Strategic Interactions in Information Decisions with a Finite Set of Players*

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Abstract

We consider a tractable class of two-player quadratic games to examine the relation between strategic interactions in actions and in information decisions. We show that information choices become substitutes when actions are sufficiently complementary. For levels of substitutability sufficiently high, information choices become complements for some initial information decisions. When attention is restricted to beauty contest games, our results contrast qualitatively with the case studied by Hellwig and Veldkamp (2009), where the set of players is a continuum. Also, we find that, for games different from beauty contests, high levels of external effects may lead to complementary information choices for any degree of complementarity in actions. We apply our theoretical results to study strategic interactions in the information choice in commonly analyzed games, including investment externalities, Cournot and Bertrand games.

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1. Introduction

The optimal action of a decision maker in a variety of environments—including oligopolistic industries, networks, investment activities, financial markets, and monopolistic competition—depends on her expectation of both an exogenous state of the world and other agents' actions. Regarding actions, the decision maker may wish either to approximate to (complementarity) or to differentiate from (substitutability) other agents' actions.[1] Models incorporating these features have been used to study particular problems in many fields.[2] In these environments, information about the underlying state is crucial in the whole decision making problem: since optimal actions and the state are correlated, information about the latter conveys information about the other agents' actions as well. In practice, the information that a decision maker has about unknown parameters depends to some extent on her decision of how much to learn. This paper studies the relation between strategic interactions in the action choice and the information choice using quadratic preferences.

Consider a group of players where each of them can choose the informativeness of a private signal about some unknown state of the world before they choose their actions. Suppose that signals are (conditionally) independent across the players. Does it follow that information choices become strategic complements (substitutes) when actions are strategic complements (substitutes)? In beautiful recent paper, Hellwig and Veldkamp (2009)—henceforth, HV—show that the answer is affirmative when there is a large number of identical small players engaged in a beauty contest game. In contrast, this paper shows that the answer is not always affirmative when the set of players is finite or relatively small. In particular, Proposition 2 shows that if actions are very complementary in a beauty contest game, then a player wants to learn less when others want to learn more.

Two reasons explain why our results differ from those in HV. The first one is related to the shape of a player's optimal action when preferences are quadratic. At least since Morris and Shin (2002), and Calvó-Armengol and de Martí (2007), it is well known that, for the class of games studied in this paper, a player's optimal action is linear in her private signal. Then, when the set of players is small, the slope of an optimal action is crucially more affected by others' information choices than in the case with a large number of players. The second reason is the assumed players' risk aversion with respect to the discrepancy between their actions, which makes them act cautiously when others' information decisions affect considerably their own optimal actions.

[1]Consider, for instance, a group of firms engaged in an oligopolistic competition game where the market demand depends on some uncertain parameter. Typically, prices are complements in a Bertrand setting while quantities are substitutes in a Cournot setting.

[2]Incentives of this nature have been considered, among many others, by: (i) Cooper and John (1988) to study coordination failures in macroeconomic models, (ii) Morris and Shin (2002), Hellwig (2005), Cornand and Heinemann (2008), and Angeletos and Pavan (2007) to study the effects of public information disclosure, (iii) Morris and Shin (2005) to study the welfare consequences of central bank transparency, (iv) Calvó-Armengol and de Martí (2007, 2009) to study efficiency properties of communication networks, and (v) Calvó-Armengol, de Martí, and Prat (2009) to study information transmission in networks.

To illustrate how our results work, consider the two-player case with complementary actions and suppose that a player, say player 2, improves her information. When the degree of complementarity is moderate, player 2's optimal action becomes more sensitive to her private signal. Besides, the change induced by the increase in player 2's information in player 2's optimal action is considerably higher than that caused in player 1's optimal action. Then, in order to align their actions, player 1 wishes to improve her information as well. However, when instead complementarity in actions is very high, player 2's optimal action becomes less sensitive to her private signal regardless her information choice. In particular, it approaches the optimal action associated with acquiring no information at all. Then, player 1 wishes to decrease the sensitivity of her optimal action to her private signal too. Furthermore, the increase in player 2's information causes now a similar change in both players' optimal actions. Given this, player 1's desire to insure against the risk that her action deviates from that of player 2 could motivate her to reduce her information.

In these games, optimal actions depend on arbitrarily higher-order iterated expectations of the state, that is, what the player expects that any other player expects that any other player expects, and so forth, about the state. As in Morris and Shin (2002), the assumption that the set of players is a continuum enables HV to use an average expectation operator to keep track of higher-order iterated expectations. For a finite set of players, this approach is appropriate only when higher-order beliefs are very homogeneous across players. But, if the players begin with heterogeneous beliefs, then the heterogeneity would not necessarily vanish unless one imposes a very restrictive symmetric information structure. Thus, an average expectation operator would be ill-suited to keep track of the required higher-order beliefs with a finite number of players and a flexible information structure.

This paper departs from the assumptions used by HV in two directions. First, we consider a finite set of players and do not use an average expectation operator to account for higher-order beliefs. Instead, we follow the approach introduced in the networks literature by Calvó-Armengol and de Martí (2007) and keep track of higher-order beliefs using a knowledge index. This index depends on the covariances between the signals received by each of the players and the state. Second, our class of quadratic games includes the beauty contest game studied by HV as a special case but allows for a richer set of external effects. For this broader class of preferences, Proposition 2 also shows that, starting from some initial information decisions, players wish to complement the information choices of others when actions are very substitutes. For tractability, we conduct our analysis through a two-player game, though our results continue to hold qualitatively so long as the set of players is finite.

Perhaps more important than extending earlier work, this paper emphasizes that understanding strategic interactions in information depends crucially on whether one considers a large or a small set of players. Because higher-order average expectation operators approximate the average of higher-order expectations as the number of players tends to infinity, our results asymptotically converge to the HV result. However, for a finite number of players, keeping track of higher-order beliefs through a knowledge index leads to conclusions different from those obtained using an average expectation operator. The implications of our results are thus more relevant in environments involving a relatively small number of players, such as oligopolistic competition, organizations or a group of agents engaged in a common task. Our results, however, do not go beyond the insights obtained by HV in markets with a large number of players, as it is typical in models of monopolistic competition. Hence, one could in principle regard the HV model as more relevant to examine the information choice in models with a general equilibrium or macro flavor and our analysis as more suitable for micro applications.

The class of preferences assumed in this paper also allows us to examine the role played by second order external effects in information decisions. Proposition 3 shows that, when actions are complements, this role goes in the opposite direction to the role played by a high degree of complementarity. High levels of the externality favor that information choices be complements even when actions are very complementary. This is intuitive since the externality gives players an additional incentive for coordination in actions. More precisely, when the externality is high, each player cares more about the suitability of others' actions with the state. This makes complementing others' information decisions more valuable. Consider, for instance, a group of firms competing in a Bertrand oligopoly where the market demand depends on some unknown parameter. The second order externality is high when the offered products are very homogenous. In such a case, each firm has an incentive to learn more about the parameter when others are well informed in order to choose a price appropriate to the market conditions and slightly below the prices set by its competitors.

Calvó-Armengol and de Martí (2009) use a key result in team theory due to Radner (1963) to demonstrate that Nash equilibrium (in pure strategies) is unique in a beauty contest game with a finite number of players endowed with an exogenously given information choice. In the quadratic game considered in this paper, the cross-derivatives (in actions) of payoffs are symmetric, which allows us to extent their argument to our broader class of preferences. Thus, for a given information choice, optimal action (pure) strategies are well defined and unique in our game. Nevertheless, in a game with endogenous information choice, multiple equilibria may arise even when equilibrium is unique in the corresponding game without information choice. When information choices are complements, this problem is indeed present in the model considered by HV because it assumes a discrete information choice. The model used in this paper assumes a continuous information choice in a compact set, which guarantees uniqueness of equilibrium for those levels of coordination in actions under which information choices are either complements or substitutes.

The question studied in this paper is central to investigate whether heterogenous beliefs can be endogenously sustained in dynamic situations where players interact repeatedly and there is a new realization of the state in each period. In particular, when information decisions are substitutes, heterogeneous beliefs, commonly assumed in problems under asymmetric information and in industrial organization models, can be endogenously sustained. In Section 4, we use our theoretical results to analyze whether heterogenous beliefs are likely to prevail in settings with information choice that inherently involve a small number of players. We apply our analysis to two specific areas of research. We begin with a typical model of production externalities where a group of investors choose how much they invest in a new sector. The productivity of the sector depends on an uncertain parameter and on the aggregate investment. For a given investment cost function, heterogeneous beliefs are more likely to persist when returns are very sensitive to other players' investment. In addition, less concavity of the return function facilitates such persistence.

The second area of research is the problem of information acquisition in oligopolistic industries. In Cournot games, heterogenous beliefs are likely to persist since the conditions required to obtain complementarity in information decisions when actions are substitutes are difficult to meet in these settings. In Bertrand games, we find that the persistence of heterogenous beliefs crucially depends on the combination of two effects of opposite sign. The first of these effects favors that firms wish to differentiate their information choices and is due to the complementarity in prices. The second effect makes firms wish to complement their information choices and comes from the second order externality derived from prices.

The rest of the paper is organized as follows. Section 2 introduces the model. We examine players' optimal actions and the nature of their strategic interactions in the information choice in Section 3. We turn to applications in Section 4 and conclude in Section 5. The proofs of Propositions 1, 2, and 3 are grouped together in the Appendix.

2. The Model

2.1. *Actions and Payoffs*

We consider two players, $i = 1, 2$, who make decisions in a two-stage game. In the first stage, nature selects a state of the world $\theta \in \mathbb{R}$, which is unobservable for the players. Then, each player makes an information choice about the state. Information decisions are taken simultaneously. In the second stage, each player i chooses an action $a_i \in \mathbb{R}$. Actions are taken simultaneously too.

