Fuzzy Political Campaigns

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Abstract

We analyze a model of political campaigns, where a challenger aims to unseat an incumbent. The challenger and the incumbent differ in their quality. All voters want to elect the candidate with higher quality. The challenger chooses the level of precision of campaign messages to affect his probability of being elected. More precise campaigns are more costly for the challenger. We characterize the equilibria when voters observe both the message and the precision. We show that only two-step non-trivial equilibira are possible. This framework allows us to investigate whether limits on campaign spending may be welfare-enhancing.

Keywords: political campaigns, campaign finance limits, information transmission, signalling.
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1 Introduction

The objective of our research is to investigate information transmission during political campaigns. In political campaigns, candidates spend much effort and money on increasing their exposure to potential voters. It is often the case that they saturate airwaves and mailboxes with campaign advertisements and make frequent appearances at rallies, town-hall meetings, and TV programs. Though political candidates carefully manage their "message," a fully rational voter will not be fooled by their spins. However, being frequently seen and heard has the effect of provoking the interests of voters, who may choose to further investigate the candidates.

In this paper, we construct a model where candidates can choose the precision of communication, in the form of affecting the variance/precision of the signal that voters receive about their quality. Voters are fully rational and choose the candidate with higher expected quality, based on the candidate's precision of communication and the actual message they receive. We abstract from the issue of negative campaigns or heterogeneous voter preferences. One of the distinct features of our research is that we allow the level of precision to be observable to the voters. Therefore, precision of communication may be used as a signal of quality by candidates.

In papers by Austen-Smith [1] and Landi [6] voters are risk averse and uncertain about the policy positions of candidates, higher variance implies lower utility for voters. Candidates affect variance by campaign advertising. In his model, Austen-Smith also considers the presence of interest groups and the possibility for candidates to change policy stands to attract contributions. However, voters are not fully rational because in equilibrium policy should be perfectly known and there is no reason why voters should be uncertain. Landi assumes that candidates have a fixed campaign budget to be allocated between positive and negative advertising. Positive advertising reduces the own variance of policy and negative advertising increases the opponent' s variance.

In both of these papers, voters' risk aversion drives candidates' behaviour– *ceteris paribus*, candidates want to decrease their own variance and increase their opponent's variance. However, low variance is not necessarily beneficial to the candidate as voters are risk neutral in our model. A lower-quality candidate would prefer to have a higher variance, but since precision is observed by the voters, the candidate has to be sufficiently precise in order to have a chance to be elected at all. In this paper we want to focus on information transmission of political campaign in a context with common value, that is, all voters value quality and there is no conflict of interest among voters. Political campaign affects the precision of the signal and is used by voters to update beliefs about candidates.

In our paper, political campaigns transmit information both directly and indirectly. This is in contrast to the previous work, where campaign advertising is assumed to be either directly informative, as by Coate [2] [3] and Degan [4], or indirectly informative ("money-burning"), as by Prat [8] [9]. A closely related paper is by Li and Li [7]. In their model, candidates may control the informativeness and the tone of campaigns. In other words, they can choose between a precise campaign and an imprecise one, and between a positive campaign and a negative one. They adopt a discrete setup and analyze the effect of competing campaigns. A key difference between our setup and theirs is that we adopt a continuous setup, which also allows a welfare analysis of campaign spending and voter choice.

2 Model

There is an election with an incumbent candidate running against a challenger. Candidates are characterized by their quality, which can be thought of as the candidate's ability to run the office, for example, the ability to gather information, the ability to build consensus, the ability to influence public opinion, and so on. For simplicity, and since voters have observed the incumbent behaviour during his past mandate, we assume that the incumbent quality, $\theta_I > 0$, is commonly known. In contrast, before any campaign takes place, voters only know the distribution of the challenger's quality. In particular, from the voters' perspective a challenger's quality is a random variable Θ, which is normally distributed with mean 0 and variance σ_θ^2. The challenger's realized quality, $\theta \in R$, is his private information.

Before the election, voters observe a signal on the challenger's quality through the challenger's campaign:

$$s = \theta + \sigma_h \varepsilon, \tag{1}$$

where ε is a random error term distributed according to the standard normal distribution $N(0,1)$ and $\sigma_h \geq 0$ is the standard deviation of the signal when the challenger chooses precision h. Therefore, σ_h is lower if the challenger chooses higher precision. We assume σ_h goes to zero as h becomes arbitrarily large, and goes to infinity as h approaches zero.

We make two fundamental assumptions: (i) the candidate has control over precision h and (ii) voters observe the candidate's precision of message, h. One may interpret h as the effort put in by the candidate to articulate his quality as a candidate, through stump speeches, media appearances, campaign ads, and so on.

In other words, although voters observe whether a candidate is vague and clear during a campaign, they may rationally infer only imperfectly his type. Affecting the precision of the signal is costly, as it requires effort and campaign funding. The cost of a level of precision h is represented by the increasing function $C(h)$. In the basic setting we assume that this cost is independent of a candidate's type. With a slight abuse of notation, we also use $C(\sigma_h)$ to denote the cost of inducing a signal of variance σ_h. We make the following assumptions about the function $C(\cdot)$.

Assumption 1. The cost function satisfies:

1. $C'(\sigma_h) < 0$ for all $\sigma_h \in (0, \infty)$;

2. $\lim_{\sigma_h \to 0} C(\sigma_h) = \infty$;

3. $\lim_{\sigma_h \to \infty} C(\sigma_h) = 0$.

Voters only care about the quality of the elected politicians. All voters have common value preferences and observe the same information.[1] Therefore, we can just consider a representative voter.

[1]We may allow differences in voters' political preferences. If the candidates' political positions are commonly known, and a voter's utility derived from a candidate's

The assumption that voters only receive signals from the challenger is, in our model, without loss of generality. In fact, since the incumbent's quality is known, and quality is the only thing that the voter cares about, there is no scope for him to exert effort during the political campaign. In reality, incumbents do spend as much, if not more, on political campaign as challengers. This could be incorporated in the model by assuming that the match between the candidates' qualities and the state of the economy can change over time, that the incumbent quality is only observed with noise also during his mandate, or that the candidates can engage in negative campaigns about their opponent's quality.

