Noncooperative Foundation of Nash Bargaining Solution in n-Person Games with Incomplete Information*

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Abstract

This paper provides a non-cooperative bargaining game model to support the n-person asymmetric Nash bargaining solution for the bargaining problem with incomplete information. We show that our bargaining game possesses a stationary sequential equilibrium in which all types of proposers offer the ex-post efficient, Bayesian incentive compatible, budget-balanced mechanism with the "full surplus extraction" property. Furthermore, the conditionally expected payoff vector in the stationary sequential equilibrium is characterized as the generalized asymmetric Nash bargaining solution under incomplete information.

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Key words: Nash bargaining solution, incomplete information, informed principal, non-cooperative bargaining

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1 Introduction

This paper present a noncooperative bargaining game model to support the n-person asymmetric Nash bargaining solution. The key feature of our model is to consider a bargaining game with incomplete information and with more than three players. We consider a variation of a noncooperative bargaining game model with random proposers by Hart and Mas-Colell (1996) and Okada (1996). The bargaining procedure is described as follows. On player is selected as a proposer according to some probability distribution among n players in each bargaining round. The selected player proposes a feasible allocation rule. If the proposal is accepted unanimously, all players plays a communication game under the mechanism. If some player rejects the proposal, the game ends with some exogenously given breakdown probability. With the complementary probability, the game goes to the next round.

In the model, each player has a private information about his type and he makes a proposal for an allocation rule, so called "mechanism," when he becomes a proposer in the bargaining game. Thus, a informed proposer designs a mechanism. Our bargaining game includes the problem of mechanism design by a informed principal in Myerson (1983).

The program of establishing a noncooperative foundation for the Nash (1950) bargaining solution was initiated by Nash (1953). The Nash bargaining solution is the payoff allocation which maximize the product of players' gain over their disagreement payoffs. He provided a simultaneous-offers demand game with two players who face the uncertainty about the feasibility of the payoff allocations. In the limit as the uncertainty is vanished, the payoff allocations in the Nash equilibrium of the game converges to the Nash bargaining solution. Rubinstein (1982) provides the alternating-offer bargaining game where the payoff allocations in every subgame perfect equilibrium converges to the Nash bargaining solution in the limit as players become perfectly patient. Binmore, Rubinstein and Wolinsky (1986) obtain the Nash bargaining solution in the limit if the exogenous risk of breakdown is vanishing. The asymmetric Nash bargaining solution is defined as a payoff allocation which maximize the weighted product of players' payoff gains. Binmore (1987) obtains the asymmetric Nash bargaining solution as a stationary subgame perfect equilibrium outcome by generalizing the recognition probability over players to be selected as a proposer.

The extensions to the n-person Nash bargaining solution has been pursued by Hart and Mas-Colell (1996) and Krishna and Serrano (1996). Krishna and Serrano (1996) provides a noncooperative bargaining game model in which players can exit after partial agreements and does not contain the chance moves and stochastic elements to realize the Nash bargaining solution. On the other hand, in Hart and Mas-Colell (1996), a proposer is randomly selected with equal probability and the proposal is agreed to by unanimous consent among the players. If the proposal is rejected by some players, players face a risk of breakdown of the negotiations to continue the next bargaining round. The bargaining game model in this paper is an extension of Hart and Mas-Colell's model to the general recognition probability distribution. Recently, some noncooperative multilateral bargaining game models are provided to support the n-person asymmetric Nash bargaining solution by Miyakawa (2006), Okada (2007), Laruelle and Valenciano (2008), Kultti and Vartiainen (2009), and Britz, Herings and Predtetchinski (2010). If there is no private information, our bargaining game model exactly coincide with the model in Miyakawa (2006).

The Nash bargaining solution for two-person bargaining problem with incomplete information has been examined by Harsanyi and Selten (1972), Myerson (1984) and de Clippel and Minelli (2004). They derive the generalized Nash bargaining solution in incomplete information bargaining problem as a Bayesian Nash equilibrium outcome of the bargaining game satisfying some axioms. We extend two-person bargaining problem with incomplete information to n-person (more than 3-person) bargaining problem.

We obtain the following results. There exists a stationary sequential equilibrium in which every player proposes the ex-post efficient, Bayesian incentive compatible, budget-balanced mechanism satisfying the "full surplus extraction" property, as examined in Cremer and McLean (1988), McAfee and Reny (1992), Kosenok and Severinov (2008) and Severionv (2008). In order to guarantee the existence of such a desirable mechanism for each proposer, we assume that a prior probability distribution over players' types satisfies *Cremer-McLean condition* and *Identifiability condition.* We are necessitated to assume that the game has more than three players and types of players are correlated in order to satisfy both conditions. Furthermore, the conditionally expected payoff vector in the stationary sequential equilibrium is characterized as that payoffs which maximize the weighted product of players' "ex-ante" expected payoff gains in the limit as a risk of the breakdown of the negotiations is vanishing. A stationarity assumption plays an important role in our bargaining game in contrast to the alternating-offer bargaining game with two-sided private information in Chatterjee and Samuelson (1987) and Cramton (1992).

This paper is organized as follows. Section 2 defines the Bayesian bargaining problem and a solution concept of our noncooperative bargaining game with incomplete information. Section 3 provides the generalized asymmetric Nash bargaining solution. Section 4 characterizes a stationary sequential equilibrium of the noncooperative bargaining game. Section 5 discusses relationships between the conditionally expected payoff allocation in the stationary sequential equilibrium and the generalized Nash bargaining solution.

2 Model

2.1 Bayesian Bargaining Problem

We consider n-person bargaining problem with $n(\geq 3)$ private informed players. We denote the set of players by $N = \{1, 2, \ldots, n\}$ and a generic element by $i \in N$. As in Myerson (1983, 1984), a n-person bargaining problem Γ is characterized by the following form

$$\Gamma = (D, d^*, \{\Theta_i\}_{i\in N}, \{v_i\}_{i\in N}, p),$$

where D is the set of public decisions or feasible outcomes and $d^* \in D$ is the disagreement point. For each player, Θ_i is the set of possible types and a generic element of Θ_i is denoted by $\theta_i \in \Theta_i$. We also denote the set of type profile by $\Theta = \prod_{j\in N} \Theta_j$ and a element by $\theta \in \Theta$. We let Θ_{-i} denote the set of types of the players other than i and $\theta_{-i} \in \Theta_{-i} = \prod_{j\neq i} \Theta_j$. We assume that D and Θ are finite sets[1].