The final payoff u_i to each player i depends on the state θ and on the action profile $a = (a_1, a_2)$. More precisely, u_i is given by a twice-differentiable, real-valued function $U : \mathbb{R}^3 \to \mathbb{R}$ whose functional form is assumed to be common for both players. Throughout the paper we shall take $i = 1$, without loss of generality, when we need to fix a given player in the analysis. Fixing player $i = 1$, and denoting partial derivatives by subscripts in the usual way, we measure the degree of strategic complementarity/substitutability in the players' actions using parameter

$$\lambda := -\frac{U_{a_1a_2}}{U_{a_1a_1}}.$$

Parameter λ above gives us information about the slope of player 1's best response with respect to player 2's action. In addition, we use parameter

$$\pi := \frac{U_{a_2a_2}}{U_{a_1a_1}}$$

to identify the (second order) externality generated on player 1 by the action chosen by player 2. This parameter π summarizes information about the slope of player 2's action strategy which is most preferred by player 1.

We assume that U is quadratic, which guarantees linearity of optimal action strategies. Besides, we impose that each player's payoff is strictly concave in her own action, which ensures that best responses in actions are well defined and, further, that there exist a Nash equilibrium in pure strategies for the corresponding game without information choice. We also assume that each player's payoff is concave in the other player's action. From these two last assumptions it follows $\pi \geq 0$. Finally, to guarantee that optimal actions are well defined and unique in each equilibrium, we need to assume that $\lambda \in (-1, 1)$. For a one-stage, beauty contest game with action choice but without information choice, Calvó-Armengol and de Martí (2009) showed that Nash equilibrium actions exist and are unique if $\lambda \in [0, 1)$. Their original argument can also be used for our class of preferences when $\lambda \in (-1, 1)$ to show that optimal action (pure) strategies are well defined and unique in each perfect Bayes-Nash equilibrium of our game.

Assumption 1. —Preferences— For each player $i = 1, 2$ such that $u_i = U(a_i, a_j, \theta)$,

(i) U is quadratic;

(ii) $U_{a_ia_i} < 0$;

(iii) $U_{a_ja_j} \leq 0$;

(iv) $\lambda = -U_{a_ia_j}/U_{a_ia_i} \in (-1, 1)$.

Fixing player $i = 1$, Assumption 1 (i) above is equivalent to impose that $U(a_1, a_2, \theta) = (a_1, a_2, \theta)' \cdot H \cdot (a_1, a_2, \theta)$, where H is the 3×3 Hessian matrix of $U(a_1, a_2, \theta)$. Other than the restrictions imposed above to make the analysis tractable, this specification of preferences is quite general and, in particular, allows for strategic complementarity ($\lambda > 0$) and substitutability ($\lambda < 0$) in actions.[3] Higher values of $\lambda > 0$ mean more complementarity and lower values of $\lambda < 0$ mean more substitutability. We impose no restrictions on how the state, or the relations between the state and actions, affect payoffs. In particular, we make no assumptions about the derivatives $U_{\theta\theta}$, $U_{a_1\theta}$, and $U_{a_2\theta}$.

The payoff structure of a beauty contest game is a particular case of the preference specification given by Assumption 1. Fixing player $i = 1$, the variant of the beauty contest

[3] Angeletos and Pavan (2007) use a similar class of quadratic preferences to analyze the social value and the efficient use of public information in a setting with a continuum of players.

game studied by HV (adapted to our two-player game) is given by the payoff function:

$$U(a_1, a_2, \theta) = -(1-\lambda)^{-2}\big[a_1 - (1-\lambda)\theta - \lambda a_2\big]^2. \tag{1}$$

This payoff function satisfies Assumption 1 on preferences. Furthermore, for the payoff specification in (1) above, we have $\pi = \lambda^2$.

2.2. *Information Choice*

We consider a Gaussian information structure for tractability. In the first stage of the game, nature draws a state realization θ from a normal distribution with mean μ and variance σ^2. This distribution summarizes the (common) priors of the players about θ. In addition, each player i observes a signal realization[4] $s_i \in \mathbb{R}$ about the state.

We assume that each player i can choose in the first stage the informativeness of her signal by choosing a value for the correlation coefficient between the random variables with realizations θ and s_i. By doing so, the player makes a decision on her own belief revision process, following Bayes' rule, and ends up with some posteriors about θ. Then, she uses those posteriors in the second stage to choose her action.[5] We use $\tilde{z}$ to denote a random variable with realization z.

Definition 1. An *information choice for player* i, $x_i \in [0,1]$, is the square of a value for the correlation coefficient between the random variables $\tilde{\theta}$ and $\tilde{s}_i$.

Assumption 2. —Information Structure— The random vector $(\tilde{\theta}, \tilde{s}_1, \tilde{s}_2)$ follows a multi-normal distribution with mean vector $(\mu, \mu, \mu) \in \mathbb{R}^3$ and variance-covariance matrix

$$\begin{pmatrix} \sigma^2 & x_1^{1/2}\sigma\gamma & x_2^{1/2}\sigma\gamma \\ x_1^{1/2}\sigma\gamma & \gamma^2 & x_1^{1/2}x_2^{1/2}\gamma^2 \\ x_2^{1/2}\sigma\gamma & x_1^{1/2}x_2^{1/2}\gamma^2 & \gamma^2 \end{pmatrix},$$

where $\sigma^2 = \mathrm{Var}[\tilde{\theta}]$, $\gamma^2 = \mathrm{Var}[\tilde{s}_1] = \mathrm{Var}[\tilde{s}_2]$, and $x_i = (\mathrm{Cov}[\tilde{\theta}, \tilde{s}_i]/\sigma\gamma)^2$ for each player $i = 1, 2$. Moreover, $0 < \sigma^2, \gamma^2 < \infty$.

Thus, we model information decision as a continuous choice. Higher values of x_i indicate higher degrees of informativeness for the signal chosen by player i. Assumption 2 above implies that $\tilde{\theta} \sim N(\mu, \sigma^2)$ and $\tilde{s}_i \sim N(\mu, \gamma^2)$ for both players $i = 1, 2$. Further, some basic algebra on normal distributions yields $\mathrm{Var}[\tilde{\theta} \,|\, s_i] = \sigma^2(1 - x_i)$, so that the posterior variance of the state is differentiable and strictly decreasing in the information

[4]We regard a signal as a random variable and a signal realization simply as a particular realization of the random variable.

[5]Modeling decisions on information by allowing players to move from a prior distribution to a posterior distribution, using Bayesian updating, is quite standard in the literature. For instance, this approach is used by Allen (1983, 1986) in a more abstract setting from the perspective of information demand theory, and by Bergemann and Välimäki (2002) in their work on mechanism design when players are allowed to acquire information.

choice $x_i \in [0,1]$. In this way, the informativeness of the signals is ranked according to the induced posterior variances of the state.[6]

Notice that assuming $\mathrm{Cov}[\tilde{s}_1, \tilde{s}_2] = x_1^{1/2} x_2^{1/2} \gamma^2$ is the same as requiring

$$\mathrm{Cov}[\tilde{s}_1, \tilde{s}_2] = \frac{\mathrm{Cov}[\tilde{\theta}, \tilde{s}_1]\mathrm{Cov}[\tilde{\theta}, \tilde{s}_2]}{\mathrm{Var}[\sigma^2]},$$

which, in turn, is equivalent to assuming that signals are conditionally independent given the state. Other than this restriction, we allow for a flexible structure of correlation between the players' signals. Let $(x_1, x_2) \in [0,1]^2$ denote an *information profile*.

The cost of information acquisition for each player i is given by a twice-differentiable function $C : [0,1] \to \mathbb{R}$ which satisfies $C' \geq 0$ and $C'' \geq 0$.[7]

Fixing player $i = 1$, we know from some basic results on normal distributions that the random variable $\tilde{\theta} \mid s_1$ has a normal distribution with mean

$$\mathbb{E}[\tilde{\theta} \mid s_1] = \mu + \frac{x_1^{1/2} \sigma}{\gamma}(s_1 - \mu) \tag{2}$$

and the random variable $\tilde{s}_2 \mid s_1$ follows a normal distribution with mean

$$\mathbb{E}[\tilde{s}_2 \mid s_1] = \mu + x_1^{1/2} x_2^{1/2}(s_1 - \mu). \tag{3}$$

This is the only piece of information about player 1's posteriors that we will need in our subsequent analysis.

For a given pair of parameters $(\lambda, \pi) \in (-1,1) \times \mathbb{R}_+$, we shall use $\Gamma_{(\lambda,\pi)}$ to denote the two-stage game that we have described.

2.3. *Equilibrium*

The backward induction process in equilibrium is as follows. Given an information profile $(x_1, x_2) \in [0,1]^2$ (selected by the players in the first stage), each player i chooses in the second stage an action a_i, for each signal realization s_i that she observes, so as to maximize her expected payoff $\mathbb{E}[U(a_i, a_j, \tilde{\theta}) \mid s_i]$. By solving this optimization problem, we obtain player i's best response in actions. Given an information choice $x_i \in [0,1]$, let

[6] In their paper, HV assume that the player can choose a subset from a finite set of signals so that the information choice is discrete. They rank the informativeness of the information choice in terms of the induced posterior variance-covariance matrix. Our model specifies the information choice in a different way and, in particular, assumes that the information choice is continuous. Nevertheless, both approaches are conceptually similar in the sense that both rank analogously the informativeness of a signal using posterior variances. More important, both papers pursue the same goal of examining whether ex-ante payoffs satisfy either increasing or decreasing differences in the information choice. Our set up allows us to do this exercise using differential calculus.

