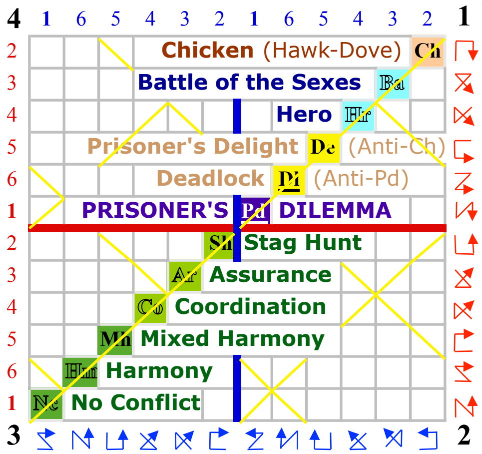

Visualizing the Topology of 2 X 2 Games

From Prisoner's Dilemma to Win-win

Bryan Bruns[1]

Abstract: As a tool for institutional analysis and design, this paper presents additional visualizations of Robinson and Goforth's topology of ordinal 2x2 games linked by swaps in adjoining payoffs, in a modified, more accessible version of their "periodic table" display, including a complete set of game families and common names. The visualizations show the elegant arrangement of game properties in the topology, and locate Prisoner's Dilemma and other games most studied by game theory research within the full set of strict ordinal 2x2 games, which are mostly asymmetric, mostly with mixed interests, and a fourth of which have win-win equilibria. Additional families of games, categorized by payoffs at Nash Equilibria, illustrate further order in the topology. The topology provides a framework for index numbers and common names to identify similar and related games, which could contribute to cumulative research and understanding of relationships among 2x2 games. For the design of institutional mechanisms, visualization of the topology can help to understand re-alignments of incentive structures that might be reached through negotiation, side payments, or changes in information, technology, preferences, or rules; mapping potential transformations into the adjacent possible.

Keywords: Game theory, topology of 2x2 games, asymmetric games, strategic moves between games, social dilemmas, institutional analysis and design

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Prisoner's Dilemma and a small number of other two-player two-move (2x2) games such as Chicken, Battle of the Sexes, and Stag Hunts have been central conceptual models in the increasing application of game theory in economics, political science, evolutionary biology and other fields. The topology developed by Robinson and Goforth (2003c, 2004g; 2004d, 2005e, 2005f; 2004a, 2009b; D Robinson, D Goforth, and Cargill 2007) goes beyond earlier taxonomy (Rapoport and Guyer 1966; Rapoport, Guyer, and Gordon 1976) and typology (Brams 1994), to provide an elegant way to understand relationships among not only these commonly studied symmetric games, but also among the much larger set of asymmetric games.[2]

The topology, including the payoff families discussed in this paper, offers a useful tool for understanding the properties of games, and their relationships, and so can aid teaching and analysis. The topology maps the strategic moves between games when payoffs shift enough to switch the ordinal ranking of payoffs. Such transmutations could occur as a result of new information, technological innovation and other changes in transaction costs, unilateral action, negotiation, or deliberate institutional redesign. The topology helps to understand the robustness or instability of game outcomes in response to changes that switch payoff ranks. It can help understand the potentials for deliberately transforming social dilemmas into win-win games, through swaps in payoff ranks that result from redesigning governance rules. For 2x2 games, the topology maps what Stuart Kauffman (2002) calls the adjacent possible, generalizing a concept from chemical reaction networks to describe the reachable system states just one step away. A better understanding of potential transformations has practical implications for the study and design of governance mechanisms, where an important goal can be understood as properly aligning incentive structures and fitting institutional arrangements to the characteristics of assets and transactions, including the risks of subsequent opportunistic behavior (Williamson 1996).[3]

The periodic table of 2x2 games prepared by Goforth and Robinson[4] illustrates the topology efficiently and elegantly, using order graphs to diagram payoffs and inducement slopes. However, the use of order graphs may make the table less than intuitive for those who have not

[2] The implications of this topological structure at the heart of game theory do not yet seem to be widely recognized. Subsequent work referring to the topology includes Perlo-Freeman (2006), Brams and Kilgour (2009, 17,18), and Simpson (2010, Ch. 3).
[3] For a review of research on social dilemmas, including sanctions and other structural solutions, see Kollock (1998).
[4] A poster and an interactive version, with a manual (DJ Goforth and DR Robinson 2009a) are available at http://www.cs.laurentian.ca/dgoforth/home.html

learned to interpret the order graphs in terms of numeric payoffs. This paper provides a modified version (Table 1), showing numeric payoffs and using additional visualization methods (mainly from Tufte 1983) to make the display more accessible and informative.

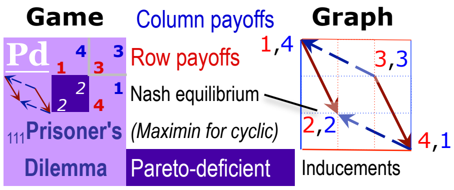
Goforth and Robinson explain how the topology conveniently groups games by important properties including number of dominant strategies and Nash Equilibria, and groups similar games such as Battles of the Sexes, Stag Hunts, Cyclic games and those with 4-4 Nash Equilibria. They extend the Battle of the Sexes games and the Coordination games (also called Stag Hunt or Assurance) to include asymmetric variations. They also identify an interesting family of games composed of Prisoner's Dilemma and its asymmetric siblings and cousins, “Alibi” games, all with poor, Pareto-inferior, outcomes (2-2 and 3-2). To assist in using the topology as a tool for institutional analysis and design, this paper categorizes additional families and subfamilies based on the distribution of outcomes at Nash Equilibria:

- *Tragic* games, which have poor (3-2) outcomes, like Alibi games, but without Pareto-superior alternatives; forming an extended Prisoner’s Dilemma Family
- *Second Best* games where both players get their second ranked payoff (3-3) at Nash equilibria, exemplified by Prisoner’s Delight (Binmore 2005: 63).
- *Biased games,* with somewhat unequal (4-3) Nash Equilibria, including three subfamilies: Battles of the Sexes, Altruistic, and Self-serving.
- *Altruistic* games, exemplified by Buchanan’s Active Samaritan’s Dilemma (1977), where a player with a dominant strategy receives their second-ranked payoff, and the other player gets their first preference, at a single 4-3 Nash equilibrium.
- *Self-serving* games where only one player has a dominant strategy, and gets their top-ranked payoff and the other gets their second choice, at a single 4-3 Nash equilibrium.
- *Unfair* games, with highly unequal 4-2 Nash Equilibria, where following dominant strategies would lead one player to get their top payoff and the other their third choice.
- *Harmonious* subfamily with a single 4-4 Nash Equilibrium. Together with Stag Hunt games with two Nash Equilibria (also called Coordination or Assurance games), these make up the family of Win-win games with 4-4 Nash Equilibria (also called “no conflict,” “trivial, or “boring”), which have been less interesting for game theorists but represent important objectives in aligning incentives for collective action.

The following sections introduce the topology and how it maps relationships among games linked by payoff swaps, and then discuss structures in the topology, categorization of payoff families, visualization methods, common names for games, and conclusions.

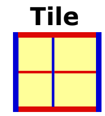
2 X 2 Games

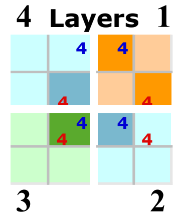
1. Visualizing the Adjacent Possible in the Topology of Two-player, Two-strategy Games



Pareto-optimal outcomes in **bold** font

Adjacent games are neighbors by payoff swaps
1↔2 swaps form tiles of 4 games
2↔3 swaps join tiles into 4 layers
3↔4 swaps link layers
Layers differ by alignment of 4s
Each layer is a torus, table is a torus
Layers scrolled to center Prisoner's Dilemma





Families	Harmonious
1. **Win-win** 4,4	Stag Hunt
2. **Biased** 4,3	Battle
Self-serving	Altruistic
3. Second Best 3,3	
4. **Unfair** 4,2	Winner Loser
5. PD Family	Tragic 3,2
Pd 2,2	Alibi 3,2
6. **Cyclic**	