Probability measure p is a common prior on Θ and $p_i(\theta_i)$ denote the marginal probability distribution of player i's type θ_i. The conditional probability of type profile θ_{-i} for other than player i by player i with type θ_i is

$$p_i(\theta_{-i}|\theta_i) = \frac{p(\theta)}{\sum_{\theta'_{-i}\in\Theta_{-i}} p(\theta'_{-i}, \theta_i)}.$$

Each v_i is a payoff function from $D \times \mathbb{R} \times \Theta$ to the real number $\mathbb{R}$. We assume that a payoff function for each player i is quasi-linear in decision d

[1] It is well-known that no a priori finite bound on the number of types exists to model a game with incomplete information (Mertens and Zamir, 1985). Moreover, it should be assumed that the type space has the "beliefs-determine-preference" property, thus, there is a one-to-one correspondence between a player's preferences and a player's beliefs about other types. Heifetz and Neeman (2006) pointed out that information structures with this property are "small" among all conceivable common prior information structure.

and transfer t_i, i.e., $v_i(d,t_i,\theta) = u_i(d,\theta) + t_i$. A payoff for each player at disagreements is normalized to zero. That is, it is assumed that $v_i(d^*,0,\theta) = u_i(d^*,\theta) = 0$ for all $\theta \in \Theta$. The choice rule is represented by the combination of the decision rule $x : \Theta \to D$ and the transfer rule $t : \Theta \to \mathbb{R}^n$. We denote that $t(\cdot) = (t_1(\cdot), \dots, t_n(\cdot))$.

We impose the following conditions on the prior distribution p. The first one is introduced by Cremer and McLean (1988), so it is called "Cremer-McLean condition."

Definition 1. A probability distribution p satisfies *Cremer-McLean condition* if there are no $i \in N$, $\theta_i \in \Theta_i$ and $\lambda_i : \Theta_i \setminus \{\theta_i\} \to \mathbb{R}_+$ such that

$$p_i(\theta_{-i}|\theta_i) = \sum_{\theta'_i \in \Theta_i \setminus \{\theta_i\}} \lambda_i(\theta'_i) p_i(\theta_{-i}|\theta'_i), \quad \text{for all } \theta_{-i} \in \Theta_{-i}.$$

This condition means that vectors $p_i(\cdot|\theta_i)$ can not be expressed as a convex combination of all other vectors $p_i(\cdot|\theta'_i)$, $\theta'_i \neq \theta_i$ with weights $\lambda_i(\theta'_i)$.

We add identifiability condition by Kosenok and Severinov (2008).

Definition 2. A probability distribution p satisfies *identifiability condition* if for all $q \in \Delta(\Theta)$; $q \neq p$, there exists $i \in N$ and $\theta_i \in \Theta_i$ such that $q_i(\theta_i) > 0$ and for any collection of nonnegative coefficients $\{\lambda_{\theta'_i,\theta_i}\}$, we have

$$q_i(\theta_{-i}|\theta_i) \neq \sum_{\theta'_i \in \Theta_i} \lambda_{\theta'_i,\theta_i} p_i(\theta_{-i}|\theta'_i)$$

for at least one $\theta_{-i} \in \Theta_{-i}$.

Note that Cremer-McLean condition rules out the case that types of players are independent and each player's conditional beliefs are independent of his type. Thus, prior p have some correlation among types. Cremer-McLean condition holds generically when the number of types for each player is less than or equal to the number of types of all other players. Moreover, as shown in Kosenok and Severinov (2008), identifiability condition holds generically when there are at least three players ($n \geq 3$) and in case that $n = 3$, at least one of the players has at least three types. Cremer-McLean condition and identifiability condition will be used to ensure the existence of an ex-post efficient, acceptable (interim individually rational), ex-post budget balanced Bayesian incentive compatible mechanism which is offered by a proposer in a bargaining game. This result has been established by Kosenok and Severinov (2008).

2.2 Non-cooperative Bargaining Game

We present a noncooperative bargaining game to realize the generalized Nash bargaining solution as an equilibrium outcome. The key feature of our bargaining game is that a player who is selected as a proposer offers a mechanism to determine a public decision and transfers among players and, then, all other players accept or reject the mechanism. Thus, negotiations about a mechanism are conducted among players. A mechanism μ is formally defined as a combination of message spaces $S_1, \ldots, S_n$ for all players and an outcome function $g : \prod_{i \in N} S_i \to D \times \mathbb{R}^n$ mapping from the set of message profiles to the set of public decisions and transfers. We write $\mu = (S_1, \ldots, S_n, g) \in \mathfrak{M}$ and $g(\cdot) = (d(\cdot), t(\cdot))$, where $\mathfrak{M}$ is the set of feasible mechanisms. Without loss of generality, we can focus on deterministic outcome functions because the payoff function is quasi-linear.

We consider the following noncooperative bargaining game $G(\Gamma, w, \rho)$ with incomplete information.

Stage 0: A nature selects a type profile $\theta \in \Theta$. Each players learn his own types θ_i privately.

Stage 1: At the beginning of each round t, one player is selected as a proposer according to a probability distribution $w \in \Delta(N)$. In other words, player i is randomly chosen as a proposer with probability w_i among N.

Stage 2: The selected proposer i offers a mechanism $\mu^i \in \mathfrak{M}$.

Stage 3: All other players accept or reject the mechanism simultaneously.

Stage 4: If all players accept it, μ^i is implemented, i.e., each player sends a message $s_i \in S_i$ and then $g(s) \in D \times \mathbb{R}^n$ is determined. If some player reject it, the game continues to the next round with probability ρ and the game returns to stage 1. Otherwise, the negotiation breaks down with probability $1 - \rho$ and the game ends. In this case, all players get their disagreement payoff of 0.

The bargaining game is regarded as an extension of the informed principal game by Myerson (1983). If a proposer is predetermined and the game always ends when the proposal is rejected, i.e., $\rho = 0$, our game is the same game in Myerson (1983). If the game is in complete information, in other words, Θ is singleton, our game is reduced to a bargaining game to realize the asymmetric Nash bargaining solution as stationary equilibria when $\rho \to 1$ in Miyakawa (2006) and Okada (2007).

We adopt a sequential equilibrium by Kreps and Wilson (1982) with a stationary property as a solution concept. When the game is in complete information, the solution solution concept corresponds to a stationary subgame perfect equilibrium.

The bargaining game model can be represented by an infinite-length extensive form game. All nodes in an information set of player i in the extensive form at round t is determined by a sequence of past actions $z = (z_1, \ldots, z_{t-1}, z_t)$, where z_t, $t = 1, 2, \ldots$, denotes the sequence of actions in round t. It describe a history about who became a proposer, what a mechanism was offered by the proposer and which of an acceptance or a rejection responders selected. A posterior belief $\beta_i(\theta_i)$ about other players' types for player i with type θ_i at round t is represented by a probability measure on Θ_{-i}. The beliefs for all players is denoted by $\{\beta_i\}_{i \in N} = \{\{(\beta_i(\theta_i))_{\theta_i \in \Theta_i}\}_{i \in N}\}$, where $\beta_i(\theta_i) \in \Delta(\Theta_{-i})$. As a result, a state at round t is given by $(z, \{\beta_i\}_{i \in N})$. We denote a strategy for player i a sequence $\sigma_i = \{\sigma_i^t\}_{t=0}^{\infty}$, where σ_i^t is the tth round strategy. A strategy combination $\sigma = (\sigma_1, \dots, \sigma_n)$ determines the payoffs for all players.

