Simultaneously Signaling and Screening with Seller Financing

Hsien-Hung Chiu*
Department of Economics
National Chi Nan University

March 2011

Abstract

In this paper, we study the relationship between consumer screening, quality assurance and seller financing. A seller sells an investment good to a continuum of buyers. The investment good is bought either for cash or credit. Credit is provided by a competitive loan market or by the seller itself. The quality of the investment good, which is privately known to the seller, differs in the expected returns and thus credit buyers have different default probabilities. We show that seller financing in the form of contractual payment terms can screen and price discriminate between cash buyers and credit buyers. It can also signal the quality of an investment good since a low- type good results in a higher default risk.

We characterize the separating equilibrium in which the low type seller offers a cash price alone, while the high type seller offers a menu of payment terms. We analyze the how the equilibrium depends on the quality difference between high type and low type good, and the relative size of cash and credit market. We discuss how signaling limits the seller's ability to price discriminate. In addition, we also compare our results with those with signaling by a money-back guarantee and show the conditions under which signaling by seller financing has a lower signaling cost.

JEL Classification: D82, L15

Keywords: Seller Financing, Liquidity Constraints, Product Signals, Screening

*Department of Economics, National Chi Nan University, Nantou, Taiwan. E-mail: hhchiu@ncnu.edu.tw

1 Introduction

Seller financing or vendor financing is referred to as a form of financial intermediation performed by the seller. In contrast to the pure financial intermediation provided by financial institutions such as banks, seller financing is an important source of financing for the purchase of many intermediate goods or investment goods. In many instances, seller financing is in a form of payment terms, which often consists of a cash price and various deferred-payment options.

For example, trade credit is an important form of seller financing in which sellers offer contractual payment terms with credit options to buyers. Seller financing also plays an important role in the sale of many investment goods, such as machines or the sale of small business. In those cases, the return of the good or the success of the business is uncertain at the time of purchase. Seller financing can convey some information to buyers about the expected return on the goods or business.

There exists some empirical evidence supporting the role of seller financing as a signal of quality assurance, especially for some goods whose quality can not be easily discovered. For example, Tirtiroglu and Laband (2004) uses data from seller-financed second mortgages and shows the quality assurance role played by seller financing.

A natural question regarding the role of seller financing is why a seller is willing to offer credits to buyers, even though there exists a competitive loan market. Another question is why the rate of seller financing is usually lower than the rate of loans borrowed from financial institutions, even though the cost of funds is higher. We abstract from the traditional explanations that focus on the financial advantages of seller financing or the use of seller financing as a price discrimination device. Instead, we show that seller financing can be used as a signal of the quality of the good.

We consider a model in which a seller sells an investment good to a continuum of buyers. The good lasts for two periods and the return of the good is uncertain at the time of sale and will be revealed only in the second period. The seller has better information about the quality of the good. Buyers differ in two different levels of budgets. Some of the buyers are liquidity-constrained with zero budget, i.e., they don't have sufficient budgets to pay at the time of purchase. Seller financing can be used as a signal because a low quality good is more likely to yield a lower return. Since the return on the investment good itself is the only collateral that buyers can pledge for their loans, the lender may bear the default risk when the realized return on the good is low.

We first consider a basic model, in which the return of the good is a constant for all buyers. In this case, the demand for the good is not elastic to prices. Thus, signaling by price alone is not costly to the low-type seller and consequently no equilibrium exists. On the contrary, if the high-type seller offers a menu of payment terms that consists of a cash price and credit price, it is more costly for the low-type seller to masquerade as the high-type one. The reason is that the low quality good is more likely to be defective and therefore this causes a loss to lenders since the good is the only collateral. We characterize the separating equilibrium in which the low quality seller offers a cash price alone, but the high quality seller offers a menu of payment terms. By adopting this strategy, the high-type seller can make it less attractive for the low-type seller to masquerade as the high quality one. By employing the refinement criterion of "intuitive criterion" by Cho and Kreps (1987), we can characterize the least cost separating equilibrium, i.e., the most efficient equilibrium to the high-type seller that yields the least profit reduction.

The benchmark model can then be extended to a more general one with heterogenous buyers. In this case, the return of the good varies across buyers. As long as the high-type good has a higher production cost, signaling by cash price alone is possible since the consequent loss of sales volume is less damaging to the seller with higher quality and higher cost. However, if the seller can offer financing to liquidity-constrained buyers, signaling by payment terms is more efficient to the high-type seller as long as the cost of funds or the cost difference is not large. The reason is that by doing so the seller can coordinate pricing and financing strategy more effectively and thus make a higher profit.

We also compare our results with other signals of product quality such as warranty and money-back guarantee. When the seller uses a warranty or money-back guarantee as a signal of product quality, it raises the reservation price of all buyers but also bears a higher risk if the good turns out to be defective. On the other hand, the seller can use payment terms as a signal and separates the buyers into two groups: cash buyers and credit buyers. In this situation, the seller charges a lower cash price but bears the default risk form credit buyers only. Under some conditions when the seller has a lower cost of funds or the good has a higher probability of being defective, signaling by seller financing might outperform signaling by a warranty or money-back guarantee.

1.1 Related Literature

There are many theoretical explanations for seller financing, even though the cost of funds is higher for the seller or the loan market is perfectly competitive. This issue was first discussed in the literature on trade credit, which refers to a common form of financing in the intermediate goods market that allows buyers a variety of payment terms with credit options. The traditional theory focuses on the financing advantages of trade credit. For example, the seller might have an advantage of providing financing in either investigating the credit worthiness of buyers or monitoring the repayments of credits[1].

Smith (1987) and Lee and Stowe (1993) both consider trade credit in relation to information asymmetry. In Smith (1987) buyer's default risk is privately known and the seller uses payment options[2] as a screening device to identify the buyers. On the contrary, in Lee and Stowe (1993) the seller uses the size of cash discounts as a signaling device of good quality. Our paper is close to Lee and Stowe (1993) in the sense that seller financing is used as a signaling device. However, our paper differs from theirs in two aspects. First, the seller in Lee and Stowe (1993) is a price taker and their payment options consist of only the size of cash discounts. In our model, we consider a menu of payment terms as a signaling device, which consists of a cash price and a credit price. Second, a major driving force of the outcome in Lee and Stowe (1993) is the risk-sharing motives of buyers. However, we consider risk neutral buyers with liquidity constraints and with access to a competitive loan market.

Brennan, Miksimovic, and Zechner (1988) first investigated the role of seller financing in the presence of a perfectly competitive loan market. They ask why the financing offered by sellers is

[1] Petersen and Rajan (1997) further discusses three different sources of cost advantage: advantage in information acquisition, advantage in controlling the buyer, and advantage in salvaging values from existing assets.

[2] The payment options consist of a credit price and a cash price with some discounts.

usually below the market interest rates, even though sellers have a higher cost of funds. They study seller financing for an investment good and buyers have different default probabilities when they borrow to buy such a good. They show that adverse selection in the credit market leaves room for seller financing to price discriminate between cash and credit buyers. Sen (1998) considers a different model in which a seller sells a durable good and buyers differ in income profiles. He demonstrates that a monopolist seller can find it optimal to offer a menu of deferred payment plans to screen buyers with different income profiles. In contrast to Brennan, Miksimovic, and Zechner (1988), Sen (1998) establishes the role of seller financing without resort to any default-related reasons[3]. In those two papers, a seller uses payment terms as a device to screen buyers, while we consider that the seller uses payment terms to signal the unobservable quality of the good.

Our paper is related to the literature on signaling unobservable quality[4]. There are mainly two branches in this line of research. The first one considers the signals that are default-independent, such as advertising. Advertising together with price is used as an effective signal of good quality. Following the seminal paper of Nelson (1974), Milgrom and Roberts (1986) shows that uninformative advertising can be used as a signal of good quality, since only the high quality good can generate repeat purchases. Linnemer (2002) considers the case in which some buyers are informed. He shows that a combination of price and advertising is a more efficient signal even if there is no repeat purchase.

The second line of research focuses on the default-dependent variables, such as warranties and money-back guarantees (see, e.g., Moorthy and Srinivasan (1995) and Balachander (2001)). The default-dependent signals are more costly to the low-type seller since a low quality good is more likely to be defective and therefore more costly to the seller when offering longer warranties or money-back guarantees.

In our model, the lender bears the liability when the good turns out to be defective. Our paper is related to the literature on warranties or money-back guarantees as signals in the sense that signals are all default-dependent. However, in our model the seller uses both a cash price and a credit price as signals, not just only the length of warranties as a signal.

The rest of the paper is organized as follows. In Section 2 we present a basic model with homogenous buyers and state the game and the equilibrium concept in our model. In Section 3, we characterize the separating equilibrium and show respectively the conditions for the existence of the first best and least cost separating equilibrium. In Section 4, we extend the basic model to a more general one with heterogenous buyers and then characterize the equilibrium when the seller signals by price alone and by offering a payment plan. In Section 5, we discuss the pooling equilibrium and compare our results with signaling by warranty or money-back guarantee. Our main results and conclusions are summarized in section 6.

[3] The analysis in Sen (1998) relies on the assumption of imperfect financial markets. However, the main point in Brennan, Miksimovic, and Zechner (1988) is to show that seller can offer financing to price discriminate among consumers even when the financial market is perfectly competitive.

[4] For a comprehensive review of the literature on signaling product quality, see Kirmani and Rao (2000).

2 Basic Model

An Investment Good Market

A monopoly seller sells an investment good to a continuum of buyers and each buyer has a unit demand for the good. The quality of this investment good is privately known to the seller. For simplicity, we assume that the quality of the good is either low or high: $q \in \{l, h\}$. We assume that an investment good of quality q lasts for one period and yields a return of v_q if it is working. On the contrary, the good yields a return of zero if it is defective. The probability that a good of quality q turns out to be defective is denoted by α_q. We say an investment good is of high quality if it yields a higher return when working and has a lower probability of being defective, i.e. $v_h > v_l$ and $\alpha_h < \alpha_l$. The production cost for a unit good of quality q is denoted by c_q. We assume that c_q is a constant and $c_h = c \geq c_l = 0$.

Buyer

There are two kinds of buyers with different budgets at the time of purchase. We say a buyer is rich if he has a sufficient budget to pay for the good. On the contrary, we say a buyer is poor if he has zero budget. A poor buyer has to be financed if he decides to buy. The fraction of rich buyers is denoted by β and is commonly known.

Financing Arrangement

There is a competitive loan market in which buyers can borrow from financial institutions to finance the purchase of the investment good. The cost of funds to each financial institution is normalized to zero. Financial institutions offer a loan schedule $r_f(L)$ to buyers, where $r_f(L)$ is the loan rate at a loan size L. In other words, if a buyer borrows L from a financial institution, he has to repay $L(1 + r_f(L))$ in the next period.