Chart

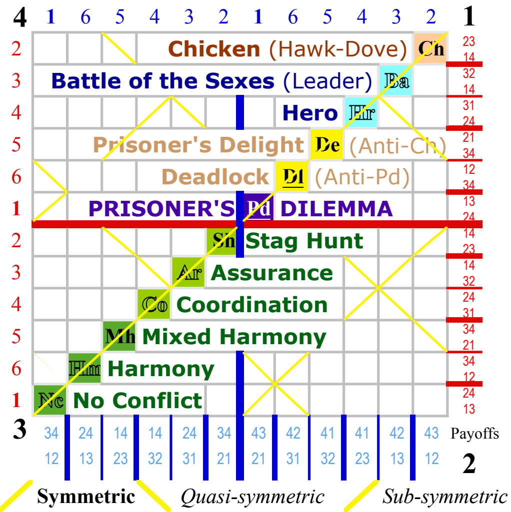
4												1
Hg 421 Hegemony	Sd 426 SamaritanD	Qt 425 QuasiAltruis	Bc 424 BiasedCycle	Ic 423 InferiorCycle	En 422 Endless	Cb 121 Called Bluff	Bu 126 Bully	Ut 125 Unfair Type	Sb 124 Skewed Ba.	Ab 123 Asym Battle	Ch 122 **Chicken**	2 3 ↑↓ 1 4 **2**
Sn 431 Samson	Ge 436 Generous	Ge 435 Generous	Qc 434 Quasi Cycle	Tc 433 TragicCycle	In 432 Inspector	Pn 131 Patron	Pr 136 Protector	Pr 135 Protector	Qb 134 QuasiBattle	Ba 133 **Battle**	Ab 132 Asym Battle	3 2 ↑↓ 1 4 **3**
Dh 441 Delilah	Ge 446 Generous	Ge 445 Generous	Pc 444 PursuitCycle	Fs 443 Fixed Sum	Mc 442 MissileCrisis	Pn 141 Patron	Pr 146 Protector	Pr 145 Protector	Hr 144 **Hero**	Qb 143 QuasiBattle	Sb 142 Skewed Ba.	3 1 ↑↓ 2 4 **4**
Hs 451 Hostage	Be 456 Benevolent	Su 455 Subsym Be	Se 454 SecondBest	Se 453 SecondBest	Bb 452 Big Bully	Tr 151 Tragedy	Ad 156 Asym Del.	**De** 155 **P. Delight**	Pr 154 Protector	Pr 153 Protector	Ut 152 Unfair Type	2 1 ↓↓ 3 4 **5**
Mt 461 BlackmailTy	Bt 466 Benev.Type	Be 465 Benevolent	Se 464 SecondBest	Se 463 SecondBest	Ha 462 Hamlet	Tc 161 TotalConflict	**Dl** 166 **Deadlock**	Ad 165 Asym Del.	Pr 164 Protector	Pr 163 Protector	Bu 162 Bully	1 2 ↓↓ 3 4 **6**
Ht 411 Hegm.Type	At 416 AltruistType	Au 415 Altruist	Re 414 Revelation	Ai 413 Alibi	Ap 412 Asym Pd	Pd 111 **Prisoner'sD**	Tc 116 TotalConflict	Tr 115 Tragedy	Pn 114 Patron	Pn 113 Patron	Cb 112 Called Bluff	1 3 ↓↓ 2 4 **1**
Lc 321 LowConflict	Al 326 Aligned	Pl 325 PureAligned	Ac 324 Asym Coord	Aa 323 Asym Assur	**Sh** 322 **Stag Hunt**	Ap 221 Asym Pd	Ha 226 Hamlet	Bb 225 Big Bully	Mc 224 MissileCrisis	In 223 Inspector	En 222 Endless	1 4 ↓↑ 2 3 **2**
Mu 331 Mutual	Pv 336 Privileged	Pv 335 Privileged	Po 334 Pure Coord	Ar 333 **Assurance**	Aa 332 Asym Assur	Ai 231 Alibi	Se 236 SecondBest	Se 235 SecondBest	Fs 234 Fixed Sum	Tc 233 TragicCycle	Ic 232 InferiorCycle	1 4 ↓↑ 3 2 **3**
Mu 341 Mutual	Pv 346 Privileged	Pv 345 Privileged	Co 344 **Coordination**	Po 343 PureCoord	Ac 342 Asym Coord	Re 241 Revelation	Se 246 SecondBest	Se 245 SecondBest	Pc 244 PursuitCycle	Qc 243 QuasiCycle	Bc 242 BiasedCycle	2 4 ↓↑ 3 1 **4**
Al 351 Aligned	Ah 356 Asym Hm	**Mh** 355 **Mixed Hmny**	Pv 354 Privileged	Pv 353 Privileged	Pl 352 PureAligned	Au 251 Altruist	Be 256 Benevolent	Su 255 Subsym Be	Ge 254 Generous	Ge 253 Generous	Qt 252 QuasiAltruist	3 4 ↑↑ 2 1 **5**
Pl 361 PureAligned	Hm 366 **Harmony**	Ah 365 Asym Hm	Pv 364 Privileged	Pv 363 Privileged	Al 362 Aligned	At 261 AltruistType	Bt 266 Benev.Type	Be 265 Benevolent	Ge 264 Generous	Ge 263 Generous	Sd 262 SamaritanD	3 4 ↑↑ 1 2 **6**
Nc 311 **No Conflict**	Pl 316 PureAligned	Al 315 Aligned	Mu 314 Mutual	Mu 313 Mutual	Lc 312 LowConflict	Ht 211 Hegm.Type	Mt 216 BlackmailTy.	Hs 215 Hostage	Dh 214 Delilah	Sn 213 Samson	Hg 212 Hegemony	2 4 ↑↑ 1 3 **1**
3 **1** 3→4 1→2	**6** 2→4 1→3	**5** 1→4 2→3	**4** 1→4 3←2	**3** 2→4 3←1	**2** 3→4 2←1	**1** 4←3 2←1	**6** 4←2 3←1	**5** 4←1 3←2	**4** 4←1 2→3	**3** 4←2 1→3	**2** 4←3 1→2	**2**

Game Numbers: Layer: Row: Column. Symmetric along SW-NE diagonal. Swapping Row and Column positions swaps indices, and Layer for 2&4

To find a game: Make ordinal 4>3>2>1. Put column with Row's 4 right, row with Column's 4 up. Find layer by alignment of 4s; then intersection of Row&Column payoffs.

see Robinson & Goforth 2005 *The Topology of the 2x2 Games: A New Periodic Table* www.cs.laurentian.ca/dgoforth/home.html www.BryanBruns.com/2x2chart.html

a. Twelve Symmetric Games on the Diagonal



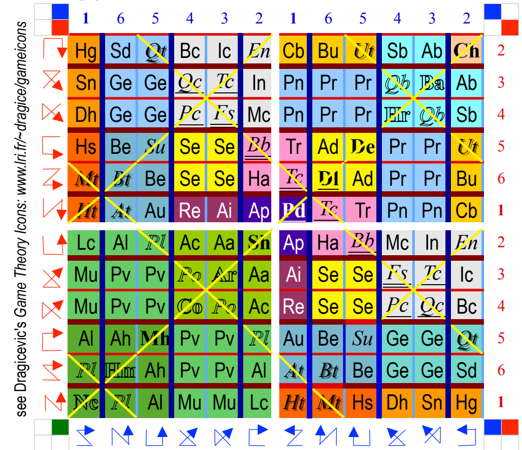
b. Dominant Strategies & Nash Equilibria

2	421	426	425	424	423	422	121	126	125	124	123	122
3	431	436	435	434	433	432	31	136	135	134	133	132
4	441	446	445	444	443	442	141	146	145	144	143	142
5	451	456	455	454	453	452	151	156	155	154	153	152
6	461	466	465	464	463	462	161	166	165	164	163	162
1	411	416	415	414	413	412	111	116	115	114	113	112
2	321	326	325	324	323	322	221	226	225	224	223	222
3	331	336	335	334	333	332	231	236	235	234	233	232
4	341	346	345	344	343	343	241	246	245	244	243	242
5	351	356	355	354	353	35	251	256	255	254	253	252
6	361	366	365	364	363	362	261	266	265	264	263	262
1	311	316	315	314	313	312	211	216	215	214	213	212

3 Dominant strategies Nash Equilibria 2

D_1 Column	D_0 None	1	0/2
D_2 Double	D_1 Row	1	1

c. 4 Layers, 12 Payoff Patterns, 144 Games



4 games per tile, 36 games and 9 tiles per layer

66 asymmetric pairs: 66 + 12 = 78 "unique" games

d. High swaps (3↔4) Link Tiles Across Layers

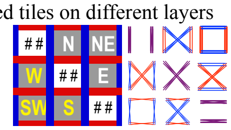
connecting equivalently-located tiles on different layers