[7] Our results do not depend on any assumptions on the cost of information acquisition since this cost does not affect the cross-derivatives of a player's ex-ante payoff in the information choices. In particular, the results regarding strategic interactions in the information choice (Propositions 1, 2, and 3) continue to hold when there is no cost of information acquisition. A sufficiently convex cost function would guarantee a concave maximization problem for the players in the first stage of the game.

$\alpha_{x_i} : \mathbb{R} \to \mathbb{R}$ be an *action strategy* for player i, so that $\alpha_{x_i}(s_i)$ is the action chosen by player i upon observing signal realization s_i, conditional on having chosen x_i in the first stage.

Given a family of pairs of action strategies $\{(\alpha_{x_1}, \alpha_{x_2}) : (x_1, x_2) \in [0,1]^2\}$ that both players follow in the second stage, each player i selects in the first stage an information choice x_i so as to maximize her expected payoff $\mathbb{E}[U(\alpha_{x_i}(\tilde{s}_i), \alpha_{x_j}(\tilde{s}_j), \tilde{\theta})] - C(x_i)$. By solving this optimization problem, we obtain player i's best response in information decisions. Let $\Delta([0,1])$ denote the set of probability distributions over the feasible set of information choices of each player.

Definition 2. A *perfect Bayes-Nash equilibrium* for the game $\Gamma_{(\lambda,\pi)}$ is a pair of action strategies (α_1^*, α_2^*) and a pair of probability distributions $(\delta_1, \delta_2) \in \Delta([0,1]) \times \Delta([0,1])$ such that, for each player $i = 1, 2$, the following conditions are satisfied:
(i) for each $s_i \in \mathbb{R}$ and each $(x_1, x_2) \in [0,1]^2$,

$$\alpha_{x_i}^*(s_i) = \arg\max_{a_i \in \mathbb{R}} \mathbb{E}[U(a_i, \alpha_{x_j}^*(\tilde{s}_j), \tilde{\theta}) \mid s_i], \tag{SR2}$$

and
(ii) $\delta_i(x_i^*) > 0$ implies

$$x_i^* \in \arg\max_{x_i \in [0,1]} \int_{[0,1]} \mathbb{E}[U(\alpha_{x_i}(\tilde{s}_i), \alpha_{x_j}(\tilde{s}_j), \tilde{\theta})] d\delta_j(x_j) - C(x_i). \tag{SR1}$$

To answer the question addressed in this paper, our object of analysis is a player's expected payoff in the first stage given that both players follow their optimal action strategies in the second stage. In particular, one needs to examine how the sign of parameter λ influence whether this expected payoff exhibits either increasing or decreasing differences. This paper does not characterize information decisions in equilibrium and does not study the properties of the set of perfect Bayes-Nash equilibrium of the game $\Gamma_{(\lambda,\pi)}$.

When the players follow their optimal action strategies in the second stage, a player's expected payoff in the first stage depends only on the information profile (x_1, x_2). Fixing player $i = 1$, we use a function $F : [0,1]^2 \to \mathbb{R}$ specified as

$$F(x_1, x_2) := \mathbb{E}[U(\alpha_{x_1}^*(\tilde{s}_1), \alpha_{x_2}^*(\tilde{s}_2), \tilde{\theta})] - C(x_1) \tag{4}$$

to denote the ex-ante payoff to player $i = 1$ in the first stage when both players follow their optimal action strategies in the second stage. Thus, the goal of this paper is to analyze the relation between the pair of parameters (λ, π) and the degree of strategic complementarity/substitutability that the ex-ante expected payoff F exhibits. We do this exercise by studying how the sign of parameter λ and the magnitude of parameter π affect the sign of the second order derivative $F_{x_1 x_2}$. To do so, we need to derive first the optimal action strategies $(\alpha_{x_1}^*, \alpha_{x_2}^*)$ and then obtain a closed expression for F using backward induction.

3. Main Results

Obtaining a closed expression for the function F defined in (4) above is constructive. We first derive optimal action strategies and then study the ex-ante expected payoff of the players when they follow such action strategies.

3.1. *Optimal Action Strategies*

Let us fix player $i = 1$ throughout this section. Consider a given information profile $(x_1, x_2) \in [0,1]^2$, selected by the players in the first stage, and a given signal realization $s_1 \in \mathbb{R}$, observed by player 1 in the second stage. Given our differentiability assumptions and the assumption that $u_1 = U(a_1, a_2, \theta)$ is strictly concave in player 1's own action, Assumption 1 (ii), player 1's optimal action strategy $\alpha^*_{x_1}(s_1)$ is characterized by the condition

$$\mathbb{E}\big[U_{a_1}(\alpha^*_{x_1}(s_1), \tilde{a}_2, \tilde{\theta}) \,\big|\, s_1\big] = 0.$$

It is useful to consider first the complete information case. Suppose that θ is known to the players (consider, e.g., $x_1 = x_2 = 1$). Then, in equilibrium both players follow the same optimal action strategy, which depends only on the state and not on signal realizations. Let $\tau(\theta)$ be the optimal action of any player under complete information when the state is θ. Thus, $\tau(\theta)$ is well defined as the unique solution to $U_{a_1}(\tau(\theta), \tau(\theta), \theta) = 0$. Furthermore, since U is quadratic, $\tau(\theta)$ must be linear in θ and, therefore, we can write $\tau(\theta) = \tau_0 + \tau_1\theta$ for some $\tau_0, \tau_1 \in \mathbb{R}$.

Now, using a first order Taylor expansion of $U_{a_1}(\alpha^*_{x_1}(s_1), a_2, \theta)$ around $(\tau(\theta), \tau(\theta), \theta)$, we obtain

$$\begin{aligned} U_{a_1}(\alpha^*_{x_1}(s_1), a_2, \theta) =& U_{a_1a_1}(\tau(\theta), \tau(\theta), \theta)[\alpha^*_{x_1}(s_1) - \tau(\theta)] \\ &+ U_{a_1a_2}(\tau(\theta), \tau(\theta), \theta)[a_2 - \tau(\theta)], \end{aligned}$$

where we have made use of $U_{a_1}(\tau(\theta), \tau(\theta), \theta) = 0$. Then, it follows that

$$\begin{aligned} &\mathbb{E}\big[U_{a_1}(\alpha^*_{x_1}(s_1), \tilde{a}_2, \tilde{\theta}) \,\big|\, s_1\big] = 0 \\ &\qquad \Leftrightarrow U_{a_1a_1}\alpha^*_{x_1}(s_1) - \big(U_{a_1a_1} + U_{a_1a_2}\big)\mathbb{E}[\tau(\tilde{\theta}) \,|\, s_1] + U_{a_1a_2}\mathbb{E}[\tilde{a}_2 \,|\, s_1] = 0. \end{aligned}$$

To ease the notational burden, let us write $\mathbb{E}_i[\cdot]$ instead of $\mathbb{E}[\cdot \,|\, s_i]$, $i = 1, 2$, when no possible confusion arises. Thus, after rewriting the last equality above, we are left with

$$\alpha^*_{x_1}(s_1) = (1-\lambda)\mathbb{E}_1[\tau(\tilde{\theta})] + \lambda\mathbb{E}_1[\tilde{a}_2].$$

Then, replacing $\tilde{a}_2$ by $\alpha^*_{x_2}(\tilde{s}_2) = (1-\lambda)\mathbb{E}_2[\tau(\tilde{\theta})] + \lambda\mathbb{E}_2[\tilde{a}_1]$ in the expression above yields

$$\alpha^*_{x_1}(s_1) = (1-\lambda)\mathbb{E}_1[\tau(\tilde{\theta})] + (1-\lambda)\lambda\mathbb{E}_1\big[\mathbb{E}_2[\tau(\tilde{\theta})]\big] + \lambda^2\mathbb{E}_1\big[\mathbb{E}_2[\tilde{a}_1]\big],$$

so that, by iterating recursively, we obtain

$$\alpha^*_{x_1}(s_1) = (1-\lambda)\left[\frac{\tau_0}{1-\lambda} + \tau_1\left(\mathbb{E}_1[\tilde{\theta}] + \lambda\mathbb{E}_1\big[\mathbb{E}_2[\tilde{\theta}]\big] + \lambda^2\mathbb{E}_1\Big[\mathbb{E}_2\big[\mathbb{E}_1[\tilde{\theta}]\big]\Big] + \cdots\right)\right].$$

Equivalently, we can write

$$\alpha^*_{x_1}(s_1) = \tau_0 + (1-\lambda)\tau_1 \sum_{k=0}^{\infty} \lambda^k \mathbb{E}_1\mathbb{E}_2\mathbb{E}_1 \cdots \mathbb{E}_{p(k)}[\tilde{\theta}], \tag{5}$$

where $\mathbb{E}_1\mathbb{E}_2\mathbb{E}_1 \cdots \mathbb{E}_{p(k)}[\tilde{\theta}]$ denotes the $(k+1)$-order iterated expectations of $\tilde{\theta}$. These nested expectations give us what player 1 expects that player 2 expects that player 1 expects, and so on up to the $k+1$ level of iteration, of the unknown state of the world θ. Here, the subindex $p(k)$ equals 1 if k is either zero or even and equals 2 if k is odd. Note that the expression (5) above for $\alpha^*_{x_1}(s_1)$ is well defined because $\lambda \in (-1,1)$, as imposed by Assumption 1 (iv).

