Searching a Bargain: Play it Cool or Haggle*

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Abstract

This paper aims to shed light on imperfectly competitive search markets where the sellers announce their initial demands prior to the buyer's visit and market participants of both sides have the opportunity of building reputation on inflexibility. The buyer facing two sellers can negotiate with only one at a time and can switch his bargaining partner with some cost. The introduction of commitment types that are inflexible in their demands, even with low probabilities, makes the equilibrium of the resulting multilateral bargaining game essentially unique. The equilibrium has a war of attrition structure. If the sellers' initial demands are the same, then the buyer will never visit one seller more than once. If instead the demands are different, a given seller may be visited twice and the buyer may choose to go first to the seller with the higher demand. Although the sellers compete in the spirit of Bertrand, the equilibrium prices are in contrast to the Bertrand's prediction.

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1. Introduction

Consider markets in which buyer is an investor who would like to buy a poorly managed but promising small business, a consumer who plans to purchase a house, a car, etc., or a high skilled labor searching for a job. As it is the case for many more, behaviors of the market participants aiming to buy (or sell) a good or service in these markets differ from those that Bertrand's (1883) price competition and Diamond's (1971) search model aim to explain. The common characteristics of the trading mechanism that we often times observe in these markets are that there is more than one potential seller (competing with each other over the buyers) and each seller usually suggests or posts his price so the buyer can pay that price, buy the good or the service, and finalize the trade.[1] However, the general practice in these markets is that the buyers negotiate with the sellers with the hope of getting a deal better than the solicited "buy-it-now" prices.

On the other hand in many instances, even the competing sellers and buyers may be inflexible in their demands during the haggling process. A car dealer, for example, may be restricted by the owner regarding how flexible he could be in his demands while negotiating with a buyer. An entrepreneur who owns a successful small business may commit to his demands while negotiating with investors to sell his business or a franchise because he might have overly optimistic expectations about the future of his business. A senior manager (or faculty) who already has a good status-quo might be willing to accept the new job offer only when he finds it remarkably superior. Thus, the sellers' posted prices (or the buyers' initial demands) can naturally give rise to the fear that the sellers (or the buyers) might not be willing to negotiate and would insist on the initial prices (demands) they announce.

Given that the sellers and the buyers have the opportunity of building "*reputation*" on inflexibility, market participants' equilibrium behavior in imperfectly competitive search markets, such as the ones that are briefly exemplified in previous paragraphs, and the impacts of their behaviors on equilibrium outcome(s) are not known to us. Providing a suggestive explanation in this venue, that may facilitate further applications in various other fields such as labor economics and market microstructure, is the main motivation of this paper.

[1] The buyers can solicit the sellers' posted prices through on-line search or by requesting quotes without visiting the sellers' stores or engaging in some negotiation process

For this reason I consider the following simple market set-up: there are two sellers having an indivisible homogeneous good and a single buyer who wants to consume only one unit.[2] All players are impatient, i.e. they discount time, and the valuation of the good is one for the buyer and zero for the sellers. There is no informational asymmetry regarding the players' valuations and time preferences. The buyer can learn the sellers' initial demands (or posted prices) for the good before visiting their stores (so the sellers compete in the spirit of Bertrand). He can get the good from the seller who is asking the lowest price by paying the seller's demand. Or he can haggle with the sellers to receive a better offer. But to get a deal better than the posted prices, the buyer has to visit a seller. The buyer can move back and forth between the stores freely. However, switching from one seller to another incurs some (small but positive) cost.

Upon arriving at a store, the buyer and the seller can negotiate according to some predetermined bargaining protocol (in discrete time); whenever a player makes an offer his opponent immediately accepts (and finishes the game) or rejects the offer. Other than this assumption, I do not impose any restriction on possible bargaining protocols that might be used in the negotiation process. However, I am interested in equilibrium outcome(s) of the resulting multilateral bargaining game in the limit as the players can make increasingly frequent offers, with the interpretation that the players can make offers at any time they want.

There are two main challenges of analyzing this market setup as a two-stage game (I call it bargaining problem): The first stage is a price competition game between two sellers and the second stage is the negotiation phase between the buyer and the sellers. These two points make the analysis of such markets (in a game-theoretical framework) rather difficult. The first one is that the second-stage multilateral bargaining game has a continuum of subgame perfect equilibria (even in the limit as time between offers converge to zero), and this set depends on the fine details of the bargaining protocol.[3] This multiplicity problem makes the current bargaining models in the literature *ad hoc* and inconclusive when they are incorporated into a price competition game. The second

[2] In Section 4, I consider the case where the number of sellers is some $N \geq 2$. I assume, without loss of generality, that there is a unique buyer since I presume that the sellers have large number of goods to sell (so no competition between the buyers) and the buyers cannot convey information to one another (no interaction between the buyers).

[3] For related literature see Osborne and Rubinstein (1990) and the references therein.

challenge is integrating the first-stage price choices with the second-stage bargaining game. Posting a price has an unambiguous meaning in, for example, Bertrand price competition or in Diamond's search model because in these models the buyer knows that he cannot attain a price lower than the posted prices. Thus, he is not inclined to bargain with the sellers. However, the core presumption of my analysis in this paper is the players' uncertainty about others' stubbornness, so the buyer believes that he can actually get a better price through haggling with the sellers.

As I will discuss it later in detail, the literature seems weak on providing suggestive answers to the following critical questions. (1) Which bargaining protocol should we pick to model the multilateral bargaining game, and when we pick a particular one, how can we rationalize it over the others? (2) Therefore, what is the feasible and tractable way of modeling the trading mechanism in such markets as a multilateral bargaining game between buyers and sellers? (3) How might the players interpret the posted prices when the sellers post price and when there is a belief that the sellers may not commit to these prices? (4) Given the uncertainty about the players' commitment abilities, how should the sellers and the buyer choose their initial demands? Furthermore, what would be the equilibrium outcome of the negotiation process?

The approach I am suggesting in this paper is a simple remedy to these questions; I show that a slight perturbation of the problem by introducing "*obstinate*" types, which allows players to build reputation for inflexibility, engenders an essentially unique equilibrium. Following Kreps and Wilson (1982) and Milgrom and Roberts (1982), I assume that each of three players suspects that the opponents might have some kind of irrational commitment forcing them to insist on a specific allocation. Even if players assign small probabilities to the obstinate types, the profusion of equilibria of the multilateral bargaining game between the buyer and the sellers reduce to a unique one.

I also show that the unique equilibrium allocation does not depend on the fine details of the bargaining protocols, nor do the sellers extract all the surplus of the buyer because of the positive search cost (in contrast to Diamond's search model). Instead, it depends on the initial demands and reputations (the probability of being the obstinate type) as well as the time preferences of the players. Moreover, even though the sellers compete in the Bertrand fashion, the equilibrium outcomes are in contrast to the Bertrand's prediction; marginal cost pricing is not necessarily the unique equilibrium outcome.

Obstinate (or commitment) types take an extremely simple form. Parallel to Myerson (1991), and Abreu and Gul (2000) a commitment player always demands a particular share and accepts an offer if and only if it weakly exceeds that share. An obstinate seller, for example, never offers a price below his original posted price, and never accepts an offer below that price. Similarly, an obstinate buyer always offers a particular amount, and will never agree to pay more. Thus, a rational player must choose either to mimic an inflexible type, or reveal his rationality and continue negotiation with no uncertainty regarding his actual type.

Analogous to Abreu and Gul (2000) I show that, given the presence of the obstinate types and the first-stage price selections, the equilibrium outcomes of the second-stage discrete-time bargaining game (between the sellers and the buyer) converge to a unique limit, independent of the fine details of the bargaining protocols, as players can make increasingly frequent offers. This limit is the unique equilibrium outcome of the following continuous-time war of attrition game. Upon arrival at a store, the buyer and the seller start to play the concession game. At any given time, a player either accepts his opponent's demand or waits for his opponent's concession. At the same time, the buyer decides whether to stay or to leave the store.

In the unique equilibrium of the continuous-time bargaining game, the buyer does not visit a given store more than once as long as the sellers' posted prices are the same. Thus, in equilibrium the buyer enters store 1, for example, at time 0, and starts playing the concession game with the seller until a specific finite time. If neither player concedes to his opponent, the buyer leaves store 1 at this time with a higher reputation, and goes directly to store 2 to continue the concession game with the second seller. The negotiation in the second store ends at some finite time with certainty, at which point the players' reputations simultaneously reach one. The equilibrium strategy of a player in the concession game is some continuous and strictly increasing distribution function with a constant hazard rate. That is, each player concedes by choosing the timing of acceptance randomly with a constant (instantaneous) acceptance rate.

