$ of students with $j_1 \in N_2$ who are each better off at $\varphi(e')(P)$ compared to $\varphi(e)(P)$. Let $i^2_{k^*} \in N_2$ be the *last* student among those in the set N_2 to apply to her permanent assignment at problem P. Then since $j_1 \in N_2$, student $i^2_{k^*}$ applies to school $\varphi_{i^2_{k^*}}(e)(P)$ at some round $r'' \geq r'$. This means that the quota of school $\varphi_{i^2_{k^*}}(e)(P)$ is full at the beginning of round r'' of the $\varphi(e)$ algorithm applied to problem P. Then there is $j_2 \in I \backslash N_2$ with $j_2 \succ_{\varphi_{i^2_{k^*}}(e)(P)} i^2_{k^*-1}$ who is rejected from school $\varphi_{i^2_{k^*}}(e)(P)$ when student $i^2_{k^*}$ is permanently admitted at round r''. This also implies that student j_2 applies to school $\varphi_{j_2}(e)(P)$ at some round $r''' > r''$. Since $r''' > r'$, clearly $j_1 \neq j_2$. Similar to before, we now check wheher j_2 is envious of $i^2_{k^*-1}$ at $\varphi(e')(P)$. Avoiding a contradiction to Lemma 2 implies there is some $j_3 \notin \{j_1, j_2\}$ who is better off at $\varphi(e')(P)$ compared to $\varphi(e)(P)$. Iterating this argument we finally reach a contradiction to the finiteness of I.

(Second part of the proof). Consider the following example.with the given priority structure $(\succ_s)_{s \in S}$ and the profile of possible preferences. Of interest to us are students i_1, i_2, and $i_3$and schools s_1, s_2, and s_3 each of which has a quota of one. Let us suppose the remaining students care about only the remaining schools for which they each have higher priority than all three students of interest. Hence suppose that we have an independent set of students and schools that guarantee that the PUS assumption holds for each student.[13]

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$*$	$\cdots$
i_1	i_2	$\vdots$	$\vdots$	$\vdots$
i_2	i_3			
$\vdots$	$\vdots$			

P_{i_1}	P_{i_2}	P'_{i_2}	P_{i_3}	$*$	$\cdots$
s_1	s_1	s_2	s_2	$\vdots$	$\vdots$
$\vdots$	s_2	$\vdots$	s_3		
	s_3		$\vdots$		
	$\vdots$				

Consider the case when $e = 1$ for problem $(P_i)_i \in I$. Clearly, student i_2 can successfully get a seat at s_2 by submitting P'_{i_2} as opposed P_{i_2} which places her at s_3. Note however that no student can ever gain by lying at problem $(P_i)_i \in I$ when $e > 1$. ∎

Proof of Proposition 2: Fix a problem. Take any two proposal-refusal mechanisms $\varphi(e)$ and $\varphi(e')$ with $e' > e$. The first e rounds of both mechanisms are identical. This means any student who has been assigned to his first choice at the end of round e under $\varphi(e)$ is permanently assigned to this school. Under $\varphi(e')$ however, such a student may be rejected from her first choice during some round $r \in \{e+1, \ldots, e'\}$ ∎

Proof of Proposition 3: (First statement) Consider the following problem with five students and five schools each with a quota of one.

[13] To be more precise, suppose for example that each $s \in S \backslash \{s_1, s_2, s_3\}$, there is $I_s \subset I$ with $|I_s| = q_s$ scuh that (1) s is the top choice of each $i \in I_s$ and (2) for each $i \in I_s$ and each $i' \in I \backslash I_s$, $i \succ_s i'$

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$	$\succ_{s_5}$
i_1	i_3	i_4	i_5	$\vdots$
$\vdots$	i_2	$\vdots$		
	$\vdots$			

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_3	s_1	s_3	s_4
$\vdots$	s_1	s_3	$\vdots$	$\vdots$
	s_2	s_4		
	s_5	s_2		
	s_4	s_5		

The outcome of the proposal-refusal (e) mechanism is the unstable matching $\mu = \left(\begin{smallmatrix} i_1 & i_2 & i_3 & i_4 & i_5 \\ s_1 & s_2 & s_5 & s_3 & s_4 \end{smallmatrix}\right)$ when $e = 3$, and the stable matching $\mu' = \left(\begin{smallmatrix} i_1 & i_2 & i_3 & i_4 & i_5 \\ s_1 & s_5 & s_2 & s_3 & s_4 \end{smallmatrix}\right)$ when $e = 2$.

