Cheap Talk with an Exit Option: A Model of Exit and Voice*

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Abstract

The paper presents a formal model of the exit and voice framework proposed by Hirschman [14]. To be more precise, we modify the cheap talk model of Crawford and Sobel [9] such that the sender of a cheap talk message has the exit option. We demonstrate that the existence of the exit option may increase the informativeness of cheap talk and improve welfare if the exit option is attractive to the sender. Moreover, it is verified that perfect information transmission can be approximated in the limit. The results suggest that the exit reinforces the voice in that the credibility of the exit increases the informativeness of the voice.

Keywords: Exit, Voice, Cheap Talk, Informativeness, Credibility

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1 Introduction

Since the publication of the book "Exit, Voice, and Loyalty" by Hirschman [14], the exit-voice perspective has been widely adopted by studies in the field of political science, and it has also been extended to various studies on relationships and organizations, such as employer-employee (or union) relationships, buyer-seller relationships, hierarchies, public services, political parties, families, and adolescent development (See Hirschman [14] and [15]).

Broadly speaking, the exit and the voice are alternative means of dealing with problems that arises within an ongoing relationship or organization. For example, consider an employer-employee relationship.[1] Suppose an employee finds himself in undesirable situation such as for conditions of employment, compensation packages, and rules of the work place. In this situation, the employee usually has two options. One is to quit the job; this is the exit option. The other is to express his dissatisfaction directly to the employer ; this is the voice option. Hirschman insists that the voice as well as the exit option is important for the sustainability of relationships and organizations—a concept that has been neglected in economics thus far.

With regard to the workings of the exit and voice options in a real economy, a point of considerable interest concerns how the exit interacts with the voice; this also constitutes the main point of Hirschman's discussion. From one perspective, the exit works as a complement to the voice. Indeed, Hirschman briefly points out in Palgrave's dictionary [15], "[t]he availability and threat of exit on the part of important customer or group of members may powerfully reinforce their voice."[2] However, it is not very clear why and how the exit can reinforce the voice. The present paper aims to clarify this by analyzing a formal model of exit and voice.

In this paper, the exit is regarded as a decision to terminate an ongoing relationship. On the other hand, the voice is interpreted as an activity involving sending a costless message that enables the improvement of the relationship. In other words, we identify the voice with "cheap talk" for transmitting useful information.[3] Among others, the model by Crawford

[1] Freeman [12].

[2] In his 1970 book [14], Hirschman appears to emphasize on a substitute aspect between exit and voice. However, in 1987 Palgrave's dictionary [15], he turned to insist that a complementarity aspect of exit and voice is also important.

[3] As we see later, Banerjee and Somanthan [2] also identify voice as an activity of sending a cheap talk

and Sobel [9] (hereinafter referred to as CS) is the most successful one describing cheap talk with private information. We employ the CS model as the basis of the environment that we consider in the present paper and extend it to the situation in which the exit option is available.

The CS model has two players. One player possesses private information about the current state of the relationship, which is randomly drawn. In order to transmit the information, she sends a costless message to her partner, and the latter responds with a decision affecting both the agents' payoffs. In CS, the latter is called the Receiver (R), while the former is called the Sender (S). CS shows that the incongruence between both agents' preferences restricts the informativeness of cheap talk; in particular, they demonstrate that perfect information transmission via cheap talk is impossible as an equilibrium behavior unless the agents' preferences completely coincide.

In the present paper, we assume that S has the exit option after he observes R's decision. When S exercises the exit option, both agents obtain their exit payoffs. The key elements of our results are the difference between agent's payoff when S chooses to stay and one when she chooses to exit. Consider the case where R's difference is large and S's difference is small but positive. In this case, R has a strong incentive to prevent S from choosing the exit option, and therefore, R will make a decision that is desirable for S even if both agents' preferences differ. Expecting R's response, S has a strong incentive to transmit more accurate information via cheap talk. It follows that the existence of S's exit option increases the informativeness of cheap talk, which in turn may increase not only S's payoff but also that of R. Moreover, we show that as S's difference approaches 0, the information transmission via cheap talk in the most informative equilibrium becomes almost perfect. In other words, the exit reinforces the voice in that the existence of the exit increases the informativeness of the voice. This is the main result of the present paper.

CS also shows that the more congruent both agents' preferences are, the more informative cheap talk is on the most efficient equilibrium. In other words, in the situation without the exit option, the informativeness of cheap talk is determined mainly by the degree of incongruence between the agents' preferences. However, in the situation with the exit option, there is another determinant of the informativeness: *the credibility of the exit*. A smaller S's difference between her maximum stay payoff and exit payoff makes her

message.

choice of the exit option more credible, which in turn enables a more informative cheap talk transmission on the equilibrium. Thus, we show that informative cheap talk transmission can be carried out even if S's preference is not exactly similar to that of R.

To the author's knowledge, the exit-voice perspective has seldom been analyzed in any formal model in economics despite the vast citations.[4] Banerjee and Somanthan [2] is one exception in that they present a game-theoretical model of voice. Like us, they consider a voice as an activity of sending a cheap talk message. However, their model differs form ours in a few respects. First, they do not consider the exit option, and therefore, they do not investigate the interplay between exit and voice, which the present paper focuses on. On the other hand, they consider the collective aspect of voice formation, which is abstracted out from our model. In this regard, the present paper can be considered as a complement to their paper. Gehlbach [13] is the paper presenting a formal model of exit and voice. In his model, voice is considered as some costly activity of gathering the members' various opinions, unifying them, and bargaining with the leader of the organization. On the other hand, in his model, there is no asymmetric information, and therefore, voice has no role of information transmission. Although his model sheds a light on one aspect of voice, in this paper, we mainly analyze an information transmission role of voice.

Apart from the exit-voice perspective, the CS model *per se* has still attracted considerable attention and has been extended to various directions.[5] However, the effect of the exit option on cheap talk has barely been analyzed. As an exception, Matthews [23] deals with a cheap talk game with a congress and a president—the receiver and sender, respectively—with veto, which is a means similar to the exit option in our model. In particular, the timing of events in his model is approximately the same as ours. However, there is a large difference with respect to what private information pertains to. In Matthews, private information concerns the sender's preference, while in our model, it pertains to the current state of the relationship. One may consider such a difference to be small, but it leads to very different outcomes: in Matthews, the informativeness of cheap talk is constrained on a strict upper bound, independent from the exit value. On the other hand, we show that in our model, an equilibrium can be close to that with perfect information transmission to any degree. In other words, Matthews does not emphasize that the existence of the exit

[4]For efforts in political science, see, for example, the survey by Dowding et al. [11].

[5]For example, see Krishna and Morgan [19], Battaglini [4], and Chen et al. [7], among others.

increases the informativeness of cheap talk, which is the main claim of the present paper.

Similarly as in CS, we do not allow the agents to design a mechanism or contract dependent upon the message sent by S. On the other hand, in the literature on delegation such as Holmström [16] and Melumad and Shibano [24], it is often assumed that R can commit to message-dependent mechanisms.[6] In this context, our result implies that in the environment with an exit option, an efficient outcome can be realized by a simple contract that allocate the joint surplus so that S's difference is small and R's difference is large even if R *cannot* commit to the message-dependent mechanisms.