Since there are two sellers, building reputation on inflexibility by haggling with the first seller is an investment for the buyer, which increases his continuation payoff in the second store. Having a higher outside option in return increases the bargaining power of the buyer in the first store. On the other hand, since the buyer discounts time, his expected payoff in store 1 is a decreasing function of his departure time from store 1; if the buyer needs more time to build his reputation before going to the other seller, then it is less likely that his opponent will concede to him earlier. Hence, the equilibrium departure time of the buyer from store 1 decreases with his own initial reputation and increases with the reputation of the sellers. When the sellers' posted prices are different, the structure of the equilibrium strategy dramatically changes. In this case, the buyer never negotiates with the seller whose posted price is higher, though he may visit this store at time 0 in order to make the "take it or I will leave you" ultimatum.

Finally, assuming that the sellers are identical and the initial reputations z_b and z_s (probabilities that the buyer and the sellers, respectively, are obstinate type) are small, I characterize the first-stage price selections of the sellers. In equilibrium both sellers must choose the same price. The set of equilibrium prices depends on the relative ratio of the players' initial reputations. However, for any given value in the interval $(0, 1)$ we can find z_b and z_s small enough to support it as an equilibrium outcome. Given that the players discount time with the same rate, the maximum equilibrium price would take values close to $\frac{1}{3}$ as players' initial reputations take decreasingly small values. Therefore, the minimum expected payoff the buyer may achieve will approximate to $\frac{2}{3}$, and the maximum expected payoff each seller can attain will converge to $\frac{1}{6}$. Namely, equilibrium outcomes are efficient in the limit.

However, this is not the unique outcome; depending on the relative ratio between the players' initial reputations, the equilibrium prices will range in the set $[0, \frac{1}{3}]$ in the limit. Since the sellers are identical, it is not surprising that both sellers must choose the same price in equilibrium. On the contrary, price undercutting is not optimal for the sellers because a deviating seller cannot attain a payoff more than the minimal demand possible (within the set of obstinate types). This is true since the deviating seller will have to accept the buyer's "take it or leave it" ultimatum regardless of the buyer's demand. Finally, I extend the results to $N(\geq 2)$ sellers and show that the highest equilibrium price is $\frac{1}{N+1}$ as the probabilities of the obstinate types vanish.

Section 2 explains the model and the bargaining problem in detail and motivates the assumptions. Equilibrium strategies of the second-stage bargaining game are characterized in Section 3. Section 4 examines the equilibrium prices and demands that the players would announce in stage 1. Finally, Section 5 makes some closing remarks.

Related Literature

A Discussion about the literature should appear here...

2. The Model

Consider a market where there are two sellers having an indivisible homogeneous good and a single buyer who wants to consume only one unit. All players are impatient, i.e. they discount time, and the valuation of the good is one for the buyer and zero for the sellers. There is no informational asymmetry regarding the players' valuations and time preferences. The buyer can learn the sellers' initial demands (or posted prices) for the good before visiting their stores. He can get the good from the seller who is asking the lowest price by paying the seller's demand. Or he can haggle with the sellers to receive a better offer. But to get a deal better than the posted prices, the buyer has to visit a seller. Switching from one seller to another incurs some (possibly small but positive) cost. The reader may wish to picture this market as an environment where the sellers' stores are located at opposite ends of a town while the buyer's position is midway in between the two. Thus, changing the bargaining partner is costly for the buyer because it takes time to move from one store to the other and the buyer discounts time.

Furthermore, each player might have some kind of irrational commitment forcing him to insist on a specific allocation. An obstinate (or commitment) type seller i is identified by a number $\alpha_i \in (0,1)$ and implementing the following strategy: He always offers α_i, rejects any price offer strictly below it and accepts any price offer weakly above it. The initial probability that a seller is obstinate is denoted by z_s. Without loss of generality, the initial prior, z_s, and the time preference, r_s, are common for both sellers.

Similarly, there is a small but strictly positive probability, z_b, that the buyer is a commitment type. Obstinate buyer with demand $\alpha_b \in (0,1)$ executes the following strategy: He always offers α_b to sellers, accepts any price offer less than or equal to α_b and rejects any price offer strictly above it. The rational buyer's time preference is r_b. Assuming that the sellers' are spatially separated, let δ denote the discount factor for the buyer that occurs due to the time, $\Delta > 0$, required to travel from one store to the other. That is, $\delta = e^{-r_b\Delta}$. Note that δ (the search friction) is the cost that the buyer incurs at each time he switches his bargaining partner.[4] Also note that, as the stores get very close to each other, δ converges to 1. Namely, the search friction vanishes.

[4]One may assume a switching cost for the buyer that is independent of the "travel time" Δ, but this

I would like to determine the equilibrium outcomes of the bargaining problem (that I will define more formally) in the limit where the search friction becomes insignificant and the set of obstinate types for each player is $[0,1]$. Namely, I am interested in the model's prediction when the sellers compete in the spirit of Bertrand and any initial demand $\alpha \in [0,1]$ chosen by a player will lead to a fear that this player might be the commitment type α. Therefore, I will first assume that the set of obstinate types for each player is some finite set $C \subset (0,1)$ and the search friction is sufficiently small. Then I will consider the limit where $C \to [0,1]$ and $\delta \to 1$ simultaneously. Furthermore, to provide a concise presentation of the analysis, I will focus my attention to the case where the initial priors z_s and z_b are sufficiently small. Thus, I make the following assumptions throughout the paper.

Assumption 1 (***Small Search Friction***)**.** *Suppose that δ is close enough to 1 so that for all $\alpha \in C$ and $\alpha' \in C$ such that $\alpha > \alpha'$ we have $1-\alpha < \delta(1-\alpha')$*

Assumption 2 (***Small Initial Priors***)**.** *Suppose that z_b and z_s are small enough so that for all $\alpha, \alpha' \in C$ and $\alpha'' \in C$ such that $\alpha \geq \alpha' > \alpha''$ we have $z_s, z_b < \bar{z}$ and $\alpha z_s < \alpha''$ where $\bar{z} = \frac{1-\alpha''-\frac{1-\alpha}{\delta}}{1-\alpha'-\delta(1-\alpha'')}$.*

Finally, let $\pi(\alpha)$ denote the conditional probability that a player is type α given that he is obstinate. That is, π is a probability distribution on C.

The Bargaining Problem

The two-stage bargaining problem in discrete-time proceeds as follows. Initially (in stage 1), each seller i simultaneously chooses and reveals his demand $\alpha_i \in C$. If he is rational, this is a strategic choice; if he is the obstinate type, then he merely declares the demand corresponding to his type. After observing both sellers' demands, α_1 and α_2, the buyer immediately accepts α (the minimum of α_1 and α_2) and finishes the game strategically if he is rational or because he is obstinate and of type α_b such that $\alpha_b \geq \alpha$. Or the buyer visits one of the sellers and makes a counter offer $\alpha_b \in C_{(\alpha_1,\alpha_2)} = \{x \in C | x < \min\{\alpha_1, \alpha_2\}\}$. Again this may be because the buyer is rational and strategically demanding α_b or because the buyer is the obstinate type α_b. After the buyer declares his

change would not affect our results. However, incorporating the search friction in this manner simplifies the notation substantially.

demand, the seller (who is currently visited by the buyer) can immediately accept the buyer's demand and finish the game or reject it, in which case the game proceeds to the second stage; the bargaining phase.

A player can convince his opponents that he is not the obstinate type by showing them that he is flexible in his demand. Therefore, players can reveal their rationality only in the negotiation phase, in stage two, by making offers different than their initial demands. In the bargaining phase, the buyer can negotiate only with the seller whom he is currently visiting. If the buyer wants to bargain with the other seller, he needs to visit that seller. The buyer can move back and forth between the sellers as much as he wants, but he will incur the travel cost at each time he switches his bargaining partner.[5] Throughout the game, both sellers can perfectly observe the buyer's moves.[6] Thus, the players' actual types remains to be the only source of uncertainty in the game.