The seller is also allowed to provide financing to buyers by offering a menu of payment terms, which consists of a cash price p_1 and a credit price p_2. A buyer can choose to pay a cash price p_1 at the time of purchase or to pay a credit price p_2 after the sale is made and the return of the good is revealed. In contrast to a financial institution with a zero cost of funds, we assume that the cost of funds to the seller r_s is strictly positive.

Following the specification in Brennan, Miksimovic, and Zechner (1988), we assume that the only collateral that buyers can pledge for their loans is the return on the investment good itself. That is, the lender faces the default risk if the good turns out to be defective and yields a return of zero.

Information Structure

We assume that the seller has private information about the quality of the good. Buyers and financial institutions can't observe the quality of the good. All other things are assumed to be commonly known among the seller, buyers and financial institutions.

2.1 The Game and Equilibrium Concept

The model stated above can be expressed as a sequential game of incomplete information as the following:

- Stage 0: Financial institutions offer a loan schedule $r_f(L)$ to buyers.
- Stage 1: Nature selects seller's type $q \in \{h, l\}$, and the seller decides to offer a cash price alone or a menu of payment terms (p_1^q, p_2^q) to buyers, where p_1^q, p_2^q denote the cash price and credit price respectively.
- Stage 2: Buyers observe the prices offered by the seller and the loan schedule offered by financial institutions. The buyers with liquidity-constraints decide whether to borrow from a financial institution or to pay the credit price (if it is offered by the seller) or not to buy. On the other hand, rich buyers have all the options that poor buyers have and also can choose to pay the cash price at stage 1.
- Stage 3: The return of the good is realized and becomes commonly known.

Equilibrium Concept

The game in our model is a sequential game of incomplete information, and we employ the Perfect Bayesian Equilibrium (PBE) concept to characterize the equilibrium of this game. This equilibrium concept requires that each player in the game is sequentially rational, i.e. the strategies of the seller and buyers must be optimal given the buyer's posterior beliefs. In addition, the buyer's posterior beliefs must be consistent with the seller's strategy and follow from Bayes' rule whenever possible.

Let $p^q = \{p_1^q, p_2^q\}$ be the pricing strategy of the seller of quality q, and $\mu(.|p) \in [0, 1]$ be a buyer's posterior belief that the good is of quality h. Let $\pi(q, \mu, p)$ denote the profit function of the seller of quality q, under the buyers' belief μ and the pricing strategy p. We restrict our attention to the PBE, which is expressed as $\{p^q, \mu(.|p)\}$ satisfying the following conditions:

- Sequential rationality:
$$p^q = \arg\max_p \pi(q, \mu(.|p), p)$$
- Beliefs are determined by Bayes' rule and players' equilibrium strategies whenever possible.
- Buyers' off-the-equilibrium beliefs should satisfy the "intuitive criterion" (Cho and Kreps (1987)).

3 Analysis

In our model, we assume that there exists a competitive loan market in which financial institutions offer a loan schedule to buyers to finance the purchase of an investment good. By observing this loan schedule, the seller then decides its pricing strategy. In this section, we first analyze the competitive loan rate in a loan market. Given this loan rate, we further characterize the Perfect

Bayesian Equilibrium in a signaling game in which the seller uses payment terms as a signal of the quality of the good.

In the subsequent analysis, we restrict our attention to the separating equilibria. Pooling equilibria will be discussed in Section 5.

3.1 A Competitive Loan Rate

We first define some notations. Let $r_f(L)$ be the loan schedule offered by a financial institution, where $r_f(L)$ specifies the loan rate at a loan size L. Let ψ be the expected probability of financing to a high quality investment good and let $\pi_f(L)$ be the expected profit of a financial institution f when the loan size is L.

Note that if financial institutions anticipate that there exists a separating equilibrium in the investment good market, i.e. buyers can distinguish the quality of the good, then buyers never default as long as the good is working[5]. Therefore, the expected profit of a financial institution can be expressed as the following:

$$\pi_f(L) = [\psi(1-\alpha_h) + (1-\psi)(1-\alpha_l)]L(1+r_f(L)) - L \tag{1}$$

Note that the first term $\psi(1-\alpha_h)$ is the probability that a financial institution finances to a high quality good which yields a return of v_h. Similarly, the second term $(1-\psi)(1-\alpha_l)$ is the probability of financing to a low quality good that yields a return of v_l. When a good of quality q yields a return of v_q, a buyer who borrows L would repay $L(1+r_f(L))$ in the next period. Following the specification in Brennan, Miksimovic, and Zechner (1988), we assume that the only collateral that buyers can pledge for their loans is the return on the investment good itself. Thus, if the good turns out to be defective, the expected payment to a lender is zero.

A competitive loan market assumption implies that the expected payoff of a financial institution should be zero at the equilibrium. Consequently, we have the equilibrium loan rate $r_f(L)$ in the competitive loan market as shown in the following lemma.

Lemma 1. *If buyers can distinguish the quality of the good, a financial institution offers a fixed rate loan schedule r_f to buyers, where r_f is*

$$r_f = \frac{(1-\psi)\,\alpha_l + \psi\alpha_h}{1-\alpha_l+\psi(\alpha_l-\alpha_h)} \tag{2}$$

The loan rate r_f is decreasing in ψ, increasing in α_h, α_l and independent of the loan size L.

Proof. See Appendix. □

In our model, the interest rate is a measure of default risk to a financial institution since the good might turn out to be defective. Financial institutions do not observe the quality of the good, so the expected default risk is decreasing in the expected probability of financing to a good of high quality ψ. On the other hand, it is increasing in the probability of being defective for each type of the good, α_h and α_l. Especially, by (2) we can see that if the good is of high quality h for sure, i.e.

[5] If a buyer can not distinguish the seller's type, then the buyer might default if the good yields a return of v_l.

$\psi = 1$, then $r_f = \frac{\alpha_h}{1-\alpha_h}$. If the good is of low quality l for sure, i.e. $\psi = 0$, then $r_f = \frac{\alpha_l}{1-\alpha l}$. To put it more precisely, we have the following:

$$\frac{\alpha_h}{1-\alpha_h} \leq r_f \leq \frac{\alpha_l}{1-\alpha_l}, \text{ for any } 0 \leq \psi \leq 1 \tag{3}$$

3.2 Good Market

In the good market, the seller sells an investment good to a continuum of buyers. Some of the buyers are liquidity-constrained and have to seek financing from either a financial institution or the seller. In this subsection, we first characterize the equilibrium when the good quality is commonly known to buyers.

Buyer's Problem

Let r_f^* denote the fixed loan rate offered by the financial institutions. For a buyer who purchases an investment good of quality q at a cash price p, the expected payoff of the buyer is

$$(1-\alpha_q)v_q - p \tag{4}$$

where $q \in \{l, h\}$. Thus, the reservation price[6] for the good q is $(1-\alpha_q)v_q$.

For a buyer who purchases a good of quality q at a cash price p and obtains financing from a financial institution, the expected payoff is

$$(1-\alpha_q)(v_q - p(1+r_f^*)) \tag{5}$$

Therefore, the reservation price for a good of quality q is $\frac{v_q}{1+r_f^*}$.

Similarly, for a buyer who buys a good of quality q at a credit price p and obtains financing from the seller, the expected payoff is

$$(1-\alpha_q)(v_q - p) \tag{6}$$

Thus, the reservation price is v_q.

Let R_a^q denote the buyer's reservation price for a good of quality q when he chooses to pay a cash price and let R_b^q denote the reservation price for a buyer when he obtains financing from a financial institution. That is, we have

$$\begin{aligned} R_a^q &= (1-\alpha_q)v_q \\ R_b^q &= \frac{v_q}{1+r_f^*} \end{aligned}$$

When the type l seller charges a cash price p alone, a poor buyer will choose to buy and borrow from a financial institution if $p \leq R_b^q$. For a rich buyer, the expected payoff if he pays a cash price p at the time $t = 1$ is $(1-\alpha_l)v_l - p$. If he borrows from a financial institution and repay at the time $t = 2$, his expected payoff is $(1-\alpha_l)(v_l - p(1+r_f^*))$. It is straightforward to verify that both rich and poor buyers will borrow from a financial institution and repay at $t = 2$ if $p \leq R_b^q$.

[6] The reservation price is the price p that makes expected payoff equal to zero.

When the type h seller offers financing, i.e. the seller offers both a cash price p_1 and a credit price p_2, a rich buyer can choose to pay a cash price at the time $t = 1$ or a credit price at $t = 2$. By (4) and (6), we can see that a rich buyer will choose to pay a cash price p_1, if and only if

$$(1 - \alpha_h)v_h - p_1 \geq (1 - \alpha_h)(v_h - p_2)$$

which translates to $p_1 \leq (1 - \alpha_h)p_2$[7]. For a poor buyer, he will choose to pay a credit price, instead of paying a cash price and obtaining financing from a financial institution, if $p_2 \leq (1 + r_f^*)p_1$. Therefore, when the type h seller offers a menu of payment terms (p_1, p_2) such that

$$\frac{p_1}{1 - \alpha_h}p_1 \leq p_2 \leq p_1(1 + r_f^*)$$

then rich buyers will choose to pay a cash price at the time $t = 1$, while poor buyers will choose to pay a credit price at $t = 2$.

3.3 Characterization of the Separating Equilibrium

We now consider the case when the quality of the good is privately known to the seller. We characterize the separating equilibrium in which the type h seller can separates itself from the type l one by adopting a menu of payment terms. Note that in any separating equilibrium, the type l seller is revealed, so the pricing strategy of the type l seller is the same as the one under complete information. Consequently, the type l seller offers a cash price alone in any separating equilibrium. That is, the type l seller offers a cash price $p_1^l = (1 - \alpha_l)v_l$.

A necessary condition for the existence of a separating equilibrium is that the low-type seller should not want to masquerade as the high-type one and the high-type seller should not want to deviate when any move would engender the worst beliefs about his type. Let $\pi(q, \mu, p)$ denote the profit function of the seller of quality q under the buyers' belief μ and the pricing strategy p. Also, let $p^h \equiv (p_1^h, p_2^h)$ be the pricing strategy of the type h seller in a separating equilibrium. The necessary and sufficient conditions to ensure a separating equilibrium are as follows:

$$\pi(l, 1, p^h) \leq \max_p \pi(l, 0, p) \tag{7}$$

$$\max_p \{\pi(h, 0, p)\} \leq \pi(h, 1, p^h) \tag{8}$$

Note that inequality (7) refers to that the type l seller has no incentive to mimic the pricing strategy of the type h seller and (8) indicates that the profit of the high-type seller has to be higher than the maximum profit reachable under the worst beliefs, i.e. the type h seller would rather choose p^h and be perceived than masquerade as the type l and optimize accordingly.