Now, we can use the earlier distributional results in (2) and (3) to obtain

$$\mathbb{E}_1[\tilde{\theta}] = \mu + x_1^{1/2}\left(\frac{\sigma}{\gamma}\right)(s_1 - \mu),$$

$$\mathbb{E}_1\big[\mathbb{E}_2[\tilde{\theta}]\big] = \mu + x_2^{1/2}(x_1x_2)^{1/2}\left(\frac{\sigma}{\gamma}\right)(s_1 - \mu),$$

$$\mathbb{E}_1\Big[\mathbb{E}_2\big[\mathbb{E}_1[\tilde{\theta}]\big]\Big] = \mu + x_1^{1/2}(x_1x_2)\left(\frac{\sigma}{\gamma}\right)(s_1 - \mu),$$

and, by iterating recursively,

$$\mathbb{E}_1\mathbb{E}_2\mathbb{E}_1 \cdots \mathbb{E}_{p(k)}[\tilde{\theta}] = \mu + x_{p(k)}^{1/2}(x_1x_2)^{k/2}\left(\frac{\sigma}{\gamma}\right)(s_1 - \mu).$$

At this point, we need an operator that allows us to keep track of the discounted $k+1$ order nested expectations of the players, in order to obtain a closed expression for optimal action strategies that includes the fixed-point equilibrium calculation. To do so, we make use of the knowledge index introduced by Calvó-Armengol and de Martí (2007) in their work on communication in networks. Consider the pair of matrices

$$\phi := \left(\frac{\sigma}{\gamma}\right)(x_1^{1/2}, x_2^{1/2})_{1\times 2}, \qquad \Omega := \begin{pmatrix} x_1^{1/2}x_2^{1/2} & 0 \\ 0 & x_1^{1/2}x_2^{1/2} \end{pmatrix}_{2\times 2}.$$

These matrices ϕ and Ω can be used to rewrite the expression above for $\mathbb{E}_1\mathbb{E}_2\mathbb{E}_1 \cdots \mathbb{E}_{p(k)}[\tilde{\theta}]$ as

$$\mathbb{E}_1\mathbb{E}_2\mathbb{E}_1 \cdots \mathbb{E}_{p(k)}[\tilde{\theta}] = \mu + \phi \cdot \Omega^k \cdot e_1(s_1 - \mu), \tag{6}$$

where $e_1 = (1,0)$. By plugging the expression in (6) above into the expression for $\alpha^*_{x_1}(s_1)$ given by (5), we obtain

$$\begin{aligned} \alpha^*_{x_1}(s_1) &= \tau_0 + \tau_1\mu + (1-\lambda)\tau_1\, \phi \cdot \sum_{k=0}^{\infty} \lambda^k \Omega^k \cdot e_1(s_1 - \mu) \\ &= \tau_0 + \tau_1\mu + (1-\lambda)\tau_1\, \phi \cdot [I - \lambda\Omega]^{-1} \cdot e_1 \cdot (s_1 - \mu), \end{aligned}$$

where I denotes the 2×2 identity matrix. The infinite sum $\sum_{k=0}^{\infty} \lambda^k \Omega^k = [I - \lambda\Omega]^{-1}$ above is well defined since we are assuming $\lambda \in (-1, 1)$. More specifically, Debreu and Herstein (1953) show that the convergence of $\sum_{k=0}^{\infty} \lambda^k \Omega^k$ is guaranteed if $|\lambda|$ is strictly less than the inverse of the largest eigenvalue of Ω. Now, it can be verified that this largest eigenvalue equals $(x_1 x_2)^{1/2}$.

Thus, the slope of player 1's optimal action strategy with respect to her private signal can be written as

$$m_1 := (1 - \lambda)\tau_1 \phi \cdot [I - \lambda\Omega]^{-1} \cdot e_1.$$

This number m_1 is known as *the knowledge index* for player 1. The knowledge index plays a key role in the analysis of higher-order beliefs in quadratic settings where a group of players is connected through a network. It can be related to standard network centrality measures used in sociology.[8]

Since $[I - \lambda\Omega]$ is a 2×2 matrix, we can compute analytically its inverse to obtain

$$m_1 = \tau_1 \left(\frac{\sigma}{\gamma}\right) \frac{(1 - \lambda)(1 + \lambda x_2) x_1^{1/2}}{(1 - \lambda^2 x_1 x_2)}.$$

Calvó-Armengol and de Martí (2009) rely on a result in team theory due to Radner (1962) to show that equilibrium action strategies are unique in a one-stage, beauty contest game with an exogenously given information choice. Their argument exploits the fact that individual payoffs in a team game can be used to solve the optimization problem of a player with quadratic payoffs. More precisely, as pointed out by Ui (2009), quadratic payoffs with symmetric cross-derivatives in actions admit a potential payoff function[9] $V(a_1, \ldots, a_n, \theta)$ which represents common interests for all n players in a team. Then, it follows from Theorem 4 in Radner (1962) that uniqueness of optimal actions in the quadratic game is guaranteed if the matrix $Q = \left(\partial^2 V(a, \theta)/\partial a_i \partial a_j\right)_{i,j=1,\ldots,n}$ is negative definite. The class of quadratic preferences considered in this paper satisfy the symmetry property $U_{a_1 a_2} = U_{a_2 a_1}$. Therefore, we can invoke Lemma 6 in Ui (2009) to conclude that the payoff function U admits a potential payoff given by

$$V(a_1, a_2, \theta) = -(1 - \lambda)\left[(a_1 - \theta)^2 + (a_2 - \theta)^2\right] - \lambda(a_1 - a_2)^2.$$

Then, we obtain

$$Q = 2 \begin{pmatrix} -1 & \lambda \\ \lambda & -1 \end{pmatrix},$$

a square matrix with eigenvalues

$$\rho_{1,2} = -1 \pm \sqrt{1 - (1 - \lambda^2)} < 0$$

[8]Ballester, Calvó-Armengol, and Zenou (2006) establish a useful relation between a centrality measure traditionally used in sociology (the Bonacich network centrality) and the set of Nash equilibrium actions for a network game with linear-quadratic payoffs.

[9]Potential games are formally defined by Moderer and Shapley (1996) and Bayesian potential games are formally defined by Heumen, Peleg, Tijs, and Borm (1996). See Ui (2009) for a general existence result of Bayesian potential games for quadratic games with symmetric cross-derivatives of payoffs.

for each $\lambda \in (-1, 1)$. In other words, the matrix Q above is definite negative so that, using the result in Theorem 4 in Radner (1962), it follows that optimal action strategies in the game $\Gamma_{(\lambda,\pi)}$ are unique for each given information profile $(x_1, x_2) \in [0, 1]^2$.

The following lemma follows by putting together all the arguments up to here.

Lemma 1. *Assume 1 and 2. Then, for each given information profile* $(x_1, x_2) \in [0, 1]^2$, *the unique pair of optimal action strategies* $(\alpha^*_{x_1}, \alpha^*_{x_2})$ *in the game* $\Gamma_{(\lambda,\pi)}$ *is given by*

$$\alpha^*_{x_i}(s_i) = \tau_0 + \tau_1\mu + m_i\,(s_i - \mu), \qquad i = 1, 2, \tag{7}$$

where $\tau_0, \tau_1 \in \mathbb{R}$, *and*

$$m_i = \tau_1\left(\frac{\sigma}{\gamma}\right)\frac{(1-\lambda)(1+\lambda x_j)x_i^{1/2}}{(1-\lambda^2 x_1 x_2)}, \qquad i = 1, 2. \tag{8}$$

As indicated earlier, the shape of the slope m_i, as a function of λ, x_1 and x_2, obtained in Lemma 1 above, plays a crucial role in our results about equilibrium interactions in the information choice.

Now, we can substitute the optimal action strategies identified in Lemma 1 into the ex-ante expected payoff F. By doing so, we are left with a one-stage game where each player i makes an information choice $x_i \in [0, 1]$ and receives her payoffs according to $F(x_i, x_j)$. In this game, each player has infinitely many (pure) information strategies. Nevertheless, since such strategies lie in the compact set $[0, 1]$ and the payoff function $F(x_i, x_j)$ is continuous in (x_i, x_j) for each $\lambda \in (-1, 1)$ and each $\pi \geq 0$, the results in Fudenberg and Levine (1986) regarding approximate games and approximate equilibrium[10] can be invoked to guarantee the existence of perfect Bayes-Nash equilibrium for our game $\Gamma_{(\lambda,\pi)}$.

3.2. *Strategic Interactions in Information Decisions*

To obtain a closed expression for the ex-ante expected payoff F, we use the expressions obtained in Lemma 1 for $(\alpha^*_{x_1}, \alpha^*_{x_2})$ to compute the expected value of u_1, as required by the definition of F given by (4). Consider a given information profile $(x_1, x_2) \in [0, 1]^2$. Using a second order Taylor expansion of $U(\alpha^*_{x_1}(s_1), \alpha^*_{x_2}(s_2), \theta)$ around $(\tau(\theta), \tau(\theta), \theta)$, we obtain

$$\begin{aligned} U(\alpha^*_{x_1}(s_1), \alpha^*_{x_2}(s_2), \theta) = {} & U(\tau(\theta), \tau(\theta), \theta) + \frac{1}{2}U_{a_1a_1}[\alpha^*_{x_1}(s_1) - \tau(\theta)]^2 \\ & + \frac{1}{2}U_{a_2a_2}[\alpha^*_{x_2}(s_2) - \tau(\theta)]^2 + U_{a_1a_2}[\alpha^*_{x_1}(s_1) - \tau(\theta)][\alpha^*_{x_2}(s_2) - \tau(\theta)], \end{aligned}$$

where we have made use of $U_{a_1}(\tau(\theta), \tau(\theta), \theta) = U_{a_2}(\tau(\theta), \tau(\theta), \theta) = 0$. Therefore, using the definition of F given in (4), we have

$$\begin{aligned} F = {} & \mathbb{E}\big[U(\tau(\tilde\theta), \tau(\tilde\theta), \tilde\theta)\big] + \frac{1}{2}U_{a_1a_1}\mathbb{E}\big[[\alpha^*_{x_1}(\tilde s_1) - \tau(\tilde\theta)]^2\big] \\ & + \frac{1}{2}U_{a_2a_2}\mathbb{E}\big[[\alpha^*_{x_2}(\tilde s_2) - \tau(\tilde\theta)]^2\big] + U_{a_1a_2}\mathbb{E}\big[[\alpha^*_{x_1}(\tilde s_1) - \tau(\tilde\theta)][\alpha^*_{x_2}(\tilde s_2) - \tau(\tilde\theta)]\big] - C(x_1). \end{aligned}$$

[10]Approximate equilibrium is formally defined by Radner (1980).