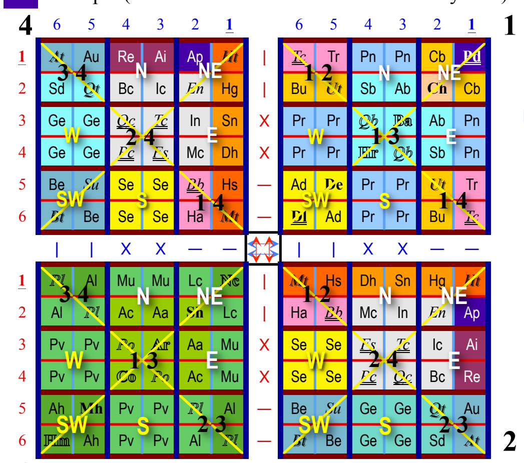
6 Hotspots double-link 2 tiles

6 Pipes link 4 tiles on 4 layers

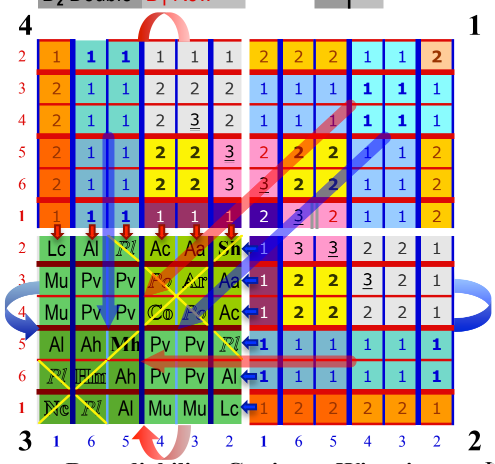
1 3 Coord.-Battle Hotspot

NE Pd Pipe (Pd scrolled to northeast corner to unify tiles)





3 High swaps realign 4s; switch row or column in tile



e. Remediability: Getting to Win-win

Swaps to reach win-win 1= single 3↔4 swap

Paths with 2 or 3 steps include 2↔3 and 1↔2 swaps

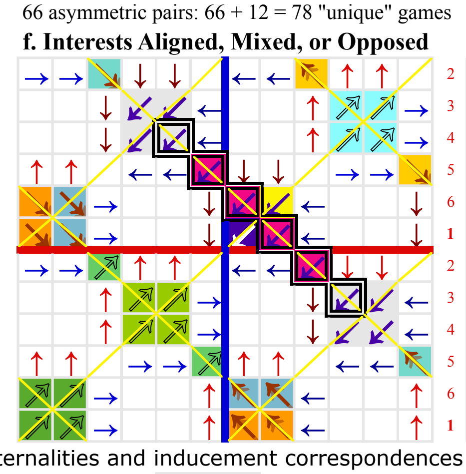
Arrows show vectors for slices of games

Bold = each player has a pathway

Pareto-efficient paths: each swap step leaves

player with same or better-ranked outcome

f. Interests Aligned, Mixed, or Opposed



Externalities and inducement correspondences

Jekyll-Hyde Type ++ Pure Cooperation

±

Pure Conflict - - Jekyll-Hyde Type

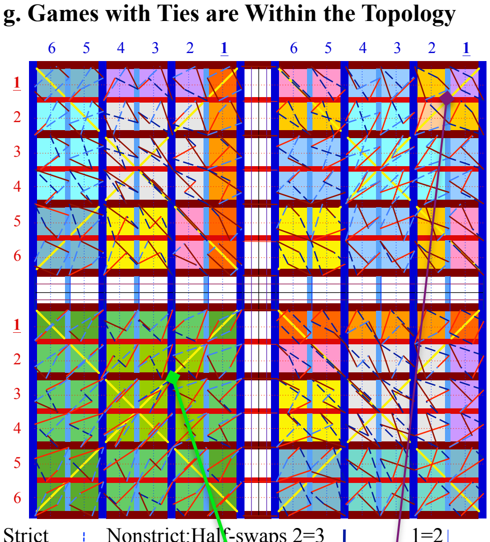
Fixed Rank-sum - - ± + +

(Zero Sum) *see* Schelling 1963 *The Strategy of Conflict*

2 Greenberg 1990 *The Theory of Social Situations*

Robinson & Goforth 2005 *The Topology of 2x2 Games*

g. Games with Ties are Within the Topology



Strict Nonstrict:Half-swaps 2=3 1=2

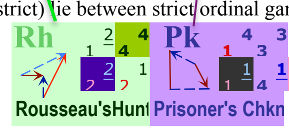
Ordinal games at grid intersections (graph nodes/vertices)

Games with ties (non-strict) lie between strict ordinal games,

(Robinson et al. 2007)

as do all normalized

2x2 games (1-4 scale)



h. Rapoport & Guyer Taxonomy

2	55	50	49	70	78	72	39	35	36	65	67	66
3	56	52	51	74	76	71	37	31	32	64	68	67
4	44	41	40	73	75	77	38	33	34	69	64	65
5	18	16	15	53	42	45	10	8	7	34	32	36
6	17	14	13	54	43	46	11	9	8	33	31	35
1	21	19	20	57	47	48	12	11	10	38	37	39
2	26	22	23	58	62	61	48	46	45	77	71	72
3	27	24	25	59	63	62	47	43	42	75	76	78
4	30	28	29	60	59	58	57	54	53	73	74	70
5	2	4	5	29	25	23	20	13	15	40	51	49
6	1	3	4	28	24	22	19	14	16	41	52	50
1	3	1	2	30	27	26	21	17	18	44	56	55

i. Brams Typology and Game Numbers

2	50	37	36	46	31	29	22	18	19	52	53	57
3	56	39	38	43	45	47	20	14	15	51	54	53
4	49	13	12	42	44	30	21	16	17	55	51	52
5	6	4	3	40	23	25	10	8	7	17	15	19
6	5	2	1	41	24	26	11	9	8	16	14	18
1	35	33	34	48	27	28	32	11	10	21	20	22
2							28	26	25	30	47	29
3							27	24	23	44	45	31
4							48	41	40	42	43	46
5							34	1	3	12	38	36
6							33	2	4	13	39	37
1							35	5	6	49	56	50

Number of non-myopic equilibria (NMEs)

3 2 1

All Nash equilibria are NMEs except pareto-deficient

3 1 6 5 4 3 2 1 6 5 4 3 2 2

	Stable	Weakly Stable	Unstable	
No Conflict N EP	D_2o D_1o D_0o	D_2c D_1c	D_0c	E=Natural outcome is Equilibrium
Mixed Motive M EP	D_2o D_1o	D_2t D_1t D_1f	D_0f D_1tf D_1ctf	2 1 0 Dominant strategies
M Ep	D_2o pareto-deficient	D_1f	D_1tf	no pressures
M e	no	D_0 2 equilibria	e_2o e_2c	competitive pressure
M e	dominance	D_0 No equilibria	e_0o e_0c	threat-vulnerable
Complete Opposition Z E	D_2o D_1o	e	eD_0c	force-vulnerable

see Rapoport et al.1976 *2x2 Games;* Robinson & Goforth 2003

	Strongly cyclic
Not cyclic	Moderately cyclic
No conflict	Weakly cyclic

see Brams 1994 *Theory of Moves;*

Difficult see Brams & Kilgour 2008

www.BryanBruns.com/2x2chart.html

An Introduction to the Topology

The structure of the topology is formed by swaps between adjoining ranked payoffs that link neighboring games (DR Robinson and DJ Goforth 2005e). The topology is composed of strict, ordinal two-person two-move games where each player has four distinct ranked preferences (d<c<b<a); these are strict in the sense that there are no ties, and ordinal in that preferences are only measured as relative ranks and are not measured on interval (ratio) or cardinal (real) scales (which would permit calculation of mixed strategies). Although only the ranks matter, the games are easier to interpret with numeric payoffs, and the discussion here follows Goforth and Robinson and others in using the numbers from 1 to 4. The games closest to each other are linked by switching the two lowest-ranked outcomes (1↔2), with swaps between the middle (2↔3) and top two payoffs (3↔4) more distant from each other.

The topology can be constructed by starting with Prisoner's Dilemma (or any other game), and swapping Row's payoffs of 1 and 2 to create a new game. Similarly, swapping Column's 1 and 2 payoffs creates a third game, and swapping both creates a fourth game (thus starting from Prisoner's Dilemma, swaps for both players lead to the game of Chicken). These four games form what Robinson and Goforth call a *tile* of four games that differ only by 1↔2 swaps.

Starting from those four games, swapping Column's 2 and 3 payoffs creates four more games. Swapping Column's 1 and 2 payoffs in these new games then completes two more tiles, on either side of the original tile. Swapping Row's 2 and 3 payoffs then creates more games above and below, and swapping Row's 1 and 2 payoffs in these games completes six more tiles, above and below. Thus, from Chicken, swapping payoffs of 2 and 3 for both players converts Chicken into a Battle of the Sexes game. These swaps of 1 and 2 and 2 and 3 form a *layer* composed of nine tiles and thirty-six games, containing all the possible changes from swapping 1 and 2 and 3. Topologically this is a torus, since further 1↔2 and 2↔3 swaps return to games already in the layer.