We consider that the seller uses a menu of payments (p_1^h, p_2^h) to signal its quality, in which the rich buyers take the cash price and poor buyers take the credit price. We have

$$\begin{aligned}
\pi(h, 1, p^h) &= \beta p_1^h + (1 - \beta)\frac{1 - \alpha_h}{1 + r_s}p_2^h \\
\pi(l, 1, p^h) &= \beta p_1^h + (1 - \beta)\frac{1 - \alpha_l}{1 + r_s}p_2^h
\end{aligned}$$

[7] Assuming that p_1 is less than his reservation price $(1 - \alpha_q)v_q$.

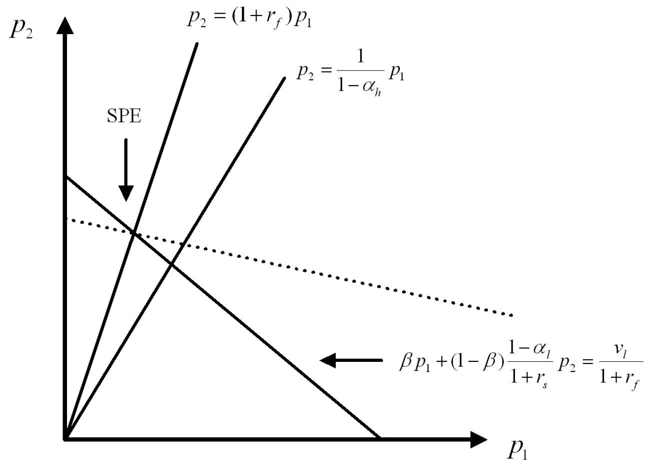

Figure 1: Separating Equilibrium with Homogenous Buyers

By (7) and (8), we can characterize the separating equilibria as the following.

Proposition 1. *Let (p_1^h, p_2^h) be a menu of payment terms offered by the type h seller. In any separating equilibrium, (p_1^h, p_2^h) must satisfy the following condition.*

$$\beta p_1^h + (1-\beta)\frac{1-\alpha_l}{1+r_s} p_2^h \le (1-\alpha_l)v_l \le \beta p_1^h + (1-\beta)\frac{1-\alpha_h}{1+r_s} p_2^h \tag{9}$$

Note that in any separating equilibrium, the type l seller charges a cash price and obtains a profit $(1-\alpha_l)v_l$. If the following inequality holds

$$\beta(1-\alpha_h)v_h + (1-\beta)\frac{1-\alpha_l}{1+r_s} v_h \le (1-\alpha_l)v_l \tag{10}$$

then it is never beneficial for the type l seller to masquerade as the type h one. The unique separating equilibrium is the first best equilibrium and each type of seller acts as if the good quality is commonly known. We summarize this result in the following proposition.

Proposition 2. *First best separating equilibrium*
If (10) holds, there exists a unique first best separating equilibrium. The strategies used are as the following: $p^l = (1-\alpha_l)v_l$ and $(p_1^h, p_2^h) = ((1-\alpha_h)v_h, v_h)$.

In contrast to the first best equilibrium, a more interesting case is to characterize the least cost separating equilibrium when the first best equilibrium is not feasible, i.e. when (10) does not hold. The least cost separating equilibrium, i.e., the separating equilibrium that satisfies the "intuitive criterion" and yields the least profit reduction to the type h seller, is the solution to the following optimization problem [MP1]:

$$[\text{MP1}] \max_{p_1,p_2} \ \beta p_1 + (1-\beta)\frac{1-\alpha_h}{1+r_s}p_2 \tag{11}$$

$$st. \ \beta p_1 + (1-\beta)\frac{1-\alpha_l}{1+r_s}p_2 \leq \frac{v_l}{1+r_f^*} \tag{12}$$

$$p_2 \leq p_1(1+r_f^*) \tag{13}$$

$$\frac{p_1}{1-\alpha_h} \leq p_2 \tag{14}$$

$$p_2 \leq v_h, \ p_1 \leq (1-\alpha_h)v_h \tag{15}$$

Note that (12) is the no mimicry condition for the type l seller and (13) ensures that the menu of payment terms is effective. If (13) does not hold, we have seen that signaling by cash price alone does not work.

Note that in the least cost separating equilibrium, the no mimicry condition for the type l seller (12) must bind. The type h seller should charge an effective credit price as high as possible. That is, $p_2 = \frac{p_1}{1-\alpha_l}$. Therefore, as shown in Figure 1, the least cost separating equilibrium occurs when both (12) and (13) bind. We summarize this result in the following proposition.

Proposition 3. *If* (10) *doesn't hold, the least cost separating equilibrium exists in the homogenous buyers model.*

- *In equilibrium, the financial institutions offer a fixed rate loan schedule* $r_f^* = \frac{\alpha_l}{1-\alpha_l}$.
- *The type l seller offers a cash price* $p^l = (1-\alpha_l)v_l$ *only.*
- *The type h seller offers a menu of payment terms* $p^h = (p_1^h, p_2^h)$*, where*

$$\begin{aligned} p_2^h &= \max\left\{\frac{v_l\,(1+r_s)\,(1-\alpha_l)}{(\alpha_l-\alpha_h)+(1-\alpha_l)(1+r_s\beta)}, v_h\right\} \\ p_1^h &= (1-\alpha_l)p_2^h \end{aligned}$$

- *The buyer's beliefs on the equilibrium path are as the following:* $\mu(p^h) = 1$, $\mu(p^l) = 0$. *In addition,* μ *is sufficiently small for any other off-the-equilibrium beliefs.*

Comparative Statics

By examining the pricing strategy of the type h seller at the least cost separating equilibrium, we have the following results:

$$\frac{\partial p_1^h}{\partial \beta} = \frac{v_l\,(1+r_s)\,(1-\alpha_l)^2 + r_s(\alpha_l - 1)}{[(-r_s+\alpha_l r_s)\beta - 1 + \alpha_l)]^2} \leq 0 \tag{16}$$

$$\frac{\partial p_1^h}{\partial r_s} = \frac{v_l\,(1-\alpha_l)^2\,(1-\alpha_l)\,(1-\beta)}{(-\beta\, r_s + \beta\,\alpha_l r_s - 1 + \alpha_l)^2} \geq 0 \tag{17}$$

In the least cost separating equilibrium, the cash price charged by the type h seller p_1^h is increasing in the cost of funds r_s and decreasing in the fraction of rich buyers β. Note that $p_2^h = p_1^h(1+r_f^*)$ in equilibrium, so the comparative results also hold for the credit price p_2^h.

If β is higher, ceteris paribus, it is more profitable for a low quality seller to mimic the pricing strategies of the high quality one. Therefore, in order to make a separating equilibrium possible, the high-type seller has to reduce the cash price. On the other hand, higher cost of funds r_s makes mimicking more costly to the low type, so the high-type seller can charge a higher price.

4 Heterogenous Buyers

In this section, we extend the basic model to a more general one with heterogenous buyers. We assume that the return of the investment good varies across buyers. For a buyer i, an investment good of quality q yields a return of $\theta_i v_q$ if the good is working, where $q \in \{l, h\}$ and θ_i has a uniform distribution with a support $[0, 1]$. This parameter θ_i is known to the buyer i and can't be observed by the seller. Similar to the specification in the homogenous case, the return of the investment good is normalized to zero if the good is defective.

Let R^q be buyer's reservation price for a good of quality q, and $D(p, R^q)$ be the demand function under a given market price p and R^q. Then, the demand function can be expressed as the following:

$$\begin{aligned} D(p, R^q) &= \Pr(\theta_i R^q > p) \\ &= (1 - \frac{p}{R^q}) \end{aligned}$$

Recall that we have defined the reservation prices R^q as follows:

$$\begin{aligned} R_a^q &= (1-\alpha_q)v_q \\ R_b^q &= \frac{v_q}{1+r_f^*} \end{aligned}$$

where R_a^q is the reservation price for a buyer when he pays a cash price, and R_b^q is the reservation price when a buyer obtains financing from a financial institution. Note also that r_f^* is the equilibrium loan rate in the financial market.

In the model with homogenous buyers, it is straightforward to see that signaling by price alone is not costly and consequently no separating equilibrium exists. We have also shown that seller financing plays a role as a signal of good quality since it is more costly for the low-type seller to provide financing to buyers. Therefore, the type l seller would rather offer a cash price alone than provide financing to buyers. In that case, the liquidity-constrained buyers will obtain financing from financial institutions.

In contrast to the previous analysis, when the return of the good varies across buyers and as a result the demand is elastic, signaling by cash price alone is possible. Essentially, the ability of a high price to facilitate signaling depends on the cost difference between the high and low-type seller[8]. Henceforth, in the following analysis we consider a case in which $c_h = c \geq c_l = 0$, and assume that $c < R_b^l$ to ensure each type of seller earns a strictly positive profit.

[8] See Bagwell and Riordan (1991) and Linnemer (2002).

4.1 Signaling by Cash Price alone

In this subsection, we first consider a situation in which the seller can not offer financing. We investigate the conditions under which signaling by price alone is possible and then we characterize the separating equilibrium within this framework.

When the seller uses cash price alone as a signal, both high quality and low quality seller can cause the buyers to go to the credit market, which implies that in equilibrium $\psi = \lambda$. Thus, we have $r_f = \frac{(1-\lambda)\alpha_l + \lambda\alpha_h}{1-\alpha_l+\lambda(\alpha_l - \alpha_h)}$ and $R_b^q = [1 - \alpha_l + \lambda(\alpha_l - \alpha_h)]v_q$.