By Assumption 1 (ii), $-U_{a_1a_1}^{-1}$ is a positive constant, so that the sign of the second order derivative F_{x_1,x_2} coincides with the sign of $-U_{a_1a_1}^{-1}F_{x_1,x_2}$. Using this, we find convenient to propose the normalization $\widehat{F} := -U_{a_1a_1}^{-1}F$ and proceed with the rest of the analysis in terms of $\widehat{F}$ instead of F. Using the definitions of λ and π, from the equation above we can derive the expression for the function $\widehat{F}$ as

$$\begin{aligned}\widehat{F} = &- U_{a_1a_1}^{-1}\mathbb{E}\big[U(\tau(\tilde{\theta}),\tau(\tilde{\theta}),\tilde{\theta})\big] - \frac{1}{2}\mathbb{E}\big[[\alpha^*_{x_1}(\tilde{s}_1)-\tau(\tilde{\theta})]^2\big] \\ &- \frac{1}{2}\pi\mathbb{E}\big[[\alpha^*_{x_2}(\tilde{s}_2)-\tau(\tilde{\theta})]^2\big] + \lambda\mathbb{E}\big[[\alpha^*_{x_1}(\tilde{s}_1)-\tau(\tilde{\theta})][\alpha^*_{x_2}(\tilde{s}_2)-\tau(\tilde{\theta})]\big] + U_{a_1a_1}^{-1}C(x_1).\end{aligned} \tag{9}$$

So, we need now to analyze the terms inside the expectation operators in equation (9) above. Using the expression provided in Lemma 1 for $\alpha^*_{x_i}(s_i)$, we obtain

$$\alpha^*_{x_i}(\tilde{s}_i) - \tau(\tilde{\theta}) = -\tau_1(\tilde{\theta}-\mu) + m_i(\tilde{s}_i-\mu) = (-\tau_1\,, m_i)\cdot\begin{pmatrix}\tilde{\theta}-\mu\\ \tilde{s}_i-\mu\end{pmatrix}.$$

From Assumption 2, we know that the pair $(\tilde{\theta}-\mu\,,\tilde{s}_i-\mu)$ is normally distributed with a mean vector of zeroes and variance-covariance matrix

$$\begin{pmatrix}\sigma^2 & x_i^{1/2}\sigma\gamma\\ x_i^{1/2}\sigma\gamma & \gamma^2\end{pmatrix}.$$

Then, using some basic results on normal distributions, we can compute the variance of the random variable $\alpha^*_{x_i}(\tilde{s}_i)-\tau(\tilde{\theta})$ as

$$\mathbb{E}\big[[\alpha^*_{x_i}(\tilde{s}_i)-\tau(\tilde{\theta})]^2\big] = \Big[\tau_1^2\sigma^2 + m_i^2\gamma^2 - 2\tau_1x_i^{1/2}m_i\sigma\gamma\Big], \qquad i=1,2. \tag{10}$$

Also, we can compute the covariance between $[\alpha^*_{x_1}(\tilde{s}_1)-\tau(\tilde{\theta})]$ and $[\alpha^*_{x_2}(\tilde{s}_2)-\tau(\tilde{\theta})]$ as

$$\begin{aligned}&\mathbb{E}\big[[\alpha^*_{x_1}(\tilde{s}_1)-\tau(\tilde{\theta})][\alpha^*_{x_2}(\tilde{s}_2)-\tau(\tilde{\theta})]\big]\\ &\qquad = \Big[\tau_1^2\sigma^2 + (x_1x_2)^{1/2}m_1m_2\gamma^2 - \tau_1(x_1^{1/2}m_1 + x_2^{1/2}m_2)\sigma\gamma\Big].\end{aligned} \tag{11}$$

By plugging the expressions obtained in (10) and (11) above into (9), and by using the expression for the slope m_i ($i=1,2$) obtained in Lemma 1, we get

$$\begin{aligned}\widehat{F} = &-U_{a_1a_1}^{-1}\mathbb{E}\big[U(\tau(\tilde{\theta}),\tau(\tilde{\theta}),\tilde{\theta})\big] + \frac{(2\lambda-\pi-1)(\tau_1\sigma)^2}{2}\\ &+ \frac{(1-\lambda)^2(\tau_1\sigma)^2}{2}\left[-x_1q_1^2 - \pi x_2q_2^2 + 2x_1q_1 - 2\left(\frac{\lambda-\pi}{1-\pi}\right)x_2q_2 + 2\lambda x_1x_2q_1q_2\right] + U_{a_1a_1}^{-1}C(x_1),\end{aligned}$$

where $q_i : [0,1]^2 \to \mathbb{R}$ denotes the function specified as

$$q_i(x_1,x_2) := \frac{1+\lambda x_j}{1-\lambda^2x_1x_2}, \qquad i=1,2.$$

All that remains then is to compute the second order derivative $\widehat{F}_{x_1x_2}$ from the expression above. For the required algebra, it is useful to take into account that, for each player $i = 1, 2$,

$$\frac{\partial q_i}{\partial x_i} = \frac{\lambda^2 x_j q_i}{1-\lambda^2 x_1 x_2}, \qquad \frac{\partial q_i}{\partial x_j} = \frac{\lambda q_j}{1-\lambda^2 x_1 x_2}, \qquad \frac{\partial^2 q_i}{\partial x_i \partial x_j} = \frac{\lambda^3 x_j q_j + \lambda^2 q_i}{1-\lambda^2 x_1 x_2}.$$

Using that, it can be verified that

$$\begin{aligned}\widehat{F}_{x_1x_2} =& \lambda \left[\frac{(1-\lambda)\tau_1\sigma}{1-\lambda^2 x_1 x_2}\right]^2 \Bigg[(2-\pi)\lambda^2 x_1 x_2 q_1 q_2 \\ &+ \lambda\Big(x_1 q_1 - \frac{\lambda-\pi}{1-\lambda} x_2 q_2 + (1-\pi) x_2 q_2^2\Big) + q_2 - \frac{\lambda-\pi}{1-\lambda} q_1 - \pi q_1 q_2\Bigg].\end{aligned} \tag{12}$$

Also, for the particular case of the beauty contest game with the preference specification given by (1), we have $\pi = \lambda^2$, and the second order derivative above becomes

$$\begin{aligned}\widehat{F}^{\mathrm{bc}}_{x_1x_2} =& \lambda \left[\frac{(1-\lambda)\sigma}{1-\lambda^2 x_1 x_2}\right]^2 \Bigg[- x_1 x_2 q_1 q_2 \lambda^4 - x_2 q_2^2 \lambda^3 \\ &+ (2x_1 x_2 q_1 q_2 - x_2 q_2 - q_1 q_2)\lambda^2 + (x_1 q_1 + x_2 q_2^2 - q_1)\lambda + q_2\Bigg].\end{aligned} \tag{13}$$

With the expressions for the ex-ante expected payoff of player 1 given by (12) and (13) above at hand, we can state our main results.

Proposition 1. *Assume 1 and 2. Then, $\widehat{F}_{x_1x_2}(x_1, x_2) = 0$ for each $(x_1, x_2) \in [0,1]^2$ when $\lambda = 0$. Moreover, for each $\pi \geq 0$ there exists some $\epsilon > 0$, bounded away from zero, such that if $\lambda \in (0, \epsilon)$, then $\widehat{F}_{x_1x_2}(x_1, x_2) > 0$ for each $(x_1, x_2) \in [0,1]^2$ while if $\lambda \in (-\epsilon, 0)$, then $\widehat{F}_{x_1x_2}(x_1, x_2) < 0$ for each $(x_1, x_2) \in [0,1]^2$.*

Hence, in our model, strategic interactions in the information choice have the same nature as those in the action choice when the degree of coordination is moderate, $\lambda \in (-\epsilon, \epsilon)$ for some $\epsilon > 0$. This is true for a more general class of games than a beauty contest game. The result above agrees with the main result in HV for the case with a continuum of players, provided that the degree of complementarity/substitutability in actions is not too high.

However, the relation between strategic motives obtained in Proposition 1 can be reversed when the degree of coordination in actions is sufficiently high. Proposition 2 below gives us precisely this result. Our results are driven by the shape of the slope of the players' optimal actions. Of particular importance is the observation that the slope of a player's optimal action (in her private signal) is quite sensitive to the information choices of both players. In contrast, the influence of a player's information choice on the sensitivity of any player's optimal action is crucially mitigated in a game with a continuum of players.

