Status-Seeking in Hedonic Games with Heterogeneous Players*

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Abstract

We study hedonic games with heterogeneous player types that reflect her nationality, ethnic background, or skill type. Agents' preferences are dictated by status-seeking where status can be either local or global. The two dimensions of status define the two components of a generalized constant elasticity of substitution utility function. In this setting, we characterize the core as a function of the utility's parameter values and show that in all cases the corresponding cores are non-empty. We further discuss the core stable outcomes in terms of their segregating versus integrating properties.

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1 Introduction

When following fashion or joining a political party, choosing a home or finding a job, individuals' choices define group membership. In such situations, individuals are often motivated by status seeking. On the one hand, all members of a given group enjoy the same social status relative to other groups. On the other hand, the status of members of the same group may differ in social status relative to each other when individual heterogeneity is taken into account. Thus, social status has a 'global' (inter-group) and a 'local' (intra-group) dimension.

In this work we study the interplay between global and local status in group formation by quality-indexed players of two distinct types. We take a player's type to capture innate characteristics such as nationality, ethnic background, or skill-type. Thus groups may be homogeneous (i.e., contain one type of players) or heterogeneous (i.e., contain both types of players) in nature. Depending on players' preferences for global and local status as represented by a constant elasticity of substitution utility function, we obtain different sets of core-stable outcomes. We further discuss these outcomes in terms of their segregating versus integrating properties. Segregated outcomes refer to partitions of the player set in which high-quality and low-quality players of each type are members of different groups. Instead, integrated outcomes refer to partitions in which the type-specific average quality of players in each group is the same.

Our work contributes to the theoretical literature in economics on socially referenced preferences inspired by Schelling (1978), on social status started by Frank (1985), and in particular is akin to models founded on constrained interdependence (cf. Cole et al. 1992).[1] The novelty of this work is in its focus on the way global and local status jointly shape group formation, and in its methodology rooted in the hedonic games tradition which allows for an arbitrary number of groups to be formed, and for groups of arbitrary size.

More closely, our study is related to Milchtiach and Winter (2002) and Watts (2007) who also discuss segregation within a status-based preferences setting. We build upon the work of Watts (2007) in defining our notions of local and global status and the properties of segregation and integration. As in Watts (2007), our agents prefer to have a higher local status measured by their relative position in the group. While we measure the relative position as the distance from the average, she captures it by the rank of the individual in the group.[2] Moreover, while global status in her work is measured by the average quality of agents in the group, here, global status is given by the average quality of group members of the other type. Therefore, an agent's quality affects the group global status directly in Watts's sense, but it affects it only in strategic terms here. Milchtiach and Winter (2002), on the other hand, define agents' preferences to be decreasing in the distance from the average quality. While there are many situations where such preferences are a good proxy for reality–e.g., voting on the level of public

[1] For a very recent extensive survey of theoretical works on social status as well as studies that provide empirical evidence for the significance of status seeking in economics, see Truyts (2010).

[2] Notice that 'relative position' is a more general notion than 'rank' as the difference in ranks of two consecutively ordered agents is the same for all distinct pairs of consecutively ordered agents, while the difference in relative positions may differ.

good–there are other situations in which having a higher than the average index is desirable, e.g., when reward is based on relative performance. A more important distinction between our work and the works of the authors mentioned above is that they study group formation with a restriction on the number of groups that may be formed when players are of a homogeneous type. As a consequence, the notion of stability used here, the core, is not applicable in their works. Finally, as we investigate various types of preference profiles in which local and global status jointly determine agents' choices, we find conditions for which integrated outcomes may be stable. In contrast, segregated outcomes are the unique type of stable outcomes in these authors' works.

This paper also has a place within the vast literature on group formation when agents' preferences over group membership depend on the identity of the other members of the group. Group formation by heterogeneous types of agents has been analyzed in a large literature on two-sided matching problems originated by Shapley and Shubik (1972). The hedonic coalition formation literature (cf. Drèze and Greenberg, 1980) studies group formation when agents are homogeneous and their preferences depend on group membership only. Our work may be viewed as marrying these two strands of the literature.[3] Another strand of the literature that combines matching and coalition formation is that on effective coalitions (cf. Kaneko and Wooders, 1982). Like that literature, we use the notion of core to study stability, however, we do not impose any restrictions on the type of coalitions

[3] In a different paper, Dimitrov and Lazarova (2008), we study the necessary and sufficient conditions that guarantee non-emptiness of the core when the preference profiles are lexicographic.

that may form.

Within the matching literature, our work is closely related to the class of papers on many-to-one matchings with peer effects (see Dutta, and Massó, 1997; and more recently Echenique and Yenmez, 2007; Pycia, 2007; and Revilla, 2007). The difference between our work and theirs is that in our framework group formation occurs on both sides of the market while in theirs it happens on one side of the market only. Our paper is also related to the work of Kaneko and Kimura (1992) who study group formation by heterogeneous types agents, black and white, whose preferences over groups depend on the size of the group. Similarly, Karni and Schmeidler (1990) study the splitting of the population which contains two types of agents into three groups when preferences depend on the relative size of each group. In contrast, in our work peer effects are not size-based.

In this paper, we use the notion of the core to study stability where identity is conceptualized as a hedonic trait, thus our work is also related to the literature on hedonic coalition formation. Banerjee et al. (2001), Bogomolnaia and Jackson (2002), and Ihlé (2007), among others, introduce various notions of stability and provide sufficient conditions for the existence of stable partitions in hedonic games. In this literature, however, identity is summarized in the index of each agent and authors do not discuss heterogeneous types of agents. Moreover, the preference profiles studied here differ from those usually analyzed in the literature such as separable, size-based, and symmetric preferences.

Finally, this paper is related to the literature on local public goods (cf. Tiebout, 1956; and, more recently, Conley and Wooders, 2001) as we, too, study group membership by heterogeneous types of agents. We, however, do not discuss public group production and the size of the partition in our model is not restricted as in the case of jurisdictions.

The rest of the paper is organized as follows. The next section introduces the basic concepts used in our analysis. In Section 3 we characterize the set of core stable outcomes for different parameter values of the constant elasticity of substitution utility function. In particular, when individuals seek only local status or when local and global status are considered to be (imperfect) substitutes, we show the generic uniqueness of the core: in all core-stable outcomes agents have zero utility. When individuals seek only global status, instead, the core stable outcomes vary in terms of players' utility levels. In this case, we provide an algorithm that characterizes the core-stable outcomes. We further provide a characterization of the core when global and local status are treated as substitutes and show the non-emptiness of the core by means of another algorithm. Finally, we conclude in Section 4 with some insights that our analysis contributes to the existing literature.