The buyer and seller i bargain in discrete time according to some protocol g^i that generalizes Rubinstein's alternating offers protocol. A bargaining protocol g^i between the buyer and seller $i \in \{1, 2\}$ is defined as $g^i : [0, \infty) \to \{1, 2, 3\}$ such that for any time $t \geq 0$, an offer is made by the buyer if $g^i(t) = 1$ and by seller i if $g^i(t) = 2$.[7] Moreover, $g^i(t) = 3$ implies a simultaneous offer. An infinite horizon bargaining protocol is denoted by $g = (g^1, g^2)$. The bargaining protocol g is discrete. That is, for any seller $i \in \{1, 2\}$ and for all $\bar{t} \geq 0$, the set $I^i := \{0 \leq t < \bar{t} | g^i(t) \in \{1, 2, 3\}\}$ is countable. Notice that this definition for a bargaining protocol is very general and accommodates non stationary, non alternating protocols.

An offer $x \in (0, 1)$ denotes the share the seller is to receive. If the proposer's opponent

[5]The buyer does not need to visit the other seller's store to re-enter the one that he previously visited. So, for example, the buyer may change his mind while he was going to the second store and may turn back to the first one to continue negotiating with the first seller. However, the buyer will never behave that way in equilibrium.

[6]Namely, I assume that the sellers are not totally isolated from each other. Instead, each seller can scrutinize his opponents' moves throughout the haggling process. One might imagine, for example, that the buyer negotiates with the sellers publicly (not behind closed doors), or through a middleman that represents both sellers. Clearly, in some circumstances the sellers may not be able to attain all the information nor does the buyer convey it perfectly. Extending the model to introduce some informational imperfections may naturally result in different equilibrium behaviors during the negotiation process. These issues deserve comprehensive considerations and transcend the focus of this particular study.

[7]Time 0 denotes the beginning of the bargaining phase.

accepts his offer, the game ends with agreement x where $u_i(x,t,i) = xe^{-tr_s}$ denotes the payoff to the seller i, $u_j(x,t,i) = 0$ is the payoff to the seller $j \in \{1,2\}$ with $j \neq i$ and finally $u_b(x,t,i) = (1-x)e^{-tr_b}$ is the payoff to the buyer. If the proposer's opponent rejects his offer, the game continues. Prior to the next offer, the buyer decides whether to stay or leave the store. If the buyer decides to stay, the next offer is made at time $t' := \min\{\hat{t} > t | \hat{t} \in I^i\}$, for example, by the buyer if $g^i(t') = 1$. The two-stage discrete-time bargaining problem is denoted by $\mathrm{G}\langle g, (C, z_i, \pi_i, r_i)_{i\in\{b,s\}}\rangle$ (or $\mathrm{G}(g)$ in short). The bargaining problem $\mathrm{G}(g)$ ends if the offers are compatible; in the event of strict compatibility the surplus is split equally.[8]

Motivating the Obstinate Types

An obstinate player is a man of unbendable perseverance. Such steadfast attitude would be in play for sellers because they might be confined to do so. A company may be inflexible in a wage negotiation due to some regulations within the company. A car dealer, a sales clerk or a realtor, for example, may be restricted by the owners regarding how flexible he could be in his demands while negotiating with a buyer. Or a fresh college graduate, who is competing with other candidates for a specific job opening, may commit to a certain salary because he wants to pay his student loan without too much financial difficulty.

Steady persistence in adhering to a course of action as assumed for an obstinate (type) buyer would be reasonable when, for example, the "buyer" is looking for a move up. A worker (negotiating with more than one firm) may accept the new job offer if it provides a significant jump in his salary or title relative to the position he is already holding. On the other hand, a successful investor (venture capitalist) who only has assets that have high profit margins in his portfolio may commit to buy a house, a land or a small business only if it is a real bargain because otherwise it may not be worth to include it into his portfolio. An entrepreneur who is running a successful small business may commit to his demands while negotiating with investors to sell his business or a franchise because he might have overly optimistic expectations about the future of his business.

Therefore, I assume that the obstinate buyer (regardless of his demand) is not enthusiastic to haggle with the sellers and to behave strenuously active in moving back

[8]This particular assumption is not crucial because simultaneous concession occurs with probability zero in equilibrium.

and forth between the sellers. That is, the obstinate buyer is a man who plays it cool. To be more specific, I assume that the obstinate buyer (1) does not discount time, (2) incurs a positive (but very small) switching cost ($\epsilon_b > 0$) every time he switches his bargaining partner, (3) understands the equilibrium and leaves his bargaining partner when he is convinced that his partner is also obstinate, and finally (4) visits each seller with equal probabilities to announce his demand if it is not compatible with the lowest price announced by the sellers.

According to (1), the time of an agreement is not a concern for the obstinate buyer, and thus he does not feel need to distinguish himself from the rational buyer who wishes to reach an agreement as quickly as possible. Since the obstinate buyer does not discount time, i.e. $r_b = 0$, we have $\delta = 1$. Therefore, ϵ_b is the only search friction that the obstinate buyer is subject to and it has no crucial impact on our analysis.[9]

The statement in (3) can be interpreted as an implication rather than an assumption. Since the obstinate buyer does not value time (statement (1)), he is indifferent between staying with his current partner or visiting the other seller at any time (ignoring the switching cost). However, if he leaves his current partner before being convinced that he is obstinate, he will revisit this seller later if he exhausts all his hope to reach an agreement with the other seller. Therefore, since the switching cost ϵ_b is positive, the obstinate buyer will switch his partner just once and thus leaves a store when he is convinced that his opponent is the obstinate type.

Moreover, since the sellers share a common initial prior of being the commitment type, the obstinate buyer is initially indifferent about which store to visit first regardless of the sellers' announced demands. Assumption made in (4), however, is a simplification assumption that can be generalized with no impact on the main message of our results.[10]

Finally, one may think that coexistence of some other commitment types for the buyer (the ones who value time and wish to reach an agreement quickly) could change our results, but this is not necessarily the case. For example, consider a house owner who is negotiating with more than one person to sell his house in order to pay his urgent depth. The buyer (the house owner in this case) may have to commit to a certain price.

[9]See Footnote 4

[10]For example, one may assume that there are multiple types for the obstinate buyer (regarding the initial store selection) such that some always chooses a fix seller and some visits the sellers according to their announcements while the rest is possibly the combination of these.

But in this case, he will clearly not fit to the obstinate buyer I described above simply because he needs to reach an agreement as quickly as possible. If the buyer, as in this case, has some commitment, he may wish to distinguish himself from the rational buyer, so he may go back and forth between the sellers multiple times. Thus, he may reach an agreement earlier. However, when time is a crucial factor for a bargainer, he usually needs to compromise between two things; time of the agreement and the share he will receive. Therefore, the commitment of the buyer who haggles with the sellers fiercely would be credible from the point of view of the sellers if he is committed to a demand relatively lower than the one who would play it cool. Thus, coexistence of such commitment types with the ones I assume here will not alter our results if we assume that the buyer's commitment to high demands is interpreted by the sellers such that the buyer must be the one who will play it cool. For this reason, I restrict my attention only to those commitment types that I described above.

3. The Bargaining Problem in Continuous Time

I am interested in equilibrium outcome(s) of the bargaining problem $\mathrm{G}(g)$ in the limit where the players can make sufficiently frequent offers. Therefore, for $\epsilon > 0$ small enough, let $\mathrm{G}(g_\epsilon)$ denote discrete-time bargaining problem where the buyer and the sellers bargain, in stage two, according to the protocol $g_\epsilon = (g_\epsilon^1, g_\epsilon^2)$ such that for all $t \geq 0$ and $i \in \{1, 2\}$, both seller i and the buyer have the chance to make an offer, at least once, within the interval $[t, t+\epsilon]$ in the bargaining protocol g_ϵ^i.[11] In this sense, the discrete-time bargaining problem $\mathrm{G}(g_\epsilon)$ converges to continuous time as $\epsilon \longrightarrow 0$.[12]

In Appendix C, I show that given the declared demands in stage 1, the second stage equilibrium outcomes of the discrete-time bargaining problem $\mathrm{G}(g_\epsilon)$ converge to a unique limit, independent of the exogenously given bargaining protocols, g_ϵ, as $\epsilon \longrightarrow 0$, and this limit is equivalent to the unique outcome of the following continuous-time war of attrition game. Suppose, without loss of generality, that the buyer visits seller i and declares α_b in stage 1 such that α_b is incompatible with the sellers' demands. Upon the beginning of the second stage (i.e. seller i does not accept the buyer's demand), the buyer and seller

[11] More formally, either $g^i(\hat{t}) = 3$ for some $\hat{t} \in [t, t+\epsilon]$, or $g^i(t') = 1$ and $g^i(t'') = 2$ for some $t', t'' \in [t, t+\epsilon]$.