Dessein [10] and Marino [21] compare the outcome of simple delegation without message-dependent mechanism with one of cheap talk. The results of the present paper are also related to them. In the present paper, we assume that R cannot delegate her action choice to S. Despite this assumption, our result implies that there is a sequence of equilibrium actions converging to the most desirable action for S. This outcome is realized if R commits to delegating the choice of the action to S. In other words, our result implies that even if a commitment to delegation is impossible, the credibility of the exit option can bring about a similar outcome.[7]

In addition, Compte and Jehiel [8] and Bester and Krähmer [5] analyzes a mechanism design problem in the existence of an an exit option. Since their environments are somehow different from ours, the logic working in our model does not appear in their models. Indeed, Compte and Jehiel show that the existence of exit option makes it more difficult to implement an efficient outcome. This is rather an opposite conclusion of our paper: the existence of exit option leads to more efficient outcome via more informative cheap talk.

The rest of the paper is organized as follows. In Section 2, we present a formal model of exit and voice. In Section 3, we analyze a specific model (so-called "uniform-quadratic model") and present the main claim of this paper. In Section 4, we present a sufficient condition for the main claim to hold in a general model. We conclude the paper in Section 5.

[6] In the more recent literature, see Baron [3], Krähmer [18], Martimort and Semenov [22], Alonso and Matouschek [1], Mylovanov [25], and Kovac and Mylovanov [17]

[7] Bester and Strausz [6], [2] and Krishna and Morgan [20] also analyze a mechanism design problem when only partial commitment of delegation is possible.

2 Setup

There are two players, namely, the sender (S) and the receiver (R). At the beginning of the game, Nature chooses a current state of the relationship between S and R, $t \in T$, according to a probability distribution $F(t)$. A realized state is observed by S but not by R. Based on this observation, S chooses a message $m \in M$ sent to R. This message is cheap talk in that it is payoff-irrelevant. After R receives S's message, R chooses an action $a \in A$ relevant to both players' payoffs. For $i = S, R$, let i's payoff be $y^i(t, a)$. We assume $T = M = [0, 1]$ and $A = \mathbb{R}$. For $i = S, R$, $y^i(t, a)$ is defined on $[0, 1] \times \mathbb{R}$. $F(t)$ has a continuous density $f(t)$, where $f(t) > 0$ for any $t \in T$. We assume that for $i = S, R$, $y^i(t, a)$ is twice continuously differentiable, and

$$\begin{aligned}
&\forall t \ \exists a \text{ such that } \frac{\partial y^i(t,a)}{\partial a} = 0, \\
&\forall t, \forall a, \ \frac{\partial^2 y^i(t,a)}{\partial a^2} < 0, \\
&\forall t, \forall a, \ \frac{\partial^2 y^i(t,a)}{\partial a \partial t} > 0.
\end{aligned}$$

Up to this point, the ingredients are the same as in CS. Now we introduce the concept of exit. After observing R's action, S chooses whether to exit or stay. If S chooses to exit, S and R's payoffs are U^S and U^R, respectively. If S chooses to stay, S and R's payoffs are given by $y^S(t, a)$ and $y^R(t, a)$.

Remark 1 Our model assumes that only S has the exit option. Even if R also has the exit option, our results would not change, provided S keeps the exit option, since R can virtually induce S to choose the exit option by choosing some extreme action in our model. However, it is easily observed that things would change dramatically if S no longer has an exit option. This consideration suggests that whether or not the agent with the voice option has the exit option is a relevant factor.

In Section 3, we employ a more specific model. *A uniform-quadratic model* is a model in which $F(t)$ is a uniform distribution function on $[0, 1]$ and S and R's stay payoffs are expressed as

$$\begin{aligned}
y^S(t, a) &= Y^S - (t + b - a)^2, \\
y^R(t, a) &= Y^R - (t - a)^2,
\end{aligned}$$

for some $b > 0$. b is called *a bias* that represents a degree of incongruence between S and R's optimal actions.[8] Y^i is the maximum stay payoff for i. A uniform-quadratic model was originally analyzed in Section 4 of CS. We show that the main results obtained in the uniform-quadratic model can be applied to more general environments in Section 4. We assume that $U^S \leq Y^S$ and $U^R \leq Y^R$. Define the difference between i's maximum stay payoff and exit payoff by $D^i = Y^i - U^i$ for $i = R, S$.

We consider a perfect Bayesian equilibrium as an equilibrium concept. We also restrict our attention to the class of equilibria with pure strategies. A pure strategy perfect Bayesian equilibrium is defined by $(\mu, P, \alpha, \epsilon)$ in which

- $\mu : T \to M$: S's message strategy,
- $P : M \times T \to [0, 1]$: R's posterior belief distribution function over T on the observation of m,
- $\alpha : M \to A$: R's action choice strategy, and
- $\epsilon : T \times A \to \{0, 1\}$: S's exit strategy. To be more precise, $\epsilon = 1$ refers to exit, and $\epsilon = 0$ refers to stay.

The equilibrium conditions are

$$\mu(t) \in \arg\max_{m \in M} \left\{\epsilon(t, \alpha(m))U^S + (1 - \epsilon(t, \alpha(m)))y^S(t, \alpha(m))\right\}, \quad \forall t \in T,$$

$$P(m, t) = \frac{\lambda(\{\tilde{t}|m = \mu(\tilde{t})\} \cap [0, t])}{\lambda(\{\tilde{t}|m = \mu(\tilde{t})\})}, \quad \forall m \text{ such that } \{\tilde{t}|m = \mu(\tilde{t})\} \neq \emptyset \ (\lambda\text{: Lebesgue measure}),$$

$$\alpha(m) \in \arg\max_{a \in A} \int_{t \in T} \left\{\epsilon(t, a)U^R + (1 - \epsilon(t, a))y^R(t, a)\right\} P(m, dt), \quad \forall m \in M,$$

$$\forall t \in T, \forall a \in A, \begin{cases} U^S > y^S(t, a) & \Rightarrow \epsilon(t, a) = 1, \\ U^S < y^S(t, a) & \Rightarrow \epsilon(t, a) = 0. \end{cases}$$

The first line refers to the condition that $\mu(t)$ is an optimal message for type t of S given R's strategy and S's exit strategy. The second line refers to the condition that R's posterior belief is updated by adhering as much as possible to the Bayesian approach. The third line refers to the condition that $\alpha(m)$ is an optimal action for R, given R's posterior updated based on the observation of m and S's exit strategy. The last line refers to the condition that $\epsilon(t, a)$ is an optimal exit choice for type t of S, given a realized action a.

[8] For interpretations of biases in the real world, see the discussion in Dessein [10], among others.

We state that an action a is *induced on the equilibrium path* if there exists $t \in T$ such that $a = \alpha \circ \mu(t)$ and $\epsilon(t,a) = 0$. For ease of exposition, given $\underline{t} < \bar{t}$, denote the uniform distribution function on interval $[\underline{t}, \bar{t}]$ by $U_{\underline{t}}^{\bar{t}}$, i.e.,

$$U_{\underline{t}}^{\bar{t}}(t) = \begin{cases} 0, & \text{if } t < \underline{t}, \\ \frac{t-\underline{t}}{\bar{t}-\underline{t}}, & \text{if } \underline{t} \leq t \leq \bar{t}, \\ 1, & \text{if } t > \bar{t}. \end{cases}$$

With a slight abuse of notation, denote the distribution function with unit mass on point $\bar{t}$ by $U_{\bar{t}}^{\bar{t}}$, i.e.,

$$U_{\bar{t}}^{\bar{t}}(t) = \begin{cases} 0, & \text{if } t < \bar{t}, \\ 1, & \text{if } t \geq \bar{t}. \end{cases}$$

3 Uniform-Quadratic Model

3.1 Preliminary: Environment without Exit

In this section we analyze a uniform-quadratic model. We first revisit CS's results in an environment without exit. If the exit option is not available, perfect information transmission via cheap talk does not occur. This is because S has no incentive to truthfully report the current state because of the fear of R exploiting the information.

