Capacity pre-commitment, price competition and forward markets

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Abstract

After the works of Kreps and Scheinkman's (1983) and, more recently, Moreno and Ubeda's (2005), the Cournot model can be seen as a reduced form of a more realistic model of capacity choice followed by price competition. We show that this is not the case if forward markets are added. Allaz and Vila introduce forward markets previous to a spot market Cournot competition and show that the strategic interaction between the two types of markets has a pro-competitive effect. However, if we replace the Cournot competition with the capacity choice plus price competition, the result no longer holds.

1 Introduction

We introduce a forward market in Moreno and Ubeda's (2005) model on capacity precommitment. Firms first build capacity, second, after observing capacity choices, they choose forward positions, and, finally, after observing choices in the forward market, they choose a reservation price at which they are willing to supply their residual capacity (capacity minus forward positions) in the spot market. Reservation prices define a supply function that, together with the market demand, determines the equilibrium quantity and the market clearing price at which all quantities are sold. A no arbitrage condition is added to make sure that, in equilibrium both forward and spot prices are equal.

In the spot market, this model is a variation of Kreps and Scheinkman's (1983) model of price competition a la Bertrand. We choose Moreno and Ubeda's version for its better properties, like the existence on unique Cournot outcomes in a pure strategies equilibrium (without our addition of forward markets) and the resemblance with the way many markets actually work. For details see Moreno and Ubeda (2005).

Our purpose is to check the robustness of the results in Allaz (1992) and Allaz and Vila (1993), where the introduction of forward markets enhances the competitiveness of a Cournot-like oligopolistic market. Since capacity precommitment plus price choice give Cournot outcomes in a more realistic setting, it is only natural to check whether the procompetitive effect is maintained in this model. We find that this is not the case. The addition of a forward market previous to the spot market has no effect in enhancing competition. The previous choice of capacity, that prevents price competition from falling beyond Cournot's, also prevents forward markets from being procompetitive.

2 The model

There are $n \geq 2$ firms in the industry. The market (inverse) demand function P is twice differentiable, strictly decreasing and concave[1] on a bounded interval $(0, X)$, where $X > 0$ satisfies $P(x) = 0$ for $x \geq X$. The market demand is denoted by $D = P^{-1}$. All firms have access to the same technology. The cost to install capacity x is $b(x)$, where $b : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is twice continuously differentiable and convex on $\mathbb{R}_+$, and satisfies $0 < b'(0) < P(0)$, and $b(0) = 0$. The marginal production up to capacity is constant, and without loss of generality it is assumed to be zero.

Competition runs in three stages. At the first stage each firm $i \in \{1, ..., n\}$ chooses its capacity x_i. After the first stage firms observe their opponents' capacity decisions. At the second stage each firm i chooses a forward position f_i. After the second stage firms observe their opponents' forward positions.

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[1] The only important property of concave demand that is used in both Kreps and Scheinkman 1983 and Moreno and Ubeda 2005 is that it gives a unique, well defined maximum in the profits maximization problem. Many non concave demands share this feature. In particular, a demand below the equilateral hyperbole with fast enough convergence to the axes will do the job. We maintain Moreno and Ubeda's model.

Finally, at the third stage each firm chooses a reservation price p_i at which to sell their entire residual capacity $x_i - f_i$. Firms remaining capacities and reservation prices are then used to form the aggregate supply which, together with the residual demand, $D - \sum_{i=1}^{n} f_i$, determines the market clearing price, p, and (using an unspecified tie-breaking rule) firms' outputs in the spot market, $(s_1, ..., s_n)$. Firm i's revenue in the spot market is ps_i. Firms revenue in the forward market is computed as $p_f f_i$, where p_f is the price in the forward market, that is determined as the spot market equilibrium price given capacities $(x_1, ..., x_n)$ and forward positions $(f_1, ..., f_n)$. This implies that, in equilibrium, $p_f = p$. This no arbitrage condition greatly simplifies the model, as arbitrageurs are not necessary[2]. Firm i's total payoff is the difference between total revenues $p_f f_i + ps_i$ and total costs, $b(x_i)$.