[12] One may assume that the travel time is discrete and consistent with the timing of the bargaining protocols so the buyer never arrives a store at some non-integer time.

i immediately begin to play the following concession game: At any given time, a player either accepts his opponent's demand or waits for a concession. At the same time, the buyer decides whether to stay or leave store i. Concession of the buyer or seller i, while the buyer is in the store, marks the completion of the game. In case of simultaneous concession, surplus is split equally.[13] If the buyer leaves store i and goes to store j, the buyer and seller j start playing the concession game upon the buyer's arrival at that store. Both sellers can perfectly observe the buyer's moves throughout the game. Thus, the players' actual types are the only source of uncertainty in the game.[14] I denote the two stage bargaining problem in continuous-time by G.

Since we have this convergence result, I will use the game G, the bargaining problem in continuous-time, in my analysis throughout the paper. In this section, I examine the second stage equilibrium strategies of the bargaining problem G. Next section characterizes the equilibrium strategies and outcomes of the first stage. I finish my analysis by investigating the equilibrium outcomes of G (demand selections in stage 1) in the limit where the set of obstinate types converges to $[0, 1]$ and the search friction vanishes.

The Case Where the Sellers' Demands are the Same

Suppose now that the sellers choose the same demand, $\alpha \in C$, in stage 1 and it is incompatible with the buyer's demand, $\alpha_b \in C$. Since the sellers are initially identical, the rational buyer is indifferent about which store to visit first in stage 1. Therefore, I assume, without loss of generality, that the rational buyer chooses each seller with equal probabilities when the sellers' demands are the same.[15] Hence, regardless of the store visited in stage 1, the posterior probability that the buyer is obstinate is equal to his initial reputation z_b.

The equilibrium of the continuous-time bargaining problem in the second stage is unique. A short descriptive summary of the equilibrium strategy is as follows (see *Figure*

[13]This particular assumption is not crucial because simultaneous concession occurs with probability zero in equilibrium.

[14]After leaving store i and traveling part way to store j, the buyer could, if he wished, turn back and enter store i again.

[15]This is the only equilibrium strategy of the rational buyer when he is strong (the definition of the term will be given in this section). Otherwise, it is one of the equilibrium strategies, all of which yield the same expected payoff to the buyer. However, picking this particular one does not affect our results in subsequent sections.

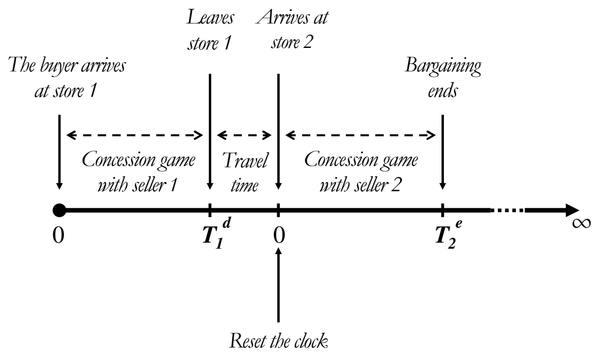

Figure 1: The time-line of the buyer's equilibrium strategy

1). At time 0, the buyer enters store 1, for example, and starts playing the concession game with the seller until time T_1^d. At this time the buyer leaves store 1, if the game has not yet ended, and goes directly to store 2. Once the buyer arrives at store 2, the buyer and seller 2 play the concession game until T_2^e, when both players' reputations reach 1. That is, by time T_2^e the game ends with certainty if one of the players is rational. Therefore, in equilibrium the buyer visits each store at most once. The departure time of the buyer from store 1, T_1^d is greater than or equal to zero depending on the primitives of the model.

Each player's equilibrium strategy in the concession game is a continuous and strictly increasing distribution function. That is, in equilibrium both the buyer and the sellers concede by choosing the timing of acceptance randomly with a constant hazard rate (or instantaneous acceptance rate). Therefore, at any moment the players are indifferent to either accepting the opponent's demand or waiting. The hazard rate of a player depends only on the demands chosen in stage 1 and his opponent's time preferences.

For the sake of simplicity in presentation and notation, I will focus for the moment on equilibrium strategy where the buyer visits each store at most once. Appendix A will consider more elaborate strategies to prove that all the results in this section hold without this restriction, which is a consequence of the equilibrium.

The buyer's strategy in the bargaining stage has two parts. The first part, σ_b, determines the buyer's location as a function of history. Assume, without loss of generality, in equilibrium the buyer visits store 1 first and then store 2. Then let T_1^d denote the time that the buyer leaves store 1 if no agreement has been reached yet. Denote by ω_i the time that the buyer starts negotiating with seller i (if agreement has not been reached yet). That is, $\omega_1 = 0$ and $\omega_2 = T_1^d + \Delta$ where Δ is the travel time between the stores. For notational simplicity, I manipulate the subsequent notation and denote ω_2 by 0. That is, I reset the clock once the buyer arrives in store 2 (but not the players' reputations).[16]

The second part is a pair of right continuous distribution functions $F_b^i : \mathbb{R}_+ \cup \infty \to [0, 1]$, $i = 1, 2$.[17] Thus, for each t, $F_b^i(t)$ is the probability that the buyer concedes to seller i by time t (inclusive). Similarly, seller i's strategy in the bargaining phase is a right continuous distribution function $F_i : \mathbb{R}_+ \cup \infty \to [0, 1]$ such that for all $t \geq 0$, $F_i(t)$ denotes the probability that seller i concedes to the buyer by time t (inclusive).[18]

Given the strategy of the buyer, let $z_b(t)$ denote the buyer's reputation (probability that the buyer is the obstinate type) at time $t \geq 0$. It is updated according to the Bayes' rule and is consistent with the buyer's strategy: For example, since the buyer visits seller 1 first, for any $t \geq 0$, we have $z_b(t) = z_b/(1 - F_b^1(t))$, and $z_b(t)$ is no less than the buyer's initial reputation z_b.[19] Furthermore, since the buyer leaves store 1 at time T_1^d, it must be that $F_b^1(T_1^d) \leq 1 - z_b$. On the other hand, the buyer visits store 2 if the players cannot reach an agreement in store 1, implying that $F_b^2(T_2^e) \leq 1 - z_b(T_1^d)$ where T_2^e denotes the time that the continuous-time bargaining problem ends in store 2. Since I consider the equilibrium strategies where the buyer visits the stores at most once, it must be that $F_1(T_1^d) \leq 1 - z_s$ and $F_2(T_2^e) \leq 1 - z_s$.[20]

Given F_b^i, seller i's expected payoff of conceding to the buyer at time $t \geq 0$ is

$$\begin{aligned} U_i(t, F_b^i) := \alpha \int_0^t e^{-r_s y} dF_b^i(y) \\ + \frac{1}{2}(\alpha + \alpha_b)[F_b^i(t) - F_b^i(t^-)]e^{-r_s t} + \alpha_b[1 - F_b^i(t)]e^{-r_s t} \end{aligned} \tag{1}$$

[16]Thus, with some manipulation of the notation, I define each player's distribution function as if the concession game in each store starts at time 0.

[17]Since the buyer leaves store 1 at time T_1^d, $F_b^1(.)$ is defined over $[0, T_1^d]$ corresponding to the time frame that the buyer is in store 1 according to σ_b.

[18]Note that F_b^i is the sellers' belief about the buyer's play during the concession game with seller i. Hence, it is the strategy of the buyer from the point of view of the sellers. For this reason, the distribution function F_b^i never reaches 1 since the buyer is the obstinate type with probability z_b, implying that $\lim_{t\to\infty} F_b^i(t) = 1 - z_b$. Similar arguments are valid for the sellers' strategies F_1 and F_2.

[19]Note that the buyer's reputation, $z_b(t)$, reaches 1 when $F_b^1(t)$ reaches $1 - z_b$.