Definition 3. A pair of a strategy combination and a belief system (σ, β) is called a *stationary sequential equilibrium* if σ is a sequential equilibrium and σ_i^t in each period t $(t = 1, 2, \dots)$ depends only on a belief system β_i and history z_t within round t.

In a stationary sequential equilibrium, every player's action does not depend on the whole history of actions. Moreover, any player's behavior in each bargaining round does not change even if agreements were rejected in past periods.

3 Mechanism and Nash Bargaining Solution

3.1 Bayesian incentive compatible mechanism

Before characterizing our solution of the bargaining game, we present some notions of mechanism and assumptions.

We assume that all mechanisms in the set of feasible mechanisms $\mathfrak{M}$ have a finite set of outcomes. It is sufficient to consider all feasible mechanisms in the case that the type space is finite. Moreover, we assume that $\mathfrak{M}$ is finite in order to apply the concept of sequential equilibrium to our bargaining appropriately[2].

Let μ^i denotes a mechanism which is proposed by player i. We call a mechanism in which the message space S_i for each player is the type space Θ_i *direct mechanism.* Thus, a direct mechanism is represented by $\mu^i = (\Theta_1 \times \cdots \times \Theta_n, g^i(\cdot))$, where $g^i : \Theta_1 \times \cdots \times \Theta_n \to D \times \mathbb{R}^n$. Moreover, $g^i(\cdot) = (d^i(\cdot), t^i(\cdot))$.

Under any direct mechanism μ^i, we can define the conditionally expected payoff for player j, given that his type is θ_j, if all players report their types truthfully as follows:

$$U_j(\mu^i|\theta_j) := \sum_{\theta_{-j} \in \Theta_{-j}} \left[u_j(d^i(\theta_{-j}, \theta_j), (\theta_{-j}, \theta_j)) + t^i_j(\theta_{-j}, \theta_j) \right] p_j(\theta_{-j}|\theta_j)$$

Moreover, the conditionally expected payoff for player j when he reports $\hat{\theta}_j \in \Theta_j$ and all other players report their types honestly is

$$U_j(\mu^i, \hat{\theta}_j|\theta_j) := \sum_{\theta_{-j} \in \Theta_{-j}} \left[u_j(d^i(\theta_{-j}, \hat{\theta}_j), (\theta_{-j}, \theta_j)) + t^i_j(\theta_{-j}, \hat{\theta}_j) \right] p_j(\theta_{-j}|\theta_j).$$

Let us introduce three notions about a direct mechanism.

Definition 4. A direct mechanism μ^i is *Bayesian incentive compatible* if for all $j \in N$ and for all $\hat{\theta} \in \Theta_j$,

$$U_j(\mu^i|\theta_j) \geq U_j(\mu^i, \hat{\theta}_j|\theta_j).$$

Definition 5. A direct mechanism $\mu^i(\cdot) = (d^i(\cdot), t^i(\cdot))$ is *budget balanced* if for $\forall \theta \in \Theta$,

$$\sum_{j \in N} t^i_j(\theta) = 0.$$

[2] The concept of sequential equilibrium by Kreps and Wilson (1982) is defined for finite games only.

Definition 6. A direct mechanism $\mu^i(\cdot) = (d^i(\cdot), t^i(\cdot))$ is *ex-post efficient* if for all $\theta \in \Theta$,

$$d^i(\theta) \in \arg\max_{d \in D} \sum_{i \in N} u_i(d, \theta).$$

3.2 The generalized Nash bargaining solution

Let us introduce some Nash bargaining solutions. Focus on the set of all incentive compatible mechanisms to define the conditionally expected payoff for each player. Given any mechanism μ, we let $U(\mu)$ denote the vector of all conditionally expected payoffs $U_i(\mu|\theta_i)$ for each type of each player. That is,

$$U(\mu) = ((U_i(\mu|\theta_i)_{\theta_i \in \Theta_i})_{i \in N}.$$

Harsanyi and Selten (1972) proposed the generalized Nash bargaining solution for games with incomplete information as a solution of

$$\max_{\mu \in \mathfrak{M}} \prod_{i \in N} \left(\prod_{\theta_i \in \Theta_i} U_i(\mu|\theta_i)^{p_i(\theta_i)} \right),$$

where $p_i(\theta_i) = \sum_{\theta_{-i} \in \Theta_{-i}} p(\theta_{-i}, \theta_i)$. Nash (1950) presented the symmetric Nash bargaining solution to a bargaining problem under complete information as a solution of

$$\max_{v \in V} \prod_{i \in N} (v_i),$$

where V is the set of feasible payoff allocations. Harsanyi and Selten's solution is one of natural generalizations of the Nash (1950) bargaining solution.

The symmetric Nash bargaining solution can be extended to the asymmetric Nash bargaining solution.

Definition 7. A payoff allocation v^* is called the *asymmetric Nash bargaining solution* with weight $w = (w_1, \ldots, w_n) \in \Delta(N)$, $w_i > 0$ if v^* is a solution of the maximization problem:

$$\max_{v \in V} \prod_{i \in N} (v_i)^{w_i}, \tag{1}$$

where V is the set of feasible payoff allocations.

Recently, some noncooperative bargaining game models has been presented to realize the asymmetric Nash bargaining solution as a stationary subgame perfect equilibrium (SSPE) outcome by Miyakawa (2006), Okada (2007), Laruelle and Valenciano (2008), Kultti and Vartiainen (2009) and Britz, Herings and Predtetchinski (2010).

We introduce another generalization of the asymmetric Nash bargaining solution with weight w under incomplete information, which is different from that by Harsanyi and Selten (1972).

Definition 8. The vector of all conditionally expected payoffs $U(\mu)$ is the *asymmetric Nash bargaining solution* with weight w to a n-person Bayesian bargaining problem Γ if $U(\mu)$ is a solution of the maximization problem:

$$\max_{\mu \in \mathfrak{M}} \prod_{i \in N} \left(\sum_{\theta_i \in \Theta_i} p_i(\theta_i) U_i(\mu|\theta_i) \right)^{w_i},$$

where $\mathfrak{M}$ is the set of all feasible mechanisms.

4 Characterization of Equilibria

4.1 Inscrutability Principle

Let us start to characterize a stationary sequential equilibrium in our bargaining game model.

First, there is no loss of generality in considering only direct incentive compatible mechanisms on the equilibrium path of $G(\Gamma, w, \rho)$ by the *revelation principle* in Myerson (1979). For any sequential equilibrium of any mechanism $\mu^i \in \mathfrak{M}$ which is proposed by player i, there exists an outcome-equivalent direct Bayesian incentive compatible mechanism.

Second, there is no loss of generality in assuming that all types of the proposer should offer the same mechanism on the equilibrium path, so that the proposer's actual choice of mechanism will convey no information about the type of the proposer to other players. This assumption is called the *inscrutability principle* by Myerson (1983).