3   Forward positions and price competition with capacity constrains

3.1   Price competition in the third stage

The game firms face in the third stage is exactly the same game firms face in the second stage in Moreno and Ubeda's (2005) model. The only thing to be noted is that we must use residual capacities and demand. This is because decisions in this third stage do not alter whatever profits were obtained in the second stage. Thus we can make use of all the results in their work, namely the lemma and Proposition A. In particular, the existence of a pure strategy equilibrium is guaranteed.

3.2   The choice of forward positions

This is the key to our result. In this Section we will make extensive use of the following equilibrium condition: whenever one firm is marginal in the spot market, it chooses zero positions in the forward market, as the effect of entering in the forward market is always negative to its interests. After this useful lemma, we will consider three relevant cases that exhaust the possible situations in the second stage. Denote by $r(x_{-j})$ Firm j's best reply against x_{-j} if it had no capacity constrain..

Lemma 0. $r(x_{-j}) < r(x_{-j} + f_j) + f_j$ for all $f_j > 0$.

Proof. By definition, $r(x_{-j}) = P^{-1}(p(x_{-j})) - x_{-j}$, where $p(x_{-j})$ maximizes $p(P^{-1}(p) - x_{-j})$, and $r(x_{-j} + f_j) = P^{-1}(p(x_{-j} + f_j)) - x_{-j} - f_j$, where $p(x_{-j} + f_j)$ maximizes $p(P^{-1}(p) - x_{-j} - f_j)$. The respective first order conditions[3] for the maximization problems are $p = -\frac{P^{-1}(p)}{(P^{-1})'(p)}$, and $p = -\frac{P^{-1}(p) - f_j}{(P^{-1})'(p)}$, whence is is

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[2]Allaz (92) shows that the arbitrage condition is equivalent to the addition of arbitrageurs in his model

[3]Conditions on the demand function ensure that first order conditions are sufficient, and that the solution exists and is unique.

clear that $p(x_{-j}) > p(x_{-j} + f_j)$, and then $P^{-1}(p) < P^{-1}(p(x_{-j} + f_j))$, and finally $r(x_{-j}) < r(x_{-j} + f_j) + f_j$. QED.

An illustration of Lemma 0 is provided in Figure 1, where $r(.)$ is the quantity where marginal revenue in the residual demand is zero (and equal to marginal cost).
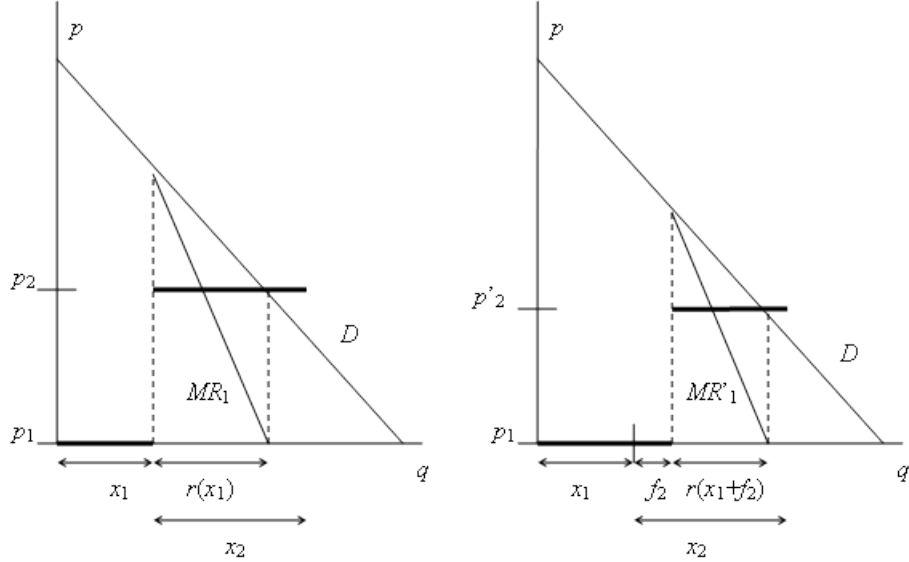

Fig. 1: Illustration of Lemma 0.

Case 1: $x_1 \geq r(x_{-1})$, $x_j < r(x_{-j})$ for all $j \in \{1, ..., i-1, i+1, ..., n\}$.