Recall that $\pi(q, \mu, p)$ denote the profit function of the seller of quality q under the beliefs μ, where $\mu \in [0, 1]$ is the buyer's beliefs that the seller is of high quality. From the analysis in the previous section with homogenous buyers, we have shown that in the separating equilibrium the type l seller will offer a cash price only. Thus, the profit function of the type l seller $\pi(l, 0, p)$ can be written as the following:

$$\pi(l,0,p) = \begin{cases} p(1 - \frac{p}{R_b^l}) & \text{, if } p \leq R_b^l \\ 0 & \text{, otherwise} \end{cases}$$

Recall that $R_b^l = [1 - \alpha_l + \lambda(\alpha_l - \alpha_h)]v_l$ is a buyer's reservation price for the good l if he borrows from a financial institution. Since the profit function stated above is strictly concave, we have the optimal price p_l and profit π_l as the following:

$$\begin{aligned} p_l^* &= \frac{1}{2}R_b^l \\ \pi_l^* &= \frac{1}{4}R_b^l \end{aligned}$$

For the type h seller, under the beliefs that it is of high quality, the profit function is

$$\pi(h,1,p) = \begin{cases} [\beta(1 - \frac{p}{R_a^h}) + (1-\beta)(1 - \frac{p}{R_b^h})](p - c) & \text{, if } p \leq R_b^h \\ \beta(1 - \frac{p}{R_a^h})(p - c) & \text{, if } R_b^h < p \leq R_a^h \\ 0 & \text{, otherwise} \end{cases}$$

Let p_h^* denote the optimal cash price of the high-type seller, i.e. $p_h^* = \max_p \pi(h, 1, p)$. By a simple calculation, we can see that

$$p_h^* \in \left\{ \frac{1}{2} \frac{((1-\beta)c + R_b^h)R_a^h + \beta c R_b^h}{(1-\beta)R_a^h + \beta R_b^h}, \frac{1}{2}(R_a^h + c) \right\}$$

Characterization of the Separating Equilibrium

Suppose that there exists a separating equilibrium in which the type h seller offers a cash price $p^h \leq R_b^h$. The condition to ensure that the type l seller must not gain by charging p^h is

$$\pi(l, 1, p^h) \leq \pi_l^*$$

which implies the following:

$$[\beta(1 - \frac{p^h}{R_a^h}) + (1-\beta)(1 - \frac{p^h}{R_b^h})]p^h \leq \frac{1}{4}R_b^l \tag{18}$$

Inequality (18) holds if and only if $p^h \notin (\underline{p}^h, \overline{p}^h)$, where $\overline{p}^h$ and $\underline{p}^h$ are defined as follows:

$$\begin{aligned}
\overline{p}^h &= \frac{1}{2}\frac{R_a^h R_b^h + \sqrt{R_a^h R_b^h [R_a^h (R_b^h - R_b^l) + \beta R_b^l (R_a^h - R_b^h)]}}{(1-\beta) R_a^h + \beta R_b^h} \\
\underline{p}^h &= \frac{1}{2}\frac{R_a^h R_b^h - \sqrt{R_a^h R_b^h [R_a^h (R_b^h - R_b^l) + \beta R_b^l (R_a^h - R_b^h)]}}{(1-\beta) R_a^h + \beta R_b^h}
\end{aligned}$$

In addition, the condition to ensure that a type h seller does not want to move from p^h when any move would engender the worst beliefs about his type is as follows:

$$\pi(h, 1, p^h) \geq \max_p \pi(h, 0, p)$$

which also implies

$$[\beta(1 - \frac{p^h}{R_a^h}) + (1-\beta)(1 - \frac{p^h}{R_b^h})](p^h - c) \geq \frac{1}{4}\frac{(R_b^l - c)^2}{R_b^l} \tag{19}$$

By examining (19), we can further show that (19) holds if and only if p falls in an interval $[p^h_{\min}, p^h_{\max}]$, where

$$\begin{aligned}
p^h_{\max} &= \frac{1}{2}\frac{1}{(1-\beta) R_a^h + \beta R_b^h}\left\{[R_b^h R_b^h + c(R_a^h - \beta R_a^h + \beta R_b^h)] + \frac{1}{R_b^l}\sqrt{AB}\right\} & (20) \\
p^h_{\min} &= \frac{1}{2}\frac{1}{(1-\beta) R_a^h + \beta R_b^h}\left\{[R_b^h R_b^h + c(R_a^h - \beta R_a^h + \beta R_b^h)] - \frac{1}{R_b^l}\sqrt{AB}\right\} & (21)
\end{aligned}$$

and the constants A and B are defined as

$$\begin{aligned}
A &= R_b^l (R_a^h R_b^h - R_a^h R_b^l + \beta R_a^h R_b^l - \beta R_b^h R_b^l) \geq 0 \\
B &= \beta c^2 (R_a^h - R_b^h) + R_a^h (R_b^h R_b^l - c^2) \geq 0
\end{aligned}$$

A necessary condition for the existence of the separating equilibrium is $p^h_{\max} \geq \overline{p}^h$. Otherwise, if $\overline{p}^h < p^h_{\max}$, then no price p can satisfy both (18) and (19) and therefore no separating equilibrium exists. By examining $\overline{p}^h$ and $p^h_{\max}$, we can further verify that $p^h_{\max} > \overline{p}^h$ for any $c > 0$. Therefore, we can conclude that for any $c > 0$, a separating equilibrium in which $\overline{p}^h \leq p^h \leq p^h_{\max}$ always exists.

If $p_h^* \geq \overline{p}^h$, then the first best equilibrium exists and we have $p^h = p_h^*$. On the contrary, if $p_h^* < \overline{p}^h$ then any $p^h \in [\overline{p}^h, p^h_{\max}]$ constitutes an equilibrium. We employ the usual "intuitive criterion" to restrict our attention to the least cost separating equilibrium, i.e. the separating equilibrium that yields the least profit reduction for the type h seller. In this case, $p^l = \frac{1}{2}R_b^l$ and $p^h = \overline{p}^h$ forms a least cost separating equilibrium. Thus, we can conclude that $p^l = \frac{1}{2}R_b^l$ and $p^h = \max\{\overline{p}^h, p_h^*\}$, together with the appropriately specified off-the-equilibrium beliefs, form a unique separating equilibrium that satisfies the intuitive criterion. We summarize our results in the following proposition.

Proposition 4. *In the model with heterogenous buyers, if $c > 0$ there exists a unique separating equilibrium, in which the seller signals good quality with a cash price alone.*

- *In equilibrium, the financial institutions offer a fixed rate loan schedule* $r_f^* = \frac{(1-\lambda)\,\alpha_l + \lambda\alpha_h}{1-\alpha_l+\lambda(\alpha_l-\alpha_h)}$.
- *The pricing strategies of the seller are:* $p^l = \frac{1}{2}[1-\alpha_l+\lambda(\alpha_l-\alpha_h)]v_l$ *and* $p^h = \max\{\overline{p}^h, p_h^*\}$, *where* $\overline{p}^h = \frac{1}{2}\frac{R_a^h R_b^h + \sqrt{R_a^h R_b^h[R_a^h(R_b^h-R_b^l)+\beta R_b^l(R_a^h-R_b^h)]}}{(1-\beta)R_a^h+\beta R_b^h}$ *and* $p_h^* = \frac{1}{2}\frac{((1-\beta)c+R_b^h)R_a^h+\beta c R_b^h}{(1-\beta)R_a^h+\beta R_b^h}$
- *The buyer's beliefs are:* $\mu(p)\begin{cases} =1 & , \textit{ if } p = \max\{\overline{p}^h, p_h^*\} \\ =0 & , \textit{ if } p = p^l \textit{ or } p \in (\underline{p}^h, \overline{p}^h) \\ \in [0,1] & , \textit{ otherwise} \end{cases}$

Proof. See Appendix. □

Proposition 4 shows that a unique separating equilibrium always exists as long as c, the production cost of the type seller, is strictly positive. When $\dot{c} > 0$, the loss of sale volume is less damaging to the high cost, high quality seller. Therefore, in the model with heterogenous buyers, signaling by price alone is feasible.

For the off-the-equilibrium beliefs, $\mu(p)$ has to be sufficiently small for $p \in (\underline{p}^h, \overline{p}^h)$. Otherwise, the type h seller has incentives to deviate to some $p \in (\underline{p}^h, \overline{p}^h)$. If $c = 0$, we can see that the profit of the high-type seller from signaling is exactly the same as its profit reachable when it is perceived as the low-type one, i.e. $\pi(h, 1, p^h) = \max_p \pi(h, 0, p)$. In this case, the high-type seller has no incentives to separate itself from the low-type one and it may prefer a pooling equilibrium.

4.2 Signaling by Seller Financing

In this section, we consider a case in which the seller is allowed to provide financing to buyers. The seller provides financing to buyers by offering a menu of payment terms such that buyers are divided into two groups: cash buyers and credit buyers. Namely, we consider the case in which the seller offers a menu of payment terms $p^h = \left(p_1^h, p_2^h\right)$ such that $\frac{1}{1-\alpha_h}p_1^h \le p_2^h \le p_1^h(1+r_f^*)$, so rich buyers take the cash price and poor buyers take the credit price. If $p_2^h > p_1^h(1+r_f^*)$, a poor buyer would rather borrow from a financial institution than pay a credit price p_2^h. Similarly, if $\frac{1}{1-\alpha_h}p_1^h > p_2^h$, a rich buyer will choose to pay a credit price p_2^h.

The profit function of the type h seller when it is recognized, denoted by $\pi(h, 1, p_2^h)$, is expressed as follows:

$$\pi(h,1,p^h) = \beta(1-\frac{p_1^h}{R_a^h})(p_1^h - c) + (1-\beta)\left[(1-\frac{p_2^h}{v_h})[\frac{(1-\alpha_h)}{(1+r_s)}p_2^h - c]\right] \tag{22}$$

In addition, p_1^h and p_2^h must meet the following conditions: $p_2^h \le v_h$ and $p_1^h \le (1-\alpha_h)v_h$. Note that the first term $(1-\frac{p_1^h}{R_a^h})(p_1^h - c)$ in (22) is the seller's profit obtained from rich buyers under a cash price p_1^h and $(1-\frac{p_2^h}{v_h})\left[\left(\frac{1-\alpha_h}{1+r_s}\right)p_2^h - c\right]$ is the seller's profit obtained from poor buyers under a credit price p_2^h and the cost of funds r_s.

Since the profit function of the type h seller is concave, we can obtain a unique optimal price $p_{1,h}^*$ and $p_{2,h}^*$ as shown below:

$$\begin{aligned} p_{1,h}^* &= \frac{1}{2}\left(R_a^h + c\right) \\ p_{2,h}^* &= \frac{1}{2}[v_h + \frac{(1+r_s)c}{1-\alpha_h}] \end{aligned}$$

We have already shown that when the good quality is known, the type l seller's optimal strategy is to offer a cash price $p^*_{1,l} = \frac{1}{2}R^l_b$. In any separating equilibrium, a type l seller adopts the same pricing strategy as the one under complete information, i.e. the type l seller offers a cash price $p^*_{1,l} = \frac{1}{2}R^l_b$ which yields a profit $\pi^*_{1,l} = \frac{1}{4}R^l_b$. We say that $(p^h_1,\, p^h_2)$ forms a separating equilibrium if the following no mimicry condition holds:

$$\pi(l, 1, p^h) \le \max_p \pi(l, 0, p) \tag{23}$$

which also implies

$$\beta(1 - \frac{p^h_1}{R^h_a})p^h_1 + (1-\beta)\left[(1 - \frac{p^h_2}{v_h})\frac{(1-\alpha_l)}{(1+r_s)}p^h_2\right] \le \frac{1}{4}R^l_b \tag{24}$$

Similar to the analysis in the model with homogenous buyers, inequality (24) says that the type l seller must not gain by mimicking the payment terms offered by the type h seller.