The timing is as follows. The challenger exerts campaign effort (h). The voter observes h and the signal s, uses them to update the expectations on the challenger's quality, and then vote. The candidate who is elected enjoys the benefit from being in office, which we normalize to 1, and the voter enjoys the elected politician's quality.

The problem can be solved backward. Given the voter's beliefs on the challenger's type, conditional on the observed precision of political campaign h and the signal s, $P^e(\theta|h,s)$, the voter must choose whether to reelect the incumbent ($v = 1$) or the challenger ($v = 0$) in order to maximize the expected utility from the electoral outcome

$$\max_{v\in\{0,1\}} U = v\theta_I + (1-v)E[\theta|s,h],$$

where the expectation is taken with respect to the probability distribution of the challenger's quality induced by the belief $P^e(\theta|h,s)$.

The solution of the voter's problem consists of re-electing the incumbent if his quality is better than the expected quality of the challenger, i.e. if $\theta_I \geq E[\theta|s,h]$, and of voting for the challenger otherwise. Implicitly, we have assumed that the voter votes for the incumbent in case of indifference.

Let $\pi(\theta,h)$ be the probability that the challenger wins the election when the challenger's quality is θ, and his campaign precision is h. The

political position and that from his quality is additively separable, then we can redefine the incumbent's quality as the sum of his quality and the incumbent's positional (dis)advantage over the challenger for the median voter.

problem for the challenger consists of choosing h in order to maximize his expected net benefits from office:

$$\max_{h \geq 0} V(\theta) = \max_{h \geq 0} \pi(\theta, h) - C(h)$$

The above description defines a game with incomplete information between the representative voter and the challenger. A strategy for the challenger is the signal precision $h : R \to R^+$. A pure strategy for a voter is a re-election rule $v : R \times R^+ \to \{0, 1\}$. The solution concept we adopt is Perfect Bayesian Equilibrium.

Definition 1. A *Perfect Bayesian Equilibrium* of the campaign game consists of a challenger's strategy, $h(\theta)$, a the voter's strategy, $v(s, h)$, together with the system of beliefs $P^e(\theta|h, s)$ such that
(i) $v(s, h)$ maximizes U given beliefs $P^e(\theta|h, s)$;
(ii) $h(\theta)$ maximizes V given the voter's strategy $v(s, h)$
(iii) Beliefs are calculated using Bayes rule (whenever applicable) and are consistent with the challenger's strategy:

$$P^e(\theta = x|h, s) = \frac{p(s|h, x)p(x|h)}{p(s|h)} = \frac{p(s|h(x), x)p(x|h)}{p(s|h(x))}$$

where $p()$ denotes the density function.

We are interested in characterizing the equilibria of this game and in seeing whether there are equilibria in which information about the challenger quality is transmitted to voters through the candidate campaign.

We will call an equilibrium *informative* if the voter is equilibrium uses the information contained in the signal, besides the information (if any) contained in the observed precision.

3 Equilibrium

The context described in a previous section constitutes a signalling game, where the sender is the challenger, the signal is the precision h, and the receiver is the representative voter. The difference between this game and a "pure" signalling game is that the voter makes his inference based not only on the the level of precision, but also on the random message generated with the level of precision. We analyze in this section the existence and characterization of separating, pooling, or semi-pooling equilibria as well as their informational content. We focus on *monotone equilibria*, in which the challenger's precision is nondecreasing in his quality.

Separating equilibria

Let us consider first separating equilibria. In a separating equilibrium, each challenger type chooses a different level of precision. Let $\widehat{h} = h(\widehat{\theta})$ be the precision chosen by type $\widehat{\theta}$.

Proposition 1. *There is no fully separating equilibrium of this game.*

Proof. See the Appendix. ∎

A fully separating equilibrium is impossible because in such an equilibrium, voters will perfectly infer the quality of the challenger from his precision alone. Therefore, the challenger is elected if and only if his quality is higher than the incumbent's quality. But this means that a challenger should not spend any effort at all to increase his precision if his quality is lower than the incumbent's quality.

Pooling equilibria

In a pooling equilibrium $h(\theta) = h^*, \forall\theta$ for some $h^* \geq 0$. The proposition below shows that the only pooling equilibrium is one that requires each type of candidate to not campaign at all.

Proposition 2. *There exists a unique fully pooling equilibrium in which $h^* = 0$.*

The pooling equilibrium with zero precision is sustained by the voters' belief that any positive precision must be from a challenger with quality lower than the incumbent's quality. In this equilibrium, since the signal is completely uninformative, the representative voter must make his decision based on his prior. As we assume the challenger's expected quality is lower than that of the incumbent, the challenger will not be elected.

There does not exist a pooling equilibrium with a positive precision. This is because for any precision, if the challenger's quality is low enough, his probability of being elected approaches zero, which means that he would rather choose zero precision.

Semi-pooling equilibria

The only other type of equilibrium is semi-pooling equilibrium, which takes the "two-step" form, a challenger whose quality is below a threshold $\widetilde{\theta}$ chooses zero precision, while one whose quality is above $\widetilde{\theta}$ chooses a common level of positive precision.

Proposition 3. *In any equilibrium, there is at most one positive level of precision. Furthermore, if a challenger of quality θ chooses positive precision, then a challenger of quality $\theta' > \theta$ also chooses positive precision. This implies that all semi-pooling equilibria must be two-step equilibria.*

We only offer an intuitive argument, in place of a formal proof. First, if a challenger of quality θ prefers a level of positive precision to zero precision, then one of quality $\theta' > \theta$ should also prefer the same level of positive precision level to zero precision, because quality θ' has a weakly higher chance of being elected than θ, given the same precision level. This implies that, in equilibrium, a challenger chooses zero precision if and only if his quality is below a threshold, $\widetilde{\theta}$. Second, there cannot be two different levels of positive precision, h and h', where $h < h'$. If there were, for precision level h, there must exist quality values above θ_I such that the challenger chooses each level. Otherwise, the voters would infer that the quality of the challenger is below θ_I and would not elect the challenger, and the challenger would be better off choosing zero precision. However, in a monotone equilibrium, this means that the higher precision is chosen only by quality levels above θ_I. Unless the precision is so high that the cost of precision is equal to 1, the challenger's payoff from winning office, a low-quality challenger who is supposed to choose zero precision would want to deviate and choose h' instead. But this means a high-quality challenger who is supposed to choose h' and earn zero payoff would strictly prefer to choose h instead. Therefore, there could not be two different levels of precision in equilibrium.