2 Notation and Definitions

Let $N^a = \{1^a, 2^a, \ldots, m^a\}$ and $N^b = \left\{1^b, 2^b, \ldots, n^b\right\}$ with $m \leq n$ be two disjoint and finite sets of agents of type a and type b, respectively. For each player $i \in N := N^a \cup N^b$ we denote by $\mathcal{N}_i = \{X \subseteq N \mid i \in X\}$ the collection of all coalitions containing i. A partition π of N is called a coalition structure. For each coalition structure π and each player $i \in N$, we denote by

$\pi(i)$ the coalition in π containing player i, i.e., $\pi(i) \in \pi$ and $i \in \pi(i)$. Further, we assume that each player $i \in N$ is endowed with a preference $\succeq_i$ over $\mathcal{N}_i$, i.e., a binary relation over $\mathcal{N}_i$ which is reflexive, complete, and transitive. Denote by $\succ_i$ and $\sim_i$ the strict and indifference relation associated with $\succeq_i$ and by $\succeq := (\succeq_1, \succeq_2, \ldots, \succeq_n)$ a profile of preferences $\succeq_i$ for all $i \in N$. A player's preference relation over coalitions canonically induces a preference relation over coalition structures in the following way: For any two coalition structures π and π', player i weakly prefers π to π' if and only if he weakly prefers "his" coalition in π to the one in π', i.e., $\pi \succeq_i' \pi$ if and only if $\pi(i) \succeq_i' \pi(i)$. Hence, we assume that players' preferences over coalition structures are purely hedonic. That means they are completely characterized by their preferences over coalitions. Finally, a *hedonic game* $(N, \succeq)$ is a pair consisting of the set of players and a preference profile. Given a hedonic game $(N, \succeq)$, a coalition structure π of N is *core stable* if there does not exist a nonempty coalition X such that $X \succ_i \pi(i)$ holds for each $i \in X$.

3 Preferences and the Core

Each each agent $i^c \in N^a \cup N^b$, $c \in \{a, b\}$, is endowed with quality level q_i^c.[4] Without loss of generality, we index the agents in such a way that $q_1^a > q_2^a > \ldots > q_m^a > 0$ and $q_1^b > q_2^b > \ldots > q_n^b > 0$; thus, 1^c is the member of N^c with the highest quality, 2^c is the member of N^c with the second highest quality, and so on.

We assume that players' choice of group membership is driven by status-

[4] One might think of the quality index as a reflection of the individual's talent or material endowment.

seeking. We distinguish between two types of status: local status which is defined by a player's relative position among the members of the group of his own type; and global status as defined by the average quality of the group members of the opposite type. For all coalitions $S \subseteq N$ and $c \in \{a, b\}$, we let $q^c(S) := \dfrac{\sum_{i^c \in S \cap N^c} q_i^c}{|S \cap N^c|}$ be the type-specific average quality of group S. We follow the convention $q^c(S) = 0$ for $S \cap N^c = \emptyset$.

Consider an agent $i^c \in N^c$, $c \in \{a, b\}$, and a group $S \in \mathcal{N}_{i^c}$. As a member of group S agent i^c derives utility according to the following constant elasticity of substitution (CES) utility function

$$u_{i^c}(S) = \left(\alpha \cdot (q_i^c - q^c(S))^{\frac{\sigma-1}{\sigma}} + \beta \cdot q^{c'}(S)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $c' \in \{a, b\}$ with $c' \neq c$. The first component of the utility function, $q_i^c - q^c(S)$, reflects a player's local status and the second component, $q^{c'}(S)$, summarizes her global status. Notice that while global status is always a positive number, local status may be negative. This will be the case for all players in a group whose quality is below that of the average quality of the players of the same type who are members of this group. The two positive parameters α and β capture the relative weight attributed to local and global status, respectively. Given this CES utility function, we further need to assume that α is an odd positive integer, otherwise for some σ values[5], there may be a coalition in which a player, whose quality is below the average quality of the players of the same type, attains a higher local status than a player of the same type with quality above this average. The

[5] In particular, we will discuss the case $\sigma \to 1$.

elasticity of substitution between the two types of status is constant and is given by σ.

Finally, we define the properties of segregation and integration on which our analysis of the core stable outcomes will focus. Following Watts (2007, Def. 3), a coalition structure π is segregated if (i) given any three agents $i^c, j^c, k^c \in N^c$ with $c = \{a, b\}$ such that $j^c \in \pi(i^c)$ and $q_k^c \in \left(q_i^c, q_j^c\right)$, we have $k^c \in \pi(i^c)$; and (ii) given any four agents $i^c, j^c, k^c, \ell^c \in N^c$ with $c = \{a, b\}$ where $q_i^c, q_j^c \geq q'$ and $q_k^c, q_\ell^c \leq q''$ with $q'' < q'$, it cannot be that $k^c \in \pi(i^c)$, $\ell^c \in \pi(j^c)$ and $j^c \notin \pi(i^c)$. A coalition structure π is fully integrated if for any two agents $i^c, j^c \in N^c$ with $c \in \{a, b\}$, we have that $q(\pi(i^c) \cap N^a) = q(\pi(j^c) \cap N^a)$ and $q(\pi(i^c) \cap N^b) = q(\pi(j^c) \cap N^b)$.

Next, we characterize the core as a function of the parameter values. We first consider the two extreme cases: where only local status matters; and where only global status matters.

3.1 Local Status

If agents look only at the groups of their own type and are guided by the distance between their own quality and the average quality of the group, their preferences over compositions of a- and b-groups may be represented by (1) with β set equal to 0. That is for all $i^c \in N^c$, $c \in \{a, b\}$, and any group $S \in \mathcal{N}_{i^c}$, agent i^c derives utility

$$u_{i^c}(S) = \alpha \cdot (q_i^c - q^c(S)).$$

Our first result is straightforward.[6] Consider the set of coalition structures in which there is at most one player of each type in a coalition structure element, i.e.,

$$\Pi^* = \{\pi : |\pi\,(i^c) \cap N^c| \leq 1 \text{ for each } c \in \{a, b\} \text{ and } i^c \in N^c\}. \tag{2}$$

It is easy to show that Π^* fully describes the core is this case. In other words, in a core stable coalition structure there are no coalitions containing at least two distinct players of the same type - if this were the case, then among those players of the same type, the one with the lowest quality would prefer to stay alone, and hence, can block the corresponding coalition structure.[7] Clearly, the set of core stable outcomes when only local status matters are all segregated in nature.