Note that the objective function of the high-type seller (22) is maximized at $p^*_{1,h} = \frac{1}{2}(R^h_a + c)$ and $p^*_{2,h} = \frac{1}{2}\left(v_h + \frac{(1+r_s)c}{1-\alpha_h}\right)$. Thus, the first best separating equilibrium exists if

$$\beta(1 - \frac{p^*_{1,h}}{R^h_a})p^*_{1,h} + (1-\beta)\left[(1 - \frac{p^*_{2,h}}{v_h})\frac{(1-\alpha_l)}{(1+r_s)}p^*_{2,h}\right] \le \frac{1}{4}R^l_b \tag{25}$$

Suppose that (25) does not hold, then we will need to further characterize the least cost separating equilibrium. In the least cost separating equilibrium, the menu of payment terms (p^h_1, p^h_2) offered by the type h seller can be expressed as the solution to the following maximization problem [MP2]:

$$[\text{MP2}]\ \max_{p^h_1, p^h_2}\ \ \beta(1 - \frac{p^h_1}{R^h_a})(p^h_1 - c) + (1-\beta)\left[(1 - \frac{p^h_2}{v_h})[\frac{(1-\alpha_h)}{(1+r_s)}p^h_2 - c]\right] \tag{26}$$

$$s.t.\ \ \beta(1 - \frac{p^h_1}{R^h_a})p^h_1 + (1-\beta)\left[(1 - \frac{p^h_2}{v_h})\frac{(1-\alpha_l)}{(1+r_s)}p^h_2\right] \le \frac{1}{4}R^l_b \tag{27}$$

$$p^h_2 \le p^h_1(1 + r^*_f) \tag{28}$$

$$\frac{p^h_1}{1-\alpha_h} \le p^h_2 \tag{29}$$

where (27) is the no mimicry condition for the type l seller, and (28), (29) are the conditions to ensure that the menu of payment terms is effective[9].

By examining the no mimicry condition (27), we can see the that $\pi(l, 1, p^h)$, the profit of the type l seller when mistaken as the type h one, is maximized at $p^h_1 = \frac{1}{2}R^h_a$ and $p^h_2 = \frac{1}{2}v_h$. For the least cost separating equilibrium, if (25) does not hold, then the least cost separating equilibrium must occur when the no mimicry condition (27) binds.

To further characterize the least cost separating equilibrium, we need to investigate the properties of iso-profit curves of $\pi(h, 1, p)$ and $\pi(l, 1, p)$. Let $\frac{dp^h_2}{dp^h_1}|_{\pi^h}$ denote the slope of the iso-profit curve of the profit function $\pi(h, 1, p)$, and we have

$$\frac{dp_2}{dp_1}|_{\pi^h} = \frac{\beta v_h\,(1+r_s)\left(2p_1 - R^h_a - c\right)}{R^h_a\,(1-\beta)\,(c - 2p_2 + v_h + 2\alpha_h p_2 - \alpha_h v_h)} \le 0 \tag{30}$$

[9] (28) and (29) ensure that poor buyers will take the credit price and rich buyers will take the cash price.

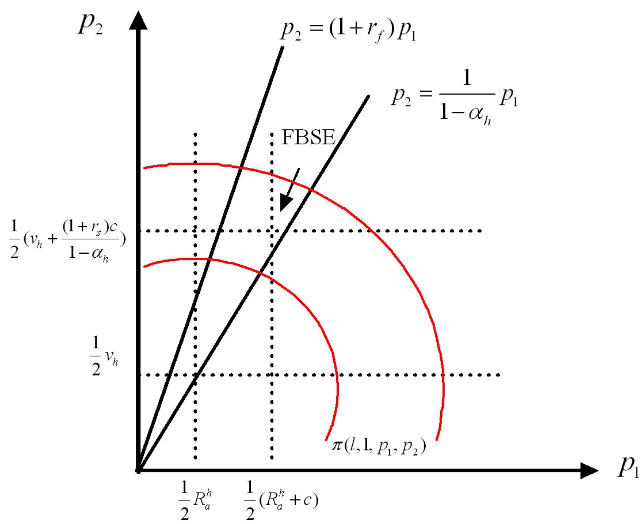

Figure 2: Separating Equilibrium with Heterogenous Buyers

for any (p_1, p_2) such that $p_1 \geq p_{1,h}^*$ and $p_2 \geq p_{2,h}^*$.

In addition, let $\frac{dp_2}{dp_1}|_{\pi^l}$ be the slope of the iso-profit curve of $\pi(l, 1, p)$, we have

$$\frac{dp_2}{dp_1}|_{\pi^l} = \frac{\beta v_h(1+r_s)(2p_1 - R_a^h)}{R_a^h(\alpha_l - 1)(1-\beta)(2p_2 - v_h)} \leq 0 \tag{31}$$

for any (p_1, p_2) such that $p_1 \geq \frac{1}{2}R_a^h$ and $p_2 \geq \frac{1}{2}v_h$. By comparing (30) and (31), we can identify the candidate solutions to the optimization problem [MP2] and then characterize the least cost separating equilibrium. We summarize our results in the following proposition.

Proposition 5. *A candidate least cost separating equilibrium exists in the model with heterogenous buyers, where*

- *In equilibrium, the financial institutions offer a fixed rate loan schedule* $r_f^* = \frac{\alpha_l}{1-\alpha_l}$.
- *the type* l *seller offers a cash price* $p_1^l = \frac{1}{2}(1-\alpha_l)v_l$ *alone.*
- *The type* h *sellers offers a menu of payment terms* (p_1^h, p_1^h), *where* (p_1^h, p_1^h) *is the solution to the optimization problem [MP2].*
- *The solutions to [MP2] are either* (27),(28) *or* (27),(29) *bind.*

Proof. See Appendix. □

4.3 Existence of the Separating Equilibrium

Recall that a separating equilibrium exists only if the type h seller does not want to move from the equilibrium strategy when any move would engender the worst beliefs about his type. In other words, (p_1^h, p_1^h) must meet the following inequality:

$$\max_p \pi(h,0,p) \leq \beta(1-\frac{p_1^h}{R_a^h})(p_1^h-c)+(1-\beta)\left[(1-\frac{p_2^h}{v_h})[\frac{(1-\alpha_h)}{(1+r_s)}p_2^h-c]\right]$$

When the high-type seller is mistaken as the low-type one, the maximum profit reachable by offering a cash price is as follows:

$$\max_{p_1} \pi(h,0,p_1) = \begin{cases} \frac{1}{4}\frac{(R_b^l-c)^2}{R_b^l} & \text{, if } c \leq R_b^l \\ 0 & \text{, otherwise} \end{cases} \tag{32}$$

Similarly, the maximum profit reachable by offering a credit price can be expressed as

$$\max_{p_2} \pi(h,0,p_2) = \begin{cases} \frac{1}{4}\frac{[v_l(1-\alpha_h)-(1+r_s)c]^2}{v_l(1-\alpha_h)(1+r_s)} & \text{, if } c \leq \frac{v_l(1-\alpha_h)}{(1+r_s)} \\ 0 & \text{, otherwise} \end{cases} \tag{33}$$

In addition, in the previous section we have shown that price alone can be used as a signal of the quality of the good when the return of the good varies across buyers and the cost difference is strictly positive. Thus, the high-type seller uses seller financing as a signal of good quality only if by doing so the seller's equilibrium profit is greater than using price alone as a signal. By Proposition 4, the equilibrium profit of the high-type seller $\pi(h,1,p^h)$ can be expressed as the following:

$$\pi(h,1,p^h) = [\beta(1-\frac{p^h}{R_a^h})+(1-\beta)(1-\frac{p^h}{R_b^h})](p^h-c) \tag{34}$$

where $p^h = \max\{\overline{p}^h, p_h^*\}$.

Therefore, we can conclude that the sufficient conditions for the existence of the least cost separating equilibrium of signaling by seller financing is that the equilibrium profit of the high-type seller must be greater than the profit in (32), (33) and (34). In the subsequent analysis we consider two cases with zero cost difference and positive cost difference.

4.3.1 Zero Cost Difference

We have shown that if the cost difference between the high-type and low-type seller is zero, i.e., $c_h = c_l = 0$, then there is no separating equilibrium in which the seller uses price alone as a signal of the quality of the good. This is because with the same cost, a higher price is not costly to the low-type seller to masquerade as the high-type one. Therefore, no separating equilibrium exists when price alone is used as a signal of the quality of the good. By (32) and (33), the sufficient condition for the existence of the separating equilibrium can be reduced to

$$\pi(h,1,p_1^h,p_2^h) \geq \max\left\{\frac{(1-\alpha_h)v_l}{4(1+r_s)}, \frac{[1-\alpha_l+\lambda(\alpha_l-\alpha_h)]v_l}{4}\right\}$$

In the following proposition we show that there always exists a least cost separating equilibrium if $c=0$ and r_s is sufficiently large.

Proposition 6. *If $c=0$ and $r_s \geq \frac{(\alpha_l-\alpha_h)(1-\lambda)}{[1-\alpha_l+\lambda(\alpha_l-\alpha_h)]}$ there always exists a separating equilibrium in which*

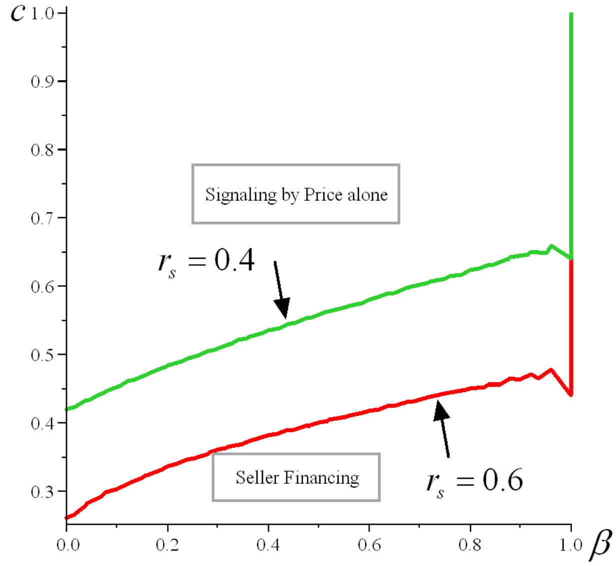

Figure 3: Existence of Separating equilibrium with Seller Financing

- *the type l seller offers a cash price $p_1^l = \frac{1}{2}(1-\alpha_l)v_l$ alone.*
- *The type h sellers offers a menu of payment terms $\left(p_1^h, p_1^h\right)$, where $\left(p_1^h, p_1^h\right)$ is the solution to the optimization problem [MP2] such that both* (27) *and* (29) *bind.*

Proof. See Appendix $\square$

If r_s is sufficiently large, then the sufficient condition for the existence is translated to

$$\pi(h, 1, p_1^h, p_2^h) \geq \frac{[1-\alpha_l+\lambda(\alpha_l-\alpha_h)]v_l}{4}$$

which is trivially satisfied by the binding no mimicry condition (27). If r_s is relatively small, the high-type seller might do better by offering a menu of payments such that all choose to be credit buyers.