To be more precise, CS shows that in any equilibrium there are finite intervals partitioning T and S informs R via cheap talk of which interval a true state is lying on. The necessary and sufficient condition for the existence of the equilibrium with N intervals is

$$b < \left\langle \frac{1}{2N(N-1)} \right\rangle, \tag{1}$$

where $\langle \cdot \rangle$ is the operator such that

$$\left\langle \frac{x}{y} \right\rangle = \begin{cases} \frac{x}{y}, & \text{if } y \neq 0, \\ \infty, & \text{if } y = 0, \ x \neq 0. \end{cases}$$

In other words, regarding N as the informativeness of the cheap talk, the informativeness is determined by the bias b. The smaller b is, the more intervals the equilibrium has. Henceforth, we restrict our attention to the most informative equilibrium or the equilibrium with the most intervals.[9] Indeed, CS shows that the equilibrium with the most intervals is Pareto superior to any other equilibrium with less intervals.

[9] Che et al. [7] present a condition that select the most informative equilibrium in uniform-quadratic models.

3.2 Characterization of No-Exit Equilibria

Hereafter, we consider the environment in which the exit option is available for S. We consider the case where R's exit value U^R is small, or equivalently D^R is large. A large D^R implies that R has a strong incentive to prevent S from choosing the exit option.

As an extreme case, suppose $Y^S = U^S$, or equivalently $D^S = 0$. In this case the following constitutes an equilibrium:

$$\begin{aligned}
&\mu(t) = t, \quad \forall t, \\
&P(m,t) = U_m^m(t), \quad \forall m \\
&\alpha(m) = m + b, \quad \forall m, \\
&\epsilon(t,a) = 0, \quad \text{iff } a = t + b.
\end{aligned}$$

In other words, perfect information transmission is realized via cheap talk.

The intuition is simple. Since $D^S = 0$, S will choose the exit option unless the best action for S is chosen. On the other hand, since D^R is sufficiently large, more precisely $D^R \geq b^2$, R wishes to continue the relationship, and therefore, R will make the greatest effort to keep S in the relationship by choosing the best action for S. Expecting this, S truthfully reports a current state without fear of exploitation by R.

This equilibrium gives S and R's ex ante payoffs, denoted by V^S and V^R, as follows:

$$\begin{aligned}
V^S &= Y^S, \\
V^R &= Y^R - b^2.
\end{aligned}$$

On the other hand, CS tell us that S and R's largest equilibrium ex ante payoffs in the environment without exit are as follows:

$$\begin{aligned}
\hat{V}^S &= Y^S - \frac{4N^2(N^2+2)b^2+1}{12N^2}, \\
\hat{V}^R &= Y^R - \frac{4N^2(N^2-1)b^2+1}{12N^2},
\end{aligned}$$

where N is the largest natural number satisfying (1). By a direct calculation, if $b < \frac{1}{2\sqrt{3}}$, then

$$Y^S > \hat{V}^S, \tag{2}$$

$$Y^R - b^2 > \hat{V}^R. \tag{3}$$

In other words, the existence of S's exit option increases the ex ante payoff of R as well as S unless the bias is very large.[10]

Even if $D^S > 0$, the smaller D^S is, the more accurate is the information sent on the equilibrium, provided D^R is sufficiently large. Indeed, we can identify a sequence of equilibria in which S and R's payoffs are approaching those in the equilibrium with perfect information, Y^S and $Y^R - b^2$. Throughout this section, we assume that $D^S > 0$ and $D^R > 0$.

Hereafter, we focus on the equilibrium in which an exit option is never exercised on the equilibrium path. We call it *No-Exit Equilibrium (NEE)*. The following result insists that any NEE is characterized by a partition of the state space consisting of finite intervals.[11]

Lemma 1 In any equilibrium, there are only finite actions induced on the equilibrium path. This implies that any NEE $(\mu, P, \alpha, \epsilon)$ is characterized by a partition $\{\tau_n\}_{n=1,\dots,N}$ of $[0,1]$ such that

- N is finite,
- τ_n is an interval for $n = 1, \dots, N$, and
- there exist $\{t_n\}_{n=0,\dots,N}$, $\{m_n\}_{n=1,\dots,N}$, $\{a_n\}_{n=1,\dots,N}$ such that
  - $\inf \tau_n = t_{n-1}$ and $\sup \tau_n = t_n$ for $n = 1, \dots, N$,
  - $0 = t_0 < t_1 < \dots < t_N = 1$,
  - $\mu(t) = m_n$ for any $t \in \tau_n$, $m_n \neq m_{n'}$ for $n \neq n'$, and therefore $P(m_n, t) = U_{t_{n-1}}^{t_n}(t)$ for $n = 1, \dots n$,
  - $\alpha(m_n) = a_n$ for $n = 1, \dots, N$, and
  - $\epsilon(t, a_n) = 0$ for $t \in \tau_n$ and $n = 1, \dots, N$.

All the proofs are relegated to Appendix. Below, we derive the equilibrium condition for NEE with N intervals. The following result shows that any interval of NEE is classified into three categories.

[10] Note that if $b \geq \frac{1}{2\sqrt{3}}$, there is no informative equilibrium in environment without exit, by (1).

[11] Note that an equilibrium may have infinite number of intervals in our model. This is because there are infinitely many actions inducing S to choose the exit option.

Lemma 2 Fix an NEE and an interval $\hat{\tau}$. Let $\inf \hat{\tau} = \underline{t}$, $\sup \hat{\tau} = \bar{t}$, and $\hat{a} = \alpha \circ \mu(t)$ for $t \in \hat{\tau}$. Then, $\bar{t} - \underline{t} \leq 2\sqrt{D^S}$. Moreover, $\hat{\tau}$ belongs to either one of the categories:

Interval $\mathcal{N}$: $\underline{t}$, $\bar{t}$, and $\hat{a}$ satisfy

- $\bar{t} - \underline{t} < 2\sqrt{D^S} - 2b$,
- $\hat{a} = \frac{\underline{t}+\bar{t}}{2}$,
- $y^S(\underline{t}, \hat{a}) > U^S$, and
- $y^S(\bar{t}, \hat{a}) > U^S$.

Interval $\mathcal{A}$: $\underline{t}$, $\bar{t}$, and $\hat{a}$ satisfy

- $2\sqrt{D^S} > \bar{t} - \underline{t} \geq 2\sqrt{D^S} - 2b$,
- $\hat{a} = \bar{t} - \sqrt{D^S} + b$,
- $y^S(\underline{t}, \hat{a}) > U^S$, and
- $y^S(\bar{t}, \hat{a}) = U^S$.

Interval $\mathcal{F}$: $\underline{t}$, $\bar{t}$, and $\hat{a}$ satisfy

- $\bar{t} - \underline{t} = 2\sqrt{D^S}$,
- $\hat{a} = \bar{t} - \sqrt{D^S} + b$, and
- $y^S(\underline{t}, \hat{a}) = y^S(\bar{t}, \hat{a}) = U^S$.

The receiver has an incentive to choose $\hat{a}$ in each case if $\sqrt{D^R} \geq \sqrt{D^S} + b$.