Lemma 1. *Any pure strategy equilibrium in Case 1 is of the following form: Firm 1 chooses $f_1 = 0$ in the forward market and sets $p_1 = P(r(x_{-1}) + x_{-1})$ in the spot market, where it is marginal and sells $s_1 = r(x_{-1})$ at p_1.*

Proof. First, let us show that there exists such an equilibrium. For that take $f_j = x_j$ for all $j \neq 1$. It is clear that the only equilibrium in the third stage given positions in the forward markets is that Firm 1 chooses a price $p_1 = P(r(x_{-1}) + x_{-1})$. The other firms have no capacity left to offer and they may set any price without affecting the supply. Firm 1 has no incentive to offer a positive position in the forward market. If it offers $f_1 > 0$, in the spot market it is still the only and marginal firm, and should choose a price $p_1' = P(r(x_{-1} + f_1) + x_{-1} + f_1)$, that makes it sell $r(x_{-1} + f_1)$, if $x_1 - f_1 > r(x_{-1} + f_1)$ and $p_1'' = P(x)$, with sales of $x_1 - f_1$ otherwise. By the no arbitrage condition, the forward and spot market prices must be the same. Thus, the combination of price p_1' and quantity $(r(x_{-1} + f_1) + f_1)$ in the first case and $x_1 = x_1 - f_1 + f_1$ in the second, can not give a higher profit than the maximum attained at p_1 and $r(x_{-1})$. Offering $f_1 < 0$ is like increasing Firm 1's capacity (it has its physical capacity of x_i plus the virtual capacity of f_1). It will still be the marginal

firm in the spot market, with no consequence in the equilibrium price and total outcome.

Each one of the non marginal firms is indifferent between selling $f_j = x_j$ or any other $0 \leq f_j < x_j$. In any case, it is still non marginal in the spot market, as $x_j < r(x_{-j})$ implies $x_j - f_j < r(x_{-j} + f_j)$ by Lemma 0 (to check: it is still the case that $x_1 \geq r'(x'_{-1})$, where r' is the best reply including virtual capacities, and x' includes virtual capacities). The marginal firm, Firm 1, sets the price to sell $x_1 = r(x_{-1} - f + f) = r(x_{-1})$, and the price will be the same regardless of the forward positions taken by the non marginal firms. Because the forward and spot market prices must be the same, Firm j will be selling the same quantity at the same price for all $f_j \leq x_j$.

We now show that all other equilibria must satisfy the conditions in the statement. (i) First notice that, if $f_1 = 0$, it is still the case that Firm 1 is the marginal firm in the equilibrium in the spot market, as $x_1 - f_1 \geq r(x_{-1} + f_1)$ is satisfied for $f_1 = 0$, and for other firms we also have $x_j - f_j < r(x_{-j} + f_j)$ by Lemma 0. Under these conditions, Moreno and Ubeda show that Firm 1 must be marginal.

Consider, then, a situation (ii) in which firms choose forward positions as in $(f_1, f_2, ..., f_n)$, with $f_1 > 0$. For this to be an equilibrium, it must be that Firm 1 does not want to deviate. (iia) If it is marginal in the subgame after $(f_1, f_2, ..., f_n)$, with $f_1 > 0$, and also in subgames after $(f'_1, f_2, ..., f_n)$ with $f'_1 = 0$, it will deviate to $f'_1 = 0$, as shown before. For the deviation not to be profitable, it is needed that another firm be marginal in the equilibrium after $(f'_1, f_2, ..., f_n)$ with $f'_1 = 0$, but we just saw that this cannot be the case.