[20]In equilibrium, it must be that $F_1(T_1^d) = 1 - z_s$, and given that the buyer visits store 2 $F_2(T_2^e) = 1 - z_s$ and $F_b^2(T_2^e) = 1 - z_b(T_1^d)$.

with $F_b^i(t^-) = \lim_{y\uparrow t} F_b^i(y)$.[21]

In a similar manner, given F_i, the expected payoff of the buyer who concedes to seller i at time $t \geq 0$ (conditional on not reaching a deal with seller j if store j was visited first) is

$$U_b^i(t, F_i) := (1-\alpha_b)\int_0^t e^{-r_b y} dF_i(y) + \frac{1}{2}(2-\alpha-\alpha_b)[F_i(t) - F_i(t^-)]e^{-r_b t} + (1-\alpha)[1-F_i(t)]e^{-r_b t} \qquad (2)$$

where $F_i(t^-) = \lim_{y\uparrow t} F_i(y)$.[22]

The next result characterizes the equilibrium strategies of the continuous-time bargaining problem G in the second stage.

Proposition 3.1. *Suppose that the sellers declare the same demand, α, and the buyer chooses α_b in stage 1 such that $\alpha_b < \alpha$. The unique equilibrium of the continuous-time bargaining problem G in stage 2 (assuming that the buyer visits seller 1 first) is the following:*

(i) *The buyer visits each store at most once.*

(ii) *The buyer's strategy in the concession game with each seller is a continuous and increasing (cumulative) distribution function; $F_b^1(t) = 1 - c_b^1 e^{-\lambda_b t}$ and $F_b^2(t) = 1 - e^{-\lambda_b t}$ where*

$$c_b^1 = \begin{cases} \frac{z_b}{X_s} e^{\lambda_b T_1^d}, & \text{if } z_b < X_s \\ 1, & \text{otherwise,} \end{cases}$$

such that $X_s = \left(\frac{z_s}{A}\right)^{\frac{\lambda_b}{\lambda}}$ and $A = \frac{1-\alpha_b-\frac{1-\alpha}{\delta}}{\alpha-\alpha_b}$. Moreover, the sellers' strategies are $F_1(t) = 1 - z_s e^{\lambda(T_1^d - t)}$ and $F_2(t) = 1 - z_s e^{\lambda(T_2^e - t)}$.

(iii) *The buyer leaves the first store when he is convinced that his opponent is the obstinate type. Therefore, the optimal time for the buyer to leave store 1 is*

$$T_1^d = \begin{cases} \min\{-\frac{\log z_s}{\lambda}, -\frac{\log(z_b/X_s)}{\lambda_b}\}, & \text{if } z_b < X_s \\ 0, & \text{otherwise,} \end{cases}$$

Moreover, if the game does not end before T_1^d, the buyer leaves store 1 at this time with probability 1.

[21] U_i is evaluated at time 0 in "real time".

[22] If the buyer visits seller i first, then U_b^i is evaluated at time 0 (in real time). Otherwise, it is evaluated at time $w_i + \Delta$ (in real time).

(iv) Finally, the concession game ends in store 2 at time

$$T_2^e = \min\left\{-\frac{\log X_s}{\lambda_b}, -\frac{\log z_b}{\lambda_b}\right\}$$

and for each $i \in \{1,2\}$, $\lambda = \frac{(1-\alpha)r_b}{\alpha-\alpha_b}$, $\lambda_b = \frac{\alpha_b r_s}{\alpha-\alpha_b}$.

I defer the proofs of all the results in this section to Appendix A.

Characterization of the distribution functions $(F_i, F_b^i)_i$ uses arguments in Hendricks, Weiss and Wilson (1988) and is analogous to the proof of Lemma 1 in Abreu and Gul (2000). In equilibrium of the concession game between the buyer and a seller, if a player's strategy, distribution function, has a discontinuity point at some time t, his opponent prefers to wait a little longer, instead of conceding in some $\hat{\epsilon}$-neighborhood of t. Therefore, if $(F_i, F_b^i)_i$ are equilibrium strategies in the interval $[0, T]$ where T is either equal to T_i^d or T_i^e depending on which store the buyer visits first, then there cannot be common discontinuity point for these distribution functions on this interval.

On the other hand, if a player does not concede to his opponent during the time interval $[t, t'] \subset [0, T]$, his opponent prefers to wait in the interval $[t, t' + \hat{\epsilon}]$ for some small but positive $\hat{\epsilon}$. Along with the previous argument, in equilibrium a player's strategy cannot have a discontinuity point in $(0, T]$. Therefore, equilibrium strategies (F_i, F_b^i) must be strictly increasing, continuous and differentiable over $(0, T]$, implying that players are indifferent between conceding and waiting at any time of the concession game.[23] A simple manipulation in the utility functions given in equations (1) and (2) gives us the functional form of these distribution functions.

In equilibrium, the buyer's continuation payoff is no more than $1 - \alpha$ if he reveals his rationality.[24] Since the obstinate buyer leaves a seller when he is convinced that his bargaining partner is the commitment type, leaving the first seller "earlier" (or "later") than this time would reveal the buyer's rationality. Therefore, in equilibrium the rational buyer never leaves a seller as long as there is positive probability that this seller is a rational type, and he immediately leaves otherwise. Clearly the buyer does not revisit a seller once he knows that this seller is the obstinate type.

Since the buyer is indifferent between conceding and waiting at all times during a concession game, his expected payoff in the concession game with, for example, seller i is

[23] Notice that F_i or F_b^i (not both) may be discontinuous at 0.

[24] Arguments similar to the proof of Proposition A.2 in the Appendix yields this result.

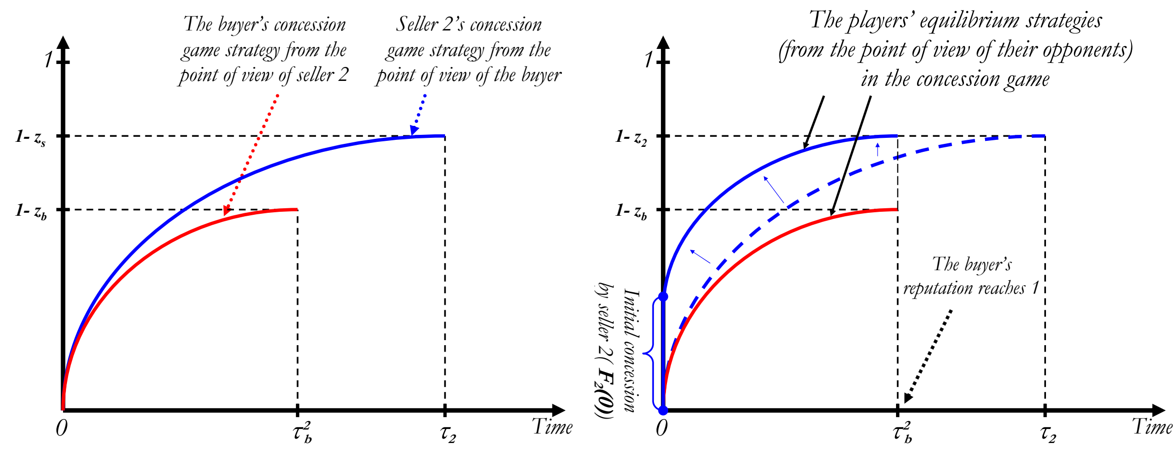

Figure 2: Concession game strategies of the buyer and seller 2 in equilibrium

equal to what he can achieve at time 0, the beginning of the concession game, i.e.[25]

$$F_i(0)(1-\alpha_b) + (1-F_i(0))(1-\alpha) \tag{3}$$

Note that in equilibrium, at most one player makes an *initial probabilistic concession*, namely $F_i(0)F_b^i(0) = 0$. I call the buyer ***strong relative to*** seller i if he receives this probabilistic gift from seller i and ***weak*** if he does not. Thus, if the buyer is weak relative to a seller, his expected payoff in the concession game with this seller is $1-\alpha$.

Players' strength in a concession game is determined within the equilibrium. Suppose for the moment that there is only one seller (seller 2) and a buyer, whose strategies in the concession game are as given in *figure 2*. If no player makes an initial probabilistic acceptance, then reputations of the buyer and seller reach 1 at time τ_b^2 and τ_2, respectively. However at time τ_b^2, seller 2 will be convinced that the buyer is the obstinate type. Thus in equilibrium, seller 2 should also finish the concession game by this time, which implies that the seller's strategy (the distribution function) must reach $1-z_s$ at this time. However, since the shape of the distribution function is determined by the constant hazard rate, seller 2's reputation reaches one at time τ_b^2 only if the seller sets $F_i(0) > 0$. Namely, seller 2 must concede to the buyer at time 0 with a positive probability, implying that in equilibrium the buyer is strong relative to seller 2.