3.2 Global Status

Consider next the other extreme case in which there are no own-type peer effects and each player seeks a group membership where the players of the opposite type have higher average quality.[8] Players' preferences are thus represented by (1) with $\alpha = 0$ that takes the form

$$u_{i^c}(S) = \beta \cdot q^{c'}(S). \tag{3}$$

[6]Notice that for this result we do not need the restriction that α is an odd integer.

[7]It is straightforward to see that the core of a corresponding hedonic game which has either a- or b-type agents contains only the partition into singletons.

[8]Note that this type of problem has not been previously studied in the matching literature. Unlike in the many-to-one matching models, here coalition formation happens on *both sides* of the market. Furthermore, it differs from the standard many-to-many model because the outcome is a partition of the player set.

The core in this case is again non-empty as for instance the following three coalition structures are core stable.

$$\pi' : \{\{1^a\} \cup N^b, N^a \setminus \{1^a\}\},$$
$$\pi'' : \left\{N^a \cup \{1^b\}, N^b \setminus \{1^b\}\right\},$$
$$\pi''' : \left\{\{1^a\} \cup \{1^b\}, \{2^a\} \cup \{2^b\}, \ldots, \{m^a\} \cup \{m^b\}, N^b \setminus \{1^b, \ldots, m^b\}\right\}.$$

Clearly, coalition structure π' is the one most preferred by the b-type agents as they are in the same coalition with the a-group with the highest average quality. Similarly, π'' is the most preferred core stable coalition structure by the a-type agents. One can think of π''', instead, as a "fair" coalition structure as the best set of a-agents is grouped together with the best set of b-agents.[9] While π''' is a segregated outcome which is in the core of any hedonic game with this type of preferences, outcomes π' and π'' have a hybrid nature: they are segregated with respect to one type of players and integrated with respect to the other.

Keeping these three examples in mind, let us now fully describe the set of core stable coalition structures for this extreme case. We precede the main result by providing an algorithm which delivers a partition π of the set of agents $N^a \cup N^b$ into compositions of a- and b-groups.

Algorithm 1

- Set $N^1 := N^a$, $N^2 := N^b$, and $\pi := \emptyset$.

- Repeat the following until $N^1 \cup N^2 = \emptyset$:

[9] In the literature on social status based on constraint interdependence, the coalition structure π''' is called 'positively assortative' (cf. Truyts 2010, p. 144).

- Find a group $A \cup B$ with $A \subseteq N^1$ and $B \subseteq N^2$ s.t.

either

$A = \left\{ i^a \in N^1 : q_i^a \geq q_j^a \text{ for all } j^a \in N^1 \right\}$ and

$B \in \left\{ B' \subseteq N^2 : q(B') \geq \max \left\{ q_i^b : i^b \in N^2 \setminus B' \right\} \right\}$,

or

$A \in \left\{ A' \subseteq N^1 : q(A') \geq \max \left\{ q_i^a : i^a \in N^1 \setminus A' \right\} \right\}$ and

$B = \left\{ i^b \in N^2 : q_i^b \geq q_j^b \text{ for all } j^b \in N^2 \right\}$.

- Set $N^1 := N^1 \setminus A$, $N^2 := N^2 \setminus B$ and $\pi := \pi \cup \{A \cup B\}$.

- Return π.

We denote by $\widetilde{\Pi}$ the set of all partitions delivered by the above algorithm.

Proposition 1 *Let $(N, \succeq)$ be a hedonic game with status-based preferences represented by the CES utility function given in (1) with $\alpha = 0$. Then a coalition structure π is core stable if and only if $\pi \in \widetilde{\Pi}$.*

Proof. Let $\pi = \{A_1 \cup B_1, A_2 \cup B_2, \ldots, A_P \cup B_P\} \in \widetilde{\Pi}$. We show that π is core stable.

Notice first that by construction the average quality of the groups A_p and B_p, $p = 1, \ldots, P$, is non-negative. Suppose now that $X \subseteq N$ is blocking π. Then it has to be the case that $X \cap N^a \neq \emptyset$ and $X \cap N^b \neq \emptyset$. Let $\underline{p} = \min \{p : (A_p \cup B_p) \cap X \neq \emptyset\}$.

Case 1 ($A_{\underline{p}} \cap X \neq \emptyset$ and $B_{\underline{p}} \cap X = \emptyset$): Take $i^a \in A_{\underline{p}} \cap X$ and let $\bar{i}^b \in X \cap N^b$ be the agent with the highest quality level in $X \cap N^b$. Since X is blocking π we have

$$q_{\bar{i}}^b \geq q(X \cap N^b) > q(\pi(i^a) \cap N^b) = q(B_{\underline{p}}). \tag{4}$$

Note in addition that $X \cap N^b \subseteq N^b \setminus \left(\cup_{p=1}^{\underline{p}} B_p \right)$ and that, by construction, we have either

$$q(B_{\underline{p}}) = q_{\tilde{i}}^b \tag{5}$$

with $\tilde{i}^b$ being the b-agent with the highest quality level in $N^b \setminus \left(\cup_{p=1}^{\underline{p}-1} B_p \right)$, or

$$q(B_{\underline{p}}) \geq \max \left\{ q_i^b : i^b \in N^b \setminus \left(\cup_{p=1}^{\underline{p}} B_p \right) \right\}. \tag{6}$$

By $\bar{i}^b \in X \cap N^b \subseteq N^b \setminus \left(\cup_{p=1}^{\underline{p}} B_p \right) \subseteq N^b \setminus \left(\cup_{p=1}^{\underline{p}-1} B_p \right)$ and combining (4) with either (5) or (6), we have a contradiction.

Case 2 ($A_{\underline{p}} \cap X = \emptyset$ and $B_{\underline{p}} \cap X \neq \emptyset$)): The proof is analogous to the one in Case 1.