Swapping Row's payoffs of 3 and 4 creates a different game, after which the same steps can be used to create another layer. Similarly, Column's 3 and 4 payoffs can be swapped, and then swaps for both, to form two more layers. Robinson and Goforth began with the most well-known game, Prisoner's Dilemma, indexing it as Game 111, located on Row 1 and Column 1 of Layer 1. However, it turned out that displaying game properties works better with Prisoner's Dilemma near the center. This is equivalent to scrolling the indices of the torus for each layer by one step in each direction, and a "periodic table" format is the main one used in the following discussion.[5]

The topology provides a convenient way to organize the set of strict ordinal 2x2 games. The table displaying the topology has the familiar symmetric games along a diagonal axis from bottom left to top right (Table 2a). Starting at the top right, there are three games with two Nash Equilibria, beginning with Chicken, also known as Hawk-Dove, and an ordinal equivalent to Snowdrift; followed by two versions of Battle of the Sexes. Then there are two games with 3-3 Nash Equilibria, one labeled by Robinson and Goforth as Anti-Chicken and the other as Anti-Prisoner's Dilemma, which are also known as Prisoner's Delight (Binmore 2005) and as Deadlock respectively. Next is Prisoner's Dilemma, where the 3-3 Pareto-optimal outcome is not a Nash Equilibrium. Following that come three Stag Hunt-type games (also referred to as Coordination or Assurance games) with two Nash Equilibria, one of which gives both players their best payoff, followed by three games with a single 4-4 Nash Equilibrium.

Moving from the set of 12 symmetric games to the full topology, Row payoffs are the same across each row, and Column payoffs the same along each column. Thus, for each symmetric game, such as Prisoner's Dilemma, the payoff pattern is extended across the row and along the column, so the asymmetric games can be understood as mixtures of payoff patterns from two symmetric games. The table is symmetric along the diagonal axis, with matching games on each side of the diagonal equivalent to swapping the positions of the row and column players, a mirror reflection. The 12 symmetric games, plus the 66 other games above or below the diagonal, constitute the total of 78 unique games identified by Rapoport and colleagues (Rapoport and

[5] For displaying individual games, Robinson and Goforth place higher values up and to the right, as is conventional with Cartesian coordinates. However, in arranging games, they placed Prisoner's Dilemma and its layer at the lower left. While their decision to start with the most well-known game is understandable, it could be described as roughly equivalent to starting the periodic table of the elements with element 92, Uranium, a particularly complex, unstable, and potentially dangerous element.

In extending the topology to non-strict games, Robinson, Goforth and Cargill (2007) arrange the different categories of preference structures in a natural order of increasing complexity up and to the right, from the simplest null game to the strict ordinal games including Prisoner's Dilemma. An equivalent natural ordering in terms of increasing complexity would put Prisoner's Dilemma and its layer in the upper right or northeastern quadrant (Bruns 2010), which is the approach used here. Thus (before scrolling Prisoner's Dilemma to the center) the chart would proceed from Harmony in the lower left corner to Prisoner's Dilemma in the upper right. However, rather than proposing new index numbers, it seems best to retain Robinson and Goforth's indexing. It may be noted that the interactive applet for the periodic table of 2x2 games on David Goforth's website allows rearranging games within the table in various ways, including relocating layers, see http://www.cs.laurentian.ca/dgoforth/home.html

Guyer 1966; Rapoport, Guyer, and Gordon 1976)). Showing the full topological structure requires displaying all 144 games. From social science point of view (Ostrom 2005), position, e.g., as Row or Column, is important if payoffs differ between players, as in asymmetric games, which provides an additional argument for looking at the full set of 144 games.

As explained by Robinson and Goforth (2005) the topology groups games by the presence of dominant strategies and number of Nash Equilibria (Table 2b). Nash Equilibria form patterns with row-dominant strategies in the three rows in each layer, and column-dominant strategies in the three columns in each layer. Therefore, nine games in one quadrant of each layer have two dominant strategies, such as Prisoner's Dilemma and its neighbors. These and most other games have a single Nash Equilibrium. Two regions contain games with two Nash Equilibria: the Battle of the Sexes games and Chicken on Layer 1 and the Stag Hunt games on Layer 3. The cyclic games with no Nash Equilibria also form two blocks of games. The family of Prisoner's Dilemma games identified by Robinson and Goforth makes an L-shaped wedge extending out from Prisoner's Dilemma.

Robinson and Goforth's book, and their periodic table display relies on order graphs, which are very useful for showing symmetries. This includes games where the numbers have quasi-symmetric or sub-symmetric patterns but are not symmetric from the point of view of the players. The display presented here includes numeric values, as a way of making the topology easier to understand and more accessible.

The order graphs do help to illustrate how the interests of players may be aligned, opposed, or mixed (Figure 2f). The slopes in order graphs can be seen as inducement alignments (Greenberg 1990; DR Robinson and DJ Goforth 2005e) showing how one player's response to their incentives, given the other player's strategy, raises or lowers payoffs to the other player, in other words the externalities that result when the a player responds to individual incentives. In pure cooperation games with positive alignments (Table 2e), each player's incentives lead to higher payoffs for the other player. Conversely, in negatively aligned pure conflict games, such as Prisoner's Dilemma, whatever one player does in response to her incentives hurts the other.

Robinson and Goforth identified an interesting, and apparently previously unrecognized set of precisely misaligned situations, which they named "Type" Games, where one player's actions always help the other player, while the second player's actions always harm the first. More colloquially, the misaligned incentive structure makes one player kind and the other cruel, depending on position and the stucture of payoffs. A literary analogy is Robert Louis Stevenson's story, *Strange Case of Dr. Jekyll and Mr. Hyde*, and this could be used as a more evocative name for such games, calling them Jekyll-Hyde Type games.

The full set of 576 2x2 strict ordinal games includes games that differ only by swapping rows or columns, or both. For the topology, these are assumed to be equivalent, reducing to 144 games. Each game in the table thus represents four possible ways of swapping rows, columns, or both. Given the multiple forms a game could take, and the number of possible games, procedures for locating a game are given at the bottom of the table. Following the convention used by Robinson and Goforth, two columns may need to be swapped to put Row's payoff of 4 in the right hand column. Similarly, the two rows may need to be swapped to put Column's payoff of 4 on the upper row.[6] This makes it possible to identify which of the four layers will contain the game, since each layer represents one possible configuration of how the two highest (4) payoffs can be aligned, either in the same cell (Layer Three), on the same row or column (Layers Two and Four) or in opposite corners (Layer One). Once the layer has been located, the configurations for the row payoff pattern can be identified from the six possibilities, and similarly for column payoffs, with the game located at their intersection.

Overall, the topology provides an elegant and convenient way of arranging the 2x2 strict ordinal games, based on minor swaps between adjoining payoffs. The table has a symmetric structure, arranging games according to the number of dominant strategies and Nash equilibria. It groups the two families of games with two Nash Equilibria, the Battle of the Sexes Games and Stag Hunt games, and includes a family of Prisoner's Dilemma games with Pareto-inferior Nash Equilibria. A more detailed and systematic introduction, discussing symmetries and explaining relevant concepts from topology and group theory, is provided in Robinson and Goforth's (2005) book

Topology Structure: Tiles, Layers, Hotspots, and Pipes

As described above, swaps in the two lowest payoffs, 1↔2, form tiles, such as the four proper Battle of the Sexes games and four proper Coordination or Stag Hunt games. The four

[6] Robinson and Goforth included an additional rule, basically to put 3-3 Nash Equilibria in the upper right for the Second-Best Games. This is not used here in order to preserve the alignment of darker-shaded Nash Equilibria in the display, and to keep the position of the 4s consistent in each layer.

Delight games with 3-3 Nash Equilibria form another tile, as do four Harmony games with a single 4-4 Nash Equilibrium. In most, but not all, cases such minor swaps do not affect the game outcomes, so that games on the same tile usually have identical outcomes.

Tiles chained together by 2-3 swaps form layers*,* (Table 2d). The 36 games with either one or two 4-4 Nash Equilibria make up Layer Three. The games that have received most attention from game theorists are located on Layer One, where the highest payoffs are diagonally opposite each other, and so on different strategies. These include Chicken, Prisoner's Dilemma, and the Battle of the Sexes Games, along with the less well-known Delight games. Layers Two and Four are mirror images of each other, containing the cyclic games and other families discussed below. Since the games and tiles in a layer are linked by 2-3 swaps, each layer in the topology is a torus.

The layers are linked by 3↔4 swaps, creating structures that Goforth and Robinson named *hotspots* and *pipes*. Since each layer is a torus, the display can be "scrolled" one step to the left, creating a symmetrical display that makes it easier to see the structure of hotspots and pipes (Table 2d). This puts Prisoner's Dilemma at the top right , indexed as Game 111, on Row 1 and Column 1 of Layer 1, in their three digit numbering system.

Hotspots link eight games, two tiles in two layers. Thus the Battle of the Sexes Tile and the Coordination Tile are linked by 3–4 swaps, forming the most easily seen hotspot. The two mirror image tiles of cyclic games form another hotspot. The four layers are linked by a total of six hotspots, each of which links two layers. These can be labeled according to the two layers they link, so the 12 Hotspot links Layers One and Two.