As a result, we can assume that on the stationary sequential equilibrium path of $G(\Gamma, w, \rho)$, all types of the proposer i offer a direct mechanism $(x^i(\cdot), t^i(\cdot))$ which is incentive compatible under the beliefs $p_j(\theta_{-j}|\theta_j)$, $j \in N$. Moreover, the equilibrium beliefs of any player at stage 2 of $G(\Gamma, w, \rho)$ are equal to $p_j(\theta_{-j}|\theta_j)$ by the inscrutability principle. We possess a stationary sequential equilibrium such that the belief system $\{\beta_i\}_{i\in N}$ on the equilibrium path remains unchanged at the initial posterior belief system $\{(p_i(\theta_{-i}|\theta_i))_{\theta_i\in\Theta_i}\}_{i\in N}$ by the Bayes's rule.

4.2 Existence of Sequential Equilibrium

In this section, we will show that our noncooperative bargaining game $G(\Gamma, w, \rho)$ has a stationary sequential equilibrium in which each proposer offers a mechanism with the "full residual surplus extraction" property. Here, the "full residual surplus extraction" property means that a proposer gets all residual surplus after giving only their expected continuation payoffs of all responders if they reject the proposal. The proposal with this property plays a key role in our noncooperative bargaining game. For example, consider the ultimatum game. In this case, the continuation payoff of a responder is zero because the game ends if he rejects the proposal. So that, the proposer offers a proposal to extract all surplus of their cooperation and this proposal consists of a subgame perfect equilibrium in the bargaining game. Even in Rubinstein's alternating-offer bargaining or other bargaining game models, a player offers a proposal to assign responders only their continuation payoffs that they can get if they reject it in equilibrium. If the residual surplus after removing the responder's continuation payoff is negative, the proposer will select a delay of agreement.

We apply the same idea to the bargaining game with incomplete information. In the context of mechanism design, the full surplus extraction has been examined by Cremer and McLean (1988), McAfee and Reny (1992) and Kosenok and Severinov (2008). They identified a necessary and sufficient condition for the full surplus extraction by the uninformed principal through Bayesian incentive compatible, individually rational, ex-post efficient mechanisms without or with ex-post budget balancing. The necessary and sufficient condition is a pair of *Cremer-McLean condition* and *identifiability condition* for a prior probability distribution, which was defined in Definition 1 and 2. Severinov (2008) showed that there exists a ex-post efficient, interim individually rational, ex-post budget balanced, Bayesian incentive compatible mechanism with full surplus extraction property if a prior distribution about types satisfies Cremer-McLean and identifiability condition even in the informed principal setting. The individually rationality constraint implies the requirement for accepting the proposal by the principal. Then, designing a mechanism with "full surplus extraction" property is the same to the "full residual surplus extraction" proposal in the ultimatum bargaining game. We will consider a sequential equilibrium of $G(\Gamma, w, \rho)$ such that every player offers the "full residual surplus extraction" proposal when she becomes a proposer.

In order to explain a proposal with the "full residual surplus extraction" property formally, let us firstly define the expected social surplus from an ex-post efficient mechanism for type θ_i of player i by

$$W_i(\theta_i) = \sum_{\theta_{-i}\in\Theta_{-i}} \left[\max_{d\in D}\sum_{j\in N} u_j(d,(\theta_{-i},\theta_i))\right] p_i(\theta_{-i} \mid \theta_i).$$

In addition, we impose an assumption about the expected social surplus as follows:

Assumption 1. It is assumed that the expected social surplus from an ex-post efficient mechanism for every type of every player is positive; i.e., $W_i(\theta_i) > 0$ for all $\theta_i \in \Theta_i$ and $i \in N$.

Applying Corollary 1 of Theorem 1 in Kosenok and Severinov (2008), we have the following theorem:

Theorem 1. (Kosenok and Severinov (2008)) *Under Identifiability and Cremer-McLean condition, there exists an ex-post efficient, Bayesian incentive compatible, budget-balanced mechanism $\mu^{i*} = (d^{i*}(\cdot), t^{i*}(\cdot))$ in which the expected payoff of type θ_i of player i (proposer i) $V_i(\theta_i)$ is equal to*

$$V_i(\theta_i) = W_i(\theta_i) - \rho \sum_{\theta_{-i}\in\Theta_i} p(\theta_{-i}|\theta_i) \sum_{j\in N, j\neq i} v_j(\theta_j)$$

and the expected payoff for each type of player $j(\neq i)$, $V_j(\theta_j)$, is $\rho v_j(\theta_j)$, where it is assumed that $V_i(\theta_i) \geq 0$ for all $i \in N$ and $\theta_i \in \Theta_i$.

Proof. See Appendix. □

If $\rho v_j(\theta_j)$, $j \neq i$, are regarded as continuation payoffs for type θ_j of player j, the mechanism μ^{i*} corresponds to player i's proposal with the "full residual surplus extraction" property.

We examine whether the following strategies and beliefs can be supported as a part of sequential equilibrium of $G(\Gamma, w, \rho)$. First, all types of player i offer the mechanism μ^{i*} in stage 2 if he is selected as a proposer. Then, all types of all other players accept the proposal in stage 3. In stage 4, all players report their types truthfully. As a result, the mechanism μ^{i*} is implemented. Beliefs in stage 3 after μ^{i*} is offered and beliefs in stage 4 after all types accept μ^{i*} when they report their types are given by the initial conditionally beliefs $p_i(\cdot|\theta_i)$ for any type $\theta_i \in \Theta_i$ of agent $i \in N$.

In conclusion, we succeed in supporting the above strategies as a stationary sequential equilibrium of $G(\Gamma, w, \rho)$.

Theorem 2. *Suppose that probability distribution p satisfies Identifiability and Cremer-McLean conditions for all $i \in N$ and that $V_i(\theta_i)$ in Theorem 1 is nonnegative for all $i \in N$ and all $\theta_i \in \Theta_i$. Then, there exists a stationary sequential equilibrium of $G(\Gamma, w, \rho)$ in which all types of player i as a proposer offers the ex-post efficient mechanism μ^{i*}.*

Proof. See Appendix. □

Applying the same argument in Severinov (2008), we obtain that the mechanism μ^{i*} for each proposer i is a neutral optimum of Myerson (1983). See Severinov (2008) for detailed proof.

5 Relationships to the Nash Bargaining Solution

We clarify a relationship between the Nash bargaining solution and the expected payoff vector which is realized in the stationary sequential equilibrium which was provided in Theorem 2.