The only remaining possibility is (iib) that Firm 1 is not marginal in the subgame after $(f_1, f_2, ..., f_n)$, with $f_1 > 0$. If no firm is marginal in the spot market, the price is $P^{-1}(x) < p_1$ and Firm 1 will be selling x_1. However, Firm 1 can maximize profits by choosing $f_1 = 0$, and then p_1 to sell $r(x_{-1})$. Say, then, that the marginal firm in the spot market equilibrium is Firm 2. In this subgame Firm 2 sets price $p_2 = P(r(x_{-2} + f_2) + x_{-2} + f_2) < p_1$, thus selling $r(x_{-2} + f_2) + f_2 < x_2$ at a price $p_2 < p_1$. Firm 2 can easily avoid both this subgame and being marginal by setting $f'_2 = x_2$ (if $f_2 = x_2$ Firm 2 cannot be marginal), and inducing the subgame $(f'_1, f'_2, ..., f_n)$ with $f'_1 = 0$ and $f'_2 = x_2$. In order for Firm 2 to be unwilling to do so (and being marginal in the equilibrium), it must be made worse off, and for that we need that the marginal firm, say Firm 3, in the equilibrium of this new subgame with $f'_1 = 0$ and $f'_2 = x_2$ sets a price $p_3 < p_2$, otherwise Firm 2 will be better off as it sells a higher quantity when it is non marginal. We can repeat the argument for Firm 3 and, by induction, for any other marginal firm. Because the number of firms is finite, eventually we will get $p_k < p_k$ for some k. A contradiction. This means that we cannot have $f_1 > 0$ in equilibrium. QED.

In Lemma 1 we saw that all firms but one can avoid being marginal by choosing $f_j = x_j$. Clearly, this may not be necessary. Any quantity f_j such that Firm 1 prefers being marginal rather than letting Firm j being marginal (if that is a possibility after j's choice of f_j) will do the job.

Case 2: $x_j < r(x_{-j})$ for all $j \in \{1, .., n\}$.

Lemma 2. *Any pure strategy equilibrium in Case 2 is of the following form: Firm j chooses $f_j \leq x_j$ in the second stage and $p_j \leq P(\sum_j x_j)$ in the third stage, where it sells $s_j = x_j - f_j$.*

Proof. The proof is straightforward. For any vector of futures positions $(f_1, ..., f_n)$ notice that, to compute the reaction function for Firm j, the residual demand is the same if we substract x_{-j} or we substract first f_{-j} and then $x_{-j} - f_{-j}$, that is $r(x_{-j}) \equiv r_{f_{-j}}(x_{-j} - f_{-j})$, where $r_{f_{-j}}$ is the reaction function in demand $P^{-1} - f_{-j}$. Then, applying Lemma 0 we have that $x_j < r(x_{-j})$ implies that $x_j - f_j < r_{f_{-j}}(x_{-j} - f_{-j} + f_j)$ for all j, so that all firms are capacity constrained in all the subgames and sell up to capacity for all $(f_1, ..., f_n)$. QED.

Case 3: There is a subset $M \subset N$ with $card(M) \geq 2$ such that $x_j \geq r(x_{-j})$ for all $j \in M$.

Lemma 3. Any pure strategy equilibrium in Case 3 is of the following form: One firm i in M chooses $f_i = 0$ in the forward market and sets $p_i = P(r(x_{-i}) + x_{-i})$ in the spot market where it is marginal.

Proof. Order firms in M according to their capacities so that $x_1 \geq x_2 \geq ... \geq x_m$. Consider three cases.

(i) There is an equilibrium in which Firm 1 is marginal in the game without forward markets and Firm 1 prefers the outcome in this equilibrium rather than the outcome in any other equilibrium (if it exists) in which any other firm in M is marginal. Clearly, it must be that $x_1 > x_2$, and no other firm in M prefers the equilibrium in which it is marginal (if it exists) to the equilibrium in which Firm 1 is marginal (the price is higher in this last case, and the quantity sold larger). Now, in the game with forward markets, firms other than 1 can avoid being marginal by playing $f_j = x_j$. If Firm 1 is marginal, it better chooses $f_1 = 0$. The complete argument is the same as in Case 1 for non-marginal firms.

(ii) More than one firm prefers an equilibrium in which it is marginal rather than any other equilibrium: It is easy to see that this case cannot occur. If Firm k prefers being marginal, it must be that $x_k > x_j$ for all other $j \in M$, but then Firm j prefers the equilibrium in which k is marginal.

(iii) For any firm k there exists another Firm j such that Firm k prefers the equilibrium in which j is marginal. As we saw in (i), if $x_1 > x_2$ all firms but 1 can avoid being marginal by playing $f_j = x_j$. By repeating the argument, Firm 1 will set $f_1 = 0$ and will be marginal in the spot market. If $x_1 = x_j$ for some $j \in M$, there will be multiple equilibria, with either 1 of j being the marginal and taking no forward positions: if $f_k > 0$ and k is marginal, the only reason not to deviate to $f_k = 0$ is that, in this subgame, the equilibrium mandates that other firm j be marginal, and one that k does not want it to be marginal. If this is the case, the price must be lower and so must be the capacity. But, then, this firm prefers that Firm k be marginal, and repeating the argument in (i) no firm with a smaller capacity will ever be marginal in equilibrium as they can effectively avoid being marginal by setting $f_j = x_j$. QED.