Pipes link sixteen games, a stack of four tiles, one tile in each layer. Using geographic coordinates, north, east, south, and west, provides a convenient way to label the pipes. In the symmetrical view, the Prisoner's Dilemma pipe discussed by Robinson and Goforth is located in the corner of each layer. It contains Prisoner's Dilemma and Chicken on Layer 1, as well as two other Called Bluff games on the same tile with unfair 4-2 Nash Equilibria. On Layers 2 and 4, the pipe includes an Alibi game along with a cyclic game, and two games with unfair 4-2 Nash Equilibria. On layer four, the Prisoner's Dilemma Pipe contains a Stag Hunt Game with a second Pareto-inferior 2-2 Nash Equilibrium, as well as three games with a single 4-4 Nash Equilibrium.

In the symmetric display with the Prisoner's Dilemma Tile in the upper right, northeast corner (flipped northeast-southwest and then northwest to southeast compared to Robinson and

Goforth's display), the pipe that Robinson and Goforth named the Alibi pipe lies to the left of the Prisoner's Dilemma tile, and is labeled N, the North Pipe. Its mirror twin lies below the Prisoner's Dilemma Tile, labeled E, for the East Pipe. These are the most diverse pipes, and so the most unstable in response to swaps. The three other pipes, West, Southwest and South lie in the lower left corner of each layer. For these three pipes, the games within each tile all have the same Nash Equilibria, and all allow the players to get their first or second choice outcome, making these pipes "nicer," as well as more stable or robust against changes. The hotspots lie in between the two sets of pipes, forming a diagonal set of three tiles slanting from upper left to lower right.

Going back to the asymmetric display with Prisoner's Dilemma scrolled to the center, it can be seen that a single 3↔4 swap converts Prisoner's Dilemma into an Alibi game, on Layer Two or Layer Four (Game 221 or Game 412). A second 3↔4 swap converts the Alibi games into a Stag Hunt (Game 322).

This structure of tiles, layers, hotspots, and pipes shows the feasible set of transmutations available for changing one game into another through swaps in adjoining payoffs. The three minor swaps (1↔2, 2↔3, and 3↔4) form the structure of the topology, showing which games are close neighbors and which are more distant from each other. The topological structure of swaps can be used to understand the potential for strategic moves between games, moving "out of the box" to transform social dilemmas into win-win games.

Payoff Families

As an additional means for understanding transformations between games and their impacts, families of similar games can be categorized according to the distribution of payoffs at Nash Equilibria for pure strategies. This extends the families discussed by Robinson and Goforth: Cyclic, Battles of the Sexes, Coordination (Stag Hunt/Assurance) and Prisoner's Dilemma Family to cover the full set of 2x2 strict ordinal games. Nash Equilibria provide a convenient and important way of identifying solutions. However, it should be remembered that Nash Equilibria are not necessarily the only or best solutions, especially under conditions of repeated play, limited information, bounded rationality, where communication is possible, or where payoffs can be measured on interval or cardinal scales that allow calculation of mixed strategies.

Categorization on the basis of payoff distributions for Nash Equilibria is relatively straightforward. In some cases, it is useful to distinguish subfamilies, based on number of Nash Equilibria, presence of Pareto-inferior outcomes, and whether the player with a dominant strategy achieves their most-preferred outcome.

The major ambiguity in categorizing by payoffs is Chicken (Game 122), which fits with its distinctiveness as the second-most unique game after Prisoner's Dilemma. Based on payoffs, Chicken clearly belongs with the games that have 4-2 Nash Equilibria (ignoring, for this purpose, other solution concepts that might offer ways of achieving the more equitable, but less stable, 3-3 outcome). In terms of structure, as in the Rapoport and Guyer taxonomy and in Goforth and Robinson's identification of "proper" and adjoining "improper" games, since Chicken has two Nash Equilibria it clearly would belong to the Battle of the Sexes (sub)family. Topologically, if Chicken is included in the Unfair games with 4-2 payoffs, it then links the four patches of Unfair games, which would otherwise be disconnected (remembering that the table itself is a torus that wraps around top and bottom and left and right). Since the categorization approach used here is defined by payoffs, and given the topological links, Chicken is here included as part of the Unfair family, while recognizing its borderline situation and similarity to the Battle of the Sexes (sub)family. It may be noted that in terms of the potential for transformation into a Win-win game (Figure 2f), Chicken resembles the adjoining Unfair games, since it requires two swaps, one for each player, in contrast to the Battles of the Sexes which can be converted into Stag Hunts through a single swap for either player.

Payoff Families	Nash Equilibria	Pareto-optimal	Dominant Strategies	Symmetric	Asymmetric	Total	
1. WIN-WIN 4-4	**1**	**1**	**1-2**	**6**	**30**	**36**	**25%**
Harmonious	1	1	1-2		24	27	19%
Stag Hunt	2	**1**	0		6		6%
2. BIASED 4-3	**1**	**2**	**0-2**	**2**	**42**	**44**	**31%**
Altruistic	1	2	1		24	24	17%
Self-serving	1	2	1		12	12	8%
Batttles	2	2	0		6		6%
3. SECOND BEST 3-3	**1**	**3**	**1-2**	**2**	**10**	**12**	**8%**
4. UNFAIR 4-2	**1**	**2-3**	**1-2**	**1**	**18**	**19**	**13%**
Chicken	2	3	0		0	1	1%
Winner	1	2-3	1-2		6	6	4%
Loser	1	2-3	1-2		12	12	8%
5. PD FAMILY	**1**	**2-3**	**1-2**	**1**	**14**	**15**	**10%**
Prisoners' Dilemma 2-2	1	2-3	1-2		2		2%
Alibi 2-3	1	2	1		4		3%
Tragic	1	2	1-2		8		6%
6. CYCLIC	**0**	**2-4**	**0**	**0**	**18**	**18**	**13%**
				12	**132**	**144**	**100%**

Prisoner's Dilemma. Robinson and Goforth found a family of neighbors to Prisoner's Dilemma which they termed "Alibi" games, in accordance with the story they developed to explain an asymmetric version of Prisoner's Dilemma, where one player has an alibi and so faces a lesser penalty if he chooses not to confess. These seven games include Prisoner's Dilemma plus a pair of asymmetric siblings with a 2-2 Pareto-inferior outcome like Prisoner's Dilemma, as well as another four cousins with Pareto-inferior 3-2 Nash Equilibria.[7]

[7] In the eleven "difficult" games identified by Brams and Kilgour (2009) that could be solved (stabilized) by democratic voting on a reduced set of payoffs, their Class 1 games are the PD family of Prisoner's Dilemma and Alibi Games. Their three Class 3 games are Cyclic, and the four Class 2 games are Unfair.

Tragic. Adjoining the Prisoner's Dilemma Family identified by Robinson and Goforth are another set of games with 3-2 outcomes. In a sense, these are even sadder than the 3-2 Alibi games, since they lack even the possibility of reaching a Pareto-superior outcome as long as the payoff ranks do not change. It seems appropriate to include these Tragic games as a subfamily in an extended Prisoner's Dilemma Family, which would then total 15 games.

Second Best. Next to the Tragic games is a tile of four Delight games with 3-3 Nash Equilibria, one of which Binmore (2005) discusses under the name "Prisoner's Delight" and the other has been called Deadlock. Two more tiles on the other side of the Prisoner's Dilemma family, on Layers Two and Four, also have 3-3 outcomes, for a total of 12 games. Given the payoff pattern, and in reference to the classic economic concept (Lipsey and Lancaster 1956), these are labeled Second Best Games. Rapoport and Guyer (1967) chose to treat these games as "trivial," since under conditions of perfect information the solution seems obvious. However, they illustrate an important truth about a solution that may be mutually satisfactory even though not the best outcome for either individual. Furthermore, under circumstances of imperfect information or other constraints, achieving such a solution could still represent a significant, and desirable, achievement, and, in some cases, might be more feasible than either achieving the cooperative 3-3 outcome in Prisoner's Dilemma or transforming Prisoner's Dilemma into a win-win Stag Hunt.

Cyclic. Games with no Nash Equilibria were recognized by Von Neumann and Morganstern (1953) and included in the Rapoport, Guyer and Gordon taxonomy, and in the Robinson and Goforth table. From each cell, one player has an incentive to move, to change strategy, making it hard to find a stable solution. Including mirror twins, there are 18 Cyclic games, equivalent to 13% of the total of 144 games. Maximin strategies, avoiding the worst payoff, have been advocated as one solution for such games, and these are shown in the table in italic font and color corresponding to the payoff families. However, as can be seen, for one third of such games, maximin fails to achieve a Pareto-optimal result. If payoffs represent values on an interval or cardinal scale, then mixed strategy solutions can be calculated for Cyclic games.

Although they cite Robinson and Goforth's book, Brams and Kilgour do not seem to have realized that their "difficult" games are located next to each other in the topology, on five tiles, in Figure 2d labeled as Northeast; East (Alibi); and South (Anti-Alibi), in other words on the Alibi Tiles and on the Prisoner's Dilemma Pipe on Layers One, Two, and Four.