Given the above proposition, we may conclude further that there are two types of semi-pooling equilibria: *uninformative*, in which the level of precision alone is instrumental to the voters' decision; *informative*, in which both the level of precision and the signal are useful.

Proposition 4. *For each $\widetilde{\theta} > \theta_I$, there exists a 2-interval (uninformative) semi-pooling equilibrium of the following form:*

$$h(\theta) = \begin{cases} \widetilde{h} > 0 & \text{if } \theta \geq \widetilde{\theta}, \\ 0 & \text{if } \theta < \widetilde{\theta}, \end{cases}$$

where

$$\widetilde{h}: \; 1 - C(\widetilde{h}) = 0,$$

$$v(s,h) = v(h) = \begin{cases} 0 & \text{if} \quad h = \widetilde{h} \\ 1 & \text{if} \quad h \neq \widetilde{h} \end{cases},$$

and the beliefs substaining such equilibrium are, among others, $P^e(\theta | h \geq \widetilde{h}) = \phi(\theta | \theta \geq \widetilde{\theta})$, $P^e(\theta | h < \widetilde{h}) = \phi(\theta | \theta < \widetilde{\theta})$.

In the above equilibrium, only relatively high quality levels will choose a positive precision of the signal transmitted by political campaigns. Since high types pool, they will choose by definition the same level of precision. This is indeed an equilibrium under any beliefs that assign zero probability to relatively high types when an out-of-equilibrium precision is observed. The equilibrium precision level is determined by the indifference of type $\widetilde{\theta}$ between choosing precision $\widetilde{h}$ (in which case he is elected with probability 1) and 0. Clearly, this is an equilibrium, as no low type challenger has an incentive to chose a positive precision, as he will not be elected but he would incur the cost. Similarly, no type higher than $\widetilde{\theta}$ has an incentive to deviate, as he is already elected with probability 1 and by deviating he would not get elected.

In this equilibrium, the possibility of campaigns increases the possibility of electing challengers who are better than the incumbent. In fact, very bad challengers ($\theta < \widetilde{\theta}$) never gets elected. And in the case in which $\widetilde{\theta} = \theta_I$ information is aggregated efficiently because types better than the incumbents are always elected and types worse than the incumbent are never elected. However, this equilibrium is "uninformative" in the sense that, paradoxically, the specific campaign message is irrelevant to the voter, who only needs to observe whether the campaign is more or less precise. Therefore, this equilibrium is qualitatively equivalent to a money burning equilibrium in a standard signalling game. Campaign advertising does not transmit any useful information above the one contained in the signal precision. In addition, the payoff of challengers is the same across types and equals zero. So this equilibrium is not strong, as although nobody has incentive to deviate they do not have any strong incentive to follow the equilibrium strategy either.

Proposition 5. *For each $\widetilde{\theta}$ that satisfies $\widetilde{\theta} < \theta_I$ and $H(\widetilde{\theta}) > \theta_I$, there exists a unique two-interval (informative) semi-pooling equilibria of the following form:*

$$h(\theta) = \begin{cases} \widetilde{h} > 0 & \text{if } \theta \geq \widetilde{\theta}, \\ 0 & \text{if } \theta < \widetilde{\theta}, \end{cases}$$

and

$$\widetilde{h}: \ \Pr_{s|\widetilde{\theta}}[E(\Theta|s, \widetilde{h}) \geq \theta_I] - C(\widetilde{h}) = 0,$$

$$v(s,h) = v(h) = \begin{cases} 1\{E(\Theta|s, \widetilde{h}) \geq \theta_I\} & \text{if} \quad h = \widetilde{h} \\ 0 & \text{if} \quad h \neq \widetilde{h} \end{cases},$$

and (unconditional) beliefs substaining such equilibrium are $P^e(\theta|h \geq \widetilde{h}) = \phi(\theta|\theta \geq \widetilde{\theta})$, $P^e(\theta|h < \widetilde{h}) = \phi(\theta|\theta < \widetilde{\theta})$ and conditional beliefs are calculated according to Bayes' rule.

The above proposition characterizes the "informative" two-step equilibria. A challenger whose quality is below a threshold, $\widetilde{\theta}$, chooses zero precision and will not be elected at all. If the challenger's quality is above the threshold, then he chooses positive precision. This precision level is determined by the threshold $\widetilde{\theta}$. Given the threshold, for each level of precision, there exists a corresponding cutoff, $\bar{s}$, such that the representative voter prefers the challenger to the incumbent if the signal he observes about the challenger's quality is above $\bar{s} \in \mathbb{R}$. Hence, the signal the voter observes is instrumental in the electoral decision he makes, that is, the signal is informative.

The probability of winning the election for a challenger who chooses positive precision is the probability that the voter observes a signal higher than $\bar{s}$. In equilibrium, at the threshold quality, $\widetilde{\theta}$, the challenger is indifferent between precision $\widetilde{h}$ and zero precision, as his probability of winning is exactly equal to the cost of creating the precision $\widetilde{h}$. The challenger's winning probability at $\widetilde{\theta}$ decreases with precision, but the campaign cost increases with precision. As the cost increases from zero to infinity when the precision goes from minimum to maximum, the threshold quality's probability of winning decreases from one to zero. Therefore, there is a unique level of precision in equilibrium for each $\widetilde{\theta}$.

Outline of the Proof of Proposition 5. By properties of the normal distribution, the conditional distribution of Θ given h and s is

$$\Theta|(s,h) \sim N(\mu, \sigma^2),$$

where

$$\frac{1}{\sigma^2} \equiv \frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_h^2}, \quad \mu \equiv (1-\lambda)s, \quad \lambda \equiv \frac{\sigma_h^2}{\sigma_\theta^2 + \sigma_h^2}.$$

Therefore,

$$E(\Theta|s,h) = \sigma H\left(\frac{\tilde{\theta} - (1-\lambda)s}{\sigma}\right) + (1-\lambda)s, \tag{2}$$

or

$$E(\Theta|s,h) = \sigma H\left(\frac{\tilde{\theta} - \mu}{\sigma}\right) + \mu, \tag{3}$$

where $H(t)$ is the conditional mean of the standard normal distribution when truncated to the right at t. The above formula simply evaluates the mean of the conditional distribution of Θ given signal s, when truncated to the right at $\tilde{\theta}$.