As seen in a proof of Theorem 2, every player i proposes the mechanism μ^{i*} and the proposal is accepted by all other players at the initial round. On the other hand, if he is not a proposer, he accepts mechanism μ^{j*} proposed by player j. Thus, type θ_i of player i gets a payoff of

$W_i(\theta_i) - \sum_{\theta_{-i}\in\Theta_{-i}} p_i(\theta_{-i}|\theta_i) \sum_{j\in N, j\neq i} \rho v_j(\theta_j)$ if he is a proposer and obtains $\rho v_i(\theta_i)$ if he is a responder. Recall that player i is selected as a proposer with probability w_i and becomes a responder with probability $1 - w_i$ in the noncooperative bargaining game. All types of player i offers the same mechanism by Inscrutability principle, so that every responders' belief is unchanged at the initial belief $p_j(\theta_{-j}|\theta_j)$. In addition, even if some player rejects the proposal, the beliefs of all players have no change because the acceptance and rejection are indifferent for every responder in the sequential equilibrium. By stationarity assumption, every player plays the same strategy in next round because the belief system is unchanged and their strategies does not depend on actions in the previous round. By the rule of the game, the expected equilibrium payoffs of type θ_i of player i should satisfy the following equation: For all $i \in N$ and for all $\theta_i \in \Theta_i$,

$$v_i(\theta_i) = w_i \left[W_i(\theta_i) - \sum_{\theta_{-i}\in\Theta_{-i}} p_i(\theta_{-i}|\theta_i) \sum_{j\in N, j\neq i} \rho v_j(\theta_j) \right] + (1 - w_i)\rho v_i(\theta_i). \tag{2}$$

This consists of $\sum_{i\in N} |\Theta_i|$ equations. We denote a solution of (2) by $v_i^\rho(\theta_i)$ for each ρ. If the above equation has a solution $v_i^\rho(\theta_i) \geq 0$, $i \in N$ for any ρ, we can show the existence of the corresponding stationary sequential equilibrium to the expected payoffs $v_i^\rho(\theta_i)$, $i \in N$ in the same way as in Theorem 2. Note that the above simultaneous equation does not necessarily have a nonnegative solution for any ρ. If the ex-post core is non-empty, in other words, if the ex-post efficient payoff allocation is consistent, then, there exists a nonnegative solution $v_i^\rho(\theta_i)$ for any ρ. However, under Assumption 1, there exists some $\bar{\rho}$ such that the simultaneous equation has a nonnegative solution for any $\rho \leq \bar{\rho}$.

We are now ready to state the main theorem. This theorem holds in the case that the ex-post core is nonempty.

Theorem 3. *Let* $((v_i^\rho(\theta_i))_{\theta_i\in\Theta_i})_{i\in N}$ *be the conditionally expected payoff vector in a stationary sequential equilibrium of the bargaining game* $G(\Gamma, w, \rho)$ *and let* $((v_i^*(\theta_i))_{\theta_i})_{i\in N}$ *be a limit point of* $((v_i^\rho(\theta_i))_{\theta_i\in\Theta_i})_{i\in N}$ *as* $\rho \to 1$. *Then,* $((v_i^*(\theta_i))_{\theta_i})_{i\in N}$ *is a solution of*

$$\max_{(v_i)_{i\in N}} \prod_{i\in N} \left(\sum_{\theta_i\in\Theta_i} p_i(\theta_i) U_i(\mu|\theta_i) \right)^{w_i} \tag{3}$$

$$\text{subject to } \sum_{i\in N} \sum_{\theta_i\in\Theta} p_i(\theta_i) U_i(\mu|\theta_i) = \sum_{\theta\in\Theta} \max_{d\in D} \sum_{i\in N} u_i(d,\theta) p(\theta),$$

$$v_i = \sum_{\theta_i\in\Theta_i} p_i(\theta_i) U_i(\mu|\theta_i), \quad \text{for } i \in N.$$

Proof. Let us define

$$U_i(\mu^{i*}|\theta_i) := W_i(\theta_i) - \sum_{\theta_{-i}\in\Theta_{-i}} p_i(\theta_{-i}|\theta_i) \sum_{j\in N, j\neq i} \rho v_j^\rho(\theta_j).$$

Rearranging (2), we have

$$U_i(\mu^{i*}|\theta_i) = \frac{1-\rho}{w_i} v_i^\rho(\theta_i) + \rho v_i^\rho(\theta_i), \quad \text{for } \theta_i \in \Theta_i, \ i \in N. \tag{4}$$

From $\lim_{\rho\to 1} v_i^\rho(\theta_i) = v_i^*(\theta_i)$ and (4), we have $\lim_{\rho\to 1} U_i(\mu^{i*}|\theta_i) = v_i^*(\theta_i)$ for all $\theta_i \in \Theta_i$ and for all $i \in N$. Moreover, $\lim_{\rho\to 1} \rho v_i^\rho(\theta_i) = v_i^*(\theta_i)$ trivially.

Because the mechanism μ^{i*} satisfies the full residual surplus property, we have

$$U_i(\mu^{i*}|\theta_i) + \sum_{\theta_{-i}\in\Theta_{-i}} p_i(\theta_{-i}|\theta_i) \sum_{j\in N, j\neq i} \rho v_j^\rho(\theta_j) = W_i(\theta_i), \quad \text{for all } \theta_i \in \Theta_i.$$

Multiplying the above equation by each $p(\theta_i)$ and adding them up together, we obtain

$$\sum_{\theta_i\in\Theta_i} p_i(\theta_i) U_i(\mu^{i*}|\theta_i) + \sum_{j\in N, j\neq i} \sum_{\theta_j} p_j(\theta_j) \rho v_j^\rho(\theta_j) = \sum_{\theta\in\Theta} \left[\max_{d\in D} \sum_{j\in N} u_j(d,\theta) \right] p(\theta). \tag{5}$$

For other $j \in N$, $j \neq i$, we also have

$$\sum_{\theta_j \in \Theta_j} p_j(\theta_j) U_j(\mu^{j*}|\theta_j) + \sum_{i \in N, i \neq j} \sum_{\theta_i} p_i(\theta_i) \rho v_i^{\rho}(\theta_i) = \sum_{\theta \in \Theta} \left[\max_{d \in D} \sum_{i \in N} u_i(d, \theta) \right] p(\theta). \tag{6}$$

Let us define the n-dimensional vector by

$$z^i(\rho) = (\sum_{\theta_i} p_1(\theta_1) \rho v_1^{\rho}(\theta_1), \ldots, \sum_{\theta_i \in \Theta_i} p_i(\theta_i) U_i(\mu^{i*}|\theta_i), \ldots, \sum_{\theta_n \in \Theta_n} p_n(\theta_n) \rho v_n^{\rho}(\theta_n)),$$

where $\sum_{\theta_i \in \Theta_i} p_i(\theta_i) U_i(\mu^{i*}|\theta_i)$ is the ith element of the vector. Using the vectors, we represent (5) and (6) by $H(z^i(\rho)) = 0$ and $H(z^j(\rho)) = 0$. Note that $\lim_{\rho \to 1} z^i(\rho) = \lim_{\rho \to 1} z^j(\rho)$ and $H(z^i(\rho)) - H(z^j(\rho)) = 0$. By Taylor's theorem, we have for some t, $0 < t < 1$,

$$\begin{aligned} & H(z^i(\rho)) - H(z^j(\rho)) \\ &= \frac{\sum_{\theta_i \in \Theta_i} p_i(\theta_i) v_i^{\rho}(\theta_i)}{w^i} \frac{\partial H}{\partial z_i}(t z^i(\rho) + (1-t) z^j(\rho)) \\ &\quad - \frac{\sum_{\theta_j \in \Theta_j} p_j(\theta_j) v_j^{\rho}(\theta_j)}{w^j} \frac{\partial H}{\partial z_j}(t z^i(\rho) + (1-t) z^j(\rho)) = 0. \end{aligned} \tag{7}$$