Case 3 ($A_{\underline{p}} \cap X \neq \emptyset$ and $B_{\underline{p}} \cap X \neq \emptyset$): The proof is again analogous to the one in Case 1 with the additional remark that $X \cap N^b \subseteq N^b \setminus \left(\cup_{p=1}^{\underline{p}-1} B_p \right)$.

We conclude that π is core stable.

Suppose now that $\overline{\pi} = \left\{ \overline{C}_1, \overline{C}_2, \ldots, \overline{C}_R \right\}$ is a core stable coalition structure but $\overline{\pi} \notin \widetilde{\Pi}$. Let $\overline{A}_r := \overline{C}_r \cap N^a$ and $\overline{B}_r := \overline{C}_r \cap N^a$ for all $\overline{C}_r \in \overline{\pi}$. W.l.o.g., let the coalition structure elements of $\overline{\pi}$ be ordered in such a way that $\overline{\pi} = \left\{ \overline{A}_1 \cup \overline{B}_1, \overline{A}_2 \cup \overline{B}_2, \ldots, \overline{A}_R \cup \overline{B}_R \right\}$ with $q(\overline{B}_r) \geq q(\overline{B}_{r+1})$ for $r = 1, \ldots, R-1$ with the average quality of the empty set being equal to zero.

Notice first that if there is a coalition structure element $\overline{C}_r \in \overline{\pi}$ s.t. $|\overline{A}_r| \geq 2$ and $|\overline{B}_r| \geq 2$, then $\overline{\pi}$ will be not core stable as the higher quality a- and b-agents in $\overline{C}_r$ would block it by forming a coalition. Thus, for all $\overline{C}_r \in \overline{\pi}$ either $\left|\overline{A}_r\right| \in \{0, 1\}$ and $\left|\overline{B}_r\right| \geq 1$, or $\left|\overline{A}_r\right| \geq 1$ and $\left|\overline{B}_r\right| \in \{0, 1\}$.

Next, take $\overline{A}_1 \cup \overline{B}_1$ and consider the following possible cases.

Case 1 ($\overline{A}_1 = \emptyset$): The coalition $\left\{1^a, 1^b\right\}$ is blocking $\overline{\pi}$. Since $q(\overline{B}_1) \geq q(\overline{B}_r)$ holds for all $r = 2, \ldots, R$, it implies $1^b \in \overline{B}_1$. In addition, $\overline{A}_1 = \emptyset$ implies that $u_{1^a}(\overline{\pi}(1^a)) < q_1^b$ and $u_{1^b}(\overline{\pi}(1^b)) = 0 < q_1^a$. Thus, we have a contradiction to the core stability of $\overline{\pi}$.

Case 2 ($\left|\overline{A}_1\right| = 1$): If $\overline{A}_1 \neq \left\{i^a \in N^a : q_i^a \geq q_j^a \text{ for all } j^a \in N^a\right\} = \{1^a\}$ and $\overline{A}_1 \notin \{A' \subseteq N^a : q(A') \geq \max\{q_i^a : i^a \in N^a \setminus A'\}\} \ni \{1^a\}$, then, by the same reasoning as in Case 1, coalition $\left\{1^a, 1^b\right\}$ can block $\overline{\pi}$. Hence, we conclude that $\overline{A}_1$ has to have the structure as indicated in the above algorithm.

Furthermore, if $\overline{A}_1 = \left\{i^a \in N^a : q_i^a \geq q_j^a \text{ for all } j^a \in N^a\right\} = \{1^a\}$ and $\overline{B}_1 \notin \left\{B' \subseteq N^b : q(B') \geq \max\left\{q_i^b : i^b \in N^b \setminus B'\right\}\right\} \ni \left\{1^b\right\}$, then coalition $\left\{1^a, 1^b\right\}$ is blocking $\overline{\pi}$ since $q_1^b > q(\overline{B}_1)$ and $q_1^a > q(\overline{A}_r)$ hold for all $r = 2, \ldots, R$ (note that $\overline{\pi}\left(1^b\right) \cap N^a = \overline{A}_r$ for some $r \in \{2, \ldots, R\}$). Thus, we have again a contradiction to the core stability of $\overline{\pi}$.

The case in which $\left|\overline{B}_1\right| = 1$ can be treated similarly. In an analogous way one can show that all elements of $\overline{\pi}$ have the structure provided by the above algorithm. We conclude that the core stability of $\overline{\pi}$ implies $\overline{\pi} \in \widetilde{\Pi}$. ■

As a corollary of Proposition 1, one can note that a fully integrated coalition structure is never in the core of a hedonic game when preference are based on global status. The reason for this is that there is at most a single representative of at least one of the players types in every coalition structure element derived by Algorithm 1.

3.3 Local and Global Status

Here we discuss those cases in which both local and global status determine players' choice of group membership.

The first case we discuss is when local and global status are (imperfect) complements. In this case we obtain a generic uniqueness of the core as in all core stable coalition structures, players obtain zero utility.

Proposition 2 *Let* $(N, \succeq)$ *be a hedonic game with status-based preferences represented by the CES utility function given in (1). If* $\sigma \to 0$ *or* $\sigma \to 1$*, then a coalition structure* π *is core stable if and only if* $\pi \in \Pi^*$ *as defined in (2).*[10]

The proof of Proposition 2 is straightforward. It is easy to show that (1) takes the form

$$u_{i^c}(S) = \min\{\alpha \cdot (q_i^c - q^c(S)), \beta \cdot q^{c'}(S)\} \tag{7}$$

when $\sigma \to 0$; and the form

$$u_{i^c}(S) = (q_i^c - q^c(S))^{\alpha} \cdot (q^{c'}(S))^{\beta} \tag{8}$$

when $\sigma \to 1$.

Equations (7) and (8) imply that no two players of the same type will be members of the same coalition in a core stable coalition structure. This is because the player with the lower quality will obtain a negative utility and therefore will block this coalition structure by staying alone (recall that in

[10] The restriction that α is an odd integer is important when $\sigma \to 1$ but not when $\sigma \to 0$.

(8) α is an odd integer). Therefore, in all core stable coalition structures each player obtains local status of 0. Finally, notice that irrespective of whether a player is in a group with any other player of the opposite type or stays alone, her utility is 0 since qualities are strictly positive.