(Second statement) Consider the following problem with seven students, four schools each with a quota of one and one school (school s_5) with a quota of three.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$	$\succ_{s_5}$
i_6	i_2	i_5	i_7	$\vdots$
i_4	i_3	i_6	i_6	
i_1	i_4	$\vdots$	$\vdots$	
i_2	$\vdots$			
$\vdots$				

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}	P_{i_6}	P_{i_7}
s_1	s_1	s_2	s_2	s_3	s_3	s_4
s_5	s_2	s_5	s_1	$\vdots$	s_4	$\vdots$
$\vdots$	$\vdots$	$\vdots$	s_5		s_1	
			$\vdots$		s_5	
					$\vdots$	

The outcome of the proposal-refusal (e) mechanism is the Pareto efficient matching $\mu = \left(\begin{smallmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 \\ s_5 & s_2 & s_5 & s_5 & s_3 & s_1 & s_4 \end{smallmatrix}\right)$ when $e = 3$, and the inefficient matching $\mu' = \left(\begin{smallmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 \\ s_5 & s_2 & s_5 & s_1 & s_3 & s_5 & s_4 \end{smallmatrix}\right)$ when $e = 2$. ∎

Proof Proposition 4:(First statement) Ergin and Sönmez (2006) establish this result for $e = 1$. So suppose $e = 2$. Let $Q = (Q_i)_{i \in I}$ be an arbitrary strategy profile and let μ be the resulting outcome of the Shanghai mechanism. Suppose μ is unstable, i.e., there is a pair (i, s) such that $s \; P_i \; \mu(i)$ and either $|\mu(s)| < q_s$ or $i \succ_s j$ for some $j \in \mu(s)$.This implies that s is not the first choice of student i at Q_i.Let Q'_i be any strategy where student i ranks s as her first choice. Now we compare the first two rounds of the Shanghai mechanism applied to problems (Q'_i, Q_{-i}) and Q. Note that each student $k \in I \backslash \{i\}$ apply to the same school in the first round of the procedure for both problems. Then at problem (Q'_i, Q_{-i}) student i is tentatively placed to school s at the end of the first round. Since the set of students who apply to each school $s' \in S \backslash \{s\}$ at the first round are the same for both problems, the set of students who are rejected from each $s' \in S \backslash \{s\}$ must also be the same. This means if a student has not applied to school s in the first two rounds of problem Q, the she does not apply to it in the first two rounds of problem (Q'_i, Q_{-i}). Then student i must be (permanently) assigned to school s at (Q'_i, Q_{-i}). Hence, Q cannot be a Nash equilibrium profile, and μ is not a Nash equilibrium outcome.

Conversely, let μ be a stable matching under true preferences P_I . Consider a preference profile $Q = (Q_i)_{i \in I}$ where each student i ranks $\mu(i)$ as her first choice at Q_i.Clealr, the Shanghai mechanism terminates at the first round when applied to Q. We claim that Q is an equilibrium profile. Consider a student i and a school s such that $s \; P_i \; \mu(i)$. Let Q'_i be an alternative strategy for i to secure a seat at school s. Since the Shanghai mechanism terminates at the end of the second round, student i needs to rank school s either first or second at Q'_i. Suppose i ranks s as her first choice. Since μ is stable, $|\mu(s)| = q_s$. Since each $j \in \mu(s)$ applies to s at the first round, student i is rejected from s at the end of the first round. Suppose i ranks s as her second choice. Since each $j \in \mu(s)$ has been tentatively placed at s at the first round, student i cannot secure a seat at s even if she applies to it in the second round either.