As a result, we obtain from (5) and (7) that as $\rho \to 1$,

$$\frac{\sum_{\theta_i \in \Theta_i} p_i(\theta_i) v_i^*(\theta_i)}{w_i} = \frac{\sum_{\theta_j \in \Theta_j} p_j(\theta_j) v_j^*(\theta_j)}{w_j}, \quad \text{for } i, j \in N, i \neq j, \tag{8}$$

$$\sum_{i \in N} \sum_{\theta_i \in \Theta} p_i(\theta_i) v_i^*(\theta_i) = \sum_{\theta \in \Theta} \max_{d \in D} \sum_{i \in N} u_i(d, \theta) p(\theta). \tag{9}$$

The vector $v_i^*(\theta_i)$ satisfies the Kuhn-Tucker condition of the maximization problem (3). □

Note that the equilibrium expected conditionally payoff for each type $v_i^*(\theta)$ as $\rho \to 1$ does not correspond to the asymmetric Nash bargaining solution with weight w to a Bayesian bargaining problem Γ in the precise sense. This is caused from a constraint that all types of player i proposes the same mechanism by Inscrutability principle. This means that it is difficult to characterize the equilibrium payoffs from the viewpoint of the Nash bargaining solution. But, by (8), we have

$$\frac{\sum_{\theta_i\in\Theta_i} p_i(\theta_i)v_i^*(\theta_i)}{w_i} = \frac{\sum_{\theta_j\in\Theta_j} p_j(\theta_j)v_j^*(\theta_j)}{w_j}, \quad \text{for } i,j\in N, i\neq j.$$

This says that the equilibrium payoff allocation is fair between players in the sense that their w-weighted ex-ante expected payoffs should be equal.

As you know, the maximization problem (3) does not determine each $U_i(\mu|\theta_i)$, but $v_i = \sum_{\theta_i\in\Theta_i} p_i(\theta_i)U_i(\mu|\theta_i)$. On the other hand, the stationary sequential equilibrium considered in Theorem 2 uniquely determine the conditionally expected payoffs of each player in the bargaining game $G(\Gamma, w, \rho)$. Because the equilibrium payoff vector $v_i^*(\theta_i)$ satisfies the "full residual surplus extraction" property and $\lim_{\rho\to 1} U_i(\mu^{i*}|\theta_i) = v_i^*(\theta_i)$, we obtain that as $\rho \to 1$, for all $\theta\in\Theta$,

$$\sum_{i\in N} U_i(\mu|\theta_i) = \max_{d\in D}\sum_{i\in N} u_i(d,\theta). \tag{10}$$

This is equivalent to the ex-post efficiency of the payoff allocation. Furthermore, this condition also means that the equilibrium payoff allocation belongs to the *ex-post core* because the sub-coalitions except the grand coalition are not allowed in the bargaining problem[3]. Actually, the ex-post core for our Bayesian bargaining problem might be empty. The above characterization cannot be applied for the case that the ex-post core is empty. By (10), the equilibrium payoffs of each type θ_i of player i as $\rho\to 1$ is given by a solution $((U_i(\mu|\theta_i))_{\theta_i\in\Theta_i})_{i\in N}$ of, for $\theta_i, \theta_i'\in\Theta_i$,

$$\max_{d\in D}\sum_{j\in N} u_j(d,(\theta_{-i},\theta_i)) - U_i(\mu|\theta_i) = \max_{d\in D}\sum_{j\in N} u_j(d,(\theta_{-i},\theta_i')) - U_i(\mu|\theta_i'),$$

[3]The relationship between core and the equilibrium payoff allocation of the bargaining game should be considered in a setting with coalition formations. The first attempt in the game with incomplete information was made by Okada (2009).

for each $\theta \in \Theta$ and each $i \in N$.

Complete information and $\rho \to 1$: Consider a case in which the bargaining problem is a complete information game, thus, the type set is singleton; $\Theta = \{\theta\}$ and $\rho \to 1$. In this case, (8) and (9) is rewritten by

$$\frac{v_i^*(\theta_i)}{w_i} = \frac{v_j^*(\theta_j)}{w_j}, \quad \text{for all} \quad i, j \in N, i \neq j,$$
$$\sum_{i \in N} v_i^*(\theta_i) = \max_{d \in D} \sum_{i \in N} u_i(d, \theta).$$

This is the Kuhn-Tucker condition of the maximization problem (1). Therefore, we get the following Corollary of Theorem 3.

Corollary 1. *Fix $\Theta = \{\theta\}$. Let $v^*(\rho) = (v_i^\rho(\theta_i))$ be a stationary subgame perfect equilibrium payoff vector in the noncooperative bargaining game with complete information $G(w, \rho)$ and let $\lim_{\rho \to 1} v(\rho) = v^* = (v_1^*(\theta_1), \ldots, v_n^*(\theta_n))$. Then, v^* is the asymmetric Nash bargaining solution with weight w.*

This case has been considered by Miyakawa (2006) and Okada (2007).

Incomplete information and $\rho = 0$: When $\rho = 0$, it is just one time that a proposer make an offer in the bargaining game. The game ends with probability one after the proposal irrespective of the acceptance or rejection of it. In this case, players play the same game as the informed principal game in Myerson (1983) after one player is selected as a proposer. Applying the same procedure as in Theorem 2, we can show that this bargaining game possesses a sequential equilibrium in which type θ_i of player i as a proposer offers an ex-post efficient, Bayesian incentive compatible, budget-balanced mechanism such that the expected payoff of any type θ_i of player i is equal to $W_i(\theta_i)$, while that for every type of any other player is zero. This proposal is accepted and, then, is implemented.

The vector of conditionally expected payoffs in the sequential equilibrium is given by

$$((v_i(\theta_i))_{\theta_i\in\Theta_i})_{i\in N} = ((w_i W_i(\theta_i))_{\theta_i\in\Theta_i})_{i\in N}\,.$$

If weight $w = (w_1, \ldots, w_n)$ is a variable on $\Delta(N)$, this is a natural extension of the *contract curve* in de Clippel and Minelli (2004) to n-person bargaining game with unverifiable information. In the case of $w_i = 1/n$, i.e., each player becomes a proposer and extracts the full social surplus, the above equilibrium expected payoff allocation satisfies *Random dictatorship axiom* in Myerson (1984).