Next, we study the core stable coalition structures when players perceive the two types of status as being substitutable. Our first set of results discusses perfect substitutability between the two types of status. For this we will need the following additional notation. For any $A \subseteq N^a$ and $B \subseteq N^b$ let $\lambda_{AB} := \alpha \cdot q^a(A) - \beta \cdot q^b(B)$ (if either A or B is empty, we set the corresponding average quality level to be equal to zero). Given a coalition structure π, we write λ_{AB}^{π} for the weighted difference in the average qualities of the groups $A \subseteq N^a$ and $B \subseteq N^b$ with $A \cup B \in \pi$. Moreover, for any coalition structure π, we let $I_0^{\pi} := \left\{ i \in N^a \cup N^b : |\pi(i)| = 1 \right\}$ be the set of players that are single under π.

Theorem 1 *Let $(N, \succeq)$ be a hedonic game with status-based preferences represented by the CES utility function given in (1). If $\alpha = \beta$ and $\sigma \to \infty$, then an individually rational coalition structure π is core stable if and only if the following two conditions are satisfied:*

(1) $I_0^{\pi} \cap N^a = \emptyset$ or $I_0^{\pi} \cap N^b = \emptyset$.

(2) For any two non-empty a- and b-groups A' and B' with $\pi(i^a) \cap N^b \nsubseteq B'$ for all $i^a \in A'$ the following two implications hold:

(2.1) $\lambda_{A'B'} > \max_{B' \cap B \neq \emptyset} \lambda_{AB}^{\pi} \Rightarrow \lambda_{A'B'} \geq \min_{A' \cap A \neq \emptyset} \lambda_{AB}^{\pi}$.

(2.2) $\lambda_{A'B'} < \min_{A' \cap A \neq \emptyset} \lambda_{AB}^{\pi} \Rightarrow \lambda_{A'B'} \leq \max_{B' \cap B \neq \emptyset} \lambda_{AB}^{\pi}$.

Proof. As $\alpha = \beta$, w.l.og., we can let $\alpha = \beta = 1$. In addition, $\sigma \to \infty$

implies that (1) takes the form

$$u_{i^c}(S) = (q_i^c - q^c(S)) + q^{c\prime}(S). \tag{9}$$

Let π be a coalition structure satisfying items (1) and (2) of Theorem 1. We show that it is core stable. Suppose not, i.e., there is $X \subseteq N$ with $X = A \cup B$ that blocks π. That is, we have

$$q_i^a - \lambda_{AB} > q_i^a - \lambda_{(\pi(i^a)\cap N^a)(\pi(i^a)\cap N^b)}$$

for all $i^a \in A$, and

$$q_i^b + \lambda_{AB} > q_i^b + \lambda_{(\pi(i^b)\cap N^a)(\pi(i^b)\cap N^b)}$$

for all $i^b \in B$.

Suppose first that $A = \emptyset$. Notice then that the lowest quality agent in B can attain at most zero utility in the blocking coalition. As π is individually rational, a coalition consisting of b-type agents only cannot be blocking π. For a similar reason, a coalition which consist of only a-type agents cannot be blocking π either.

Next, suppose that the blocking coalition consists of both a- and b-type agents, and that there are $i^a \in A$ and $i^b \in B$ such that $i^b \in \pi(i^a)$. Simple algebra shows that the above two inequalities cannot hold simultaneously for these two agents.

Last, suppose that the blocking coalition consists of both a- and b-type agents such that there are no two agents of two distinct types who are grouped together under π. Such blocking possibilities are ruled out by item (2) in the statement of the theorem. To see this, notice that agent i^a gets under π exactly $q_i^a - \lambda_{(\pi(i^a)\cap N^a)(\pi(i^a)\cap N^b)}$. Similarly, any agent i^b gets $q_i^b + \lambda_{(\pi(i^b)\cap N^a)(\pi(i^b)\cap N^b)}$ under π. Hence, for the incentives of agents i^a and i^b to be part of the blocking coalition $X = A \cup B$, it must be that $\lambda_{(\pi(i^b)\cap N^a)(\pi(i^b)\cap N^b)} < \lambda_{AB} < \lambda_{(\pi(i^a)\cap N^a)(\pi(i^a)\cap N^b)}$. Therefore, item (2) guarantees that there is an a-agent (condition (2.1)) or a b-agent (condition (2.2)) for which such λ_{AB} cannot be found.

As to show that items (1) and (2) are also necessary for a coalitional matching to be core stable, let π be core stable and do not satisfy (1). This implies the existence of $i^a \in N^a$ and $i^b \in N^b$ with $\pi(i^a) = \{i^a\}$ and $\pi(i^b) = \{i^b\}$. Notice however that the pair $\{i^a, i^b\}$ is blocking π in contradiction to its core stability.

Suppose finally that π is core stable and does not satisfy (2). Consider first the case in which there are a- and b-groups A' and B' with $\pi(i^a) \cap N^b \nsubseteq B'$ for all $i^a \in A'$ such that $\lambda_{A'B'} > \max_{B'\cap B\neq\emptyset} \lambda_{AB}^{\pi}$ and $\lambda_{A'B'} < \min_{A'\cap A\neq\emptyset} \lambda_{AB}^{\pi}$ hold (i.e., (2.1) is violated). Consider then the coalition $A' \cup B'$. To see that this coalition blocks π, notice that all $i^b \in B'$ get in $A' \cup B'$ exactly $q_i^b + \lambda_{A'B'} > q_i^b + \lambda_{(\pi(i^b)\cap N^a)(\pi(i^b)\cap N^b)}$ (as $\lambda_{A'B'} > \max_{B'\cap B\neq\emptyset} \lambda_{AB}^{\pi}$ holds). Furthermore, all $i^a \in A'$ get $q_i^a - \lambda_{A'B'} > q_i^a - \lambda_{(\pi(i^b)\cap N^a)(\pi(i^b)\cap N^b)}$ because of $\lambda_{A'B'} < \min_{A'\cap A\neq\emptyset} \lambda_{AB}^{\pi}$. Similarly, one can show how A' and B' can be used to form a blocking coalition if condition (2.2) is violated. ■

The significance of Condition (2) in Theorem 1 is illustrated in the example below.