(Second statement) Consider the following priority profile and true preferences of students for a problem with three students and three schools each with a quota of one.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$
i_3	i_2	i_2
i_2	$\vdots$	i_1
i_1		i_3

P_{i_1}	P_{i_2}	P_{i_3}
s_1	s_1	s_3
s_2	s_2	s_1
s_3	s_3	s_2

Consider a strategy profile $Q = (Q_1, Q_2, Q_3)$ where $Q_1 = P_{i_1}$, $Q_3 = P_{i_3}$, and Q_2 is a strategy in which student i_2 ranks school s_2 as her first choice. It is easy to check that for any preference revelation game induced by a proposal refusal mechanism (e) with $e \geq 3$, Q is a Nash equilibrium profile which leads to the unstable matching $\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix}$.[14]

(Third statement) Consider the following priority profile and true preferences of students $P = (P_{i_1}, P_{i_2}, P_{i_3})$ for a problem with three students and three schools each with a quota of one.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$
i_3	i_2	i_2
i_2	$\vdots$	i_1
i_1		i_3

P_{i_1}	P_{i_2}	P'_{i_2}	P_{i_3}
s_1	s_1	s_3	s_3
s_3	s_2	s_1	s_1
s_2	s_3	s_2	s_2

Suppose $e = 3$. The outcome of the mechanism for the profile $P = (P_{i_1}, P_{i_2}, P_{i_3})$ is the stable matching $\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_3 & s_2 & s_1 \end{pmatrix}$. However, P is not an equilibrium since student i_2 can secure a seat at school s_1 by deviating to P'_{i_2}. A similar example can easily be constructed for all e with $3 < e < \kappa$ by using a school choice problem satisfying the PUS assumption. ∎

8 Appendix B: Nash Equilibrium Outcomes in the 6-School Environment

We first rewrite Table 4 as a preference profile, where, for each student, the underlined school is her district school:

[14] Clearly, (i_2, s_1) is a blocking pair for μ. To see that Q is indeed an equlibrium profile, it suffices to consider possible deviations by student i_2. If she ranks s_2 first, clearly her assignmnet does not change. If she ranks s_3 first, she is assigned to s_3. If she ranks s_1 first, she gets rejected from s_1 at the third round

P_1	P_2	P_3	P_4	P_5	P_6
b	a	b	a	a	a
d	d	f	$\underline{d}$	b	b
c	$\underline{b}$	d	f	c	c
$\underline{a}$	e	a	c	$\underline{e}$	$\underline{f}$
e	f	$\underline{c}$	b	d	e
f	c	e	e	f	d

We now examine the 6 different priority queues and compute the Nash equilibrium outcomes under Boston and Shanghai, which are the same. Since the outcomes are stable, the analysis is simplified by first computing the student optimal DA outcome μ^S and the college optimal μ^C and checking if there are any stable allocations in between the two in case they are different. Note that since school e is worse for each student than his district school, student 5 always gets matched to school e in all stable matchings. An allocation below μ^C is always the same regardless of the priority order since it simply assigns each student to his district school.

Every stable matching (with respect to the given profile and the corresponding priority order) is a Nash equilibrium outcome of DA. That is, the Nash equilibrium outcomes of DA is a superset of the stable set. This means any Nash equilibrium we compute for Boston (or Shanghai) is also a Nash equilibrium of DA. But there may be other unstable Nash equilibrium outcomes. In what follows, we present the Nash equilibrium outcomes for each block.

Block 1: $f = 1-2-3-4-5-6$.
There are two Nash equilibrium outcomes that are stable:

$\mu^S = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & a & c & d & e & f \end{pmatrix}$ and $\mu^C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & c & d & e & f \end{pmatrix}$

There are three unstable Nash equilibrium outcomes:

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ d & b & c & a & e & f \end{pmatrix}$, $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ c & b & a & d & e & f \end{pmatrix}$, and $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & f & d & e & c \end{pmatrix}$.

Block 2: $f = 6-1-2-3-4-5$
There are three Nash equilibrium outcomes that are stable:

$\mu^S = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ c & b & f & d & e & a \end{pmatrix}$, $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & f & d & e & c \end{pmatrix}$, and $\mu^C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & c & d & e & f \end{pmatrix}$

There are three other unstable Nash equilibrium outcomes:

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ d & b & c & a & e & f \end{pmatrix}$, $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ c & b & a & d & e & f \end{pmatrix}$, and $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & a & c & d & e & f \end{pmatrix}$.