Step 1. Given $\tilde{\theta}$ and σ_h, there exists a unique $\bar{s}$, which satisfies

$$\sigma H\left(\frac{\tilde{\theta} - (1-\lambda)s}{\sigma}\right) + (1-\lambda)s = \theta_I. \tag{4}$$

To see this, note that $\sigma H\left(\frac{\tilde{\theta}-(1-\lambda)s}{\sigma}\right)+(1-\lambda)s$ is continuous and increasing in s, converges to $\tilde{\theta}$ when $s \to -\infty$, and converges to ∞ as $s \to \infty$, which can shown by using Lemma 1 below.

Step 2. Now, given the signalling strategy of the challenger above, a challenger of quality θ wins the election with probability

$$W(\theta, \widetilde{\theta}, \sigma_h) = 1 - \Phi\left(\frac{\bar{s} - \theta}{\sigma_h}\right) \tag{5}$$

So for each $\tilde{\theta}$, the equilibrium can be characterized by (4) and the following condition:

$$W(\widetilde{\theta}, \widetilde{\theta}, \sigma_h) = 1 - \Phi\left(\frac{\bar{s} - \tilde{\theta}}{\sigma_h}\right) = C(\sigma_h), \tag{6}$$

where we have abused notation slightly and turned C into a function of σ_h, the noise of the challenger's message.

The proof is complete if we can show that for each $\tilde{\theta}$, there exists a σ_h which satisfies the above equation.

When $\sigma_h \downarrow 0$, we have $C(\sigma_h) \uparrow \infty$ by Assumption 1, and $W(\widetilde{\theta}, \widetilde{\theta}, \sigma_h)$ goes to 0 for any $\tilde{\theta} < \theta_I$ as $\bar{s}$ goes to θ_I.

When $\sigma_h \uparrow \infty$, $C(\sigma_h) \downarrow 0$ by Assumption 1. On the other hand, $E(\Theta|s, h)$ converges to $E(\Theta|\Theta \geq \tilde{\theta}) = H(\tilde{\theta})$ for all $s \in \mathbb{R}$. Therefore, $W(\widetilde{\theta}, \widetilde{\theta}, \sigma_h)$ goes to 0 if $H(\tilde{\theta}) < \theta_I$ and goes to 1 if $H(\tilde{\theta}) > \theta_I$.

In the case of $H(\tilde{\theta}) < \theta_I$, the existence of a semi-pooling equilibrium is not assured. It exists if there exists σ_h such that $W(\widetilde{\theta}, \widetilde{\theta}, \sigma_h) \geq C(\sigma_h)$.

In the case of $E(\Theta|\Theta \geq \tilde{\theta}) = H(\tilde{\theta}) > \theta_I$, the proof is done in the rest of this section: it suffices to show that $W(\widetilde{\theta}, \widetilde{\theta}, \sigma_h)$ is increasing in σ_h, which we state as Lemma 5. ∎

In the discussion that follows, without loss of generality,[2] we assume $\sigma_\theta = 1$. Thus,

$$\lambda = \frac{\sigma_h^2}{\sigma_\theta^2 + \sigma_h^2} = \frac{1}{1/\sigma_h^2 + 1} = \frac{1}{1/\sigma_h^2 + 1/\sigma_\theta^2} = \sigma^2.$$

[2] We may normalize both the quality and the signal by a common factor so that the variance of the quality distribution becomes unity. Our analysis remains the same as long as we perform the corresponding monotonic transformation of the cost function, without violating Assumption 1.

Note that σ strictly increases with σ_h from 0 to 1, as the latter increases from 0 to ∞.

As a convention for notation, we use barred symbols to indicate equilibrium values of the variables. As we have defined, $\bar{s}$ is the level of signal s such that (4) holds. That is, the voter elects the challenger if and only if the he receives a more optimistic signal than $\bar{s}$. Using (3), condition (4) can be rewritten

$$\sigma H\left(\frac{\tilde{\theta}-\mu}{\sigma}\right)+\mu=\theta_I. \tag{7}$$

We denote the μ that solves this equation by $\bar{\mu}$.

To facilitate our analysis, we collect some properties of the function H in the following lemma.[3]

Lemma 1. *Let Θ be a random variable with the standard Normal distribution. Let ϕ and Φ be respectively its density and distribution functions. Let $H(t)$ be the conditional mean of the standard Normal distribution, when truncated to the right of t, and let $M(t) = H(t) - t$. Then, the following properties hold:*

1. *The conditional mean of the standard normal distribution when truncated to the right of t is equal to the conditional density of the truncated standard Normal distribution at t:*

$$H(t)=\frac{\phi(t)}{1-\Phi(t)};$$

2. *The function H is increasing and strictly convex and M is decreasing and strictly convex, and satisfy*

$$M(t)>0, \quad \lim_{t\to-\infty} M(t)=+\infty, \quad \lim_{t\to+\infty} M(t)=0;$$
$$H'(t)=H(t)[H(t)-t]=M(t)[M(t)+t]=M'(t)+1\in(0,1);$$
$$\lim_{t\to-\infty} H'(t)=0, \quad \lim_{t\to+\infty} H'(t)=1;$$
$$\lim_{t\to-\infty} M'(t)=-1, \quad \lim_{t\to+\infty} M'(t)=0;$$
$$M''(t)=H''(t)>0;$$
$$Var(\Theta|\Theta\geq t)=1-H'(t)=-M'(t).$$

[3] The inverse of H is often called the Mills' Ratio in the statistics literature. Please see [5] for a proof of the facts in the lemma.

The following two lemmas are needed in the proof of Lemma 5.

Lemma 2.

$$\lim_{t\to\infty} \frac{M'(t)}{M(t)} = 0.$$

Proof. See the Appendix. ∎

An immediate corollary is that

$$\lim_{t\to\infty} \left(\frac{M'(t)}{M(t)}\right)^k = 0,$$

for all $k > 0$.