When there is no cost difference, in the least cost separating equilibrium the inequality (29) binds because by doing so the profit obtained from credit buyers is higher and thus makes it more costly for the low type to masquerade as the high type.

4.3.2 Positive Cost Difference

If $c > 0$, a general analysis becomes much more complicated due to the complexity of our parameter values. Below we use a numerical example to show the parameter values that ensure the existence of the least cost separating equilibrium by seller financing.

We consider an example with the following parameters: $v_h = 2, v_l = 1, \alpha_l = 0.4, \alpha_h = 0.2, \lambda = 0.5$. In Figure 3, we characterize the parameter values of β and c that ensure the existence of separating equilibrium. We can see that if the cost of the high-type seller is smaller, ceteris paribus, it is more likely that a separating equilibrium with seller financing exists. Note that we assume that the cost of low-type seller is normalized to zero, so c is the cost difference between the seller of two types. For a smaller cost difference, it is more costly for the high-type seller to prevent the low-type one from mimicking higher prices. Therefore, seller financing can be used as a complement signal to prices and reduce the signaling cost.

In addition, we can also see that for a smaller cost of funds r_s, a separating equilibrium with signaling by seller financing is also more likely to exist. This is obviously because that a lower cost of funds reduces the signaling cost when seller financing is used as a signal. Especially, when r_s is close to zero, signaling by seller financing is always better than signaling by price alone to the high-type seller since the signaling cost is reduced because of a low cost of funds.

5 Discussion

In the previous analysis, we restrict our attention to the analysis of separating equilibria. In this section, we will discuss the possibility of pooling equilibrium in our model. We show that there exists no pooling equilibrium that satisfies the intuitive criterion. In addition, we will compare the performance of signaling by seller financing with other signals of good quality, such as warranty and money-back guarantee.

5.1 Pooling Equilibria

We consider a model in which the return of the investment good varies across buyers and the seller uses price alone as a signal of good quality. Suppose there exists a pooling equilibrium p^* and let R_r^*, R_p^* be the reservation price of a rich buyer and poor buyer respectively. We have

$$\begin{aligned} R_r^* &= \lambda R_a^h + (1-\lambda) R_a^l \\ R_p^* &= \lambda R_b^h + (1-\lambda) R_b^l \end{aligned}$$

where λ is the prior probability that a good is of high quality. Let π_l^p, π_h^p be the equilibrium profit of the type l and type h seller in a pooling equilibrium, i.e.,

$$\begin{aligned} \pi_l^p &= [\beta(1-\frac{p^*}{R_r^*}) + (1-\beta)\,(1-\frac{p^*}{R_p^*})]p^* \\ \pi_h^p &= [\beta(1-\frac{p^*}{R_r^*}) + (1-\beta)\,(1-\frac{p^*}{R_p^*})](p^*-c) \end{aligned}$$

For simplicity, we consider a special case in which all buyers are liquidity-constrained, i.e., $\beta = 0$[10]. Suppose that there exists a price p at which a type l seller can make a higher profit in this pooling equilibrium than its profit under the most favorable beliefs. That is, the price p must satisfy the following inequality

[10] We first show that no pooling equilibrium exists in this special case, and then this non-existence result can be generalized to a general case.

$$\pi(l,1,p) = (1 - \frac{p}{R_b^h})p \leq \pi_l^p$$

This also implies that p has to satisfy

$$p \geq p_a \text{ or } p \leq p_b$$

where

$$\begin{aligned} p_a &= \frac{R_b^h}{2} + \frac{R_b^h}{2}\sqrt{1 - \frac{4\pi_l^p}{R_b^h}} \\ p_b &= \frac{R_b^h}{2} - \frac{R_b^h}{2}\sqrt{1 - \frac{4\pi_l^p}{R_b^h}} \end{aligned}$$

Similarly, suppose there exists a price p at which a type h seller can make a higher profit when it is recognized, i.e., the buyer's belief $\mu = 1$, than what it makes in a pooling equilibrium. In other words,

$$\pi(h,1,p) = (1 - \frac{p}{R_b^h})\,(p - c) \geq \pi_h^p$$

This also implies

$$p \in (p_c, p_d)$$

where

$$\begin{aligned} p_c &= \tfrac{(R_b^h + c)}{2} - \tfrac{R_b^h}{2}\sqrt{1 - \tfrac{4\pi_l^p}{R_b^h} + (\tfrac{c}{R_b^h})^2[1 + \tfrac{2R_b^h}{c}(1 - \tfrac{2p^*}{R_p^*})]} \\ p_d &= \tfrac{(R_b^h + c)}{2} + \tfrac{R_b^h}{2}\sqrt{1 - \tfrac{4\pi_l^p}{R_b^h} + (\tfrac{c}{R_b^h})^2[1 + \tfrac{2R_b^h}{c}(1 - \tfrac{2p^*}{R_p^*})]} \end{aligned}$$

If $c > 0$, then we can see that $p_d > p_a$, and there exist a price $p \in [p_a, p_d]$ at which the type h seller would like to deviate to p, while the type l one prefers the pooling equilibrium. Thus, p is equilibrium dominated for the type l but not type h. Thus, we can conclude that as long as $c > 0$, there exists no pooling equilibrium that satisfies the intuitive criterion.

Proposition 7. *For any $c > 0$, there exists no pooling equilibrium that satisfies the intuitive criterion.*

By showing that no pooling equilibrium exists in this special case, we can show, by a similar argument, that there exists no pooling equilibrium in a more general setting of our model.

5.2 Comparison with a Money-Back Guarantee

In our model, seller financing can be used as a signal of the quality of an investment good because a low quality good is more likely to be defective, i.e., yielding a zero return. Thus, it is more costly for a low quality seller to masquerade as a high quality one. The role of seller financing as a signal is similar to that of a warranty or money-back guarantee, since all these signals are all default-related. In this subsection, we will compare our results with those in Moorthy and Srinivasan (1995), where a seller uses a money-back guarantee as a signal of product quality.

In Moorthy and Srinivasan (1995), they compare the signaling performance of (1) price alone and (2) price with a money-back guarantee. They show that when buyers have heterogenous valuations for the product, signaling by price alone is possible. However, under some conditions signaling by price with a money-back guarantee can perform better; i.e., a money-back-guarantee is a useful supplement signal to prices.

If a seller provides a money-back guarantee to buyers, the buyer's reservation price for a high quality good is the return when it is working, i.e., v_h. Note that when a seller provides a money-back guarantee, the assumption of competitive loan market ensures that the loan rate of financing is zero, since the lender bears no default risk under a money-back guarantee provided by the seller. Let $\pi_m(h, 1, p)$ denote the profit function of the high-type seller when it is correctly identified. Then we have the following :

$$\pi_m(h, 1, p) = (1 - \frac{p}{v_h})((1 - \alpha_h)p - c)$$

Using the similar argument in the previous section, we can see that in the least cost separating equilibrium the high-type seller charges a price p^h, where p^h is the solution to the following optimization problem [MP3].

$$\begin{aligned} \text{[MP3]} \ \max_p \ & (1 - \frac{p}{v_h})((1 - \alpha_h)p - c) \\ s.t. \ & (1 - \frac{p}{v_h})((1 - \alpha_l)p) \leq \frac{1}{4} R_b^l \\ & p \leq v_h \end{aligned}$$

Let $p_m^* = Arg\max_p \pi_m(h, 1, p) = \frac{1}{2}\left(v_h + \frac{c}{1-\alpha_h}\right)$ be the optimal price of the high-type seller when good quality is commonly known. In addition, define

$$\overline{p}_m = \frac{1}{2}(v_h + \frac{\sqrt{(1 - \alpha_l)v_h[(1 - \alpha_l)v_h - R_b^l]}}{(1 - \alpha_l)})$$

By a similar argument in the previous analysis, we can see that in the least cost separating equilibrium a high-type seller charges a price p^h such that

$$p^h = \max\{p_m^*, \overline{p}_m\}$$

In our model, a seller offers a menu of payment terms which ensures that rich buyers take the cash price while credit buyers take the credit price. Compared to signaling by a money-back warranty, the seller in our model bears the default risk only from poor buyers. However, buyers' reservation price for the good is lower when a money-back guarantee is not provided by the seller.

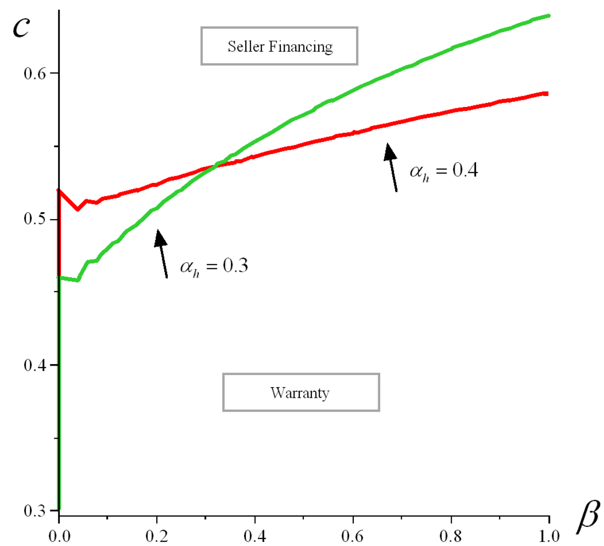

Figure 4: Comparision with signaling by a money-back guarantee

5.2.1 Numerical Example

Below we compare the equilibrium profit of the high-type seller under two cases with signaling by a money-back guarantee and signaling by seller financing. We consider a numerical example with the following parameters: $v_h = 2, v_l = 1.5, \alpha_l = 0.5, r_s = 0, \lambda = 0.5$. In Figure 4, we can see the parameter values that ensure signaling by seller financing outperforms signaling by a warranty or money-back guarantee.