Biased. Somewhat surprisingly, the largest single family is composed of 44 Biased games with 4-3 outcomes, for which three subfamilies can be distinguished: altruistic, self-serving and battles.

Altruistic. Active Samaritan's Dilemma ((Buchanan 1977), Game 262, (and the game which originally stimulated the work reported in this paper), is one of 24 games in the Altruistic subfamily where a player has a dominant strategy, but following that strategy leads them to get their second choice outcome. The structure of the game thus leads them to act in ways which could be characterized as generous or altruistic, or as trapped in a "Samaritan's Dilemma," or as a host whose symbiotic parasite gets the best of the relationship. In sixteen of these games, only one player has a dominant strategy, fully illustrating how their pursuit of a dominant strategy leads to a situation where the second player expecting such a strategy then makes a choice which results in the first player getting their second-ranked outcome. In another eight games, both players have dominant strategy, so from the perspective of one player, it produces a Samaritan's Dilemma, while the other player follows their dominant strategy and gets their top payoff (like the Self-serving games discussed below). On Layer Two, there are three tiles, and twelve games, which are all part of the Altruistic subfamily. Together, with their mirror image reflections on Columns 5 and 6 of Layer 4, these make up the largest single subfamily, of 24 games, 17% of the 144 games.

Self-serving. Conversely, there are 12 games, all on Layer One, where only one player has a dominant strategy, and gets their top payoff, and the other gets their second-ranked payoff. The relative rarity of this pattern is somewhat surprising, at least given the common, cynical assumption of the prevalence of self-serving relationships. The Self-serving games compose 12/144, 8%; if the doubly dominant Altruistic games that combine the Altruistic and Self-serving patterns are included, this rises to 20/144, 14%, still a modest proportion). Robinson and Goforth applied the name Protector to all the games on the pair of tiles with such games. The name Patron games is proposed here for the similar adjoining games.

Battle of the sexes. The two symmetrical Battle of the Sexes games with Nash Equilibria where one or the other player gets their top payoff and the other their second preference, are well known in the literature . This structure is also discussed under the name of Bach or Stravinsky (Osborne and Rubinstein 1994, 15-16) (allowing the same abbreviation, BoS), or as the Battle of the Two Cultures (Dixit and Skeath 2004).[8] One version is also known under the name of Hero (Rapoport 1967). Their asymmetric siblings, and their cousins that combine 4-3 and 4-2 Nash Equilibria have received less attention. This includes the game that Buchanan (1977) named Passive Samaritan's Dilemma (Game 132), which adjoins Chicken, and is next to the proper Battle of the Sexes tile, combining the payoffs for Chicken for one player with those of Battle of the Sexes for the other. As discussed earlier, a categorization based purely on number of Nash Equilibria would put Chicken into the Battle of the Sexes family. If categorization is based strictly on payoff distributions, and given the topological connection to the other Unfair games, Chicken is here categorized in the Unfair games, which leaves a total of eight Battle of the Sexes games.

Unfair. The games with the most unequal (4-2) Nash Equilibria total 19 games, if Chicken is included (the term unfair follows Robinson and Goforth's usage, though they did not explicitly identify this as a distinct family). As noted earlier, Nash Equilibria provide a useful basis for categorization. However this does not mean this is the only or best solution, especially in repeated play, where communication is possible, where payoffs represent ratio or cardinal values (for which mixed strategies can be calculated), or where players are considering how they might move from an initial status quo position.

Even for strictly ordinal games, Stephen Brams, in his *Theory of Moves (1994)* argues that the 3-3 outcome to Game 213, which he calls *Samson and Delilah*, is not only more just but ought to be more feasible, at least if play begins from a particular starting point. For conventional analysis using Nash Equilibria, this game presents the somewhat perverse result that a player following a dominant strategy ends up getting their third choice, even though they could choose

[8] If the goal is to avoid stereotyping, then replacing sexist stereotypes with cultural ones seems somewhat contradictory. One could use the alternative name of Bach or Stravinsky (Osborne and Rubinstein 1994) though this assumes familiarity with classical music. Alternatively a blander name such as Battle of Favorites could be used, as in choosing a movie to watch together. A more relaxed and playful approach might be to reverse the gender stereotypes in presenting the story, thus the male would prefer to watch a romantic comedy, or go to the opera, while female wants to watch an action thriller, or the boxing match. This paper will retain the common name that has historically been most used in game theory research. As discussed later, the structure of the typology and index numbers also provides way of uniquely identifying games that could be used to clarify whether the same or similar games are being discussed under different names, and thereby reduce confusion and aid cumulative analysis and understanding, without insisting on unanimity in common names.

a strategy that ought to lead the other player to choose the fairer 3-3 outcome, especially if at least one-way communication were possible.

If a fair game represents what is desirable, then the Unfair games are to be avoided. The potential for transforming them into well-aligned harmonious games, through changing relative payoffs, deserves attention. As with Altruistic and Self-serving games in the Biased family, the Unfair family could be divided according to whether one or both players have dominant strategies. However, to avoid unnecessary proliferation of subfamilies, and also given the problematic role of dominance as a solution concept for some these games, this subdivision has not been explicitly used to categorize subfamilies. However, it is useful to note that, as with the Biased games, there are some Unfair Games, located on Layer 1, where the player with the dominant strategy Wins, while in others, on Layers 2 and 4, the player with the dominant strategy gets their third-ranked preference.

Win-win. The thirty-six games on Layer Three where both players' top preference is for the same outcome have received less attention from game theorists, as indicated by their being referred to as "no conflict" or "boring." As shown by Robinson and Goforth (2005, in Chapter 8 on Classifying Conflicts), most of the games on Layer Three are actually games of mixed interests (Figure 2e). Calling these games "no conflict" is a misnomer. Movement by one player to increase their payoff may lower the other's payoff. Only ten of the thirty-six games on the layer are games of pure cooperation, where one player's response to their incentives (for a given strategy by the other player) always helps both players. And out of those ten, four games (Coordination, 344; Assurance, 333; and the pair of Asymmetrical Assurance Games, 323/332) have a second, Pareto-inferior Nash Equilibrium that is, arguably, risk dominant. A player seeking to ensure that they avoided their worst payoff, regardless of what the other player does, would end up choosing the strategy that leads to the Pareto-inferior Nash Equilibrium. (Interestingly, in the Pure Coordination games (334/343) and Asymmetric Coordination Games (324/342), a maximin strategy to avoid the worst payoff leads to an outcome that is not either of the Nash Equilibria.)

Given the potential of the topology in understanding transformations that would swap payoffs and transmute one game into another, it seems suitable to have a name for this layer that reflects the desirability of finding ways to create games where both can achieve their top-ranked result. The term Win-Win reflects the potential for a mutually optimal outcome, and so is proposed for the entire family of 36 games on Layer 3 with one or two 4-4 Nash Equilibria. Using this name need not assume that the ideal outcome is always achieved, only that it exists as one possibility.

Stag Hunt. The two well-known coordination games are accompanied by asymmetric siblings, and by “improper” cousins where the second Nash Equilibrium has a lower payoff, of 3-2 for four games, and of 2-2 for the game that Robinson and Goforth label as Stag Hunt. For this family of games, the name Stag Hunt seems more memorable and meaningful than “coordination," the term used by Robinson and Goforth. Furthermore, other games, including Battle of the Sexes, may involve elements of coordination, so it seems appropriate to use Stag Hunt as the more general name for the entire family. Some, but not all games on this layer are what Sen (1967) defined as Assurance Games, where if the other player does not choose the cooperative strategy that would achieve the 4-4 outcome, then it is best to also not cooperate.

The Stag Hunt game itself is an example of how different authors have used the same name for somewhat different games. This includes the non-strict variant, in accordance with Rousseau’s original story (Skyrms 2001, Skyrms 2004), where the player who chooses to hunt for rabbit gets a rabbit whatever the other players do (and so is indifferent between the outcomes of their strategies), but causes the other player to lose out on getting the stag if they choose to hunt stag. The topology can be extended to include such games with ties (Robinson et al. 2007). In such a larger topology, Rousseau’s Stag Hunt (with ties) lies between the ordinal Stag Hunt (Game 322) and Assurance (Game 333) (Figure 2g). The diversity of games called Stag Hunts is an example of where more precise specification of game names and payoff structures could aid accurate communication and cumulative research, as well as directing attention to the asymmetric variations on the basic Stag Hunt/Coordination/Assurance structure, which may deserve further exploration in simulation and experimental research.

Harmonious. The game labeled Harmony offers a meaningful basis for naming the corresponding tile, the asymmetric neighboring tiles, and the sub-family of win-win games with a single 4-4 Nash Equilibrium. Calling this subfamily, or the larger family of games with 4-4 outcomes, “boring” may reflect the pre-occupation of game theorists with games that may produce more frustrating results, but does little justice to the value of such mutually agreeable outcomes. Therefore, the name Harmonious is proposed as suitable for the subfamily with a single 4-4 Nash Equilibrium.