Lemma 3. *The expression* $M(t)M''(t)/M'(t)^2$ *is in* $(0,2)$, *and*

$$\lim_{t\to-\infty} \frac{M(t)M''(t)}{M'(t)^2} = 0;$$
$$\lim_{t\to\infty} \frac{M(t)M''(t)}{M'(t)^2} = 2.$$

Proof. See the Appendix. ∎

Now, to investigate how $\bar{s}$ depends on σ, we make use of the Implicit Function Theorem. Let us define the expression on the left-hand side of (4) as F, a function of σ and s (note that $\lambda = \sigma^2$). Thus,

$$\begin{aligned}\frac{\partial F}{\partial s} &= (1-\lambda)[1 - H'(t)] > 0,\\ \frac{\partial F}{\partial \sigma} &= H(t) + \sigma H'(t)\frac{\partial t}{\partial \sigma} + \sigma H'(t)\frac{\partial t}{\partial \mu}\cdot\frac{\partial \mu}{\partial \sigma} + \frac{\partial \mu}{\partial \sigma}\\ &= [H(t) - H'(t)t] - 2\sigma s[1 - H'(t)],\end{aligned}$$

where t refers to the argument of H in (4), that is,

$$t = \frac{\tilde{\theta} - \mu}{\sigma}. \tag{8}$$

The effect of an increase in σ affects the truncated conditional mean of Θ in two ways: first, it increases the conditional variance of Θ given s, which results in an increase in the truncated conditional mean, which is reflected in the first term of $\partial F/\partial\sigma$; second, it shifts the conditional mean of the pre-truncation distribution of Θ towards 0, hence the direction of the shift is opposite to the sign of s. As a result of the Implicit Function Theorem,

Lemma 4. *Fixing a cutoff $\widetilde{\theta}$, the equilibrium values, $\bar{t}$ and $\bar{\mu}$, satisfy*

1. *$\bar{t}$ is strictly increasing in σ;*
2. *$\bar{\mu}$ is strictly decreasing and strictly concave in σ.*

Proof. 1. From (7) and (8), we obtain

$$\sigma[H(\bar{t})-\bar{t}]=\theta_I-\tilde{\theta}, \tag{9}$$

which by the Implicit Function Theorem implies

$$\frac{d\bar{t}}{d\sigma}=-\frac{H(\bar{t})-\bar{t}}{\sigma[H'(\bar{t})-1]}=-\frac{M(\bar{t})}{\sigma M'(\bar{t})}>0. \tag{10}$$

2. With a slight abuse of notation, we continue to refer to the expression on the left-hand side of (7) as F, with the understanding that it is now a function of μ and σ.

$$\frac{\partial F}{\partial\mu}=1-H'(t)>0;$$
$$\frac{\partial F}{\partial\sigma}=H(t)+\sigma H'(t)\frac{\partial t}{\partial\sigma}=H(t)-H'(t)t>0.$$

By the Implicit Function Theorem, we have

$$\frac{d\bar{\mu}}{d\sigma}=-\frac{H(\bar{t})-H'(\bar{t})\bar{t}}{1-H'(\bar{t})}<0, \tag{11}$$
$$\frac{d^2\bar{\mu}}{d\sigma^2}=-\frac{H''(\bar{t})[H(\bar{t})-\bar{t}]}{[1-H'(\bar{t})]^2}\cdot\frac{d\bar{t}}{d\sigma}<0, \tag{12}$$

where we have used Lemma 1 and the fact that $d\bar{t}/d\sigma>0$. ∎

Lemma 5. *If $H(\tilde{\theta})>\theta_I$, then the probability that the cutoff quality wins the election, $W(\widetilde{\theta},\widetilde{\theta},\sigma_h)$, is strictly increasing in σ_h.*

Proof. For the cutoff quality $\tilde{\theta}$, the probability of winning the election is

$$W(\widetilde{\theta},\widetilde{\theta},\sigma_h)=1-\Phi\left(\frac{\bar{s}-\tilde{\theta}}{\sigma_h}\right).$$

Note that

$$\frac{\bar{s}-\tilde{\theta}}{\sigma_h}=\frac{\frac{\tilde{\theta}-\sigma\bar{t}}{1-\lambda}-\tilde{\theta}}{\sigma_h}=\frac{\lambda\tilde{\theta}-\sigma\bar{t}}{(1-\lambda)\sigma_h}.$$

Using the facts

$$\frac{\lambda}{1-\lambda}={\sigma_h}^2 \text{ and } \lambda=\sigma^2,$$

the probability of winning can be rewritten

$$W(\widetilde{\theta},\widetilde{\theta},\sigma_h)=1-\Phi\left(\sigma_h\tilde{\theta}-\frac{\sigma_h\bar{t}}{\sigma}\right).$$

Now, we consider how W depends on σ_h. Observe that

$$\begin{aligned}\frac{\partial W}{\partial\sigma_h}&=-\phi\left(\sigma_h\tilde{\theta}-\frac{\sigma_h\bar{t}}{\sigma}\right)\cdot\left[\tilde{\theta}-\frac{d\sigma_h/\sigma}{d\sigma_h}\cdot\bar{t}-\frac{\sigma_h}{\sigma}\cdot\frac{d\bar{t}}{d\sigma_h}\right],\\&=-\phi\left(\sigma_h\tilde{\theta}-\frac{\sigma_h\bar{t}}{\sigma}\right)\cdot\left[\tilde{\theta}-\sigma\bar{t}-\frac{\sigma_h}{\sigma}\cdot\frac{d\sigma}{d\sigma_h}\cdot\frac{H(\bar{t})-\bar{t}}{\sigma[1-H'(\bar{t})]}\right],\\&=-\phi\left(\sigma_h\tilde{\theta}-\frac{\sigma_h\bar{t}}{\sigma}\right)\cdot\left[\bar{\mu}-\frac{\sigma}{{\sigma_h}^2}\cdot\frac{H(\bar{t})-\bar{t}}{1-H'(\bar{t})}\right],\\&=-\phi\left(\sigma_h\tilde{\theta}-\frac{\sigma_h\bar{t}}{\sigma}\right)\cdot\left[\bar{\mu}+\left(\frac{1}{\sigma}-\sigma\right)\frac{M(\bar{t})}{M'(\bar{t})}\right],\end{aligned}$$

Now, we show that the expression $\bar{\mu}+\left(\frac{1}{\sigma}-\sigma\right)\frac{M(\bar{t})}{M'(\bar{t})}$ is always negative.

