Contractually Stable Coalition Structures with Externalities [†]

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Abstract

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1 Introduction

The organization of individual agents into groups has an important role in the determination of the outcome of many social and economic interactions. In many interesting social and economic situations, group formation creates either negative externalities or positive externalities for nonmembers. Examples of negative externalities are research coalitions and customs unions. Examples of positive externalities include output cartels and public goods coalitions. To predict the coalition structures that are going to emerge at equilibrium we use the concept of contractual stability (Drèze and Greenberg, Econometrica 1980) which requires that any change made to the coalition structure needs the consent of both the deviating players and their original coalition partners. The word "contractual" is used to reflect the notion that coalitions are contracts binding all members and subject to revision only with consent of coalition partners. One example are rules governing entry and exit in labor cooperatives. A new partner will enter the cooperative only if (i) he wishes to come in; (ii) his new partners wish to accept him; and (iii) he obtains from his former partners permission to withdraw (only if he was before member of another cooperative). Two different decision rules for consent are analyzed: simple majority or unanimity. We investigate whether requiring the consent of group members may help to reconcile stability and efficiency.

2 Association of firms

Cooperation among competing firms is increasingly common on oligopolistic markets. More and more often, competing firms agree to share information, build common facilities or launch common research programmes in order to decrease their production costs. Bloch (1995) proposed a simple model to analyze the formation of associations of firms where the benefits from cooperation increase linearly in the size of the association.

Consider a market with n symmetric firms indexed by $i = 1, 2, ..., n$, where $n \geq 4$. The interactions among firms is modelled as a two-stage game. In stage one, associations are formed. In stage two, given the association structure, firms compete on the market.

Once associations are formed, firms behave as competitors on the market and maximize individual profits. Demand is linear and given by

$$p = \alpha - \sum_{i=1}^{n} q_i.$$

The parameter α measures the absolute size of the market. Firms have a constant marginal cost of production, which is decreasing in the size of the association they belong to. The cost of a firm i in an association S of size s is thus given by $c_i = \lambda - \mu\, s$. The parameter values α, λ and μ are chosen in such a way that for any coalition structure, all firms are active in a Cournot equilibrium. Once associations are formed on the market, firms select non-cooperatively the quantities they offer on the market. A coalition structure $P = \{S_1, S_2, ..., S_m\}$ is a partition of the player set $N = \{1, 2, ..., n\}$, $S_i \cap S_j = \emptyset$ for $i \neq j$ and $\bigcup_{i=1}^{m} S_i = N$. Let p be the cardinality of P. A coalition structure P is symmetric if and only if $s_i = s_j$ for all $S_i, S_j \in P$. Let $P^* = \{N\}$ be the grand coalition.

For a coalition structure $P = \{S_1, S_2, ..., S_k\}$, it is easy to see that there exists a unique Cournot equilibrium on the market, and that each firm's profit $\Pi_i(P)$ is a monotonically increasing function of the following valuation:

$$V_i(P) = \alpha - \lambda + \mu\ (n+1)\, s(i) - \mu \sum_{j=1}^{k} (s_j)^2\,,$$

where $s(i)$ denotes the size of the association firm i belongs to. In fact, $V_i(P) = (n+1)\sqrt{\Pi_i(P)}$.

3 Contractual stability

How does the coalition formation proceed? A coalition structure P is obtainable from P' via S, $S \subseteq N$, if (i) $\{S_i \setminus (S_i \cap S) \mid S_i \in P\} = \{S'_i \in P' \mid S'_i \subseteq N \setminus S\}$ and (ii) $\exists \{S'_1, ..., S'_l\} \subseteq P'$ such that $\bigcup_{j=1}^{l} S'_j = S$. Condition (i) simply means that no simultaneous deviations are possible. If the players in S deviate leaving their coalition(s) in P, the non-deviating players do not move. Nevertheless, once S has moved, the players not in S can react to the deviation of S. Condition (ii) simply allows the deviating players in S to form one or several coalitions in the new coalition structure P'. Non-deviating players do not belong to those new coalitions.