Example 1 *Let* $N^a = \{1^a, 2^a, 3^a\}$ *and* $N^b = \{1^b, 2^b\}$ *with* $q_1^a = 4$*,* $q_1^b = 3$*,* $q_2^a = q_2^b = 2$*, and* $q_3^a = 1$*. Let agents' preferences be represented by the CES utility function given in (1) with* $\sigma \to \infty$ *and* $\alpha = \beta = 1$*.*

Consider the coalition structure π *with* $\pi(1^a) = \pi(1^b) = \{1^a, 1^b\}$*,* $\pi(2^a) = \pi(2^b) = \{2^a, 2^b\}$*, and* $\pi(3^a) = \{3^a\}$*. This coalition structure is not stable as it is blocked by the coalition* $\{1^a, 3^a, 2^b\}$*. Clearly,* $u_{1^a}(\{1^a, 3^a, 2^b\}) = q_1^a - \dfrac{q_1^a + q_3^a}{2} + q_2^b = 3.5 > 3 = q_1^a - q_1^a + q_1^b = u_{1^a}(\{1^a, 1^b\})$*. Similarly, one can show that both agents* 3^a *and* 2^b *strictly prefer* $\{1^a, 3^a, 2^b\}$ *over their corresponding coalitions under* π*.*

Special classes of core stable partitions can be derived as corollaries to Theorem 1.

Corollary 1 *Let* $(N, \succeq)$ *be a hedonic game with status-based preferences represented by the CES utility function given in (1). Let* $\alpha = \beta$ *and* $\sigma \to \infty$*. Furthermore, let* $\lambda \in \left[-q_n^b, q_m^a\right]$ *and* π *be a partition of* $N^a \cup N^b$ *s.t.* $\lambda_{AB} = \lambda$ *for all* $A \subseteq N^a$ *and* $B \subseteq N^b$ *with* $A \cup B \in \pi$*. Then* π *is core stable.*

The proof is easy to see. The condition $-q_n^b \leq \lambda \leq q_m^a$ ensures that π is individually rational, while the fact that the corresponding a- and b-groups have equal average quality ($= \lambda$) guarantees that conditions (1) and (2) of Theorem 1 hold.

Furthermore, Corollary 1 describes conditions under which a segregating coalition structure is in the core. It states that such segregated coalition structures are in the core if the difference between the average quality of the a- and b-groups in each coalition is the same for all elements in the partition. This result implies that it is not only that higher ranked agents of each type are grouped together under this condition, but also that a certain fairness requirement is satisfied: the average quality of each a-group belonging to a coalition in the partition exceeds/falls under the average quality of the b-group in this coalition by the same amount.

To illustrate the significance of Corollary 1 for the stability of segregating outcomes, we refer again to Example 1 above. In this example, we study a segregated outcome in which the highest ranked individuals from each type are grouped together, the second highest individuals of each type are also grouped together, and the lowest ranked a-agent remains single. As the analysis shows this segregated matching is not in the core, and indeed Corollary 1's condition, the differences between the average quality of a- and b-groups belonging to the same coalition must be equal, is not satisfied for this partition: $\lambda_{\{1^a\}\{1^b\}} = 1$, and $\lambda_{\{2^a\}\{2^b\}} = 0$. The following example shows a coalition formation problem in which the core contains a segregated outcome.

Example 2 *Let $N^a = \{1^a, 2^a, 3^a\}$ and $N^b = \{1^b, 2^b\}$ with $q_1^a = 4$, $q_1^b = 3$, $q_2^a = 2$, and $q_2^b = q_3^a = 1$. Let agents' preferences be represented by the CES utility function given in (1) with $\sigma \to \infty$ and $\alpha = \beta = 1$.*

Consider the coalition structure π with $\pi(1^a) = \pi(1^b) = \{1^a, 1^b\}$, $\pi(2^a) = \pi(2^b) = \{2^a, 2^b\}$, and $\pi(3^a) = \{3^a\}$. It is easy to see that π is core stable as there exists no blocking coalition. Notice that $\lambda_{\{1^a\}\{1^b\}} = \lambda_{\{2^a\}\{2^b\}} = \lambda_{\{3^a\}\emptyset} = 1$.

The next corollary describes conditions under which a fully integrated coalition structure is stable.

Corollary 2 *Let $(N, \succeq)$ be a hedonic game with status-based preferences represented by the CES utility function given in (1). Let $\alpha = \beta$ and $\sigma \to \infty$. Furthermore, let $q^a_m - q(N^a) + q(N^b) \geq 0$, $q^b_n - q(N^b) + q(N^a) \geq 0$, and $K \leq m$. Let $\pi = \{A_1 \cup B_1, \ldots, A_K \cup B_K\}$ be a partition of $N^a \cup N^b$ s.t. $q(A^k) = q(A^{k+1})$ and $q(B^k) = q(B^{k+1})$ for all $k = 1, \ldots, K-1$. Then π is core stable.*

Notice here that the condition that all a- and b-groups in the partition π have the same average quality implies that this average quality equals the (positive) average quality of N^a and N^b, respectively. Therefore, the conditions $q^a_m - q(N^a) + q(N^b) \geq 0$ and $q^b_n - q(N^b) + q(N^a) \geq 0$ imply that this type of partition is individually rational. Furthermore, $q(A_k) = q(A_{k+1})$ and $q(B_k) = q(B_{k+1})$ for all $k = 1, \ldots, K-1$ guarantees that condition (2) of Theorem 1 is satisfied as well. In other words, condition (2) of Theorem 1 is satisfied for all fully integrated coalition structures, and, therefore for such a partition to be in the core, only the individually rationality condition may be a constraining factor.

As an example of a coalition formation problem for which a fully integrated outcome is in the core, consider again Example 1. The coalition structure $\{\{N^a \cup N^b\}\}$ is fully integrated and it is in the core.

Our next result shows that under perfect substitutability of the a- and b-groups when $\alpha = \beta$, there always exists a core stable coalition structure.

Theorem 2 *Let $(N, \succeq)$ be a hedonic game with status-based preferences represented by the CES utility function given in (1) with $\alpha = \beta$ and $\sigma \to \infty$. Then a core stable coalition structure exists.*

Proof. Consider the following algorithm for delivering a coalition structure.