To illustrate how the relation between information decisions and the slope of the players' optimal actions drives our results, consider the case with complementary actions, $\lambda \in (0,1)$. Note that, using the expressions for m_1 and m_2 given by Lemma 1, one obtains

$$\frac{\partial m_2}{\partial x_2} = \tau_1 \left(\frac{\sigma}{\gamma}\right) \frac{(1-\lambda)(1+\lambda x_1)(x_2^{-1}+\lambda^2 x_1)x_2^{1/2}}{2(1-\lambda^2 x_1 x_2)^2},$$

$$\frac{\partial m_1}{\partial x_2} = \tau_1 \left(\frac{\sigma}{\gamma}\right) \frac{(1-\lambda)\lambda(1+\lambda x_1)x_1^{1/2}}{(1-\lambda^2 x_1 x_2)^2},$$

and

$$\frac{\partial m_2}{\partial x_2} - \frac{\partial m_1}{\partial x_2} = \tau_1 \left(\frac{\sigma}{\gamma}\right) \frac{(1-\lambda)(1+\lambda x_1)}{2x_2^{1/2}(1-\lambda^2 x_1 x_2)^2} \left[1-\lambda x_1^{1/2} x_2^{1/2}\right]^2.$$

The logic behind the result in Proposition 1 is as follows. When λ is close to zero, we see from the expression for $\partial m_2/\partial x_2$ above that an increase in x_2 causes a significative change in m_2. Therefore, it is valuable for player 1 to change m_1 in such a way so as to respond to the variation in m_2. From the expression for $\partial m_1/\partial x_2$ above, we observe that m_1 changes already in the required direction due only to the increase in x_2. However, this induced change is small since λ is close to zero. In other words, the difference $\partial m_2/\partial x_2 - \partial m_1/\partial x_2$ is relatively large in this case. In particular, note that

$$\lim_{\lambda \to 0^+} \left[\frac{\partial m_2}{\partial x_2} - \frac{\partial m_1}{\partial x_2}\right] = \tau_1 \left(\frac{\sigma}{\gamma}\right) \frac{1}{2x_2^{1/2}}.$$

As a consequence, to reach the required change in m_1, player 1 must increase m_1 further than the variation induced solely by the increase in x_2. To do this, she needs to increase x_1 as well.

On the other hand, suppose that instead λ is close to one. Then, m_2 approaches zero and player 2's optimal action approaches $\mathbb{E}[\tau(\tilde{\theta})] = \tau_0 + \tau_1 \mu$. In other words, player 2 behaves as if she acquires no information at all. Furthermore, we see from the expression for $\partial m_2/\partial x_2$ above that any increase in x_2 causes almost no change in m_2. In this case, note that

$$\lim_{\lambda \to 1^-} \left[\frac{\partial m_2}{\partial x_2} - \frac{\partial m_1}{\partial x_2}\right] = 0.$$

That is, when actions are almost perfect complements, player 1 hardly needs to increase x_1 to compensate for the change in player 2's optimal action.

Player 1 knows that one way of matching m_1 with m_2 is achieved by acquiring little amount of information herself. This makes it valuable for her to decrease x_1. Furthermore, recall that, given our class of preferences, player 1 is risk-averse with respect to the difference of actions. Player 1 wishes to insure herself against the risk of m_1 deviating from m_2. As a consequence, she might find valuable to reduce x_1 when player 2 increases x_2. This is the intuition behind the results in Proposition 2.

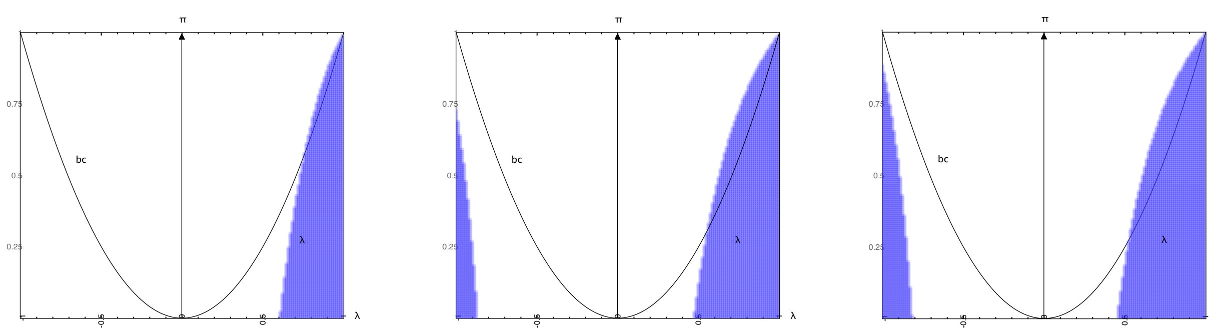
Proposition 2. *Assume 1 and 2. Then,*

(i) for each $\pi \in [0,1)$ *there exists some* $\kappa(\pi) \in (0,1)$*, bounded away from* 1*, such that if* $\lambda \in (\kappa(\pi), 1)$*, then* $\widehat{F}_{x_1x_2}(x_1, x_2) < 0$ *for each* $(x_1, x_2) \in (0,1)^2$*,*

(ii) for each $\pi \in [0,1)$ *there exists some* $\varepsilon(\pi), \delta(\pi) > 0$ *and some* $\kappa(\pi) \in (-1, 0)$*, bounded away from* -1*, such that if* $\lambda \in (-1, \kappa(\pi))$*, then* $\widehat{F}_{x_1x_2}(x_1, x_2) > 0$ *for each* $0 \leq x_1 < \varepsilon(\pi)$ *and each* $(1 - \delta(\pi)) < x_2 \leq 1$*,*

(iii) for the beauty contest game given by the payoff function in (1), there exists some $\kappa \in (0,1)$*, bounded away from* 1*, such that if* $\lambda \in (\kappa, 1)$*, then* $\widehat{F}^{bc}_{x_1x_2}(x_1, x_2) < 0$ *for each* $(x_1, x_2) \in (0,1)^2$*.*

Proposition 2 (i) says that information choices are strategic substitutes if the degree of complementarity in actions is sufficiently high. In other words, the value of additional information to each player is strictly decreasing in the other player's information. Proposition 2 (ii) shows that, starting from a situation in which player i acquires little amount of information while the other player acquires a large amount of information, a sufficiently high level of substitutability in actions implies that player i's information choice complements that of the other player. Figure 1 below illustrates these results.

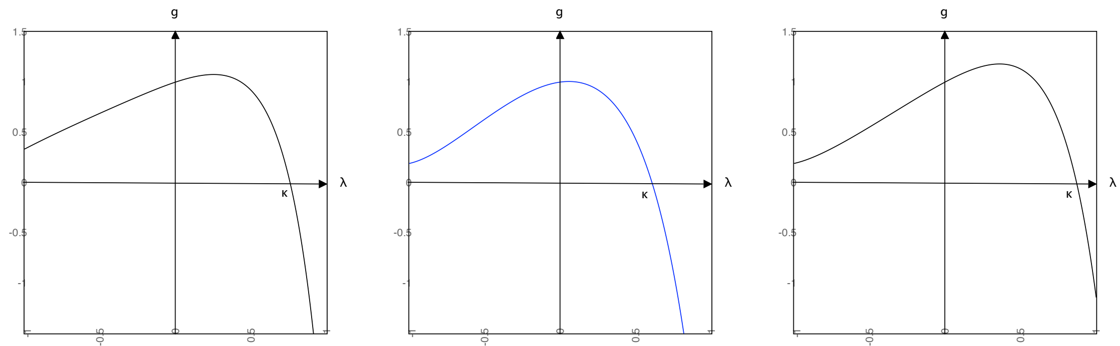


(a) $x_1 = x_2 = 0.5$ (b) $x_1 = 0.1$, $x_2 = 0.9$ (c) $x_1 = 0.05$, $x_2 = 0.95$

FIGURE 1

For (λ, π) in the shadowed region, the information choice does not have the same strategic coordination motives as the action choice. The line bc is $\pi = \lambda^2$ and corresponds to the beauty contest game.

The result provided by Proposition 2 (iii) contrasts qualitatively the main result in HV for a game with a continuum of players. In our model, for the beauty contest game considered by HV, information choices are strategic substitutes if the degree of complementarity in actions is high enough. This result is also illustrated in Figure 2 below.



(a) $x_1 = x_2 = 0.5$ (b) $x_1 = 0.2$, $x_2 = 0.8$ (c) $x_1 = 0.8$, $x_2 = 0.2$

FIGURE 2

Beauty contest game. For $\lambda < 0$, $g(\lambda) > 0$ implies $\widehat{F}^{bc}_{x_1x_2} < 0$ (information choices are substitutes). For $\lambda > 0$, $g(\lambda) > 0$ implies $\widehat{F}^{bc}_{x_1x_2} > 0$ (information choices are complements) and $g(\lambda) < 0$ implies $\widehat{F}^{bc}_{x_1x_2} < 0$ (information choices are substitutes).

Propositions 1 and 2 identify values for the degree of coordination in actions under which the value of additional information to each player (in the other player's information) is either strictly increasing or strictly decreasing. These results, combined with the result that optimal action strategies are unique for each information profile (Lemma 1) and with the fact that information is a continuous choice in a compact set, ensure that the game $\Gamma_{(\lambda,\pi)}$ has a unique perfect Bayes-Nash equilibrium when information decisions are either complements or substitutes. To see this, suppose that player 1 increases her initial information choice x_1 by a certain amount $\Delta x_1 > 0$. For the given $x_1 + \Delta x_1$, let

$$\xi(x_2) = \mathbb{E}[U(\alpha^*_{x_1+\Delta x_1}(\tilde{s}_1), \alpha^*_{x_2}(\tilde{s}_2), \tilde{\theta})] - \mathbb{E}[U(\alpha^*_{x_1}(\tilde{s}_1), \alpha^*_{x_2}(\tilde{s}_2), \tilde{\theta})]$$

be the value of this additional information Δx_1 to player 1. Then, two different information choices of player 2, $x_2, x_2' \in [0, 1]$, can be part of a perfect Bayes-Nash equilibrium of the game $\Gamma_{(\lambda,\pi)}$ only if $\xi(x_2') = C(x_1 + \Delta x_1) - C(x_1)$ and $\xi(x_2) = C(x_1 + \Delta x_1) - C(x_1)$. However, both equalities cannot hold simultaneously for those values of (λ, π) under which propositions 1 and 2 imply that ξ is either strictly increasing or strictly decreasing in x_2. Consequently, for those values of (λ, π), there must be a unique $x_2^* \in [0, 1]$ that satisfies the equilibrium condition

$$\mathbb{E}[U(\alpha^*_{x_1+\Delta x_1}(\tilde{s}_1), \alpha^*_{x_2^*}(\tilde{s}_2), \tilde{\theta})] - \mathbb{E}[U(\alpha^*_{x_1}(\tilde{s}_1), \alpha^*_{x_2^*}(\tilde{s}_2), \tilde{\theta})] = C(x_1 + \Delta x_1) - C(x_1).$$

Most of our results hold regardless the second order external effect captured by parameter π. However, external effects play also an important role in the analysis of the question studied in this paper. The shape of the shadowed regions in Figure 1 suggests that, when actions are complements, the discrepancy in the nature of strategic interactions between information choices and actions (i) decreases when the (second order) external effect increases and, in particular, (ii) disappears when the external effect is high enough so that $\pi \geq 1$. This is formally stated in the next proposition.