Note that the equilibrium profit of the high-type seller from signaling by seller financing and by a money-back guarantee is obtained by solving the optimization problems [MP2] and [MP3] respectively. By Comparing two optimization problems [MP2] and [MP3] and using the envelop theorem, we can see that

$$\begin{aligned}
\frac{\partial \pi(h, 1, p_1^h, p_2^h)}{\partial \alpha_h} &= -(1-\beta)p_2^h(1 - \frac{p_2^h}{v_h}) \leq 0 \\
\frac{\partial \pi_m(h, 1, p^h)}{\partial \alpha_h} &= -p^h(1 - \frac{p^h}{v_h}) \leq 0
\end{aligned}$$

Note that when the seller provides a menu of payment terms to buyers, the seller can charge different prices to rich and poor buyers and the gap between the signaling price and optimal price would be smaller. That is, we have

$$\frac{1}{2}(v_h + c) \leq p_2^h \leq p^h$$

where $\frac{1}{2}v_h$ is the optimal price when the buyer's reservation price for the good is v_h. By concavity of the profit function, we can see that

$$p_2^h(1 - \frac{p_2^h}{v_h}) > p_h(1 - \frac{p^h}{v_h})$$

If β is sufficiently large, then $(1-\beta)p_2^h(1-\frac{p_2^h}{v_h})$ is smaller relative to $p^h(1-\frac{p^h}{v_h})$, and thus a higher probability that the good might be defective is less damaging to the seller by using seller financing as a signal.

Note that α_h is the probability that a high quality good might be defective. For a smaller α_h, it is better for a high-type seller to provide warranty and therefore raise the reservation prices of all buyers to v_h. On the contrary, for a sufficiently large α_h, signaling by seller financing has a smaller signaling cost and therefore outperforms signaling by warranty or money-back guarantee under some conditions. This can be also easily understood by the fact the payment terms enable rich buyers to be cash buyers and reduce the seller's risk when the good yields a zero return.

6 Conclusion

In this paper, we investigate the role of seller financing as quality assurance. We study the questions about why a seller is willing to provide financing to buyers when there exists a competitive loan market. We abstract from traditional explanations that focus on the financial advantages of seller financing or the use of seller financing as a price discrimination device. Instead, we show that seller financing can be used as a signal of unobserved quality.

If the good quality is privately known to the seller, we show that seller financing in the form of contractual payment terms can be used as a signal of the quality of an investment good when the return is uncertain at the time of purchase and buyers are liquidity-constrained. We assume that good itself is the only collateral that the credit buyers can pledge for their loans and the low quality good is more likely to be defective than the high quality one. Therefore, seller financing can be used as a signal of good quality because it is more costly for the low quality seller to offer financing. We characterize the Perfect Bayesian Equilibrium using the "intuitive criterion" as a refinement criterion. We show that signaling by seller financing is necessary for the existence of separating equilibrium in the model with homogenous buyers. In the model with heterogenous buyers, even though signaling by cash price alone is possible, signaling by seller financing is more efficient, as long as the cost of funds to the seller or the cost difference is not large.

Some extensions from our model are possible. For example, we assume that financial institutions do not observe seller's strategy in our model. A more interesting case is to further investigate whether seller financing can be an equilibrium outcome if financial institutions are acting players of the game. In addition, to consider other general default arrangements when the good turns out to be defective is another possible direction for future research.

Appendices

A Proofs of Lemmas and Propositions

A.1 Proof of Lemma 1

Note that if financial institutions anticipate that there exists a separating equilibrium in the investment good market, i.e., buyers can distinguish the quality of the good, then buyers never default as long as the good is working. Therefore, the expected profit of each financial institution can be expressed as the following:

$$\pi_f = [\psi(1-\alpha_h) + (1-\psi)(1-\alpha_l)]p(1+r_f(L)) - p \tag{A.1}$$

By the assumption of the competitive loan market, the expected profit of a financial institution in (A.1) should be zero. Thus, the equilibrium loan rate is as follows.

$$r_f = \frac{(1-\psi)\alpha_l + \psi\alpha_h}{1-\alpha_l+\psi(\alpha_l-\alpha_h)}$$

Note that r_f is independent of the loan size L. By simply taking the derivative of r_f with respect to α_h, α_l, and ψ and due to that $0<\psi<1$ and $\alpha_h<\alpha_l$, we have the following results:

$$\frac{\partial \pi_f}{\partial \alpha_h} = \frac{\psi}{(\psi\alpha_h - 1 + \alpha_l - \psi\alpha_l)^2} \geq 0 \tag{A.2}$$

$$\frac{\partial \pi_f}{\partial \alpha_l} = \frac{1-\psi}{(\psi\alpha_h - 1 + \alpha_l - \psi\alpha_l)^2} \geq 0 \tag{A.3}$$

$$\frac{\partial \pi_f}{\partial \psi} = \frac{\alpha_h-\alpha_l}{(\psi\alpha_h - 1 + \alpha_l - \psi\alpha_l)^2} \leq 0 \tag{A.4}$$

By (A.2), (A.3), and (A.4) we can conclude that π_f is increasing in α_h,α_l,decreasing in ψ and independent of the loan size L.

A.2 Proof of Proposition 4

Recall that the conditions to ensure the existence of separating equilibria are as the following:

$$[\beta(1-\frac{p^h}{R_a^h}) + (1-\beta)(1-\frac{p^h}{R_b^h})]p^h \leq \frac{1}{4}R_b^l \tag{A.5}$$

$$\frac{1}{4}\frac{(R_b^l-c)^2}{R_b^l} \leq [\beta(1-\frac{p^h}{R_a^h}) + (1-\beta)(1-\frac{p^h}{R_b^h})](p^h-c) \tag{A.6}$$

For any p^h that satisfies $(A.5)$, p^h has to meet one of the two inequalities: $p^h \geq \overline{p}^h$ or $p^h \leq \underline{p}^h$, where

$$\begin{aligned}
\overline{p}^h &= \frac{1}{2}\frac{R_a^hR_b^h + \sqrt{R_a^hR_b^h[R_a^h(R_b^h-R_b^l)+\beta R_b^l(R_a^h-R_b^h)]}}{(1-\beta)R_a^h+\beta R_b^h} \\
\underline{p}^h &= \frac{1}{2}\frac{R_a^hR_b^h - \sqrt{R_a^hR_b^h[R_a^h(R_b^h-R_b^l)+\beta R_b^l(R_a^h-R_b^h)]}}{(1-\beta)R_a^h+\beta R_b^h}
\end{aligned}$$

We can also show that (A.6) holds if and only if $p \in [p^h_{\min}, p^h_{\max}]$, where

$$p^h_{\max} = \frac{1}{2}\frac{1}{(1-\beta)R^h_a + \beta R^h_b}\left\{[R^h_b R^h_b + c(R^h_a - \beta R^h_a + \beta R^h_b)] + \frac{1}{R^l_b}\sqrt{AB}\right\} \tag{A.7}$$

$$p^h_{\min} = \frac{1}{2}\frac{1}{(1-\beta)R^h_a + \beta R^h_b}\left\{[R^h_b R^h_b + c(R^h_a - \beta R^h_a + \beta R^h_b)] - \frac{1}{R^l_b}\sqrt{AB}\right\} \tag{A.8}$$

and

$$A = R^l_b(R^h_a R^h_b - R^h_a R^l_b + \beta R^h_a R^l_b - \beta R^h_b R^l_b) \geq 0 \tag{A.9}$$

$$B = \beta c^2(R^h_a - R^h_b) + R^h_a(R^h_b R^l_b - c^2) \geq 0 \tag{A.10}$$

If $p^h_{\max} > \overline{p}^h$, we can see that both $(A.5)$ and $(A.6)$ hold for any $p^h \in [\overline{p}^h, p^h_{\max}]$. By $(A.7)$ and $(A.8)$, we can verify that when $c = 0$,

$$\begin{aligned} p^h_{\max} &= \overline{p}^h = \frac{1}{2}\frac{R^h_a R^h_b + \sqrt{R^h_a R^h_b[R^h_a(R^h_b - R^l_b) + \beta R^l_b(R^h_a - R^h_b)]}}{(1-\beta)R^h_a + \beta R^h_b} \\ p^h_{\min} &= \underline{p}^h = \frac{1}{2}\frac{R^h_a R^h_b - \sqrt{R^h_a R^h_b[R^h_a(R^h_b - R^l_b) + \beta R^l_b(R^h_a - R^h_b)]}}{(1-\beta)R^h_a + \beta R^h_b} \end{aligned}$$

Furthermore, we can show that $p^h_{\max}$ is increasing in c. By $(A.7)$, we have

$$\frac{dp^h_{\max}}{dc} = -\frac{1}{2}\frac{1}{\sqrt{AB}}c(R^h_a R^h_b - R^h_a R^l_b + \beta R^h_a R^l_b - \beta R^h_b R^l_b) + \frac{1}{2} \tag{A.11}$$

Let the term $c(R^h_a R^h_b - R^h_a R^l_b + \beta R^h_a R^l_b - \beta R^h_b R^l_b)$ be denoted by K. Then, by $(A.9)$ we have $A = R^l_b(R^h_a R^h_b - R^h_a R^l_b + \beta R^h_a R^l_b - \beta R^h_b R^l_b) \geq K = c(R^h_a R^h_b - R^h_a R^l_b + \beta R^h_a R^l_b - \beta R^h_b R^l_b)$, due to that $c \leq R^l_b$. Similarly, by $(A.10)$, we have

$$\begin{aligned} B - K &= \beta c^2(R^h_a - R^h_b) + R^h_a(R^h_b R^l_b - c^2) - c(R^h_a R^h_b - R^h_a R^l_b + \beta R^h_a R^l_b - \beta R^h_b R^l_b) \\ &= -\left(c - R^l_b\right)\left(R^h_a R^h_b + cR^h_a - c\beta R^h_a + c\beta R^h_b\right) \geq 0, \text{ for } c \leq R^l_b \end{aligned}$$

Thus, we have shown that $\frac{1}{\sqrt{AB}}c(R^h_a R^h_b - R^h_a R^l_b + \beta R^h_a R^l_b - \beta R^h_b R^l_b) \leq 1$, and consequently $\frac{dp^h_{\max}}{dc} \geq 0$.

The fact that $p^h_{\max}$ is increasing in c implies that $p^h_{\max} > \overline{p}^h$ for any $c > 0$ and therefore there exists some p^h satisfying both $(A.5)$ and $(A.6)$. This completes the proof of the existence of separating equilibria.