Algorithm 2

We initialize the algorithm by setting $A_0 = N^a$, $B_0 = N^b$, $\overline{A}_0 = \emptyset$, and $\overline{B}_0 = \emptyset$. In the k^{th} step of the algorithm, we set $A_k = A_{k-1} \setminus \overline{A}_{k-1}$, $B_k = B_{k-1} \setminus \overline{B}_{k-1}$, $\overline{A}_k = \overline{A}_{k-1} \cup \{i^a \in A_k : q_i^a - q(A_k) + q(B_k) < 0\}$, and $\overline{B}_k = \overline{B}_{k-1} \cup \{i^b \in B_k : q_i^b - q(B_k) + q(A_k) < 0\}$. The algorithm stops when $\overline{A}_\ell = \overline{A}_{\ell-1}$ and $\overline{B}_\ell = \overline{B}_{\ell-1}$ and we set $K = \ell$. Define the coalition structure π by $\pi(i^c) = A_K \cup B_K$ for all $i^c \in A_K \cup B_K$, $\pi(i^a) = \{i^a\}$ for all $i^a \in \overline{A}_K = N^a \setminus A_K$, and $\pi(i^b) = \{i^b\}$ for all $i^b \in \overline{B}_K = N^b \setminus B_K$.

We show that π is core stable. First, we will show that K is finite, and, in particular that it is an integer at most equal to $n+1$. Notice that either $\overline{A}_1 = \emptyset$ or $\overline{B}_1 = \emptyset$; otherwise there is an agent with negative quality, which is not possible. For ease of exposition, suppose that $\overline{A}_1 = \emptyset$. Since $q_i^b - q(B_1) + q(A_1) < 0$ for some $i^b \in N^b$, it is clear that $q_i^b < q(B_1)$ and, therefore, $q(B_2) \geq q(B_1)$. This is why for all a-agents $q_i^a - q(A_2) + q(B_2) \geq 0$. Similarly, one can show that $A_K = N^a$ and $\overline{A}_K = \emptyset$. The above analysis and the fact that N^b is finite proves that K is finite. Moreover, as $q(N^a) > 0$ and $q(N^b) > 0$, implies that $A_K \neq \emptyset$ and $B_K \neq \emptyset$, and, therefore $K \leq n+1$.

Next, we will show that there is no coalition X that blocks the constructed partition π. Suppose, on the contrary, that such a coalition exists. First, suppose that X consists of homogeneous type agents, i.e., $X \subseteq N^a$ or $X \subseteq N^b$. Notice that by construction all agents in A_K and B_K have at least zero utility under π. Furthermore, all agents in $\overline{A}_K$ and $\overline{B}_K$ have also zero utility under π. Since the agents with the lowest quality in X can obtain at most zero utility in X, the coalition X cannot be blocking π.

Suppose next that there are at least two agents $i^a, i^b \in X$ who belong to the same coalition in π. For X to be blocking π it must be that

$$q_i^a - q(X \cap N^a) + q(X \cap N^b) > q_i^a - q(A_K) + q(B_K)$$

and

$$q_i^b - q(X \cap N^b) + q(X \cap N^a) > q_i^b - q(B_K) + q(A_K).$$

Simple algebra shows that the above two inequalities cannot hold simultaneously.

Last suppose that there are at least two agents $i^a, i^b \in X$ who belong to different coalitions in π. W.l.o.g., suppose that $i^a \in \overline{A}_K$ and $i^b \in B_K$. It is easy to see that the agent with the highest quality level in $\overline{A}_K$, is one who is in $\overline{A}_K$ (and therefore in $\overline{A}_{K-1}$) but not in $\overline{A}_{K-2}$. Denote this agent by $\bar{i}^a$. Then, by construction, we have

$$q_{i^a} \leq q_{\bar{i}}^a < q(A_{K-2}) - q(B_{K-2}) < q(A_K) - q(B_K). \tag{10}$$

Furthermore, notice that by definition of $\bar{i}^a$, $q(\widetilde{A}) \leq q_{\bar{i}}^a$ for all $\widetilde{A} \subseteq \overline{A}_K$. Therefore, for X to be blocking π it must be that for the b-agent in X with the lowest quality, denoted by $\underline{i}^b$, it must hold that

$$q_{\underline{i}}^b - q(B_K) + q(A_K) < q_{\underline{i}}^b - q(X \cap N^b) + q(X \cap N^a) \leq q_{\underline{i}}^b + q_{\bar{i}}^a, \tag{11}$$

where the last inequality follows from $X \cap N^a \subseteq \overline{A}_K$ (note that $X \cap A_K \neq \emptyset$

would mean that there are a b-agent ($\underline{i}^b$) and an a-agent who belong to the same coalition in π implying, as shown above, that X is not blocking π). Clearly, expressions (10) and (11) lead to a contradiction. ■

Last, we address the question under what distribution of qualities and values of the parameters of the CES utility function, we can obtain the segregated outcome which has been found in the literature as the unique core stable coalition structure.[11] For this result we need an additional notation and a supplementary result. Let us denote the minimal difference in qualities of any two consecutive players of each type as q^a_{min} and q^b_{min} where formally $q^a_{min} := \min_{k\in\{1,\ldots,m-1\}} \left\{q^a_k - q^a_{k+1}\right\}$ and $q^b_{min} := \min_{k\in\{1,\ldots,n-1\}} \left\{q^b_k - q^b_{k+1}\right\}$. First, we present a technical result.

Lemma 1 *Let $X \subseteq N^c$, $c \in \{a, b\}$, be such that $|X| \geq 2$ and let $\underline{i}$ be the lowest quality member of X. Then*

$$q_{\underline{i}} - q(X) \leq -\frac{q^c_{min}}{2}. \tag{12}$$

Proof. Let X and $\underline{i}$ be as above. Then,

$$\begin{aligned} q_{\underline{i}} - q(X) &= q_{\underline{i}} - \frac{q_{\underline{i}} + \sum_{j\in X\setminus\{\underline{i}\}} q_j}{|X|} \\ &= -\frac{\sum_{j\in X\setminus\{\underline{i}\}} q_j - (|X|-1)q_{\underline{i}}}{|X|} \\ &\leq -\frac{|X|-1}{|X|} q^c_{min} \qquad (13) \\ &\leq -\frac{1}{2} q^c_{min}, \qquad (14) \end{aligned}$$

[11] Milchtaich and Winter (2002) and Watts (2007) find these types of outcome to be the only stable outcomes in their framework. In a related literature, Eeckhout (2000), Shimer and Smith (2000), and Atakan (2006) study positively assortative outcomes.

where inequality (13) follows from the definition of q^c_{min} and inequality (14) follows from $|X| \geq 2$. ■

Now we are ready to present our final result.