Proposition 3. *Assume 1 and 2. Then, the critical threshold $\kappa(\pi)$ identified in Proposition 2 (i) is (i) nondecreasing in π for each $\pi \in [0, 1]$, and (ii) $\kappa(1) = 1$.*

High levels of the second order externality offset the effect caused by high levels of complementarity in actions. This favors that strategic interactions in information end up having the same nature as strategic interactions in actions even for high levels of complementarity in actions. The second order externality serves as an additional incentive for the players to coordinate their actions. Then, a player, say player 1, finds more valuable to learn about the state when player 2 does so as a mean to be well informed about the relation between the state and player 2's optimal action. In particular, Proposition 3 (ii) says that, when the externality is sufficiently high ($\pi \geq 1$), information choices are strategically complements whenever actions are complements. In this case, the level of the externality is not compatible with a beauty contest. Nevertheless, for these high levels of the externality, the implications agree with those in HV.

4. Applications

From the results provided by Proposition 2, one concludes that, if the degree of complementarity in actions is high enough, heterogeneous beliefs can be sustained endogenously in dynamic settings where there is a new realization of the state in each period. Also, starting from some initial information decisions, heterogenerous beliefs maybe reduced for high levels of substitutability. Presumably, a wide class of models with a finite (or a relatively small) number of players might meet the conditions leading to our results. To illustrate this, we discuss in this section some strategic settings where the results in Propositions 1, 2, and 3 are of interest. Recall from Proposition 2 that, for a given value $\pi \geq 0$ of the second order externality, $\kappa(\pi)$ denotes the critical threshold for λ above which information choices become substitutes when actions are complements.

4.1. *Investment Complementarities*

Consider the canonical model of production externalities where a_i is interpreted as the amount of investment chosen by investor i. The payoff to investor $i = 1$ is given by

$$U(a_1, a_2, \theta) = R(a_2, \theta)a_1 - c(a_1),$$

where $c(a_1)$ is a twice-differentiable cost function and $R(a_2, \theta)$ is a twice-differentiable return function that measures the externality to investor 1 due to the adequacy of agent 2's investment to the underlying state. A typical example would be that of an R&D activity. Assume $c'' > 0$, $R_{a_2}, R_\theta > 0$, $R_{a_2 a_2} < 0$, and $R_{a_2}/c'' < 1$. Thus, the coordination motive in actions has the form of a complementarity.

Using our results, one directly obtains that heterogenous beliefs can be endogenously sustained if the ratio $\lambda = R_{a_2}/c''$ exceeds the critical threshold $\kappa(\pi)$. In addition, we have $\pi = -a_1 R_{a_2 a_2}/c''$ so that, from the result in Proposition 3 (i), threshold $\kappa(\pi)$ increases as $-R_{a_2 a_2}$ increases. In other words, for a given cost function c, more sensitivity and less concavity of the return function R (with respect to the other investor's action) facilitate the persistence of heterogeneous beliefs. Nevertheless, to obtain the result in Proposition 3 (ii) that information decisions are complements for any degree of complementarity in actions, one needs $-R_{a_2 a_2} > (1/a_1)c''$ for each $a_1 > 0$. Then, since $-R_{a_2 a_2}$ must be bounded, there is a set of values for R_{a_2} under which information decisions are substitutes. Heterogeneous beliefs are endogenously sustained for those high levels of sensitivity of the return function R in the other agent's investment (measured by R_{a_2}).

Corollary 1. *In the investment model described above, (i) investor* 1 *wishes to learn less about the state when investor* 2 *learns more if* $R_{a_2} > \kappa(\pi)c''$*, and (ii) for investor* 1*,* $\kappa(\pi)$ *is nondecreasing with* $-R_{a_2 a_2}$*.*

4.2. *Cournot Duopoly*

Consider a model of Cournot competition where a_i is interpreted as the quantity of the good offered by firm i. The market price of the good is given by $P = d_0 + d_1\theta - d_2(a_1 + a_2)$ (with $d_0, d_1, d_2 > 0$) and the cost for firm $i = 1$ is given by $c_0 a_1 + c_1 a_1^2$ (with $c_0, c_1 > 0$). Then, the payoff function for firm $i = 1$ can be expressed as

$$U(a_1, a_2, \theta) = \big[(d_0 - c_0) + d_1\theta - (d_2 + c_1)a_1 - d_2 a_2\big]a_1.$$

One obtains $\lambda = -d_2/2(d_2 + c_1) < 0$ so that actions are substitutes. The assumptions of our model are satisfied since $c_1 > 0$ implies $\lambda > -1$. Also, we have $\pi = 0$. From our earlier results it follows that heterogeneous beliefs are likely to prevail in this setting. Note that, to obtain the result in Proposition 2 (ii) that information decisions become complements, one must start from an initial situation where firm 1 acquires little amount of information while firm 2 is very well informed. Furthermore, one needs that λ be close enough to -1. However, we see that $-1/2$ is the lowest value that λ can achieve in this duopoly.

4.3. *Bertrand Duopoly*

Consider a model of Bertrand competition with heterogenous goods where a_i is interpreted as the price set by firm i. The market demand for firm $i = 1$ is given by $Q_1 = e_0 + e_1\theta - e_2(a_1 - a_2)$ (with $e_0, e_1, e_2 > 0$) and its cost is given by $c_0 Q_1 + c_1 Q_1^2$ (with $c_0, c_1 > 0$). Then, the payoff function for firm $i = 1$ can be expressed as

$$U(a_1, a_2, \theta) = \big[e_0 + e_1\theta - e_2(a_1 - a_2)\big]\big[a_1 - c_0 - c_1\big(e_0 + e_1\theta - e_2(a_1 - a_2)\big)\big].$$

One obtains $\lambda = (1+2c_1e_2)/2(1+c_1e_2) \in (0,1)$ so that actions are complementary. We also have $\pi = c_1e_2/(1+c_1e_2)$. Therefore, we see that the product c_1e_2 is the key parameter to analyze whether heterogenous beliefs will be sustained endogenously in this model. For this to happen, our results tell us that c_1e_2 must be sufficiently high but below a certain bound. As c_1e_2 increases, both λ and π increase and get closer to one. Thus, the result in Proposition 2 (i) will be obtained when c_1e_2 is high enough, so that λ exceeds the required threshold $\kappa(\pi)$, but not too high so as to avoid that π gets too close to one. We see that in this Bertrand model, the effects on strategic interactions in the information choice of the complementarity in pricing decisions and the second order externality have opposite sign. For particular situations, we should be interested in knowing which of these two effects dominates. Nevertheless, as in the Cournot case, since c_1 and e_2 must be bounded, the result in Proposition 3 (ii) does not follow as we have $\kappa(\pi) \to 1$ only if $c_1e_2 \to \infty$. Consequently, it is not guaranteed that homogenous beliefs will be endogenously sustained for any degree of complementarity associated to the parameters c_1 and e_2.

Corollary 2. *In the Bertrand game described above, (i) firm* 1 *wishes to learn less about the state when firm* 2 *learns more if* $c_1e_2 > [2\kappa(\pi) - 1]/2[1 - \kappa(\pi)]$, *and (ii) for firm* 1, $\kappa(\pi)$ *is nondecreasing with* c_1e_2.

5. Concluding Remarks

This paper investigated the relation between endogenous interaction in information decisions and strategic interactions in actions for a tractable class of games with complementary or substitutive actions, externalities, and a fairly general information structure.

Our analysis highlighted the differences in the nature of interactions when the set of players is finite with respect to the case with a continuum of players. From a methodological viewpoint, keeping track of players' higher-order beliefs through a knowledge index leads to conclusions different to those obtained using an average expectation operator. The main reason behind this discrepancy is in the fact that, with a small number of players, information acquisition affects considerably the sensitivity of each player's optimal action to her private signal.

Our restriction to two-player games is for tractability reasons. With a larger number of players, computing the required inverse of matrix $[I - \lambda\Omega]$ is exceedingly challenging and one must resort to computational numerical methods. This inverse is a crucial ingredient in the slope of a player's optimal action in her private signal. However, the functional form of the entries of the inverse $[I - \lambda\Omega]^{-1}$ is not affected by increasing the number of players. Each of these entries is a polynomial in λ whose degree increases with the number of players. Then, increasing the number of players does not affected the form of ratio between polynomial functions (with respect to λ) of the slope of a player's optimal action. Consequently, our results continue to hold qualitatively so long as the number of players is finite.

As the number of players increase, our results (for the beauty contest game) converge asymptotically to the main result obtained by HV and, therefore, complementarity (substitutability) in actions tends to induce complementarity (substitutability) in information decisions. Our results are relevant in environments with a relatively small number of players, such as industrial competition settings, but lose importance for the case of markets with many traders. For such large markets, usually studied in general equilibrium and macro models, the insights obtained by HV are very useful for the analysis of strategic interactions in information decisions.