We prove the claim in two steps. First, we show that in the limit ($\sigma\to 0$ or 1) the expression is nonpositive. Then, we show that the expression is monotonically increasing in σ.

Step 1. Note that when $\sigma\to 0$,

$$\begin{aligned}\bar{\mu}&\to\theta_I,\\\bar{t}\to-\infty,\ \text{hence } M(\bar{t})/M'(\bar{t})&\to-\infty,\\\frac{1}{\sigma}-\sigma&\to+\infty,\end{aligned}$$

which implies

$$\lim_{\sigma\to 0} \bar{\mu} + \left(\frac{1}{\sigma} - \sigma\right) \frac{M(\bar{t})}{M'(\bar{t})} = -\infty.$$

On the other hand, when $\sigma = 1$,

$$\bar{\mu} < 0,$$
$$\bar{t} \text{ is finite, hence } M(\bar{t})/M'(\bar{t}) \text{ is finite,}$$
$$\frac{1}{\sigma} - \sigma = 0,$$

which implies

$$\lim_{\sigma\to 1} \bar{\mu} + \left(\frac{1}{\sigma} - \sigma\right) \frac{M(\bar{t})}{M'(\bar{t})} < 0.$$

Step 2. Now, we show that the expression $\bar{\mu} + \left(\frac{1}{\sigma} - \sigma\right) \frac{M(\bar{t})}{M'(\bar{t})}$ is strictly increasing in σ. Taking its derivative with respect to σ, we obtain

$$\begin{aligned}
&\frac{M(\bar{t})}{M'(\bar{t})} - \bar{t} + \left(-\frac{1}{\sigma^2} - 1\right) \frac{M(\bar{t})}{M'(\bar{t})} + \left(\frac{1}{\sigma} - \sigma\right) \frac{dM(\bar{t})/M'(\bar{t})}{dt} \cdot \frac{d\bar{t}}{d\sigma}, \\
&= -\bar{t} + \frac{1}{\sigma}\frac{d\bar{t}}{d\sigma} + \left(\frac{1}{\sigma} - \sigma\right) \frac{dM(\bar{t})/M'(\bar{t})}{dt} \cdot \frac{d\bar{t}}{d\sigma}, \qquad \text{[by (10)]} \\
&= \sigma\frac{d\bar{t}}{d\sigma} - \bar{t} + \left(\frac{1}{\sigma} - \sigma\right) \left(\frac{dM(\bar{t})/M'(\bar{t})}{dt} + 1\right) \frac{d\bar{t}}{d\sigma},
\end{aligned}$$

where we have used

$$\frac{d\bar{\mu}}{d\sigma} = -\frac{H(\bar{t}) - \bar{t}}{1 - H'(\bar{t})} - \bar{t} = \frac{M(\bar{t})}{M'(\bar{t})} - \bar{t},$$

as implied by (11). To show the derivative is positive, it suffices to show

$$A \equiv \sigma\frac{d\bar{t}}{d\sigma} - \bar{t} > 0;$$
$$B \equiv \frac{dM(\bar{t})/M'(\bar{t})}{dt} + 1 > 0.$$

First, we show $A > 0$.

$$\begin{aligned}
A &= \sigma \cdot \frac{M(\bar{t})}{-\sigma M'(\bar{t})} - \bar{t}, \\
&= \sigma \cdot \frac{M(\bar{t}) + M'(\bar{t})\bar{t}}{-M'(\bar{t})}, \\
&> \frac{M(\bar{t})(1 + M'(\bar{t})) + M'(\bar{t})(M(\bar{t}) + \bar{t})}{-M'(\bar{t})}, \\
&= \frac{M''(\bar{t})}{-M'(\bar{t})}, \\
&> 0,
\end{aligned}$$

where we have repeatedly used the facts $M(\bar{t}) > 0$ and $M'(\bar{t}) < 0$.

To show $B > 0$, observe that

$$\begin{aligned} B &= \frac{M'(\bar{t})^2 - M(\bar{t})M''(\bar{t})}{M'(\bar{t})^2} + 1, \\ &= 2 - \frac{MM''}{M'^2}. \end{aligned}$$

Thus, $B > 0$, is implied by Lemma 3.

Now we have shown both $A > 0$ and $B > 0$. Therefore, the claim is proved. ∎

4 Equilibrium Properties and Welfare Analysis

In this section, we investigate properties of the equilibria and perform welfare analysis. Note that in the previous section, we have shown that there exist two types of equilibria: uninformative, where the act of campaigning alone indicates that the challenger is better than the incumbent, and informative, where both the act of campaigning and the signal generated by campaigning are used by the voters to make their choice. From Proposition 4, in the former, the challenger chooses the level of campaigning that costs him as much as the benefit from winning the election. As long as the cutoff for campaigning, $\tilde{\theta}$, is above the incumbent's quality, θ_I, the level of campaigning in equilibrium is the same. However, from Proposition 5, in the latter, the level of campaigning depends on the cutoff, $\tilde{\theta}$.

Corollary 1. *In an informative equilibrium, the level of campaigning by the challenger, is increasing in the cutoff, $\tilde{\theta}$.*

The intuition for this result is as follows. In an informative equilibrium, if the cutoff for campaigning is higher, then the quality of a challenger who campaigns is higher. This implies that the voter demands a lower minimum signal to elect the challenger, which makes the challenger at the cutoff quality level easier to be elected. The equilibrium condition then requires that a higher level of precision must prevail in equilibrium.

Now, we analyze how social welfare varies with the cutoff $\tilde{\theta}$. As a starting point, we take social welfare to be the sum of the voter's expected payoff, the challenger's expected utility, the incumbent's expected utility, minus the cost of campaigning. Therefore,

$$SW = \int_{-\infty}^{\tilde{\theta}} \theta_I \phi(\theta) d\theta + \int_{\tilde{\theta}}^{\infty} [\theta_I(1 - W(\theta)) + \theta W(\theta)] \phi(\theta) d\theta$$
$$+1 - C(\sigma_h)[1 - \Phi(\tilde{\theta})], \tag{13}$$

where we have used $W(\theta)$ as a shorthand for $W(\theta, \tilde{\theta}, \sigma_h)$. The term 1 is the sum of the challenger's and the incumbent's utility from holding office. The last term is the expected cost of the political campaign. Note that by (6), we may replace $C(\sigma_h)$ with $W(\tilde{\theta}, \tilde{\theta}, \sigma_h)$, or $W(\tilde{\theta})$ in short, evaluated at the equilibrium value of σ_h. Thus, we may rewrite the expression for social welfare as

$$SW = \int_{\tilde{\theta}}^{\infty} [(\theta - \theta_I) W(\theta) - W(\tilde{\theta})] \phi(\theta) d\theta + \theta_I + 1, \tag{14}$$

where the first term in the integral measures the gain from electing the challenger, while the second term measures the cost of campaigning.