A classic question in Linnaean and other taxonomy concerns how detailed to make categories, a matter which is often discussed in terms of philosophical differences between "lumpers and splitters." The choice here has been to keep the main groupings small enough to be easily remembered, six families, with only a few subfamilies in each group. Thus, the tragic games have been grouped (or lumped) together with the Prisoner's Dilemma games. The Unfair and Biased games on Layers Two and Four could be further categorized according to whether they have dominant strategies for both players, or only for one. As mentioned, Biased and Unfair Games can also be classified using a cross-cutting distinction according to whether the player with the dominant strategy ends up better off than the other player or not, i.e., whether they receive a higher-ranked payoff, as in the "Selfish" subfamily, and in the "Winning" Unfair games on Layer one, or, as on Layers Two and Four, only get their second choice (in Altruistic games) or third choice ("Losers" in Unfair) games. Such an approach would still be different from that used by Rapoport, Guyer, and Gordon (Figure 2h), whose taxonomy, for games with one equilibrium and no win-win outcome, emphasizes the threats available to a player.

Families of games have been categorized according to payoffs at Nash Equilibria, yielding six families of games, three of which have distinct subfamilies. This adds new families and subfamilies to the Cyclic, Battle of the Sexes, Coordination (Stag Hunt/Assurance), and No Conflict categories already recognized in the literature, at least in their symmetric forms. This extends the Prisoner's Dilemmas Family identified by Robinson and Goforth, and provides a complete categorization into families and subfamilies for the full set of 2x2 strict ordinal games.

Visualization Methods

The use of colors, lines, fonts, spatial relationships, and other visualization techniques (Tufte 1983) make the table more accessible and informative. Goforth and Robinson used order graphs to provide an efficient and elegant way to display game payoffs and inducement slopes. However, relying purely on diagrams makes their "Periodic Table" rather inaccessible for newcomers and anyone else who is not able to quickly interpret the diagrams. Showing numeric payoffs makes the table easier to understand, as does following the convention of having column payoffs in each cell located somewhat higher than the row payoffs. Locating games, according to the procedures discussed above, is simplified by showing the row and column payoff patterns at the right and bottom edges of the table, and instructions at the bottom for how to locate a game. To reduce distraction, this is done in a smaller font.

Color is used to identify game families and subfamilies, which helps divide the table into major regions. Names of symmetric games along the diagonal axis are in bold. Darker shades highlight cells with Nash Equilibria. White lines separate layers, which are also marked by numbers at the corners, larger, and in bold. Verdana font has been used for clear display of payoffs on-screen, while Arial Narrow is used to fit in game names and numbers, and Times New Roman is also used for other titles and explanations.

For numeric payoffs, Pareto-optimal payoffs are shown in bold. Payoffs are in white for Pareto-inferior outcomes, as in Prisoner's Dilemma and adjoining Alibi games. Maximin strategies, where the player can at least avoid their worst payoffs, were part of the Rapoport taxonomy. These can be easily found by choosing the row or column where a player does not get a payoff of 1, and are shown with italics. This solution concept is most relevant for cyclic games, and for those games (which, of course have no Nash Equilibrium) the maximin solution is shown with similar but lighter colors than used for Nash Equilibria. (The use of white font to indicate Pareto-inferior solutions reveals how the maximin strategy yields Pareto-inferior solutions one third of the time even for cyclic games).

Tiles are edged by thicker lines, red for boundaries between rows and blue for boundaries between columns, while narrower lines separate games within tiles. Conceptually, these thin lines also show the tiles that are linked by high swaps to equivalently located games on another layer. The intersections of thick and thin lines, together with lines drawn through the middle of each game (Figure 2g) also can be seen as marking the locations of games with ties (D Robinson, D Goforth, and Cargill 2007). They also delineate slices of games on either side, vertically or horizontally, which are linked by high swaps to an equivalent set of games on another layer (Figure 2e). Thus, all the games next to a line running through the proper Battles of the Sexes on Layer One are linked to games on either side of an equivalent line running through the proper Coordination games on Layer Three. The direction of such links, the layer to which they go, is structured according to the pipes and hotspots, as shown in Figure 2d.

Overall, the approach is to show major features more prominently, while making detailed information available on closer inspection. As explained by Tufte, perhaps even more important than the ways to emphasize major information in a table are ways to de-emphasize other information, while still making it available on closer examination. A smaller font has been used for game names, which helps the display fit onto a single letter-size page, in accordance with Tufte's (1983) recommendation not to be afraid to use small fonts in tables and figures, where small size does not pose the readability problems it would if used for long blocks of text. In contrast to common bimatrix displays of games, there are no lines between cells, instead the viewer is allowed to infer lines from the colored boxes created by cells shaded to show Nash Equilibria, or maximin outcomes for cycle games. This follows Tufte's admonition to eliminate non-data ink. It would have been possible to use separate colors for each player's payoffs, as in the legend. However this would have made the table visually much "noisier" and harder to interpret. Instead numbers were colored with a darker shade of the same color used for the families, to give a more unified look.

Abbreviations are used to facilitate identifying locations of games (and, of course, following the example of many versions of Mendeleev's Periodic Table of the Elements). The font variations used for the abbreviations show the alignment of interests, so games of pure cooperation are in an outline font; *Jekyll-Hyde Type* games, where one player is kind and the other cruel, are **shadowed**, games of pure conflict have a single underline, and games with fixed rank-sums (the ordinal equivalents of zero-sum games) have a double underline. In this context, one may recall that game theory expanded from an initial concentration on fixed-sum games to analysis by Schelling and others of the larger set of games with mixed interests. Greater attention to asymmetric games could provide a further area for expanding the scope of analysis, in ways that might help reflect a greater variety of real-life situations.

While symmetry is central to the formal understanding of the topology developed by Robinson and Goforth in their book, it is less crucial to practical understanding and use of the table, and so receives less attention in this display than in Robinson and Goforth's periodic table of 2x2 games. The first supplementary diagram does show the diagonal axis of symmetric diagrams, and also lightly indicates the quasi-symmetric and sub-symmetric axes. Robinson-Goforth Game numbers are shown before game names in a small font, as well as in Figure 2b, while, as a way of avoiding clutter in the main chart, game numbers specified by Rapport, Guyer, and Gordon, and by Brams are shown in two smaller tables in Figures 2h and 2i. This visualization follows and expands on Robinson and Goforth's approach of showing structures in the topology with smaller diagrams, using the principle that Tufte (1983) call "small multiples."

In contrast to how the order diagrams are displayed in Robinson and Goforth's table, since the order diagrams already span the full range of values, the axes can be removed. Separate shading for the box containing the order diagram is also unnecessary. Arrows are added to the order diagrams, to make them more intuitive to interpret. While it is not difficult intellectually to understand that Row always wants to go to the right for higher payoffs, and Column always wants to go up, learning to read the diagrams this way may take time, while arrows make the vectors more obvious. Adding darker hues for inducement slopes that lower payoffs to the other player, and lighter hues for those that make the other player better off, subtly distinguishes the different players and the impacts of their induced moves, and the combinations of inducement correspondences.

Following the example of Dragicevic's icons for games,[9] simplified icons are used for the payoff patterns (Figure 2c) that very concisely show the ordinal relationships between the four payoffs for each game. These make it easier to see the structure of the topology within which the patterns are flipped horizontally or vertically between different layers, and rotated according to position as row or column. As mentioned earlier, the asymmetric games can be seen as combining payoff patterns from different symmetric games. All the payoff pattern icons can be generated from three basic patterns (resembling Z, K and C respectively) with various rotations and flips.

In order to make the table easier and more likely to be used, it has been designed to fit on standard letter size paper, rather than the tabloid (17"x11") size recommended by Robinson and Goforth for their original version of the table. The pages can be printed double-sided and laminated together for convenient reference, if desired. As a way of encouraging use and further improvement, the table is made available under a Creative Commons license (BY-SA), which allows printing, reuse, and modification, as long as attribution is given and any modified version is made available under the same share-alike terms.

[9] http://www.lri.fr/~dragice/gameicons/

The visualization of the topology using numeric payoffs provides an accessible display that does not require first learning to interpret order graphs. Marking Nash Equilibria with darker colors helps to show the patterns of dominant strategies, and number of Nash Equilibria. Including the row and column payoff patterns and the procedure for locating a game also makes the table easier to understand and use. Payoff families help to divide the table into relatively small, meaningful subgroups, and add visual appeal.

Once games have been located in a grid, a smaller display can show additional properties, as done in Robinson and Goforth's periodic table display. Figure 2 starts with the diagonal of symmetric games as an easily understood introduction; followed by patterns in the number of dominant strategies and Nash Equilibria; a condensed display of the chart structure with abbreviations; the structure of tiles, pipes and hotspots; remediability to reach win-win; the combinations of aligned, opposed or mixed interests and inducement coefficients, a schematic illustration of how games with ties are contained within the topology, and earlier taxonomy, typology and game numbers.