The applications analyzed in Section 4 give a rough feeling of how to avoid heterogeneous beliefs in dynamic settings where there is a new realization of the state in each period. With investment complementarities, it is tempting to suggest that many investors take part in the new sector. This would facilitate that the second order externality from the investment of each of them be small enough. Also, the application to Bertrand oligopoly could suggest that the offered goods tend to be homogeneous as a mean to increase the effect of the second order externality. However, it would not be wise to give a normative connotation to the results obtained in this paper since it lacks an analysis of the welfare consequences of endogenous information decisions. Such a welfare analysis requires one to compute equilibrium information choices and to conduct a statics comparative exercise when λ varies. Unfortunately, to do this, we need first to understand well the properties of the set of perfect Bayes-Nash equilibria of the proposed game $\Gamma_{(\lambda,\pi)}$. Angeletos and Pavan (2007) have successfully examined the comparative statics of welfare with respect to the information structure in a setting with a continuum of players and without endogenous information choice. With endogenous information choice, a comparative statics exercise along those lines seems a challenging and interesting direction for future research.

Appendix

For a given $(x_1, x_2) \in (0,1)^2$, let $h : [-1,1] \times \mathbb{R}_+ \to \mathbb{R}$ be the function specified as

$$\begin{aligned} h(\lambda, \pi) :=& (1-\lambda)\left[x_1 x_2 q_1 q_2 (2-\pi)\lambda^2 + x_1 q_1 \lambda + (1-\pi) x_2 q_2^2 \lambda + q_2 - \pi q_1 q_2\right] \\ & - (\lambda - \pi)\left[x_2 q_2 \lambda + q_1\right]. \end{aligned}$$

Then, for each $\lambda \in (-1, 1)$, we have

$$\widehat{F}_{x_1 x_2} = \left[\frac{(1-\lambda)\tau_1 \sigma}{1-\lambda^2 x_1 x_2}\right]^2 \left[\frac{\lambda}{1-\lambda}\right] h(\lambda, \pi).$$

Analogously, for a given $(x_1, x_2) \in (0,1)^2$, let $g : [-1,1] \to \mathbb{R}$ be the function specified as

$$g(\lambda) = -x_1 x_2 q_1 q_2 \lambda^4 - x_2 q_2^2 \lambda^3 + (2 x_1 x_2 q_1 q_2 - x_2 q_2 - q_1 q_2)\lambda^2 + (x_1 q_1 + x_2 q_2^2 - q_1)\lambda + q_2.$$

Then, we have

$$\widehat{F}^{\mathrm{bc}}_{x_1 x_2} = \lambda \left[\frac{(1-\lambda)\sigma}{1-\lambda^2 x_1 x_2}\right]^2 g(\lambda),$$

so that the sign of $\widehat{F}^{\mathrm{bc}}_{x_1 x_2}$ coincides with the sign of g for each $\lambda \in (0,1)$. With the functions $h(\lambda, \pi)$ and $g(\lambda)$ at hand, we proceed with the proofs of the propositions.

Proof of Proposition 1. The first claim in the proposition follows directly from the specification of the function $\widehat{F}_{x_1 x_2}$ given by (12).

As for the second claim, take a given $(x_1, x_2) \in [0,1]^2$. We obtain

$$h(0, \pi) = q_2 - \pi q_1 q_2 + \pi q_1 = 1.$$

The result follows since $h(0,\pi) > 0$, $\widehat{F}_{x_1 x_2} = 0$ for $\lambda = 0$, $\widehat{F}_{x_1 x_2}$ is a continuous function in $\lambda \in (-1,1)$, and $\lambda/(1-\lambda) > 0$ if $\lambda \in (0,1)$ while $\lambda/(1-\lambda) < 0$ if $\lambda \in (-1, 0)$. □

Proof of Proposition 2. Take a given $(x_1, x_2) \in (0,1)^2$. It can be checked that $h(0, \pi) = 1$ for each $\pi \in [0, 1)$.

(i) Take a given $\pi \in [0,1)$. Then,

$$\begin{aligned} h(1, \pi) &= (\pi - 1)[x_2 q_2 + q_1] \\ &= (\pi - 1)\left[\frac{x_2(1+x_1) + (1+x_2)}{1 - x_1 x_2}\right] < 0. \end{aligned}$$

Since $h(0,\pi) > 0$, $h(1,\pi) < 0$, and $h(\cdot, \pi)$ is a continuous function in λ, there is some $\kappa(\pi) \in (0,1)$, bounded away from 1, such that $h(\lambda, \pi) < 0$ for each $\lambda \in (\kappa(\pi), 1)$. The result follows since $\lambda/(1-\lambda) > 0$ for each $\lambda \in (0,1)$.

(ii) Take a given $\pi \in [0,1)$, and consider $x_1 = 0$ and $x_2 = 1$. Then,

$$h(-1,\pi) = 2\left[-(1-\pi)q_2^2 + q_2 - \pi q_1\right] - (-1-\pi)\left[-q_2 + q_1\right] = \pi - 1 < 0.$$

Since $h(0,\pi) > 0$, $h(-1,\pi) < 0$ for $x_1 = 0$ and $x_2 = 1$, and $h(\cdot,\pi)$ is a continuous function in λ, in x_1, and in x_2, then there is some $\varepsilon(\pi), \delta(\pi) > 0$ and some $\kappa(\pi) \in (-1,0)$, bounded away from -1, such that $h(\lambda,\pi) < 0$ for each $\lambda \in (-1,\kappa(\pi))$, each $0 \leq x_1 < \varepsilon(\pi)$, and each $(1-\delta(\pi)) < x_2 \leq 1$. The result follows since $\lambda/(1-\lambda) < 0$ for each $\lambda \in (-1,0)$.

(iii) Take a given $(x_1,x_2) \in (0,1)^2$. It can be checked that

$$\begin{aligned} g(1) &= (x_1x_2 - 1)q_1q_2 + (x_1 - 1)q_1 - (x_2 - 1)q_2 \\ &= \frac{(x_1 - 1) - (x_1 + 3)x_2}{1 - x_1x_2} < 0. \end{aligned}$$

The result follows since g is a continuous function in $\lambda \in (-1,1)$, and $g(0) > 0$ while $g(1) < 0$. □

Proof of Proposition 3. For $\lambda \in (0,1)$, let $\psi(\lambda)$ be the value of π such that $h(\lambda,\psi(\lambda)) = 0$. From the result in Proposition 2 (i), we know that there exists some $\kappa(0) \in (0,1)$ such that $\widehat{F}_{x_1x_2}(x_1,x_2) = 0$ for each $(x_1,x_2) \in (0,1)^2$. This happens if and only if $h(\kappa(0),0) = 0$ so that $\psi(\kappa(0)) = 0$ with $\kappa(0) < 1$. Also, it can be easily checked that $h(1,1) = 0$ so that $\lim_{\lambda \to 1^-} \psi(\lambda) = 1$. Suppose that the mapping $\psi : (0,1) \to \mathbb{R}_+$ is not a correspondence but a function. Then, since $h(\lambda,\pi)$ is continuous for each $(\lambda,\pi) \in (0,1) \times \mathbb{R}_+$, it must be the case that $\psi(\lambda)$ is nondecreasing in $\lambda \in (0,1)$. Therefore, we only need to verify that ψ is indeed a function.

To show that ψ is a function, take $\pi,\pi' \in \mathbb{R}_+$ such that $\pi \neq \pi'$ and suppose that, for some given $\lambda \in (0,1)$, we have $\psi(\pi) = \psi(\pi')$. This happens if and only if $h(\lambda,\pi) = h(\lambda,\pi') = 0$. Then, on the one hand, $h(\lambda,\pi) = h(\lambda,\pi')$ implies

$$(\pi - \pi')\left[(1-\lambda)[x_1x_2q_1q_2\lambda^2 + x_2q_2^2\lambda + q_1q_2] - [x_2q_2\lambda + q_1]\right] = 0.$$

So, since $\pi \neq \pi'$, it must be the case that

$$[x_1x_2q_1q_2\lambda^2 + x_2q_2^2\lambda + q_1q_2] = \frac{1}{1-\lambda}[x_2q_2\lambda + q_1].$$

On the other hand, $h(\lambda,\pi) = 0$ implies

$$\frac{1}{\lambda - \pi}\left[x_1x_2q_1q_2(2-\pi)\lambda^2 + x_1q_1\lambda + (1-\pi)x_2q_2^2\lambda + q_2 - \pi q_1q_2\right] = \frac{1}{1-\lambda}\left[x_2q_2\lambda + q_1\right]$$

Therefore, by combining the two equalities above, it follows that $\lambda \in (0,1)$ must necessarily satisfy the equation

$$x_1x_2q_1q_2(2-\lambda)\lambda^2 + x_2q_2^2(1-\lambda)\lambda + x_1q_1\lambda + q_2 - \lambda q_1q_2 = 0.$$

Now, using the definition of the functions $q_i(x_1, x_2)$, $i = 1, 2$, we know that if the equation above is satisfied, then we must necessarily have

$$\begin{aligned}(1+&\lambda x_2)(1+\lambda x_1)\lambda\big[(2-\lambda)\lambda x_1 x_2 - 1\big] \\ &+ x_2(1+\lambda x_1)^2(1-\lambda)\lambda + [x_1(1+\lambda x_2)\lambda + (1+\lambda x_1)](1-\lambda^2 x_1 x_2) = 0.\end{aligned} \tag{14}$$

But equation (14) above cannot hold for $\lambda \in (0, 1)$ because

$$(1+\lambda x_2)(1+\lambda x_1)\lambda\big[(2-\lambda)\lambda x_1 x_2 - 1\big] > -\lambda$$

and

$$x_2(1+\lambda x_1)^2(1-\lambda)\lambda + [x_1(1+\lambda x_2)\lambda + (1+\lambda x_1)](1-\lambda^2 x_1 x_2) > 1.$$

The last claim of the proposition holds since $\lim_{\lambda\to 1^-} \psi(\lambda) = 1$ implies $\kappa(1) = 1$. $\square$

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