3.3 Equilibria in the full game

Lemmas 1-3 before show that all the pure strategy equilibria in the game without forward markets have a counterpart in the game with forward markets in the following sense. Take an equilibrium in the game without forward marktes in which Firm i is marginal. Total quantity in this equilibrium is $x_{-i} + r(x_{-i})$. Then, in the game with forward markets, there is an equilibrium in which Firm i is marginal and total quantity is also $x_{-i} + r(x_{-i})$. The converse is also true.

Proposition 1. All equilibria in pure strategies yield the Cournot outcome. Moreover, the Cournot outcome can be sustained by a subgame perfect equilibrium in pure strategies.

The second and third stages of the game have the same outcomes as a game without a forward market. The proof of Theorem B in Moreno and Ubeda that shows the equilibrium choices of capacity given the continuation of the game remains exactly the same.

4 Discussion

There is nothing in the proofs that relies on the fact of there being just one period of forward markets. All the arguments rely on the already taken positions. Therefore, they apply for any number, finite or infinite, of forward market openings.

Allowing firms to buy in the futures markets does note make any change: Lemmas 0 and 2 do not require $f_i \geq 0$. Lemmas 1 and 3 consider only deviations to positive forward positions, but deviations to negative positions will give the same results.

The model has two features of interest. First, it shows that two models that give the Cournot outcome, namely quantity competition and capacity choice plus price competition, give contrasting results when forward markets are added to them. Thus, in the presence of forward markets, the former cannot be taken as a reduced form of the later.

Second, in the models of quantity choice (Allaz and Vila 1994, Ferreira 2003, Liski and Montero 2005) the use of forward markets implied greater competition. In those models, the only way to obtain the Cournot outcome in equilibrium implied that firms avoid the use of forward markets (in Ferreira 2003 the Monopolistic outcome can be obtained in equilibrium if firms are allowed to buy in the forward market, and in Liski and Montero 2005 collusion is achieved in equilibrium more easily in the presence of forward markets). Our model offers the novel result by which firms do not need to avoid completely the use of forward markets to avoid competition beyond Cournot. Rather, the use of the forward markets by all firms except the marginal one is compatible with the Cournot outcome.

Murphy and Smeers (2010) study a model of capacity choice and Cournot competition in both the forward and the spot market, and of uncertain demand. They find that forward market can enhance or mitigate market power when

capacities are endogenous and demand is unknown at the time of investment. When demand is known, the model offers the same equilibrium outcome as Cournot with no forward makets.

Price competition has been studied in the context of forward markets in a model of differenciated products but no capacity choice in Mahenc and Salanie (2004). The results show is that, contrary to the model in Allaz and Vila (1993), forward markets soften competition.

REFERENCES

Allaz, B. 1992. "Oligopoly, uncertainty, and strategic forward markets and efficiency," *International Journal of Industrial Organization* 10 297–308.

Allaz, B., J.-L. Vila. 1993. "Cournot competition, forward markets and efficiency," *Journal of Economic Theory* 59 1–16.

Ferreira J.L. 2003. "Strategic interaction between futures and spot markets," *Journal of Economic Theory* 108 141-151.

Kreps D., J.Scheinkman. 1983. "Quantity precommitment and Bertrand competition yield Cournot outcomes," *Bell Journal of Economics* 14 326–337.

Liski M., J.P. Montero.2006. "Forward trading and collusion in oligopoly," *Journal of Economic Theory* 131 212-230.

Mahenc P., F. Salanie. 2004. "Softening competition through forward trading," *Journal of Economic Theory* 116 282–293.

Moreno D., L. Ubeda. 2006.. "Capacity precommitment and price competition yield the Cournot outcome," *Games and Economic Behavior* 56 323-332.

Murphy F., Y. Smeers. 2010. "On the Impact of Forward Markets on Investments in Oligopolistic Markets with Reference to Electricity," *Operations Research* 58 515–528.