When there are two sellers, building reputation on inflexibility by haggling with the first seller is an investment for the buyer, which increases his continuation payoff in the second store. Having a higher outside option in return increases the bargaining power of

[25]Similarly, seller i's expected payoff in the concession game is $F_b^i(0)\alpha + (1-F_b^i(0))\alpha_b$.

the buyer in the first store. More formally, the existence of the second store gives the buyer a valuable opportunity to credibly threaten his opponent so that seller 1 has to offer a probabilistic gift at time 0. The buyer can force seller 1 to adjust his strategy and increase the amount of this initial gift by choosing the departure time earlier. As the buyer is expected to leave store 1 earlier, seller 1 has to offer a bigger gift, and as the gift increases, the buyer's payoff increases. However, the buyer cannot impel seller 1 to increase this gift as much as he wants, because the buyer cannot credibly threaten seller 1 by leaving before T_1^d, equilibrium departure time, since his initial reputation is not high enough.

In equilibrium, the buyer would not threaten seller 1 to leave at time T_1, for example see *Figure 3-(a)*, because his initial reputation is not high enough to build up the required reputation to become the strong player in store 2. In this case, the buyer needs to haggle with seller 1 little longer. If the buyer leaves seller 1 at time $T_2 > T_1$, for example *Figure 3-(c)*, then the buyer arrives at store 2 with a reputation that is high enough to make himself strong relative to seller 2. Hence, the equilibrium departure time T_1^d resolves the rational buyer's trade off: He wants to leave seller 1 early in order to increase his (expected) payoff but cannot live too early because he may need to build up his reputation to make his outside option credible.

If the buyer's initial reputation is small, i.e. $z_b \leq z_s^{\lambda_b/\lambda} X_s$, then he cannot build enough reputation before time τ_1 –which is the time that seller 1's reputation reaches 1 if he does not concede to the buyer at time 0– to force seller 1 for probabilistic concession at time 0. Thus, the buyer's expected payoff during the entire bargaining phase, and so at time zero, is $1 - \alpha$. Therefore, I call the buyer ***weak*** if $z_b \leq z_s^{\lambda_b/\lambda} X_s$, and ***strong*** otherwise.

However, if the buyer's initial reputation is low so that the above inequality holds, then he may have to offer a probabilistic gift to seller 1 at time 0. The amount of this gift, c_b^1, is as given in Proposition 3.1. The gift cannot be less than this particular amount because in such a case the buyer strictly prefers accepting seller 1's demand to finish the game at time τ_1 instead of moving to the second store to play the concession game with seller 2. This contradicts the fact established in Proposition 3.1 that in equilibrium the buyer's strategy, F_b^1, cannot have a discontinuity point in $(0, T_1^d]$. On the other hand, the initial gift cannot exceed this specific amount because in this case, before time τ_1 the

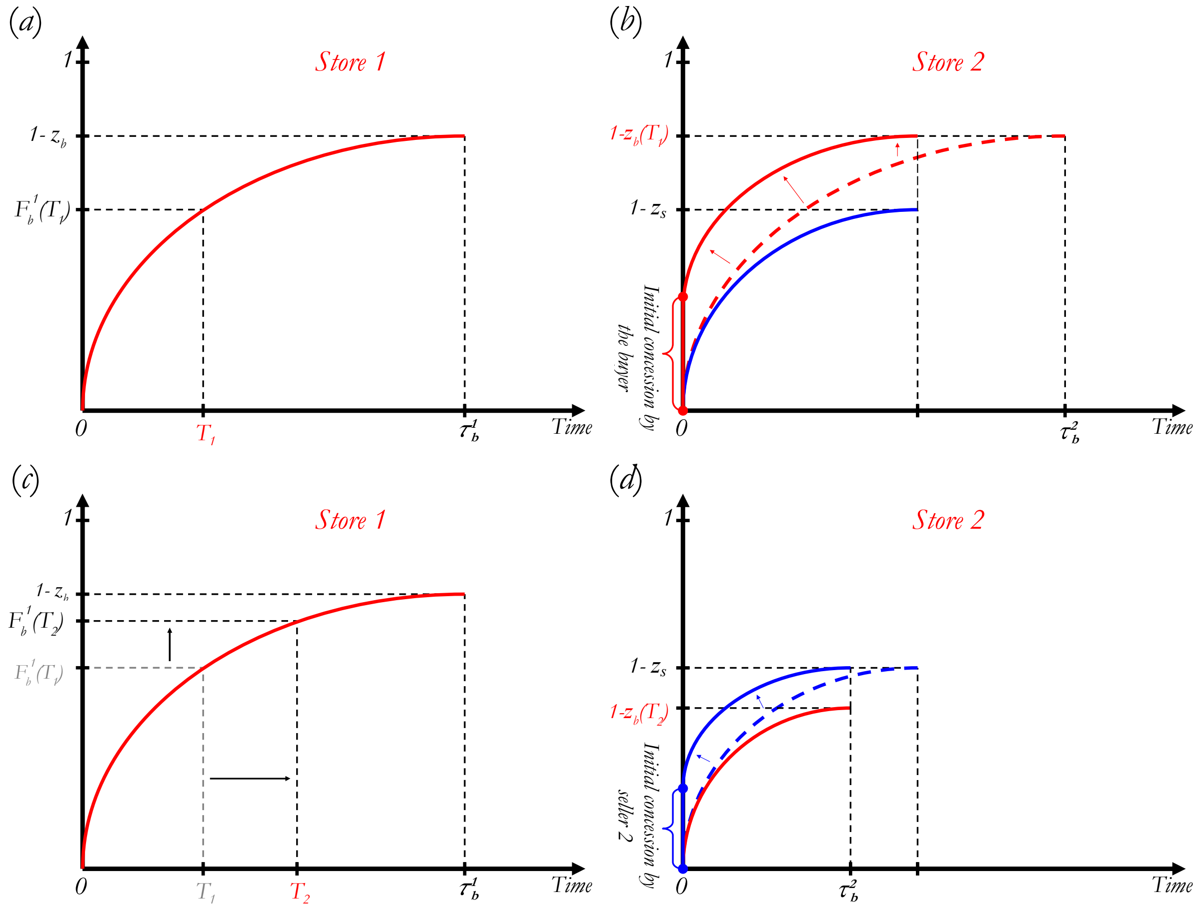

Figure 3: As the buyer continues to play the concession game in store 1, he builds up his reputation, which increases his continuation payoff in the second store. However, since the buyer is impatient, there should be an optimal departure time from store 1.

buyer's reputation will reach to the point, where it is optimal for the buyer to leave store 1. Then, seller 1 would have to adjust his strategy by making a positive probabilistic concession at time 0. That would contradict the fact that in equilibrium F_b^1 and F_1 cannot have a common discontinuity point in their domain.

On the other hand, when the buyer's initial reputation is high, i.e. $z_b \geq X_s$, then the buyer is strong relative to seller 2 even with his initial reputation z_b. In this case the buyer prefers going to store 2 and playing the concession game with this seller over conceding to seller 1 at time 0. Thus, in equilibrium the buyer leaves store 1 immediately at time 0. Since the rational seller 1 knows that the buyer does not need to build reputation but rather plans to leave his store immediately, he accepts the buyer's demand at time 0. Therefore, I call the buyer ***distance-corrected strong*** when $z_b \geq X_s$. If the buyer is distance-corrected strong, then his continuation payoff in the game G evaluated at time

0 is given by

$$(1-z_s)(1-\alpha_b)+\delta z_s\left[(1-z_se^{\lambda T^e})(1-\alpha_b)+z_se^{\lambda T^e}(1-\alpha)\right] \tag{4}$$

If the buyer is strong but not distance-corrected strong, then the buyer receives an initial probabilistic concession from the first seller he visits at time 0. That is, the concession game between the buyer and, for example, seller 1 lasts until the time of departure T_1^d, which is strictly positive in this case because the buyer needs to build his reputation in store 1 before it becomes optimal for him to go to store 2. In this particular case, the buyer's continuation payoff (in equilibrium) evaluated at time 0 is

$$\left(1-\frac{z_s^2}{Az_b^{\lambda/\lambda_b}}\right)(1-\alpha_b)+\frac{z_s^2}{Az_b^{\lambda/\lambda_b}}(1-\alpha) \tag{5}$$

In equilibrium the buyer is indifferent between conceding and waiting in the first store he visits until the departure time T^d and it is exactly at this time that he is indifferent between conceding to and leaving this seller. Moreover, according to Proposition 3.1 the buyer leaves the first store with probability 1 and from the time that the concession game starts in the second store to the time it ends, T^e, he is indifferent between conceding to the second seller and waiting in his store. As a result, the buyer's instantaneous payoff is $1-\alpha$ at all times. Therefore, the buyer's expected payoff is the same in each store and is equal to what he can achieve at time 0 in the first store, implying the functional form given in (5).