Appendix

Proof of Theorem 1: Kosenok and Severinov (2008) have established the following surprising result as a Corollary of their main Theorem (Theorem 1):

Corollary 1. (Kosenok and Severinov) *Consider any ex-ante socially rational decision rule $d(\theta)$, and suppose that the prior p is identifiable and Cremer-McLean condition holds for all agents. Then for any collection of nonnegative constants $v_j(\theta_j)$ satisfying:*

$$\sum_{j\in N}\sum_{\theta_i\in\Theta_i} v_j(\theta_i)p_j(\theta_j) = \sum_{j\in N}\sum_{\theta\in\Theta} u_j(d(\theta),\theta)p(\theta), \tag{11}$$

there exists an IC (Bayesian incentive compatible), BB (ex-post budget balanced), and IR (individually rational) Bayesian mechanism $(d(\theta), t(\theta))$ such that the expected surplus of type θ_i of agent i in this mechanism is equal to $v_i(\theta_i)$.

We check that the expected payoff vector $((V_i(\theta_i))_{\theta_i\in\Theta_i})_{j\in N}$ in our Theorem 1 satisfies the above condition (11). We have

$$
\begin{aligned}
&\sum_{i\in N}\sum_{\theta_i\in\Theta} V_i(\theta_i)p_i(\theta_i)\\
&=\sum_{\theta_i\in\Theta_i}\left[W_i(\theta_i)-\rho\sum_{\theta_{-i}\in\Theta_{-i}} p_i(\theta_{-i}|\theta_i)\sum_{j\in N,j\neq i} v_j(\theta_j)\right]p_i(\theta_i)+\sum_{j\in N,j\neq i}\rho v_j(\theta_j)p_j(\theta_j)\\
&=\sum_{\theta_i\in\Theta_i}\sum_{\theta_{-i}\in\Theta_{-i}}\left[\max_{d\in D}\sum_{j\in N} u_i(d,(\theta_{-i},\theta_i))p(\theta_{-i},\theta_i)\right]-\rho\sum_{\theta_i\in\Theta_i}\sum_{\theta_{-i}\in\Theta_{-i}} p(\theta_{-i},\theta_i)\sum_{j\in N,j\neq i} v_j(\theta_j)\\
&\qquad\qquad+\rho\sum_{j\in N,j\neq i} v_j(\theta_j)p_j(\theta_j)\\
&=\sum_{\theta\in\Theta}\left[\max_{d\in D}\sum_{j\in N} u_j(d,(\theta))p(\theta)\right]=\sum_{j\in N}\sum_{\theta\in\Theta} u_j(d^*(\theta),(\theta))p(\theta),
\end{aligned}
$$

where $d^*(\theta)\in\arg\max_{d\in D}\sum_{j\in N}u_j(d,\theta)$. Thus, $V_j(\theta_j)$ satisfies condition (11). Therefore, it leads to Theorem 1.

Proof of Theorem 2: In our candidate equilibrium strategies, all types of each player i offer the same mechanism to give each type of other player $j\in N$, $j\neq i$, the expected payoff $\rho v_j(\theta_j)$ in the first round. Then, the belief for every type of the responder j is unchanged. He is indifferent between the choices of acceptance and rejection because he gets the same payoff of $\rho v_j(\theta_j)$. Thus, the acceptance of the proposal in stage 3 is sequentially rational for every type of any player $j\in N$, $j\neq i$. Furthermore, the acceptance decision by the responders does not transmit any information about players' types. Because the mechanism μ^{i*} is Bayesian incentive compatible under the initial belief $p_j(\theta_{-j}|\theta_j)$ for all $j\in N$, it is sequentially rational for all players to report their true type in stage 4. Therefore, it is sufficient to show that there exists a profile of sequentially rational strategies and consistent beliefs off the equilibrium path such that no type of player i has an incentive to deviate from μ^{i*} to other some mechanism μ^i. We have to assume that the vector $(v_i(\theta_i))_{i\in N}$ satisfies the condition

$$W_i(\theta_i)-\rho\sum_{\theta_{-i}\in\Theta_{-i}} p_i(\theta_{-i}|\theta_i)\sum_{j\in N,j\neq i} v_j(\theta_j)\geq\rho v_i(\theta_i)$$

for the proposer i to offer an acceptable proposal. However, the equilibrium payoff vector would satisfy the condition for sufficiently small ρ under Assumption 1. I will check it later.

Fix an arbitrary mechanism $\mu=(S_1,\ldots,S_n;x^\mu(\cdot),t^\mu(\cdot))$, where S_i is a message space for player i and consider a finite game $G^i(\mu)$ as follows. Given $\theta=(\theta_j)_{j\in N}\in\Theta$. Player i

has been already chosen as a proposer. In the first stage of $G^i(\mu)$, which is corresponding to stage 2 of the original game $G(\Gamma, w, \rho)$, the proposer i has two choices. We also call this stage stage 2 even if it is the first stage. If she chooses the exit of the game and her type is θ_i, she gets $W_i(\theta_i) - \sum_{\theta_{-i}\in\Theta_{-i}} p_i(\theta_{-i}|\theta_i) \sum_{j\in N, j\neq i} \rho v_j(\theta_j)$ immediately. Otherwise, she offers the mechanism μ. Let the payoff when she chooses the exit option be denoted by $U_i(\mu^{i*}|\theta_i)$. In the next stage (stage 3), all other players accept or reject the proposal μ. In the last stage (stage 4), the mechanism μ is implemented if all players accept it. If some player rejects the proposal, each type θ_j including player i get the payoff of $\rho v_j(\theta_j)$.

Since the game $G^i(\mu)$ has only finite periods and the set of feasible mechanism $\mathfrak{M}$ is assumed to be finite, there exists a sequential equilibrium (τ, γ, β) of $G^i(\mu)$, as shown in Kreps and Wilson (1982). We let (τ, γ, β) denote the probability $\tau_i(\mu|\theta_i)$ with which type θ_i of the proposer i offers mechanism μ, the probability $\tau_j(\mu|\theta_j)$ with which type θ_j accept μ, the probability measure $\gamma_i(\cdot|\theta_i, \mu)$ on S_i^μ representing the strategy for type θ_i under mechanism μ, the belief $\beta_j^R(\cdot|\theta_j, \mu)$ of type θ_j about θ_{-j} at stage 3 after mu is offered by player i and the belief $\beta_j^I(\cdot|\theta_j, \mu)$ about other types at implementation stage 4.