Proposition 3 *Let* $(N, \succeq)$ *be a hedonic game with status-based preferences represented by the CES utility function given in (1). If* $\sigma \to \infty$, $-\alpha \cdot \frac{q^a_{min}}{2} + \beta \cdot q^b_1 < 0$ *and* $-\alpha \cdot \frac{q^b_{min}}{2} + \beta \cdot q^a_1 < 0$, *then* $\pi = \{\{1^a, 1^b\}, \{2^a, 2^b\}, \ldots, \{m^a, m^b\}, \{(m+1)^b\}, \ldots, \{n^b\}\}$ *is the unique core stable coalition structure.*

Proof. Notice that $\sigma \to \infty$ implies that (1) takes the form

$$u_{i^c}(S) = \alpha \cdot (q^c_i - q^c(S)) + \beta \cdot q^{c'}(S). \tag{15}$$

First we consider coalition structure π as defined above and show that it is core stable. As there is at most one representative of each type in a coalition structure element, each player derives 0 utility from local status. As individual qualities are strictly positive, it is clear from (15), that all players derive a non-negative utility in the coalition structure, and, therefore it is individually rational. Next, suppose that there is a blocking coalition X such that $|X \cap N^c| \geq 2$ for some $c \in \{a, b\}$. Let $\underline{i}^c \in X$ be the player with lowest quality in $X \cap N^c$. For some $c' \in \{a, b\}$ with $c' \neq c$, the utility player

$\underline{i}^c$ can derive in X is given by

$$
\begin{aligned}
u_{\underline{i}^c}(X) &= \alpha \cdot (q^c_{\underline{i}} - q^c(X)) + \beta \cdot q^{c'}(X) \\
&\leq -\alpha \cdot \frac{q^c_{min}}{2} + \beta \cdot q^{c'}(X) \qquad (16)\\
&\leq -\alpha \cdot \frac{q^c_{min}}{2} + \beta \cdot q^{c'}_1 \qquad (17)\\
&< 0 \qquad (18)
\end{aligned}
$$

where inequality (16) follows from Lemma 1, inequality (17) follows by definition, and inequality (18) follows by assumption. Therefore, X cannot block π. Last, suppose that there is a blocking coalition X s.t. $|X \cap N^c| = 1$ for each $c \in \{a, b\}$. W.l.o.g, suppose $X \cap N^b = \{i^b_k\}$ for some $k \in \{1, \dots, n\}$. This player's utility in π must equal $\beta \cdot q^a_{i_k}$ if $k \leq m$ and 0 otherwise. Hence, for player i^b_k to attain higher utility in X, $X \cap N^a = \{i^a_\ell : \ell \in 1, \dots, \min\{m, k-1\}\}$. Player i^a_ℓ utility in π, however, is $\beta \cdot q^b_{i_\ell}$ which is higher than $\beta \cdot q^b_{i_k}$ that is the utility she can achieve in X. This establishes a contradiction.

Last, we show that there is no other coalition structure which is core stable. From the analysis above (i.e., inequalities (16-18)), it is clear that the only individually rational coalition structures are those for which there is at most one player of each type in a coalition structure element. Suppose, that there is an individually rational coalition structure π' which is core stable and suppose that $\{1^a, 1^b\} \notin \pi'$. Then π' can be blocked by coalition $X = \{1^a, 1^b\}$ as $u_{1^a}(\pi'(1^a)) = \beta \cdot q^b_j < \beta \cdot q^b_1$ for all $j^b \in N^b \setminus \{1^b\}$, and, similarly, $u_{1^b}\left(\pi'(1^b)\right) = \beta \cdot q^a_i < \beta \cdot q^a_1$ for all $i^a \in N^a \setminus \{1^a\}$. Similarly by iteration, we can show that if π' is core stable, then it must contain the coalitions $\{i^a, i^b\}$ for all $i \in 1, \ldots, m$. Finally notice that the only individually rational partition of the player set $N^b \setminus \{1^b, \ldots, m^b\}$ is that into singletons. This implies that π' and π must coincide. ■

4 Conclusion

We study group formation when agents' preferences are dictated by the identity of the other agents in the group and in particular by the local and global status they may achieve by being members of a group. Our theoretical results show that in all four cases: when agents only care about their local status; when the agents only care about their global status; when local and global status are treated as substitutes; and when the two types of status are treated as complements; there exists a core stable outcome.

Furthermore, we can identify the types of outcomes which are stable in light of segregation and integration. As Truyts (2010, p. 158) points out segregated outcomes have received the most attention in the literature as they are often the more efficient, integrated outcomes, however, may sometimes be more realistic or preferred from the welfare point of view. We define as segregated those outcomes in which the higher quality agents of each type are grouped together and there are at least two groups of agents containing each type. When only local status matters and when local and global status are (imperfect) complements all core-stable outcomes are of the segregated type. When only global status matters there is a segregated outcome in the core of every hedonic game. In contrast, when local and global status are substitutes Corollary 1 shows that such segregated outcomes may be stable if and only if the difference in average quality between the groups of a- and b-agents is the same for all elements in the partition. Whether or not this condition is satisfied hinges crucially on the distribution of qualities of agents of each type. Corollary 2, instead, may be viewed as describing coalition structures characterized by full integration since all groups in the partition have the same average quality of their a-members and the same average quality of their b-members. This coalition structure can also be interpreted in the light of 'social equality' between groups as one which is envy-free. Notice that the coalition structure derived by the algorithm in the proof of Theorem 2 can be one of the type of partitions described in Corollary 2 in case the grand coalition is individually rational for all agents. When this is not the case, this algorithm derives a stable outcome of what we may call a 'hybrid' construct. In this coalition structure all agents of one type are grouped together with a strict subset of the agents of the other type, hence, these agents are in an integrated state. The other type of agents, instead, are in a segregated state because there is a quality threshold such that all agents of this type whose quality is higher are grouped together with all the agents of the opposite type and all those whose quality is lower stay single.

Finally, our results may be seen as providing an alternative mechanism to the one discussed by Frank (1985) for gluing individuals together in social groups when they care for local status. Frank argues that what keeps a low-ranked individual in a group with higher ranked individuals are transaction costs (see Frank, 1985, p. 10). These transaction costs outweigh the gains such an individual might reap from moving to another group where her local status will be higher. In our setting transaction costs are zero. What keeps low-ranked individuals in a group with higher ranked individuals is the access to a group with another type of agents that this membership provides.

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