Block 3: $f = 5-6-1-2-3-4$
There is one stable Nash equilibrium outcome:

$\mu^S = \mu^C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & c & d & e & f \end{pmatrix}$

There are four other unstable Nash equilibrium outcomes:

$\binom{1\ 2\ 3\ 4\ 5\ 6}{d\ b\ c\ a\ e\ f}$, $\binom{1\ 2\ 3\ 4\ 5\ 6}{c\ b\ a\ d\ e\ f}$, $\binom{1\ 2\ 3\ 4\ 5\ 6}{a\ b\ f\ d\ e\ c}$, and $\binom{1\ 2\ 3\ 4\ 5\ 6}{b\ a\ c\ d\ e\ f}$.

Block 4: $f = 4 - 5 - 6 - 1 - 2 - 3$.
There are two stable Nash equilibrium outcomes:

$\mu^S = \binom{1\ 2\ 3\ 4\ 5\ 6}{d\ b\ c\ a\ e\ f}$ and $\mu^C = \binom{1\ 2\ 3\ 4\ 5\ 6}{a\ b\ c\ d\ e\ f}$

There are three other unstable Nash equilibrium outcomes:

$\binom{1\ 2\ 3\ 4\ 5\ 6}{a\ b\ f\ d\ e\ c}$, $\binom{1\ 2\ 3\ 4\ 5\ 6}{c\ b\ a\ d\ e\ f}$, and $\binom{1\ 2\ 3\ 4\ 5\ 6}{b\ a\ c\ d\ e\ f}$.

Block 5: $f = 3 - 4 - 5 - 6 - 1 - 2$.
There is one stable Nash equilibrium outcome:

$\mu^S = \mu^C = \binom{1\ 2\ 3\ 4\ 5\ 6}{a\ b\ c\ d\ e\ f}$

There are three other unstable Nash equilibrium outcomes:

$\binom{1\ 2\ 3\ 4\ 5\ 6}{d\ b\ c\ a\ e\ f}$, $\binom{1\ 2\ 3\ 4\ 5\ 6}{c\ b\ a\ d\ e\ f}$, and $\binom{1\ 2\ 3\ 4\ 5\ 6}{b\ a\ c\ d\ e\ f}$.

Block 6: $f = 2 - 3 - 4 - 5 - 6 - 1$
There is one stable Nash equilibrium outcome:

$\mu^S = \mu^C = \binom{1\ 2\ 3\ 4\ 5\ 6}{a\ b\ c\ d\ e\ f}$

There are four other unstable Nash equilibrium outcomes:

$\binom{1\ 2\ 3\ 4\ 5\ 6}{d\ b\ c\ a\ e\ f}$, $\binom{1\ 2\ 3\ 4\ 5\ 6}{c\ b\ a\ d\ e\ f}$, $\binom{1\ 2\ 3\ 4\ 5\ 6}{a\ b\ f\ d\ e\ c}$, and $\binom{1\ 2\ 3\ 4\ 5\ 6}{b\ a\ c\ d\ e\ f}$.

9 Appendix C: Experimental Instructions

References

Abdulkadiroğlu, Atila and Tayfun Sönmez, "School Choice: A Mechanism Design Approach," *American Economic Review*, June 2003, *93* (3), 729–747.

__ **, Parag A. Pathak, Alvin E. Roth, and Tayfun Sönmez**, "The Boston public school match," *American Economic Review*, 2005, *95* (2), pp. 368–371.

__ **,** __ **,** __ **, and** __ , "Changing the Boston School Choice Mechanism: Strategy-proofness as Equal Access," 2006.

__ **,** __ **, and** __ , "The New York City high school match," *American Economic Review*, 2005, *95* (2), 364–367.

__ , __ , **and** __ , "Strategy-proofness versus efficiency in matching with indifferences: Redesigning the NYC high school match," *American Economic Review*, 2009, *99*, 1954–1978(25).

__ **, Yeon-Koo Che, and Yosuke Yasuda**, "Resolving Conflicting Preferences in School Choice: The "Boston Mechanism" Reconsidered," *American Economic Review*, 2011, *101* (1), 399–410.