Proposition 6. *In the uninformative equilibrium, social welfare is maximized at $\tilde{\theta} = \theta_I + 1$. Furthermore, the maximum social welfare is always higher than that under the pooling equilibrium.*

In the uninformative equilibrium, the challenger always wins the election if he campaigns at all. The cost of campaigning is equal to the benefit of winning the election. From the perspective of social welfare, it is only efficient for the challenger to campaign when the challenger's quality exceeds that of the incumbent by the cost of campaigning. Hence, the threshold that maximizes social welfare is equal to the incumbent's quality plus the benefit of winning the office.

The welfare analysis for the informative equilibrium is more involved, which we hope to deal with in future research.

5 Conclusion and discussions

In this paper, we have constructed a model in which a candidate may affect the precision of his message through costly campaigning effort. The voters observe the level of precision, as well as the campaign message itself. We characterize three types of equilibria of the game: pooling, uninformative, and informative. We show that the uninformative equilibria, in which campaigning is pure money burning and the signal is not useful in the voter's decision making, generate higher social welfare than the pooling equilibrium.

Our model can be extended in a number of ways. First, we may consider the scenario in which the incumbent's quality is not perfectly observed. Second, we may assume voters are heterogenous in terms of their preferences for the candidates. Finally, we may consider the scenario where candidates differ in both their quality and positions.

6 Appendix: Proofs

Proof of Proposition 1. In a separating equilibrium, by observing h the voter can perfectly infer the challenger's type. , then $P^e(\widehat{\theta}|\widehat{h}, s) = 1$, and $P^e(\widetilde{\theta}|\widehat{h}, s) = 0, \forall.s, \widetilde{\theta} \neq \widehat{\theta}$. In this context the signal s is therefore irrelevant, that is $\pi(\widehat{\theta}, s) = \pi(\widehat{\theta})\ \forall s$. In particular, the probability that a challenger with quality $\widehat{\theta}$ is elected $\pi(\widehat{\theta})$ is 1 if $\widehat{\theta} > \theta_I$ and 0 otherwise. The payoff function of a challenger of type $\widehat{\theta}$, is

$$V(\widehat{\theta}) = 1\{\widehat{\theta} > \theta_I\} - C(h(\widehat{\theta}))$$

where $1\{\cdot\}$ is an indicator function taking the value 1 when the expression inside the brakets is true. Clearly, this cannot be an equilibrium because all challengers with $\widehat{\theta} < \theta_I$ would profitably deviate by setting $h(\widehat{\theta}) = 0$. ∎

Proof of Proposition 2. The pooling equilibrium with $h^* = 0$ is sustained by beliefs that any $h^* > 0$ comes from a bad types ($\theta < \theta_I$). In this case, any effort will not be rewarded by a positive probability of winning and, as a result, no type finds it optimal to exert any positive effort.

Now, we show that there does not exist any fully pooling equilibrium with $h^* > 0$. Suppose all candidate types choose $h^* > 0$. The equilibrium probability that a challenger of quality θ is elected is $\pi(\theta, h^*) = \Pr_{s|\theta}[E_{P^e}[\Theta|s, h^*] > \theta_I]$, where $E_{P^e}[\Theta|s, h^*]$ is the posterior expectation about the challenger quality given signal s and precision h^*, where the expectation is taken with respect to the posterior distribution $P^e(\cdot|s, h^*)$, which in this case is just the prior distribution of the challenger quality. The probability of election $\pi(\theta, h^*)$ is the integral of this posterior expectation with respect to hte distribution of the signal conditional induced by a challenger with quality θ. Using the properties of the normal distribution we have that $E_{P^e}[\Theta|s, h^*] = \lambda^* s$, where $\lambda^* = \frac{\sigma_\theta^2}{\sigma_\theta^2 + (\sigma^*)^2}$ and $\sigma^* = 1/\sqrt{h^*}$. Rearranging, one obtains that $\pi(\theta_I, \theta, h^*) = \Pr_{s|\theta}[(\lambda^* s > \theta_I)]$. Since $s = \theta + \varepsilon\sigma^*$, conditional on the quality θ and the equilibrium precision h^* the signal is distributed according to $N(\theta, \sigma^{*2})$ and $\pi(\theta, h^*) = \Pr_{s|\theta}[(s - \theta)/\sigma^* > (\theta_I - \lambda^*\theta)/(\lambda^*\sigma^*)] = \Phi((\lambda^*\theta - \theta_I)/(\lambda^*\sigma^*))$. This probability tends to 0 when θ goes to minus infinity, it tends to 1 when θ goes towards plus infinity and it is increasing in θ. Hence, for any given h^*, there exists a type θ^* such that $\pi(\theta^*, h^*) - C(h^*) = 0$, and when $\theta < (>)\theta^*$, $\pi(\theta_I, \theta^*, h^*)B - C(h^*) < (>)0$. It follows that challengers of types $\widehat{\theta} < \theta^*$ prefer to deviate to $\widehat{h} < h^*$ independent of the off-equilibrium beliefs. ∎

Proof of Lemma 2. Let C represent the limit. For simplicity, we suppress the argument t from the expressions whenever there is no confusion. Note that M and M' both go to 0 as t goes to $-\infty$. By l'Hôpital's rule, we have

$$\begin{aligned} C &= \lim_{t\to\infty} \frac{M''}{M'}, \\ &= \lim_{t\to\infty} \frac{M'(M+t) + M(M'+1)}{M'}, \\ &= \lim_{t\to\infty} (M+t) + \frac{M(M'+1)}{M'}, \\ &= \lim_{t\to\infty} t + \lim_{t\to\infty} \frac{M}{M'}. \end{aligned}$$

Note that $M'/M < 0$ for all t, therefore $C \leq 0$. If $C < 0$, then the above equation gives

$$C - \frac{1}{C} = \infty,$$

which is impossible. Therefore, $C = 0$. ∎

Proof of Lemma 3. Again, we suppress the argument t when there is no confusion. Let $E = MM''/M'^2$.