Interval $\mathcal{N}$ stands for a non-accommodating interval in the sense that R can choose her best action without fear of S's exit. Interval $\mathcal{A}$ stands for an accommodating interval in the sense that the constraint for no exit is binding at the right end of the interval and R chooses an action more favorable for S than R's best one. Interval $\mathcal{F}$ stands for a fully accommodating interval in the sense that the constraint for no exit is binding at both ends of the interval and any other action than $\hat{a}$ induces some types of S to exit. This Lemma also insists that any NEE interval cannot be longer than $2\sqrt{D^S}$, for otherwise some types of S will choose to exit no matter which action R chooses.

On the boundary point of two adjoining intervals, S must be indifferent between sending actions corresponding to the intervals. Possible equilibrium configurations of intervals are restricted. For example, an interval $\mathcal{F}$ cannot be directly connected to an interval $\mathcal{N}$, for otherwise S has a strict incentive to choose the action corresponding to the interval $\mathcal{N}$ at any state sufficiently close to the boundary point. The following is the formal statement:

Lemma 3 Given any NEE with N intervals.

(i) For $N = 1$, the equilibrium condition for the sender is $\sqrt{D^S} \geq \frac{1}{2N}$.

(ii) For $N \geq 2$, a configuration of intervals is either of the following five patterns:

(I) $\mathcal{N}, \ldots, \mathcal{N}$,

(II) $\mathcal{N}, \ldots, \mathcal{N}, \mathcal{A}$,

(III) $\mathcal{N}, \ldots, \mathcal{N}, \mathcal{A}, \mathcal{F}, \ldots, \mathcal{F}$,[12]

(IV) $\mathcal{A}, \mathcal{F}, \ldots, \mathcal{F}$, or

(V) $\mathcal{F}, \ldots, \mathcal{F}$.

The equilibrium condition for the sender in each case is the following:[13]

(I) $b < \left\langle \frac{1}{2N(N-1)} \right\rangle$ and $\sqrt{D^S} > \frac{1}{2N} + Nb$.

(II) $\frac{1}{2N} + \frac{(N-1)^2}{N} b < \sqrt{D^S} \leq \frac{1}{2N} + Nb$ and $\sqrt{D^S} < 1 - (2N^2 - 4N + 1)b$.

(III) $\frac{1}{2N} + \frac{(i-1)^2}{N} b < \sqrt{D^S} \leq \frac{1}{2N} + \frac{i^2}{N} b$ and $\sqrt{D^S} < \frac{1-(2i^2-4i+1)b}{2N-2i+1}$ for some $i = 2, \ldots, N-1$.

(IV) $\frac{1}{2N} < \sqrt{D^S} \leq \frac{1}{2N} + \frac{1}{N} b$ and $\sqrt{D^S} < \left\langle \frac{1}{2(N-1)} \right\rangle$.

(V) $\sqrt{D^S} = \frac{1}{2N}$.

These conditions are illustrated in Figure 1. Combined with these lemmas, we derive the equilibrium condition for NEE.

Theorem 1 Suppose $\sqrt{D^R} \geq \sqrt{D^S} + b$. Then, an NEE with N intervals exists if and only if both (1) and (2) hold:

(1) Either one of (1-1)-(1-3) holds:

(1-1) $b < \left\langle \frac{1}{2N(N-1)} \right\rangle$,

[12] This case occurs only if $N \geq 3$.

[13] For the definition of the operator $\langle \cdot \rangle$, see Section 3.1.

(1-2) $\sqrt{D^S} < \frac{1-(2i^2-4i+1)b}{2N-2i+1}$ for some $i = 2, \dots, N$, or

(1-3) $\sqrt{D^S} < \left\langle \frac{1}{2(N-1)} \right\rangle$.

(2) $\sqrt{D^S} \geq \frac{1}{2N}$.

This Theorem insists that an NEE with sufficiently large number of intervals exists if and only if b is sufficiently small and/or D_S is sufficiently small as long as D^R is sufficiently large. The smallness of b is extensively discussed by CS and other literature, which means that the degree of incongruence between S and R's preferences is an important determinant of informativeness of cheap talk.

On the other hand, the smallness of D^S is a newly found determinant of informativeness. We interpret the smallness of D^S as a degree of *S's credibility of exit.* In other words, The smaller D^S is, the more credible S's threat of exit is and the more informative information cheap talk can convey. This result—the exit reinforces the voice in that the credibility of the exit increases the informativeness of the voice—is consistent with Hirschman [15]'s view.

By the direct implication of the previous theorem, we obtain the following Corollaries:

Corollary 1 Suppose that $\sqrt{D^R} > b$. Then, as U^S approaches Y^S (equivalently D^S approaches 0), there exists a sequence of equilibria in which $\alpha \circ \mu(t)$ pointwisely converges to $t + b$.

Corollary 2 Suppose that $\sqrt{D^R} > b$ and $b < \frac{1}{2\sqrt{3}}$. Then, if U^S is sufficiently close to Y^S, there exists an equilibrium in the environment with exit, in which S and R's ex ante payoffs are both larger than those in any equilibrium in the environment without exit.

Corollary 1 has two implications. One is that approximately perfect information transmission is possible via cheap talk in the presence of S's exit option. Another is that the existence of S's exit option approximately realizes the outcome in the case that R could commit to delegating her action choice to S. Corollary 2 implies that the existence of S's exit option increases the ex ante payoff of R as well as S.

3.3 Simple Contract of Allocating Y^S and Y^R

CS cheap talk model is often used in the literature on delegation such as Holmström [16] and Melumad and Shibano [24]. Basically, they assume that R can commit to the mechanism depending upon the message sent by S. In this context, our result implies that in the environment with an exit option, an efficient outcome can be realized by a simple contract even if R *cannot* commit to the message-dependent mechanism.

To see this, let us consider the following situation. Let Y be the gross surplus that S and R can jointly generate. Before Nature chooses a state, R proposes a contract specifying an allocation of Y between S's share Y^S and R's share Y^R. This contract does not depend upon the message. If $Y > U^S + U^R + b^2$, an allocation with $Y^S = U^S + \varepsilon$ and $Y^R = Y - Y^S$ for sufficiently small ε makes an almost perfect information transmission possible because it satisfies the premise of Theorem 1 for a large N.

4 General Model

In this section, in order to show that our main result holds in a more broad environment, we analyze a general setting beyond the uniform-quadratic one.

By the single-peakedness, we can find a unique maximizer of $y^i(t, \cdot)$ for any t. It is denoted by $\sigma^i(t)$. By the assumptions on y^i, it is verified that $\sigma^i(t)$ is strictly increasing in t. We consider the following somehow technical assumptions:

Assumption 1 There exists $\overline{\delta}$ such that $t > t'$ implies $\sigma^R(t) - \sigma^R(t') \leq \overline{\delta}(t - t')$ (i.e., σ^R is Lipschitz continuous in t) and there exists $\underline{\delta}$ such that $t > t'$ implies $\sigma^S(t) - \sigma^S(t') \geq \underline{\delta}(t - t')$.

Assumption 2 There exists $b > 0$ such that

$$\sigma^S(0) - \sigma^R(0) \geq b, \text{ or,}$$
$$\sigma^R(1) - \sigma^S(1) \geq b.$$

These assumptions clearly hold in uniform-quadratic models.