Finally, note that the unique equilibrium outcome in the second stage is always inefficient and this inefficiency is due to delay in agreement and uncertainty about the types of the players. For example, if the buyer is strong ($z_b > z_s^{\lambda_b/\lambda}X_s$), then the seller 1's payoff (the seller who is visited by the buyer first at time 0) is simply α_b. Moreover, as the search friction vanishes, i.e. $\delta \to 1$, the buyer's continuation payoff in the bargaining phase converges to a limit that is strictly less than $1-\alpha_b$.[26]

The Case Where the Sellers' Demands are Different

This section characterizes the unique equilibrium strategy of the continuous-time bargaining problem G in stage 2 when the sellers choose different demands in stage 1. Without loss of generality, I assume that $\alpha_2 < \alpha_1$. In this case, the structure of the equilibrium

[26]Note that the second seller's expected payoff in the game is less than $z_se^{-r_sT_1^d}\alpha_b$.

strategy drastically changes (relating to the case where $\alpha_1 = \alpha_2$). In equilibrium, the bargaining phase never ends with the buyer's concession to the seller who has the higher demand (seller 1). If the buyer ever visits store 1, the rational seller 1 concedes to the buyer (upon the buyer's arrival at this seller) because the buyer has the tendency to opt out instantly from the concession game in store 1.

More formally, consider the case where the buyer is in store 1 and playing the concession game with this seller. This means that the buyer is indifferent between accepting seller 1's demand, so receiving the instantaneous payoff of $1 - \alpha_1$, and waiting for the concession of the seller. However, if the buyer (immediately) leaves seller 1 and goes directly to the second store to accept the demand of seller 2, his discounted (instantaneous) payoff will be $\delta(1 - \alpha_2)$. Thus, if the buyer ever visits store 1 in equilibrium, then he will never accept seller 1's demand because by Assumption 2 we have $(1 - \alpha_1) < \delta(1 - \alpha_2)$. Therefore, in equilibrium the buyer does not concede to nor spend time with seller 1 given that he ever visits store 1. As a result, it must be the case that rational seller 1 instantaneously accepts the buyer's demand with probability one upon his arrival, and the buyer immediately leaves store 1 if seller 1 does not concede to him.

Since the buyer and seller 1 play an equilibrium strategy that impels seller 1 to reveal his type immediately, the buyer's expected payoff of visiting this seller is $(1 - z_s)(1 - \alpha_b) + \delta z_s v_b^2$. I Denote by v_b^2 the buyer's expected payoff in store 2 when he visits this store knowing that seller 1 is the obstinate type. Thus, if the buyer initially chooses to visit seller 2 first, then he concedes to this seller, and receives the instantaneous payoff of $1 - \alpha_2$, if and only if $1 - \alpha_2 \geq \delta[(1 - z_s)(1 - \alpha_b) + \delta z_s v_b^2]$.

This inequality holds when $z_s \geq \bar{z}$ holds.[27] However, by Assumption 2 we have $z_s < \bar{z}$, implying that if the buyer visits seller 2 first in equilibrium, then the buyer strictly prefers leaving this seller immediately upon his arrival (given that seller 2 does not accept the buyer's demand and finish the game). Hence, rational seller 2 must concede to the buyer at time 0 with probability one. Next result characterizes the second stage equilibrium strategies of the bargaining problem G.

Proposition 3.2. *Suppose that the players' declared demands in stage 1 are such that $\alpha_b < \alpha_2 < \alpha_1$. Then the unique equilibrium of the continuous-time bargaining problem G in stage 2 is the following:*

[27] See the proof of Proposition 3.2 in Appendix A

(*i*) *If the buyer visits seller* 1 *first, then rational seller* 1 *immediately accepts the buyer's demand and finishes the game at time* 0 *with probability one. In case seller* 1 *does not concede to the buyer, the buyer infers that seller* 1 *is the obstinate type, so he immediately leaves store* 1 *and never comes back to this store again. The buyer directly goes to seller* 2 *to play the concession game with this seller. The concession game with seller* 2 *may continue until the time* $T_2^e = \min\{-\frac{\log z_b^1}{\lambda_b}, -\frac{\log z_s}{\lambda_2}\}$ *where* $\lambda_2 = \frac{(1-\alpha_2)r_b}{\alpha_2-\alpha_b}$, $\lambda_b = \frac{\alpha_b r_s}{\alpha_2-\alpha_b}$ *and* z_b^1 *is the posterior probability that the buyer is the obstinate type conditional on seller* 1 *is visited first.*[28] *Moreover, players concede according to the following strategies:* $F_2(t) = 1 - z_s e^{\lambda_2(T_2^e - t)}$ *and* $F_b^2(t) = 1 - z_b^1 e^{\lambda_b(T_2^e - t)}$ *for all* $t \geq 0$.[29]

(*ii*) *If the buyer visits seller* 2 *first, then rational seller 2 immediately accepts the buyer's demand upon his arrival. Otherwise, the buyer leaves seller* 2 *immediately at time* 0 *(knowing that seller 2 is the obstinate type), and goes directly to seller* 1*. Rational seller* 1 *instantly accepts the buyer's demand with probability one upon the buyer's arrival. In case seller* 1 *does not concede, the buyer immediately leaves this seller, directly returns to seller* 2*, accepts the seller's demand* α_2 *and finalizes the game.*

Therefore, in equilibrium, when the buyer visits seller 1 first, he sends a *take it or I will leave you* ultimatum to this seller. If seller 1 does not accept the buyer's demand, then the buyer will go to the second seller. In this case, an agreement might be reached with seller 2, but possibly after some delay. On the other hand, when the buyer visits seller 2 first, he sends the same ultimatum to both sellers (first to seller 2 and then to 1). If no seller accepts the buyer's demand, then the buyer will come back to seller 2 and accept his demand α_2.[30] Hence, the buyer visits seller 1 first only when he is strong relative to seller 2 (according to the initial reputations z_b^1 and z_s) so that the initial probabilistic concession he will receive from seller 2 is high enough. This implies that z_b^1 (the posterior probability that the buyer is the obstinate type conditional on seller 1 is visited first) must be sufficiently high. The following result summarizes the last argument

[28]Suppose that the rational buyer employs a strategy such that he visits seller 1 first with probability $\sigma_b^0(1) \in [0,1]$. Then $z_b^1 = \frac{1/2 z_b}{1/2 z_b + (1-z_b)\sigma_b^0(1)}$.

[29]Note that with some manipulation of the notation, I reset the clock once the buyer enters store 2.

[30]Seller 2's immediate concession to the buyer (and receiving the payoff of α_b) is optimal because otherwise the seller can achieve at most $\alpha_2 z_s$ (since the buyer re-visits seller 2 only if seller 1 is the obstinate type) and we have $\alpha_2 z_s < \alpha_b$ by Assumption 2.

formally.

Proposition 3.3. *Suppose that the players' declared demands in stage 1 are such that* $\alpha_b < \alpha_2 < \alpha_1$*. In the unique equilibrium of the continuous-time bargaining problem G in stage 2, the buyer visits seller* 1 *first if and only if the buyer is sufficiently strong. That is,* $z_b^1 \geq \left(\frac{\alpha_2-\alpha_b}{1-\alpha_b-\delta(1-\alpha_2)}\right)^{\lambda_b/\lambda}$.