First, we show that E converges to 0 as t goes to $-\infty$. As t goes to $-\infty$, the following facts are true from Lemma 1:

$$M + t \to 0, \qquad M' \to -1.$$

Using the fact

$$M'' = M'(M + t) + M(M' + 1),$$

we have

$$\begin{aligned}
\lim_{t\to-\infty} E &= \lim_{t\to-\infty} MM'(M + t) + M^2(M' + 1), \\
&= \lim_{t\to-\infty} -M(M + t) + M^2M(M + t), \\
&= \lim_{t\to-\infty} (-M + M^3)\frac{\phi(t)}{1 - \Phi(t)}, \\
&= \lim_{t\to-\infty} -(H - t)[1 - (H - t)^2]\frac{\phi(t)}{1 - \Phi(t)}, \\
&= 0,
\end{aligned}$$

where in the last equality we have used the fact $H \to 0$ and $t^k\phi(t) \to 0$ for all k as t goes to $-\infty$.

Now, we proceed to show the other statements. Observe that

$$\begin{aligned}
E &= \frac{M[M'(M + t) + M(M' + 1)]}{M'^2}, \\
&= \frac{(M' + M^2)(M' + 1)}{M'^2}, \\
&= \frac{1 + \frac{M^2}{M'}}{\frac{M'}{M'+1}}.
\end{aligned}$$

The derivatives of the denominator and numerator with respect to t are respectively

$$\frac{d\frac{M'}{M'+1}}{dt} = \frac{M''}{(M' + 1)^2} > 0,$$

and

$$\begin{aligned}
\frac{d(1 + \frac{M^2}{M'})}{dt} &= \frac{2MM' \cdot M' - M^2M''}{M'^2}, \\
&= M(2 - \frac{MM''}{M'^2}), \\
&= M(2 - E).
\end{aligned}$$

Since $E>0$ and $M'/(M'+1)<0$, and therefore $1+M^2/M'<0$. Now suppose $E>2$, then

$$\frac{d(1+\frac{M^2}{M'})}{dt}<0,$$

which then implies $E'>0$. In other words, whenever there is a t_0 such that $E(t_0)\geq 2$, $E(t)$ will always increase for $t>t_0$. Given this, as long as

$$\lim_{t\to\infty} E=2,$$

there would be a contradiction. Hence,

$$\lim_{t\to\infty} E=2,$$

implies that $E\in(0,2)$ for all t.

Now, we show

$$\lim_{t\to\infty} E=2.$$

To show this, we consider the limits of M^2/M' and M'/M^2 as t goes to ∞. Note that M^2 and M' both go to 0 as t goes to ∞. By l'Hopital's Rule, we have

$$\lim_{t\to\infty}\frac{M^2}{M'}=\lim_{t\to\infty}\frac{2MM'}{M''}=\lim_{t\to\infty}\frac{2}{E}\cdot\frac{M^2}{M'},$$

$$\lim_{t\to\infty}\frac{M'}{M^2}=\lim_{t\to\infty}\frac{M'/M}{M}=\lim_{t\to\infty}\frac{\frac{M''M-M'^2}{M^2}}{M'}=\lim_{t\to\infty}(E-1)\frac{M'}{M^2}.$$

From the first equation, either $\lim_{t\to\infty}E=2$ or $\lim_{t\to\infty}\frac{M^2}{M'}=0$. From the second equation, either $\lim_{t\to\infty}E=2$ or $\lim_{t\to\infty}\frac{M'}{M^2}=0$. But, $\lim_{t\to\infty}\frac{M^2}{M'}=0$ and $\lim_{t\to\infty}\frac{M'}{M^2}=0$ cannot both hold. Therefore, we conclude

$$\lim_{t\to\infty} E=2. \blacksquare$$

Proof of Corollary 1. From equilibrium condition (4), fixing any σ (hence λ), an increase in $\tilde{\theta}$ leads to a decrease in $\bar{s}$, which in turn leads to an increase in the cutoff type's probability of winning, $W(\tilde{\theta},\tilde{\theta},\sigma_h)$. By equilibrium condition (6), this results in a decrease in the equilibrium variance, σ_h, or an increase in the precision of the political campaign. ∎

Proof of Proposition 6. In the uninformative equilibrium, the challenger always gets elected if he campaigns at all. From (14), we obtain

$$SW = \int_{\tilde{\theta}}^{\infty} [\theta - \theta_I - 1]\phi(\theta)d\theta + \theta_I + 1,$$

which is clearly maximized at $\tilde{\theta} = \theta_I + 1$.

In the pooling equilibrium, $SW = \theta_I + 1$. However, when $\tilde{\theta} = \theta_I + 1$, the integral is positive. Therefore, the uninformative equilibrium dominates the pooling equilibrium in terms of social welfare.

Note that the result may hold even if $\theta_I < 0$, which means that the challenger gets elected in the pooling equilibrium, that is, $SW = E(\Theta) + 1 = 1$. In the uninformative equilibrium, if $\tilde{\theta} = \theta_I + 1$, we have

$$\begin{aligned} SW &= \int_{\tilde{\theta}}^{\infty} \theta\phi(\theta)d\theta + \int_{-\infty}^{\tilde{\theta}} (\theta_I + 1)\phi(\theta)d\theta, \\ &= H(\tilde{\theta})[1 - \Phi(\tilde{\theta})] + (\theta_I + 1)\Phi(\tilde{\theta}), \\ &= \phi(\theta_I + 1) + (\theta_I + 1)\Phi(\theta_I + 1). \end{aligned}$$

Note that when $\theta_I = -1$, $SW = \phi(0) < 1$. When $\theta_I = 0$, $SW = \phi(1) + \Phi(1) > 1$. Also,

$$\frac{d[\phi(\theta) + \theta\Phi(\theta)]}{d\theta} = \Phi(\theta) > 0.$$

Therefore, there exists $\theta_I^0 \in (-1, 0)$ such that social welfare in the optimal uninformative equilibrium is better than the pooling equilibrium if and only if $\theta_I > \theta_I^0$. ∎

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