Let $Y^S(t) = y^S(t, \sigma^S(t))$ be S's maximum stay payoff at state t. Define $\underline{Y}^S = \min Y^S(t)$. Suppose $U^S < \underline{Y}^S$. Then, we can define uniquely $\gamma_-(t)$ and $\gamma_+(t)$ such that

$$\gamma_-(t) < \gamma_+(t),$$
$$y^S(t, \gamma_-(t)) = y^S(t, \gamma_+(t)) = U^S.$$

By the assumptions on y^S, it is verified that γ_+ is continuous and strictly increasing in t, and

$$\gamma_-(t) < \sigma^S(t) < \gamma_+(t)$$

holds for any t. Then, we can show that if $\gamma_+(t) - \gamma_-(t)$ is sufficiently small, there exists an NEE with many intervals.

Theorem 2 Suppose U^R is sufficiently small, Assumptions 1 and 2 hold, and $U^S < \underline{Y}^S$. Then, for any natural number $N \geq \underline{N}$, there exists an NEE with N or more intervals if $\gamma_+(t) - \gamma_-(t) < \overline{\gamma}$ holds for any t where

$$\underline{N} = \frac{\underline{\delta} + 2\overline{\delta}}{2b},$$
$$\overline{\gamma} = \frac{\underline{\delta}}{2N}.$$

Moreover, the length of each interval can be made less than or equal to $\frac{1}{N}$.

Note that the theorem does not depend upon specifications of distribution functions on t.

If S's maximum stay payoff $Y^S(t)$ is independent of t, we can make $\gamma_+(t) - \gamma_-(t)$ small to any degree by approaching U^S to $\underline{Y}^S$, which implies that perfect information transmission via cheap talk can be approximated.

Corollary 3 Suppose U^R is sufficiently small and Assumptions 1 and 2 hold. If $\underline{Y}^S = Y^S(t)$ for any t, then there exists a sequence of equilibria in which $\alpha \circ \mu(t)$ pointwisely converges to $\sigma^S(t)$ as U^S approaches $\underline{Y}^S$.

In the proof of the theorem, the equilibrium is constructed in a similar way as in the uniform-quadratic model ((IV) or (V) in Lemma 3). Consider the following example:

Example 1 [14] We assume

$$y^S = Y^S - (t-a)^2,$$
$$y^R = Y^R - (ct - b - a)^2,$$

[14] This case is a variant of Melumad and Shibano [24].

where $b > 0$ and $c > 0$. Then, since

$$\sigma^S(t) = t,$$
$$\sigma^R(t) = ct - b,$$

it is verified that Assumptions 1 and 2 are satisfied. Note that when $c > 1 + b$, we obtain

$$\sigma^S(0) > \sigma^R(0),$$
$$\sigma^S(1) < \sigma^R(1).$$

In other words, the sign of the incongruence between S and R's preferences is reversed.

Define N such that

$$\frac{1}{2(N-1)} > \sqrt{D^S} \geq \frac{1}{2N}. \tag{4}$$

Then, as U^S approaches Y^S, N goes to infinity.

We define $t_0 = 0$, and

$$t_n = 1 - 2(N-n)\sqrt{D^S}, \quad n = 1, \ldots, N,$$
$$a_n = 1 - (2N - 2n + 1)\sqrt{D^S}, \quad n = 1, \ldots, N.$$

We construct a candidate for an equilibrium as follows:

$$\mu(t) = m_n, \quad t \in (t_{n-1}, t_n),$$
$$P(m_n, t) = U_{t_{n-1}}^{t_n}(t),$$
$$\alpha(m_n) = a_n,$$
$$\epsilon(t, a) = 0, \quad \text{iff } y^S(t, a) \geq U^S.$$

Since $y^S(t, \alpha \circ \mu(t)) \geq U^S$, the exit option is never chosen on the equilibrium path. On the other hand, for $n = 2, \ldots, N$, since $y^S(t_{n-1}, a_n) = y^S(t_n, a_n) = U^S$, if R would choose $\tilde{a} \neq a_n$, some types of S belonging to (t_{n-1}, t_n) would choose the exit option. Therefore, R with a sufficiently small U^R has no incentive to deviate from the equilibrium action a_n.

Consider R's incentive after receiving a signal m_1. Since $y^S(t_1, a_1) = U^S$ and $a_1 > \sigma^S(t_1)$, if R would choose $\tilde{a} < a_1$, some types of S close to t_1 would choose the exit option. Therefore, R with sufficiently small U^R has no incentive to choose $\tilde{a} < a_1$. On the other hand, R's expected payoff function *when the exit option is never chosen* is single-peaked in which the maximum is attained at

$$a^* = c\mathbb{E}[t|t \in [t_0, t_1]] - b.$$

Suppose

$$N \geq \frac{c}{b}.$$

Then, by (4),

$$\begin{aligned}
a^* &= c\mathbb{E}[t|t \in [t_0, t_1]] - b \\
&< c\left[1 - 2(N-1)\sqrt{D^S}\right] - b \\
&\leq c\left(1 - \frac{N-1}{N}\right) - b \\
&\leq 0 \\
&< 1 - (2N-1)\sqrt{D^S} \\
&= a_1.
\end{aligned}$$

This implies that a deviation $\tilde{a} > a_1$ is never beneficial for R with a sufficiently large Y^S. Then, it is evident that the above candidate indeed constitutes an equilibrium.

Theorem 3 tells us that there may be an NEE with sufficiently many intervals even if S's maximum stay payoff $Y^S(t)$ is not constant. Consider the following example:

Example 2 We assume

$$\begin{aligned}
y^S &= Y^S(t) - (t-a)^2, \\
y^R &= Y^R(t) - (t-a)^2.
\end{aligned}$$

It is easily verified that Assumptions 1 and 2. We obtain

$$\begin{aligned}
\gamma_+(t) &= t + \sqrt{Y^S(t) - U^S}, \\
\gamma_-(t) &= t - \sqrt{Y^S(t) - U^S}.
\end{aligned}$$

Then, $\gamma_+(t) - \gamma_-(t) < \overline{\gamma}$ holds for any t if and only if

$$Y^S(t) - U^S < \frac{1}{16N^2} \quad \forall t.$$

It follows that if

$$\max_t |Y^S(t) - \underline{Y}^S| < \frac{1}{16N^2}$$

holds, then Theorem 3 guarantees the existence of NEE with N or more intervals wheneverU^S is sufficiently close to $\underline{Y}^S$ and U^R is sufficiently small compared to $Y^R(t)$.

The smallness of upper bound on $\gamma_+ - \gamma_-$ is crucial condition for Theorem 3. Consider the following example:

Example 3 [15] We assume

$$y^S = (1+t)\sqrt{a} - \frac{1}{\sqrt{1+4b}}a,$$
$$y^R = (1+t)\sqrt{a} - a.$$

Then, we obtain

$$\sigma^S(t) = \left(\frac{1+t}{2}\right)^2 (1+4b),$$
$$\sigma^R(t) = \left(\frac{1+t}{2}\right)^2,$$

Assumptions 1 and 2 are satisfied. On the other hand, it is verified that as long as $U^S < \underline{Y}^S$, for any t

$$\gamma_+(t) - \gamma_-(t) \geq \gamma_+(1) - \gamma_-(1) > \sqrt{1+4b}$$

holds. Then, the presupposition of Theorem is not satisfied for sufficiently large N.

In this example, if $y^S(\hat{t}, \hat{a}) = U^S$, then for $t < \hat{t}$, $y^S(t, \hat{a}) < U^S$. This implies that S receives a positive payoff in any boundary point except at $t = 1$. Therefore, the informativeness is mostly restricted by a bias b and not by the credibility of the exit.