Balinski, Michel L. and Tayfun Sönmez, "A Tale of Two Mechanisms: Student Placement," *Journal of Economic Theory*, 1999, *84*, 73–94.

Calsamiglia, Caterina, Guillaume Haeringer, and Flip Klijn, "Constrained School Choice: An Experimental Study," *American Economic Review*, September 2010, *100* (4), 1860–74.

__ , __ , **and** __ , "A comment on "School choice: An experimental study" [J. Econ. Theory 127 (1) (2006) 202231]," *Journal of Economic Theory*, January 2011, *146* (1), 392–396.

Cason, Timothy N., Tatsuyoshi Saijo, Tomas Sjöström, and Takehiko Yamato, "Secure implementation experiments: Do strategy-proof mechanisms really work?," *Games and Economic Behavior*, 2006, *57* (2), 206 – 235.

Chen, Yan and Tayfun Sönmez, "School Choice: An Experimental Study," *Journal of Economic Theory*, 2006, *127*, 202–231.

Dubins, Lester E. and Deborah A. Freedman, "Machiavelli and the Gale-Shapley Algorithm," *American Mathematical Monthly*, 1981, *88*, 485–494.

Ergin, Haluk I. and Tayfun Sönmez, "Games of school choice under the Boston mechanism," *Journal of Public Economics*, 2006, *90* (1-2), 215 – 237.

Featherstone, Clayton and Muriel Niederle, "Ex ante efficiency in school choice mechanisms: an experimental investigation," 2008. Working Paper.

Fischbacher, Urs, "z-Tree: Zurich Toolbox for Ready-made Economic Experiments," *Experimental Economics*, 2007, *10* (2), 171–178.

Gale, David and Lloyd S. Shapley, "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, 1962, *69*, 9–15.

Hoxby, Caroline M., "School Choice and School Productivity: Could School Choice Be a Tide that Lifts All Boats?," in Caroline M. Hoxby, ed., *The Economics of School Choice*, Chicago: University of Chicago Press, 2003.

Jr., Peter W. Cookson, *School choice: The struggle for the soul of American education*, New Haven: Yale University Press, 1994.

Miralles, Antonio, "School Choice: The Case for the Boston Mechanism," in Ozgur Akan, Paolo Bellavista, Jiannong Cao, Falko Dressler, Domenico Ferrari, Mario Gerla, Hisashi Kobayashi, Sergio Palazzo, Sartaj Sahni, Xuemin (Sherman) Shen, Mircea Stan, Jia Xiaohua, Albert Zomaya, Geoffrey Coulson, Sanmay Das, Michael Ostrovsky, David Pennock, and Boleslaw Szymanksi, eds., *Auctions, Market Mechanisms and Their Applications*, Vol. 14 of Lecture Notes of the Institute for Computer Sciences, Social Informatics and Telecommunications Engineering, Springer Berlin Heidelberg, 2009, pp. 58–60.

Nie, Haifeng, "A game theoretic analysis of college admissions," *China Economic Quarterly*, 2007, *6* (3).

Ockenfels, Axel and Alvin E. Roth, "Last-Minute Bidding and the Rules for Ending Second-Price Auctions: Evidence from eBay and Amazon Auctions on the Internet," *American Economic Review*, September 2002, *92* (4), 1093–1103.

Özek, Umut, "Equal treatment as a means of evaluating public school assignment mechanisms," 2008. Working Paper.

Pais, Joana and Ágnes Pintér, "School choice and information: An experimental study on matching mechanisms," *Games and Economic Behavior*, 2008, *64* (1), 303 – 328.

Pathak, Parag A. and Tayfun Sönmez, "School Admissions Reform in Chicago and England: Comparing Mechanisms by their Vulnerability to Manipulation," 2011. NBER Working Paper 16783.

__ **and** __ , "Leveling the playing field: sincere and sophisticated players in the Boston mechanism," *American Economic Review*, September 2008, *98*, 1636–1652(17).

Roth, Alvin E., "The Economics of Matching: Stability and Incentives," *Mathematics of Operations Research*, 1982, *7*, 617–628.