Game Names

While there is no necessity for each game to have a name, common names (and the stories that go with them) make game easier to remember and discuss, for students and researchers. A more neutral labeling in terms of numbers may be useful in some circumstances. One of the advantages of the topology is that index numbers can provide a unique way to identify games, equivalent to scientific names in Linnaean taxonomy. Rapoport and Guyer showed payoff patterns for the 78 games, (including non-ordinal variants), but without the common names, a choice which seems to make it unnecessarily difficult to locate and discuss games.

Stephen Brams (1994) developed a separate listing of game numbers, (although it omitted the "boring" games with a single 4-4 Nash Equilibrium). In contrast to the focus of most researchers on symmetric games, Brams paid substantial attention to asymmetric games. He named some of them based on Biblical and Shakespearean stories used in his analysis. In some cases he applies the same name to several games that are equivalent for his purposes, although they may differ in payoffs and other properties. For example, in his book, and in the Goforth and Robinson table that includes his names, there are several games labeled "Cuban Missile Crisis." Game names, based on Goforth and Robinson (2005) and Brams ((1994), and additional names to complete the full set of strict ordinal 2x2 games, are shown in Table 3. As mentioned, the smaller supplemental diagrams show equivalent game numbers in the Brams (Figure 2g) and Rapoport and Guyer listings (Figure 2h).

Evocative, meaningful and easily remembered names deserve to be preferred to more abstract ones. Thus Binmore's (2005) name of Prisoner's Delight (Game 155) or simply Delight seems more desirable than Goforth and Robinson's name of Anti-Chicken, and Deadlock to Anti-Prisoner's Dilemma (which can be understood by looking at the order diagrams, but otherwise may be somewhat obscure).

In accordance with Goforth and Robinson's approach of treating games on the same tile, linked by 1-2 swaps, as neighbors and usually highly similar, in cases where games do not already have individual names, it seems appropriate to apply a single name to the set of similar games on the same tile. This is what Robinson and Goforth already did for the Protector Game (Games 153, 154, 163, and 164, and also Games 135, 136, 145, and 146). This would also provide a logical name for the entire tile. Within tiles, games can be distinguished by game numbers. Assigning names based on tiles also simplifies naming by reducing the number of names needed, though new common names could develop if specific games are found to be particularly interesting.

In some cases a game may have been singled out in earlier research. Robinson and Goforth label Game 414 as one of several Alibi games. However, Brams (1994) had discussed this specifically as a game he called Revelation. Game 262 is what Buchanan (1977) called Active Samaritan's Dilemma (and stimulated the work that led to this paper). Furthermore, it seems useful to distinguish the pair of games most similar to Prisoner's Dilemma, sharing a 2-2 outcome, with the name of Asymmetric Prisoner's Dilemma (even though these were the games originally used by Robinson and Goforth for their story about an alibi game). As mentioned, a whole subfamily of games shared the characteristic that the player with a dominant strategy gets their second-choice outcome. These are labeled as altruistic, generous, and benevolent, with all the benevolent and the Altruist and Altruist Type games all having two dominant strategies.

Since the other two fixed rank-sum games, Total Conflict and Big Bully already had names, it seems appropriate to name the remaining game Fixed Sum, which is more precise than Zero Sum, which would be more familiar but harder to understand. Since names were already given for three-fourths of the Tragic games, it seems suitable to suggest the name Tragedy for the remaining game.

From the point of view of social organization, finding mutually satisfactory arrangements is an important achievement. Robinson and Goforth had labeled one of the symmetric games with a single Nash Equilibrium as Harmony (366). The next symmetric game has mixed interests, and so could be called Mixed Harmony (355), while the remaining games on the tile are given the proposed names of Asymmetric Harmony (356/365). For the two games (312/321) sharing a tile with No Conflict (311) and Stag Hunt ((322) the name Low Conflict is proposed, given their similarity with No Conflict.

One of the two symmetric games that Robinson and Goforth call Coordination is the most acute example of Sen's definition of an Assurance game, where cooperation is mutually reinforcing, but if the other player does not play the cooperative strategy then it is also better to defect. In this case, playing cooperatively while the other does not results in getting the worst outcome, rather than the best. Thus, it seems appropriate to label Game 333 as Assurance.

The games on the Aligned tiles are all asymmetric, but the players either have the same payoffs in each cell (Pure Aligned), or the same top two preferences. By contrast, in the remaining pair of tiles, interests are not as well-aligned. In these, the player with a dominant strategy can still get their second choice, even if the other player does not follow the cooperative choice. Dominance solvability in these games means that the structure of incentives leads one player to choose a cooperative strategy, and it then makes sense for the other player to also choose the cooperative strategy. These represents the simple two-player version of what Mancur Olson ((1971)) calls a privileged group, where one player has a sufficient interest to take the lead in providing the collective good. Two other games, that share tiles with the Asymmetric Coordination and Assurance games, also have the same privileged structure. Each player has one negative and one positive inducement correspondence. The name mutual is suggested, as in biological mutualism, mutual gains, and mutual aid.

The visualization presented here gives names for all of the 144 strict ordinal games, using common names where these are available in the literature, and proposing new names based on the structure of tiles and game properties, as well as a consistent set of abbreviations. This follows Robinson and Goforth's set of names where feasible. Where they used the same name, e.g., Total Conflict or Cuban Missile Crisis for several games (following Brams 1994 terminology) this chart disambiguates, giving separate names for each game. Games in tiles sharing the same name can be distinguished using game numbers in the Robinson-Goforth numbering system. Game numbers provide a way of uniquely identifying games, which can function like scientific names for species, while common names facilitate remembering and discussing games.

Conclusions

The topology of 2x2 games elegantly arranges games according to important properties and shows how they are linked by swaps in adjoining payoff ranks. Categorization into payoff families provides an additional way to understand the abundance and diversity of asymmetric games, most of which have unequal outcomes. Three of these families, and three subfamilies, seem to have received little or no previous recognition as distinct groups of games sharing a meaningful set of properties: the families of Second Best (3-3), Biased (4-3), and Unfair (4-2) games, and the subfamilies of Tragic, Altruistic, and Self-serving games.

Given the prevalence of asymmetries in social life, the topology of 2x2 games, including payoff families, may be useful in directing greater attention to the systematic study of asymmetric situations, and to analyzing the potential to solve social dilemmas through realigning incentive structures to create Win-win games. As Goforth and Robinson say, their topology stops where most work in game theory starts, at the analysis of specific games. The topology does not resolve previous debates or decide between alternative approaches to analysis and solution. Nevertheless, it may improve understanding of diversity and similarities within the 2x2 ordinal games.

The study of governance could benefit from this systematic framework for understanding strategic shifts between games. Switches in payoff ranks from changes in common knowledge, in communication, and in rules, can, under some circumstances, open feasible options for institutional redesign that converts social dilemmas and inequitable games into win-win games. The topology of 2x2 games can help understand the similarities and differences in behavior between games that differ by only by a single payoff swap, as well as the potential for transforming games. Even for games with ratio or cardinal payoff values, the ordinal case can help in understanding how if values shift past the threshold that changes the ordinal ranks, the structure of the game and its likely outcome may change.

The 2x2 games are elementary, “toy” models, highly simplified compared to the complexity of most real-world situations. Furthermore, real life decisions are usually made under conditions of bounded rationality, by fallible actors with imperfect information relying on various heuristics rather than comprehensive analysis. Nevertheless, Harmony, Prisoner’s Dilemma, Chicken, Stag Hunts, and other games capture some key elements of how different incentive structures and strategies interact. Game theory has largely grown beyond the pursuit of unique, deterministic solution concepts, and become one of many tools for understanding social behavior, analyzing evolutionary dynamics in social science and biology, and empirically examining how people interact in making interdependent decisions, including learning from experiments in the laboratory and in the field. Nevertheless, the 2x2 games still provide a fundamentally important source of concepts, particularly for designing and analyzing simulations and experiments.

The topology could be a valuable tool for teaching and learning about game theory. Availability of a more accessible display of the topology, which does not require understanding order diagrams, (but does include them) may help to expand awareness and understanding of the topology. Common names can also aid learning about and discussing games, and so it may be useful to have a full set of names for the ordinal 2x2 games, based on the structure of tiles of similar games. The names proposed here are provisional, and common names can evolve based on usage. The numbering system developed by Robinson and Goforth does provide a logical and unique way to identify games, analogous to scientific names for species. Such unique identifiers could be used to link research on games which may have been given different names, or not named, but which are identical, ordinally equivalent, or similar, contributing to cumulative synthesis of social research.

The topology of 2x2 ordinal games, including payoff families and game names as shown in the visualization presented here, offers a useful tool for improving institutional analysis and design, including understanding the variety of asymmetric games, and the obstacles and opportunities for transmuting Prisoner’s Dilemma and other conflicts into win-win games.

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