5 Conclusion

This paper investigates the interplay between exit and voice by analyzing a modified version of the Crawford and Sobel [9] model in which the sender has the exit option after the

[15] This case is a special case of Marino [21].

receiver makes a decision. The key elements of our results are the difference between agent's maximum stay payoff and exit payoff. We obtain the result that in the case where the receiver's difference is large and the sender's difference is small but positive, the latter's exit is so credible that the former makes a decision desirable to the latter so as to prevent her from exercising the exit option; through this, accurate information can be transmitted via cheap talk on the equilibrium. In other words, it is shown that the informativeness of cheap talk is determined by not only the degree of incongruence between both agents' preferences but also the credibility of the sender's exit, which is measured by the inverse of the sender's difference. To the author's knowledge, this result is unprecedented in the literature on cheap talk with private information.

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Appendix: Proofs

A Proof of Lemma 1

On the contrary, suppose that there exists an equilibrium with infinite actions induced on the equilibrium path. Then, there must be actions a_1, a_2, and a_3 induced on the equilibrium path such that $a_1 < a_2 < a_3$ and $a_3 - a_1 < \min\{\sqrt{D^R}, b\}$. Taking S's incentive into consideration, the following must hold:

$$\{t|\mu(t) = a_2\} \subseteq (\max\{a_1 - b, 0\}, a_3 - b) \neq \emptyset.$$

However, it can be easily verified that for any $t \in (\max\{a_1 - b, 0\}, a_3 - b)$,

$$Y^S - (t + b - a_1)^2 > U^S,$$
$$Y^S - (t + b - a_2)^2 > U^S.$$

It follows that there is no type of S who sends a message inducing a_2 on the equilibrium and would choose the exit option if R chooses a_1 or a_2. Since

$$-\int_{\{t|\mu(t)=a_2\}} (t - a_1)^2 dt > -\int_{\{t|\mu(t)=a_2\}} (t - a_2)^2 dt,$$

R would deviate to choosing a_1 after receiving a message inducing a_2. This is a contradiction. The later part of the statement is easily verified from the assumptions on y^S. ∎

B Proof of Lemma 2

First of all, if $t < a - b - \sqrt{D^S}$ or $t > a - b + \sqrt{D^S}$, then $\epsilon(t, a) = 1$. This implies that the length of any NEE interval must be less than or equal to $2\sqrt{D^S}$. From now on, we focus on the case where $\bar{t} - \underline{t} \leq 2\sqrt{D^S}$.

Denote R's expected equilibrium payoff of choosing a condition on the belief that $t \in \hat{\tau}$

by $\tilde{V}^R(a)$. Then we obtain the following expression:

$$W(a) = (\bar{t} - \underline{t})(\tilde{V}^R(a) - U^R) =$$

$$\begin{cases} 0 & \text{if } a \in A_1 = (-\infty, \underline{t} + b - \sqrt{D^S}], \\ (a - b + \sqrt{D^S} - \underline{t})D^R - \int_{\underline{t}}^{a-b+\sqrt{D^S}} (t-a)^2 dt & \text{if } a \in A_2 = (\underline{t} + b - \sqrt{D^S}, \bar{t} + b - \sqrt{D^S}), \\ (\bar{t} - \underline{t})D^R - \int_{\underline{t}}^{\bar{t}} (t-a)^2 dt & \text{if } a \in A_3 = [\bar{t} + b - \sqrt{D^S}), \underline{t} + b + \sqrt{D^S}], \\ (\bar{t} - a + b + \sqrt{D^S})D^R - \int_{a-b-\sqrt{D^S}}^{\bar{t}} (t-a)^2 dt & \text{if } a \in A_4 = (\underline{t} + b + \sqrt{D^S}, \bar{t} + b + \sqrt{D^S}), \\ 0 & \text{if } a \in A_5 = [\bar{t} + b + \sqrt{D^S}, \infty). \end{cases}$$

In any NEE, the optimal action must be lying on A_3 for otherwise some type of S would choose an exit option. The unique local maximizer on A_3, denoted by a^*, is

$$a^* = \begin{cases} \frac{\underline{t}+\bar{t}}{2} & \text{if } \bar{t} - \underline{t} < 2\sqrt{D^S} - 2b, \\ \bar{t} + b - \sqrt{D^S} & \text{if } \bar{t} - \underline{t} \geq 2\sqrt{D^S} - 2b. \end{cases}$$

Note that $a^* = \hat{a}$ in any case. In the case of $\bar{t} - \underline{t} < 2\sqrt{D^S} - 2b$ it is easily verified that $y^S(\underline{t}, a^*) > U^S$ and $y^S(\bar{t}, a^*) > U^S$ hold. In the case of $\bar{t} - \underline{t} \geq 2\sqrt{D^S} - 2b$ it is easily verified $y^S(\bar{t}, a^*) = U^S$ and

$$y^S(\underline{t}, a^*) \begin{cases} > U^S & \text{if } \bar{t} - \underline{t} < 2\sqrt{D^S}, \\ = U^S & \text{if } \bar{t} - \underline{t} = 2\sqrt{D^S}, \end{cases}$$

hold.

In both cases, it is verified that $\sqrt{D^R} \geq \sqrt{D^S} + b$ implies that $W(a^*) \geq 0$ and W has no local maximum on A_2 and A_4. It follows that a^* is a global optimal action. ∎

C Proof of Lemma 3

(i) is immediately proved from Lemma 2. Throughout the proof, we consider $N \geq 2$. On the boundary point of two adjoining intervals, S must be indifferent between sending actions corresponding to the intervals. By Lemma 2, they must be either of the following cases:

- $\mathcal{N}$-$\mathcal{N}$,
- $\mathcal{N}$-$\mathcal{A}$,
- $\mathcal{A}$-$\mathcal{F}$, or

- $\mathcal{F}$-$\mathcal{F}$.

This implies that possible configurations of intervals of NEE is restricted to (I)-(V).

Consider (I). In this type of equilibrium, by the analysis in CS (see Section 3.1),

$$t_n = nt_1 + 2n(n-1)b, \quad n = 0, \dots, N,$$

where

$$t_1 = \frac{1 - 2N(N-1)b}{N}.$$

The equilibrium condition is

$$\begin{aligned} &t_1 - t_0 > 0, \\ &t_N - t_{N-1} < 2\sqrt{D^S} - 2b. \end{aligned}$$

Then, we obtain

$$\begin{aligned} b &< \left\langle \frac{1}{2N(N-1)} \right\rangle, \\ \sqrt{D^S} &> \frac{1}{2N} + Nb. \end{aligned}$$

Consider (II). In this type of equilibrium,

$$t_n = \begin{cases} nt_1 + 2n(n-1)b, & n = 0, \dots, N-1, \\ 1, & n = N, \end{cases}$$
$$a_n = \begin{cases} \frac{t_{n-1}+t_n}{2}, & n = 1, \dots, N-1, \\ 1 - \sqrt{D^S} + b, & n = N. \end{cases}$$

The equilibrium condition is

$$\begin{aligned} &y^S(t_{N-1}, a_{N-1}) = y^S(t_{N-1}, a_N), \\ &2\sqrt{D^S} > t_N - t_{N-1} \geq 2\sqrt{D^S} - 2b, \\ &t_1 - t_0 > 0, \\ &t_N - t_{N-1} > 0. \end{aligned}$$