4. First Stage Equilibrium Demand Decisions

I first characterize the rational buyer's equilibrium strategy on store selection. Then I examine the set of equilibrium prices (and demands) that would be chosen by the players in the first stage of the bargaining game G. Finally, I present some limit results regarding the cases where the set of obstinate types, C, converge to $[0,1]$ and the initial probabilities of the obstinate types, z_s, z_b, vanish. For any $\alpha \in C$, let $C_\alpha = \{x \in C | x < \alpha\}$ denote the set of demands that are incompatible with α. Thus, suppose still that π is the (common) probability distribution on C and the set of obstinate types, C, is finite and reach enough; let α_{min} denote the minimum element of the set C, i.e. $\alpha_{min} = \min C$, so that we have $\{\alpha_{min}\} \neq C_\alpha$.

Assumption 3. *For any* $\alpha \in C$ *such that* $\alpha_{min} < \alpha$ *we have* $\alpha_{min} \leq \frac{1}{2}\sum_{x\in C_\alpha} x\pi(x)$*.*

Assumption 3 implies, for example, that the minimal and the second minimal element of the set C are distant enough so that half of the mean of these two is no less than the minimal element of C. This assumption is not necessary for the results, however it simplifies the subsequent analysis substantially.

A strategy for seller i in the first stage of the continuous-time bargaining problem G is some $\alpha_i \in C$ whereas a strategy for the buyer is defined as a collection (μ_b, σ_b^0). Here given the announced demands (α_1, α_2), μ_b is a probability distribution over $C \cup \{Q\}$ describing the buyer's choice between Q (immediate acceptance) and $\alpha_b \in C$. Recall that $C_{(\alpha_1,\alpha_2)} = \{x \in C | x < \min\{\alpha_1, \alpha_2\}\}$ denotes the set of demands that are incompatible with α_1 and α_2. Without loss of generality, I require that $\mu_b(Q) = 0$ for all $\alpha_b \in C_{(\alpha_1,\alpha_2)}$ and $\mu_b(Q) = 1$ otherwise. That is, both conceding at $t = 0$ (the beginning of stage 2) and choosing Q (in stage 1) corresponds to immediate concession. Finally, σ_b^0 is a probability measure over the sellers such that $\sigma_b^0(i)$ denotes the probability that the buyer visits seller i first.

Recall that the rational buyer chooses each seller with equal probabilities when the sellers' demands are the same. Next result characterizes the rational buyer's equilibrium strategy $\sigma_b^0(i)$ if the sellers choose different demands in stage 1.

Proposition 4.1. *Suppose that the sellers' declared demands in stage 1 are such that $\alpha_{min} < \alpha_2 < \alpha_1$. Then in equilibrium, the rational buyer declares his demand as α_{min} in both stores and visits seller 1 first with probability $\sigma_b^0(1) = \frac{z_b(1-\mu)}{2\mu(1-z_b)}$ where $\mu = \left(\frac{\alpha_2-\alpha_{min}}{1-\alpha_{min}-\delta(1-\alpha_2)}\right)^{\lambda_b/\lambda}$.*

Fix the search friction δ. For any $z_b, z_s \in (0,1)$, let $\mathrm{G}(z_b, z_s)$ denote the continuous-time bargaining problem G where the initial reputations of the sellers and the buyer are z_b and z_s, respectively. Denote by $C^E \subseteq C \times C$ the set of equilibrium prices (of the sellers). More formally, a pair of demands $(\alpha_1, \alpha_2) \in C^2$ is an element of C^E if and only if there exit $z_b, z_s \in (0,1)$ small enough (i.e. satisfying assumptions 1-3) such that α_1 and α_2 are equilibrium demand selections of the sellers in the first stage of the bargaining problem $\mathrm{G}(z_b, z_s)$.

Proposition 4.2. $C^E = \{(\alpha, \alpha) | \alpha \in C\}$.

Since the sellers are *ex-ante* identical, it is natural to suspect that in equilibrium both sellers should choose the same demand. However, it is surprising that any obstinate type's demand in C can be supported in equilibrium, even though the sellers compete in the Bertrand fashion. Given that both sellers choose the same demand, α, that is higher than α_{min}, the buyer's optimal strategy is such that he chooses each seller with equal probabilities and declares a demand randomly chosen from the set C_α; since π is uniform on C, each member of C_α has equal chance to be drawn. Therefore, each seller's ex-ante expected payoff of declaring the demand α is $\frac{1}{2}\sum_{x \in C_\alpha} x\pi(x)$, which is above α_{min} by Assumption 3.

On the other hand, if a seller price undercuts his opponent, then the buyer would infer that this seller is the obstinate type with certainty. In this case, the buyer uses the deviating seller's price as an "outside option" to increases his bargaining power against the other seller. Thus, the buyer prefers to visit (first) the seller whom he knows he can negotiate and possibly get a much better deal. Hence, price undercutting is not an optimal strategy for a deviating seller because it would yield payoff strictly less than $z_s\alpha_{min}$ (the buyer will visit deviating seller's store if the other seller is an obstinate type).[31]

Fix the search friction δ. Let C^E_∞ denote *the set of equilibrium prices of the bargaining problem* G *as initial priors vanish.* Therefore, any $\alpha \in C$ with $(\alpha, \alpha) \in C^E$ is also in C^E_∞ if and only if for any $z_s, z_b \in (0,1)$ (where α is the equilibrium price of the bargaining problem $\mathrm{G}(z_b, z_s)$) we have the following: Take any sequences $\{z_s^n\}$ and $\{z_b^n\}$ (where $z_s^0 = z_s$, $z_b^0 = z_b$ and for all $n \geq 0$, $z_s^n = K z_b^n$ for some finite $K > 0$) of the prior beliefs converging to zero.[32] Then α is the equilibrium price of the bargaining problem $\mathrm{G}(z_b^n, z_s^n)$ for all $n \geq 0$.

For any $\alpha \in C$ define α_b^{max} be the maximal element in the set C_α, i.e. $\alpha_b^{max} := \max\{\alpha_b \in C | \alpha_b < \alpha\}$. The following result characterizes the set of equilibrium prices as the initial priors vanish.

Proposition 4.3. $C^E_\infty = \{(\alpha, \alpha) \in C^E \mid 2\alpha_b^{max} + \alpha \leq 1\}$.

The final result in this section presents the limit set of the set of equilibrium demands supported in stage 1 of the continuous-time bargaining problem G as the set of obstinate types converges to the unit interval and the initial priorities regarding the players' types vanish. The maximum and the minimum demands that we can approximate are 1/3 and 0 respectively. According to Proposition 3.2, in equilibrium where both sellers choose demand $x \in [0, \frac{1}{3}]$, the expected payoff to the buyer is $1 - x$ whereas the expected payoff to the sellers approximates to $\frac{x}{2}$. Namely, the outcome is efficient in the limit.

Corollary 4.1. *As the set of obstinate types, C, converges to $[0,1]$ while the initial priors, z_b and z_s, vanish, the set of equilibrium demands announced in stage 1 of the continuous-time bargaining problem G converges to the set $[0, \frac{1}{3}]$.*

Proposition 4.4. *Let G_N denote the continuous-time bargaining problem where the number of sellers is some $N \geq 2$. The game G_N is identical to G except the number of players. Then as the set of obstinate types, C, converges to $[0,1]$ while the initial priors, z_b and z_s, vanish, the set of equilibrium demands announced in stage 1 of G_N converges to the set $[0, \frac{1}{N+1}]$.*

5. Concluding Remarks

[31] By assumption 3, price undercutting is still not optimal even if the deviating seller deviates to α_{min}.

[32] That is, $\forall \epsilon > 0,\ \exists M > 0$ such that $|z_s^m - 0| < \epsilon,\ \forall m > M$.

This paper develops a reputation-based model to highlight the influence of posted prices and bargaining postures on imperfectly competitive search markets. The introduction of obstinate types that are completely inflexible in their demands and offers, even with low probabilities, makes the equilibrium of the multilateral bargaining game essentially unique. The equilibrium allocation does not depend on the fine details of the bargaining protocols, nor do the sellers extract all the surplus of the buyer because of the positive search friction. Instead, it depends on the posted prices and initial reputations as well as the time preferences of the players. The equilibrium has a war of attrition structure that engenders inefficiency due to possible delay in reaching an agreement. Although the sellers compete in the spirit of Bertrand, the equilibrium predictions are in contrast to it.

Appendix

Appendix should appear here...

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