Let $s = (s_1, \ldots, s_n)$ be the profile of messages in implementation of the mechanism. We will show that the probability with which the proposer i offers μ is zero; $\tau_i(\mu|\theta_i) = 0$ in the sequential equilibrium. The expected payoff for type θ_i of player i conditional on μ being offered in equilibrium (τ, γ, β) is given by

$$U_i(\tau, \gamma, \beta|\mu, \theta_i) = \sum_{\theta_{-i}\in\Theta_{-i}} \beta_i^R(\theta_i|\theta_i, \mu) \left[\sum_{s\in S} (u_i(x^\mu(s), \theta) + t_i^\mu(s)) \prod_{j\in N} \gamma_j(s_j|\theta_j, \mu) \right] \prod_{j\in N\setminus i} \tau_j(\mu|\theta_j).$$

From the sequential rationality of player i's proposal, it follows that

$$\tau_i(\mu|\theta_i) = \begin{cases} 1 & \text{if} \quad U_i(\tau, \gamma, \beta|\mu, \theta_i) > U_i(\mu^{i*}|\theta_i), \\ 0 & \text{if} \quad U_i(\tau, \gamma, \beta|\mu, \theta_i) < U_i(\mu^{i*}|\theta_i). \end{cases} \tag{12}$$

We will show that $U_i(\mu^{i*}|\theta_i) \geq U_i(\tau, \gamma, \beta|\mu, \theta_i)$ for all $\theta_i \in \Theta_i$. The proof is given by contradiction. Suppose that there exists $\hat{\theta}_i \in \Theta_i$ such that $U_i(\hat{\theta}_i|\mu, \tau, \gamma, \beta) > U_i(\mu^{i*}|\theta_i)$. The sequential rationality implies $\tau_i(\mu|\hat{\theta}_i) = 1$. By the Bayes rule, the beliefs of type θ_j at stage 3 is

$$\beta_j^R(\theta_{-j}|\theta_j, \mu) = \frac{\tau_i(\mu|\theta_i) p(\theta_{-j}, \theta_j)}{\sum_{\theta_i'\in\Theta_i} \tau_i(\mu|\theta_i') p_{ij}(\theta_i', \theta_j)}, \quad \text{for} \quad j \neq i \in N,$$

where $p_{ij}(\theta_i, \theta_j)$ is the marginal probability distribution of type θ_i and θ_j.

We have the following inequalities:

$$
\begin{aligned}
&\sum_{j\in N, j\neq i}\sum_{\theta_j\in\theta_j}\rho v_j(\theta_j)\frac{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_{ij}(\theta'_i,\theta_j)}{\sum_{\theta'\in\Theta_i}\tau_i(\mu|\theta'_i)p_i(\theta'_i)}\\
&+\sum_{\theta_i\in\Theta_i}U_i(\mu^{i*}|\theta_i)\frac{\tau_i(\mu|\theta_i)p(\theta_i)}{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_i(\theta'_i)}\\
&>\sum_{j\in N, j\neq i}\sum_{\theta_j\in\theta_j}U_j(\tau,\gamma,\beta|\mu,\theta_j)\frac{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_{ij}(\theta'_i,\theta_j)}{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_i(\theta'_i)}\\
&+\sum_{\theta_i\Theta_i}U_i(\tau,\gamma,\beta|\mu,\theta_i)\frac{\tau_i(\mu|\theta_i)p_i(\theta_i)}{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_i(\theta'_i)}\\
&>\sum_{j\in N, j\neq i}\sum_{\theta_j\in\theta_j}U_j(\tau,\gamma,\beta|\mu,\theta_j)\frac{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_{ij}(\theta'_i,\theta_j)}{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_i(\theta'_i)}\\
&+\sum_{\theta_i\in\Theta_i}U_i(\mu^{i*}|\theta_i)\frac{\tau_i(\mu|\theta_i)p_i(\theta_i)}{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_i(\theta'_i)}.
\end{aligned}
$$

The first inequality is satisfied because μ^{i*} is the ex-post efficient, budget-balanced mechanism with the full residual extraction property for each $\theta_i \in \Theta$, and the second inequality is derived from (12). Then, there exists some $\theta_j \in \Theta_j$, $j \neq i$, such that $U_i(\tau,\gamma,\beta|\mu,\theta_j) < \rho v_j(\theta_j)$. Moreover, it should be satisfied that $\tau_j(\mu|\theta_j) > 0$. Player j can get $\rho v_j(\theta_j) > 0$ by rejecting μ. This implies that it is not sequentially rational for type θ_j to accept μ. Thus, $\tau_j(\mu|\theta_j) = 0$. This is a contradiction. We can conclude that $U_i(\tau,\gamma,\beta|\mu,\theta_i) \leq U_i(\mu^{i*}|\theta_i)$ for all $\theta_i \in \Theta_i$.

Then, if $U_i(\tau,\gamma,\beta|\mu,\theta_i) > U_i(\mu^{i*}|\theta_i)$, it implies that $\tau_i(\mu|\theta_i) = 0$ for $\theta_i \in \Theta_i$ by sequential rationality. Even if $U_i(\tau,\gamma,\beta|\mu,\theta_i) = U_i(\mu^{i*}|\theta_i)$, we can construct a new sequential equilibrium with $\tilde{\tau}_i(\mu|\theta_i) = 0$. Therefore, we can say that $\tau_i(\mu|\theta_i) = 0$ for any $\theta_i \in \Theta_i$. This means that every type $\theta_i \in \Theta_i$ select the exit option and gets $U_i(\mu^{i*}|\theta_i)$ with probability one in game $G^i(\mu)$. We can construct the corresponding $(\tau_j(\mu|\theta_j),\gamma,\beta)$ satisfying the consistency of beliefs and the sequential rationality as a sequential equilibrium of $G^i(\mu)$ by considering completely mixed strategy about the proposal of θ_i.

Consider the following strategies and beliefs of game $G(\Gamma, w, \rho)$. In every round of the bargaining game, all types of player i offer the mechanism μ^{i*} with probability one. All types of player i accept μ^{j*}, $j \neq i$ and report their types truthfully with probability one. The beliefs in stage 3 and 4 of every round are given by $p_j(\theta_{-j}|\theta_j)$ after μ^{i*} is proposed. If player i proposes mechanism $\mu \in \mathfrak{M}$ such that $\mu \neq \mu^{i*}$, each player plays (τ,γ,β) which was considered in $G^i(\mu)$ above. The finite game $G^i(\mu)$ is "embedded" in the original bargaining game $G(\Gamma, w, \rho)$. It is not difficult to check this combination of strategies and beliefs is a sequential equilibrium of $G(\Gamma, w, \rho)$. The key step of the proof is that the expected payoff for type θ_j of player j after rejecting any mechanism is regarded as $\rho v_j(\theta_j)$ by the stationarity assumption of the sequential equilibrium.

By the stationarity and the rule of the game, the expected payoff vector in the above equilibrium satisfies, for all $i \in N$ and for all $\theta_i \in \Theta_i$,

$$v_i(\theta_i) = w_i \left[W_i(\theta_i) - \sum_{\theta_{-i} \in \Theta_{-i}} p_i(\theta_{-i}|\theta_i) \sum_{j \in N, j \neq i} \rho v_j(\theta_j) \right] + (1 - w_i)\rho v_i(\theta_i). \qquad (13)$$

If $v_i(\theta_i)$, for all θ_i, is nonnegative, it holds that

$$W_i(\theta_i) - \rho \sum_{\theta_{-i} \in \Theta_{-i}} p(\theta_{-i}|\theta_i) \sum_{j \in N, j \neq i} v_j(\theta_j) \geq \rho v_i(\theta_i).$$

This implies that no player makes an unacceptable proposal. Thus, no delay of agreements occurs in the stationary sequential equilibrium. Under Assumption 1, the equation (13) possesses a nonnegative solution for sufficiently small ρ.

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