Then, we obtain

$$\begin{aligned} &\frac{1}{2N} + \frac{(N-1)^2}{N} b < \sqrt{D^S} \leq \frac{1}{2N} + Nb, \\ &\sqrt{D^S} < 1 - (2N^2 - 4N + 1)b, \end{aligned}$$

where

$$t_1 = \frac{2 - 2\sqrt{D^S} - 2(2N^2 - 4N + 1)b}{2N - 1}.$$

Consider (III). Given any $i = 2, \dots, N-1$, consider the following configuration:

$$\underbrace{\mathcal{N}, \dots, \mathcal{N}}_{i-1 \text{ times}}, \mathcal{A}, \underbrace{\mathcal{F}, \dots, \mathcal{F}}_{N-i \text{ times}}.$$

In this type of equilibrium,

$$t_n = \begin{cases} nt_1 + 2n(n-1)b, & n = 0, \dots, i-1, \\ 1 - 2(N-n)\sqrt{D^S}, & n = i, \dots, N, \end{cases}$$
$$a_n = \begin{cases} \frac{t_{n-1}+t_n}{2}, & n = 1, \dots, i-1, \\ t_n - \sqrt{D^S} + b, & n = i, \dots, N. \end{cases}$$

The equilibrium condition is

$$y^S(t_{i-1}, a_{i-1}) = y^S(t_{i-1}, a_i),$$
$$2\sqrt{D^S} > t_i - t_{i-1} \geq 2\sqrt{D^S} - 2b,$$
$$t_1 - t_0 > 0,$$
$$t_i - t_{i-1} > 0.$$

Then, we obtain

$$\frac{1}{2N} + \frac{(i-1)^2}{N}b < \sqrt{D^S} \leq \frac{1}{2N} + \frac{i^2}{N}b,$$
$$\sqrt{D^S} < \frac{1 - (2i^2 - 4i + 1)b}{2N - 2i + 1},$$

where

$$t_1 = \frac{2 - 2(2N - 2i + 1)\sqrt{D^S} - 2(2i^2 - 4i + 1)b}{2i - 1}.$$

Consider (IV). In this type of equilibrium,

$$t_n = \begin{cases} 0, & n = 0, \\ 1 - 2(N-n)\sqrt{D^S}, & n = 1, \dots, N. \end{cases}$$

The equilibrium condition is

$$2\sqrt{D^S} > t_1 - t_0 \geq 2\sqrt{D^S} - 2b,$$
$$t_1 - t_0 > 0.$$

Then, we obtain

$$\frac{1}{2N} < \sqrt{D^S} \leq \frac{1}{2N} + \frac{1}{N}b,$$
$$\sqrt{D^S} < \frac{1}{2(N-1)}.$$

Derivation of the equilibrium condition for (V) is immediate, since in this type of equilibrium,

$$t_n = 2\sqrt{D^S}n, \quad n = 0, \ldots, N,$$

then, it must hold that $t_N = 1$, or equivalently

$$\sqrt{D^S} = \frac{1}{2N}.$$

∎

D Proof of Corollary 1

It is obtained directly form Theorem 1 and the fact that each interval has length $2\sqrt{D^S}$ or less (Lemma 2). ∎

E Proof of Corollary 2

By Corollary 1, it is obvious that the sequence of S's ex ante equilibrium payoffs V^S converges to Y^S as $D^S \to 0$. Similarly, as for R's ex ante equilibrium payoff V^R,

$$V^R - V^S = Y^R - Y^S + b\int_0^1 (2t + b - 2\alpha \circ \mu(t))^2 dt \to Y^R - Y^S - b^2$$

as $D^S \to 0$. Then, V^R converges to $Y^R - b^2$. By (2) and (3), this completes the proof. ∎

F Proof of Theorem 3

In this proof, we suppose that there exists $b > 0$ such that $\sigma^S(0) - \sigma^R(0) \geq b$. With regard to the case of $\sigma^R(1) - \sigma^S(1) \geq b$, we can prove the proposition by reversing all the variables in the following proof at the center of point 1/2.

First, we prove the following Lemma:

Lemma 4 Given any $\ell > 0$ and suppose $\gamma_+(t) - \gamma_-(t) < \frac{\delta \ell}{2}$ for any t. Then, for any $\tilde{t} \geq \ell$, there exists a unique $\hat{t}$ such that $\gamma_+(\hat{t}) = \gamma_-(\tilde{t})$ and $\hat{t} < \tilde{t}$.

Proof:
By Assumption 1, $\sigma^S(\tilde{t}) - \sigma^S(0) \geq \underline{\delta}\ell$. On the other hand, by the presupposition,

$$\gamma_-(t) > \sigma^S(t) - \frac{\delta \ell}{2},$$
$$\gamma_+(t) < \sigma^S(t) + \frac{\delta \ell}{2}$$

hold for any t. Therefore,

$$\gamma_-(\tilde{t}) - \gamma_+(0) > \sigma^S(\tilde{t}) - \sigma^S(0) - \underline{\delta}\ell \geq 0.$$

Since γ_+ is continuous and strictly increasing in t, and $\gamma_-(\tilde{t}) < \gamma_+(\tilde{t})$, there is a unique $\hat{t}$ satisfying $\gamma_-(\tilde{t}) = \gamma_+(\hat{t})$ on the interval $[0, \tilde{t})$. ∎

We recursively define a sequence $\{\hat{t}_n\}_{n=0}^{N}$ in T as follows: first, we define $\hat{t}_0 = 1$. For $n \geq 1$,

1. if $\hat{t}_{n-1} = 0$, we stop the recursive process and name $n-1$ as $\hat{N}$,
2. if $\hat{t}_{n-1} > 0$ and there exists $\hat{t} \in T$ such that $\gamma_+(\hat{t}) = \gamma_-(\hat{t}_{n-1})$, we define $\hat{t}_n = \hat{t}$, and
3. if $\hat{t}_{n-1} > 0$ and there exists no $\hat{t} \in T$ such that $\gamma_+(\hat{t}) = \gamma_-(\hat{t}_{n-1})$, then we define $\hat{t}_n = 0$.

By Lemma 4, it is verified that $\{\hat{t}_n\}_{n=0}^{N}$ is a strictly decreasing sequence. Lemma 4 also implies that if $\gamma_+(t) - \gamma_-(t) < \overline{\gamma}$ for any t, then $\hat{t}_{n-1} - \hat{t}_n \leq \frac{1}{N}$ for any n, and therefore $\hat{N} \geq N$.

By the construction of $\{\hat{t}_n\}$, we obtain the following result:

Lemma 5

$$\forall n = 1, \dots, \hat{N},\ \forall t \in [\hat{t}_n, t_{n-1}],\ y^S(t, \gamma_-(\hat{t}_{n-1})) \geq U^S,$$
$$\forall n = 1, \dots, \hat{N}-1,\ \forall \hat{a} \neq \gamma_-(\hat{t}_{n-1}),\ \exists \hat{t} \in [\hat{t}_n, \hat{t}_{n-1}] \text{ such that } y^S(\hat{t}, \hat{a}) < U^S.$$

Let $V^R(a; \underline{t}, \overline{t})$ be R's expected payoff of choosing a conditional on the information that $t \in [\underline{t}, \overline{t}]$. Denote $\mathcal{E}(\underline{t}, \overline{t}) = \{\tilde{a} | y^S(t, \tilde{a}) \geq U^S\ \forall t \in [\underline{t}, \overline{t}]\}$. Then we obtain the following result:

Lemma 6 Under the presupposition of Theorem,

$$\gamma_-^S(\hat{t}_{\hat{N}-1}) \in \arg \max_{a \in \mathcal{E}(\hat{t}_{\hat{N}}, \hat{t}_{\hat{N}-1})} V^R(a; \hat{t}_{\hat{N}}, \hat{t}_{\hat{N}-1})$$

holds.