A.3 Proof of Proposition 5

Recall that the optimization problem of the type h seller is

$$[\text{MP2}] \quad \max_{p_1,p_2} \quad \beta(1-\frac{p_1}{R_a^h})(p_1-c)+(1-\beta)\left[(1-\frac{p_2}{v_h})\left[\frac{(1-\alpha_h)}{(1+r_s)}p_2-c\right]\right] \tag{A.12}$$

$$s.t. \;\; \beta(1-\frac{p_1}{R_a^h})p_1+(1-\beta)\left[(1-\frac{p_2}{v_h})\left[\frac{(1-\alpha_l)}{(1+r_s)}p_2\right]\right] \le \frac{1}{4}R_b^l \tag{A.13}$$

$$p_2 \le \left(\frac{1}{1-\alpha_l}\right)p_1 \tag{A.14}$$

$$\left(\frac{1}{1-\alpha_h}\right)p_1 \le p_2 \tag{A.15}$$

The slope of the iso-profit curve of $\pi(l,1,p_1,p_2)$, denoted by $\frac{dp_2}{dp_1}|_{\pi^l}$, is as the following:

$$\begin{aligned} \frac{dp_2}{dp_1}|_{\pi^l} &= \frac{\beta v_h\left(2p_1+2p_1r_s-R_a^h-R_a^hr_s\right)}{R_a^h\left(-2p_2+2\alpha_hp_2+2\beta p_2-2\beta\alpha_hp_2+v_h-\alpha_hv_h-\beta v_h+\beta\alpha_hv_h\right)} \\ &= \frac{\beta v_h(1+r_s)(2p_1-R_a^h)}{R_a^h(\alpha_l-1)\left(1-\beta\right)\left(2p_2-v_h\right)} \end{aligned} \tag{A.16}$$

Note that $\frac{dp_2}{dp_1}|_{\pi^l} \le 0$ for any (p_1,p_2) such that $p_1 \ge \frac{1}{2}R_a^h$ and $p_2 \ge \frac{1}{2}v_h$.

Similarly, the slope of the iso-profit curve $\pi(h,1,p_1,p_2)$, denoted by $\frac{dp_2}{dp_1}|_{\pi^h}$, is as the following:

$$\begin{aligned} \frac{dp_2}{dp_1}|_{\pi^h} &= -\frac{\beta\, v_h\left(2p_1+2p_1r_s-c-cr_s-R_a-R_a^hr_s\right)}{R_a^h\left(2p_2-2\,\alpha_hp_2-v_h+\alpha_hv_h-c-2\beta p_2+2\beta\alpha_hp_2+\beta v_h-\beta\alpha_hv_h+\beta c\right)} \\ &= \frac{\beta v_h\left(1+r_s\right)\left(2p_1-R_a^h-c\right)}{R_a^h\left(1-\beta\right)\left(c+cr_s-2p_2+v_h+2\alpha_hp_2-\alpha_hv_h\right)} \end{aligned} \tag{A.17}$$

Note that $\frac{dp_2}{dp_1}|_{\pi^h} \le 0$ for any (p_1,p_2) such that $p_1 \ge \frac{1}{2}(R_a^h+c)$ and $p_2 \ge \frac{1}{2}(v_h+\frac{(1+r_s)c}{1-\alpha_h})$.

By (A.16) and (A.17), we have

$$\begin{aligned} &\frac{dp_2}{dp_1}|_{\pi^l}-\frac{dp_2}{dp_1}|_{\pi^h} \\ = \;& \frac{\beta(1+r_s)}{(1-\alpha_h)(1-\beta)}\left[\frac{(2p_1-R_a^h)}{(\alpha_l-1)\left(1-\beta\right)\left(2p_2-v_h\right)}-\frac{\left(2p_1-R_a^h-c\right)}{\left(c+cr_s-2p_2+v_h+2\alpha_hp_2-\alpha_hv_h\right)}\right] \end{aligned} \tag{A.18}$$

If $\frac{dp_2}{dp_1}|_{\pi^l} \le \frac{dp_2}{dp_1}|_{\pi^h}$, the iso-profit curve of $\pi(h,1,p_1^h,p_2^h)$ is flatter than that of $\pi(h,1,p_1^h,p_2^h)$. The solution to the optimization problem and thus least cost separating equilibrium occurs when $(A.13)$ and $(A.15)$ bind. This completes the proof of this proposition. On the contrary, $\frac{dp_2}{dp_1}|_{\pi^l} \ge \frac{dp_2}{dp_1}|_{\pi^h}$, then $(A.13)$ and $(A.14)$ bind in equilibrium.

A.4 Proof of Proposition 6

If $c=0$, by (A.18) we have

$$\begin{aligned} &\frac{dp_2}{dp_1}|_{\pi^l}-\frac{dp_2}{dp_1}|_{\pi^h} \\ = \;& \frac{\beta(1+r_s)}{(1-\alpha_h)(1-\beta)}\left[\frac{\left(2p_1-R_a^h\right)}{(1-\alpha_h)(2p_2-v_h)}-\frac{(2p_1-R_a^h)}{(1-\alpha_l)\left(1-\beta\right)\left(2p_2-v_h\right)}\right] \end{aligned} \tag{A.19}$$

We can see that for any $p_1 \geq \frac{1}{2}R_a^h$ and $p_2 \geq \frac{1}{2}v_h$, $\frac{dp_2}{dp_1}|_{\pi^l} - \frac{dp_2}{dp_1}|_{\pi^h} \leq 0$. This implies that the iso-profit curves of $\pi(h,1,p_1,p_2)$ is flatter than that of $\pi(l,1,p_1,p_2)$. By observing the optimization problem [MP2], we can see that in the solution to [MP2], both (27) and (29) bind.

Besides, the sufficient condition for the existence of the separating equilibrium is translated to

$$\begin{aligned} \pi(h,1,p_1^h,p_2^h) &\geq \max\left\{\frac{(1-\alpha_h)v_l}{4(1+r_s)}, \frac{v_l}{4(1+r_f)}\right\} \\ &= \max\left\{\frac{(1-\alpha_h)v_l}{4(1+r_s)}, \frac{1}{4}[1-\alpha_l+\lambda(\alpha_l-\ \alpha_h)]v_l\right\} \end{aligned}$$

By the no mimicry condition, we can see that $\pi(h,1,p_1^h,p_2^h) \geq \frac{1}{4}[1-\alpha_l+\lambda(\alpha_l-\ \alpha_h)]v_l$ in any candidate solutions to [MP2]. If r_s is sufficiently large such that

$$\frac{(1-\alpha_h)v_l}{4(1+r_s)} \leq \frac{1}{4}[1-\alpha_l+\lambda(\alpha_l-\ \alpha_h)]v_l \tag{A.20}$$

then there always exists a least cost separating equilibrium. $(A.20)$ can be translated to

$$r_s \geq \frac{(\alpha_l-\alpha_h)(1-\lambda)}{[1-\alpha_l+\lambda(\alpha_l-\ \alpha_h)]}$$

and this completes the proof.

References

BAGWELL, K. (1988): "Advertising and limit pricing," *Rand Journal of Economics*, 19(1), 59–71.

BAGWELL, K., AND M. RIORDAN (1991): "High and Declining Prices Signal Product Quality," *The American Economic Review*, 81(1), 224–239.

BALACHANDER, S. (2001): "Warranty Signalling and Reputation," *Management Science*, 47(9), 1282–1289.

BOOM, A. (2004): "Download for Free - When Do Providers of Digital Goods Offer Free Samples?," Discussion Papers 70, Free University of Berlin.

BRENNAN, M., V. MIKSIMOVIC, AND J. ZECHNER (1988): "Vendor Financing," *The Journal of Finance*, 43(5), 1127–1141.

CHE, Y., AND I. GALE (2000): "The Optimal Mechanism for Selling to a Budget-Constrained Buyer," *Journal of Economic Theory*, 92(2), 198–233.

CHO, I., AND D. KREPS (1987): "Signaling Games and Stable Equilibria," *The Quarterly Journal of Economics*, 102(2), 179–222.

DYNAN, K., K. JOHNSON, AND S. SLOWINSKI (2002): "Survey of Finance Companies, 2000," *Federal Reserve Bulletin*, 18.

GAL-OR, E. (1989): "Warranties as a Signal of Quality," *The Canadian Journal of Economics*, 22(1), 50–61.

IOSSA, E., AND G. PALUMBO (2004): "Product Quality, Lender Liability, and Consumer Credit," *Oxford Economic Papers*, 56, 331–343.

KIRMANI, A., AND A. RAO (2000): "No Pain, No Gain: A Critical Review of the Literature on Signaling Unobservable Product Quality," *Journal of Marketing*, 64(2), 66–79.

LEE, Y., AND J. STOWE (1993): "Product Risk, Asymmetric Information, and Trade Credit," *The Journal of Financial and Quantitative Analysis*, 28(2), 285–300.

LINNEMER, L. (1998): "Entry Deterrence, Product Quality: Price and Advertising as Signals," *Journal of Economics & Management Strategy*, 7(4), 615–645.

——— (2002): "Price and Advertising as Signals of Quality when Some Consumers are Informed," *International Journal of Industrial Organization*, 20(7), 931–947.

LONG, M., I. MALITZ, AND S. RAVID (1993): "Trade Credit, Quality Guarantees, and Product Marketability," *Financial Management*, 22(4), 117–127.

MILGROM, P., AND J. ROBERTS (1982): "Limit Pricing and Entry under Incomplete Information: An Equilibrium Analysis," *Econometrica*, 50(2), 443–59.

Milgrom, P., and J. Roberts (1986): "Price and Advertising Signals of Product Quality," *The Journal of Political Economy*, 94(4), 796–821.

Moorthy, S., and K. Srinivasan (1995): "Signaling Quality with a Money-Back Guarantee: The Role of Transaction Costs," *Marketing Science*, 14(4), 442–466.

Nelson, P. (1974): "Advertising as Information," *The Journal of Political Economy*, 82(4), 729–754.

Petersen, M., and R. Rajan (1997): "Trade credit: theories and evidence," *Review of Financial Studies*, 10(3), 661–691.

Riley, J. (2001): "Silver Signals: Twenty-Five Years of Screening and Signaling," *Journal of Economic Literature*, 39(2), 432–478.

Sen, A. (1998): "Seller Financing of Consumer Durables," *Journal of Economics & Management Strategy*, 7(3), 435–460.

Smith, J. (1987): "Trade Credit and Informational Asymmetry," *The Journal of Finance*, 42(4), 863–872.

Soberman, D. (2003): "Simultaneous Signaling and Screening with Warranties," *Journal of marketing research*, 40(2), 176–192.

Tirtiroglu, D., and D. N. Laband (2004): "The quality assurance role of seller financing: evidence from second mortgages," *Journal of Housing Economics*, 13(3), 208–225.