Proof:
It is clear for the case where $\gamma_+(\hat{t}_{\hat{N}}) = \gamma_-(\hat{t}_{\hat{N}-1})$, since $\mathcal{E}(\hat{t}_{\hat{N}}, \hat{t}_{\hat{N}-1}) = \{\gamma_-(\hat{t}_{\hat{N}-1})\}$. Consider the case where $\gamma_+(\hat{t}_{\hat{N}}) \neq \gamma_-(\hat{t}_{\hat{N}-1})$. By the construction of $\{\hat{t}_n\}$ and Lemma 5, $\mathcal{E}(\hat{t}_{\hat{N}}, \hat{t}_{\hat{N}-1}) = [\gamma_-(\hat{t}_{\hat{N}-1}), \gamma_+(\hat{t}_{\hat{N}})]$. Then it suffices to show that $V^R(a; \hat{t}_{\hat{N}}, \hat{t}_{\hat{N}-1})$ is decreasing in a on $[\gamma_-(\hat{t}_{\hat{N}-1}), \gamma_+(\hat{t}_{\hat{N}})]$, which implies that it suffices to show that $\gamma_-(\hat{t}_{\hat{N}-1}) \geq \sigma^R(\hat{t}_{\hat{N}-1})$ since $V^R(a; \hat{t}_{\hat{N}}, \hat{t}_{\hat{N}-1})$ is decreasing in a on $[\sigma^R(\hat{t}_{\hat{N}-1}), \infty)$.

By the presupposition,

$$\gamma_-(\hat{t}_{\hat{N}-1}) > \sigma(\hat{t}_{\hat{N}-1}) - \frac{\delta}{2N}$$

holds. Meanwhile, by Assumptions 1 and 2 and Lemma 4,

$$\sigma^S(\hat{t}_{\hat{N}-1}) - \sigma^R(\hat{t}_{\hat{N}-1}) \geq \sigma^S(\hat{t}_{\hat{N}}) - \sigma^R(\hat{t}_{\hat{N}}) - \overline{\delta}(\hat{t}_{\hat{N}-1} - \hat{t}_{\hat{N}}) \geq b - \frac{\overline{\delta}}{N}$$

holds. Then, we obtain

$$\begin{aligned} \gamma_-(\hat{t}_{\hat{N}-1}) &> \sigma^S(\hat{t}_{\hat{N}-1}) - \frac{\delta}{2N} \\ &\geq \sigma^R(\hat{t}_{\hat{N}-1}) + b - \frac{\delta}{2N} - \frac{\overline{\delta}}{N} \\ &\geq \sigma^R(\hat{t}_{\hat{N}-1}). \end{aligned}$$

This complete the proof. ∎

Return to the proof of Theorem 3. We define $\{a_n\}_{n=1}^{\hat{N}}$ as follows:

$$\hat{a}_n = \gamma_{-}^{S}(\hat{t}_{n-1}).$$

Then, we construct a candidate for an equilibrium, $(\mu, P, \alpha, \epsilon)$, as follows:

$$\begin{aligned}
&\mu(t) = m_n, \quad \text{if } t \in [\hat{t}_n, \hat{t}_{n-1}],\\
&P(m_n, t) = U_{\hat{t}_n}^{\hat{t}_{n-1}}(t),\\
&\alpha(m_n) = \hat{a}_n,\\
&\epsilon(t, a) = 0, \quad \text{iff } y^S(t, a) \geq U^S.
\end{aligned}$$

From Lemmas 5 and 6, R has no incentive to deviate from α as long as R's equilibrium payoff is sufficiently larger than her exit payoff. Moreover, it is easily verified that S in $t \in [\hat{t}_n, \hat{t}_{n-1}]$ has no incentive to send message $m_{\tilde{n}}$ for $\tilde{n} \neq n$. Then, $(\mu, P, \alpha, \epsilon)$ constitutes an equilibrium. ∎

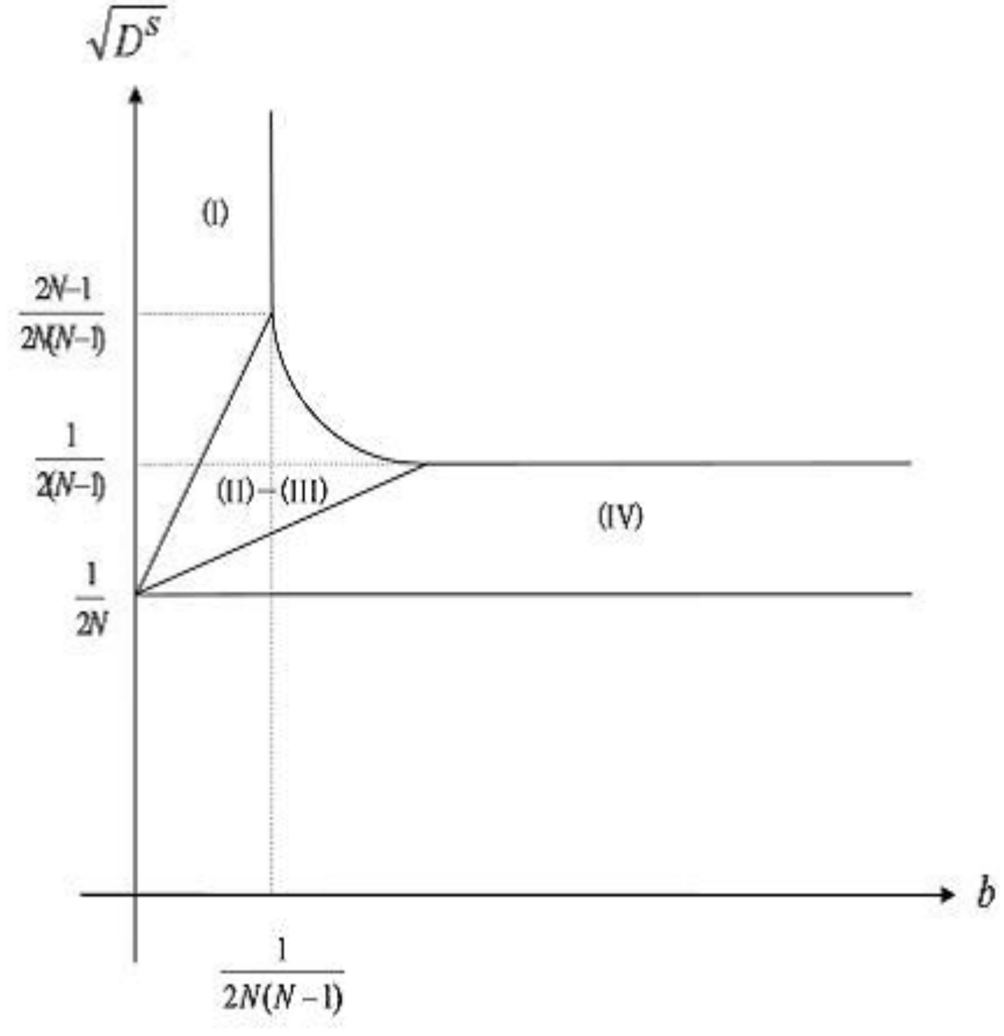

Figure 1: Equilibrium with N intervals