Definition 1. A coalition structure P is contractually stable under the unanimity decision rule if for any $S \subseteq N$, P' obtainable from P via S and $i \in S$ such that

$V_i(P') > V_i(P)$, there exists $k \in S(j)$ with $S(j) \in P$ and $j \in S$ such that $V_k(P') \leq V_k(P)$.

Under the unanimity decision rule, the move from a coalition structure P to any obtainable coalition structure P' needs the consent of every deviating player and the consent of every member of the initial coalitions of the deviating players. Then, a coalition structure is contractually stable under the unanimity decision rule if any deviating player or any member of the former coalitions of the deviating players is not better off from the deviation to any obtainable coalition structure P'.

Definition 2. A coalitional structure P is contractually stable under the simple majority decision rule if for any $S \subseteq N$, P' obtainable from P via S and $i \in S$ such that $V_i(P') > V_i(P)$, there exists

(*i*) $l \in S$ such that $V_l(P') \leq V_l(P)$, or

(*ii*) $\widehat{S} \subseteq S(j)$ with $S(j) \in P$ and $j \in S$ such that $V_k(P') \leq V_k(P)$ for all $k \in \widehat{S}$ and $\widehat{s} \geq s(j)/2$.

Under the simple majority decision rule, the move from a coalitional structure P to any obtainable coalitional structure P' needs the consent of every deviating player and the consent of more than half members of each initial coalition of the deviating players. Then, a coalitional structure P is contractually stable under the simple majority decision rule if any deviating player or half members of some former coalition of the deviating players are not better off from the deviation to any obtainable coalitional structure P'.

Obviously, a coalitional structure that is contractually stable under the simple majority decision rule is contractually stable under the unanimity decision rule. In fact each decision rule requires the consent of coalitional partners above some quota for a deviation not to be blocked. For instance, the simple majority decision rule reverts to a quota $q = (s+2)/2 - \text{mod}[s, 2]$ while the unanimity decision rule reverts to a quota $q = s$. The relationship between contractual stability under any decision rule embodied by a quota is obvious: a quota $q' < q$ refines stability. That is, the set of contractually stable coalitional structures under q' is (weakly) included in the set of contractually stable coalitional structures under q. Indeed, the probability to block a deviation is greater the higher the quota q. When the quota approaches zero ($q \rightarrow 0$), coalitional membership has no matter in terms of consent and the concept of contractual stability reverts to Hart and Kurz (1983) notion of Δ-stability. A coalition structure P is Δ-stable if for any $S \subseteq N$, P' obtainable from P via S and $i \in S$ such that $V_i(P') > V_i(P)$, there is $j \in S$ such that $V_j(P') \leq V_j(P)$.

The idea of contractual stability is that leaving a coalition may request the consent of former coalition partners. As in Drèze and Greenberg (1980) the word "contractual" is used to reflect the notion that coalitions are contracts binding all members and subject to revision only with consent of coalitional partners. One example mentioned by Drèze and Greenberg (1980) are rules governing entry and exit in labor cooperatives. A new partner will enter the cooperative only if (i) he wishes to come in; (ii) his new partners wish to accept him; and (iii) he obtains from his former partners permission to withdraw (only if he was before member of another cooperative).

Lemma 1. *$V_i(P \setminus \{S_1, S_2\} \cup \{S_1 \cup S_2\}) > V_i(P)$ for all $i \in S_1 \cup S_2$ if $s_1 < (n+1)/2$ and $s_2 < (n+1)/2$.*

Proof. Consider the incentive of members of a coalition S_1 to merge with a coalition S_2 when no other occurs in the coalition structure. This is given by $V_j(P \setminus \{S_1, S_2\} \cup \{S_1 \cup S_2\}) - V_j(P)$, where $P \setminus \{S_1, S_2\} \cup \{S_1 \cup S_2\}$ is the coalition structure formed by merging S_1 and S_2 in P and j is any member of S_1.

$$
\begin{aligned}
V_j(P \setminus \{S_1, S_2\} \cup \{S_1 \cup S_2\}) - V_j(P) &= [\mu(s_1 + s_2) - \mu s_1](n+1) \\
&\quad -\mu(s_1 + s_2)^2 - \mu(s_1)^2 - \mu(s_2)^2 \\
&= \mu s_2(n + 1 - 2s_1).
\end{aligned}
$$

So, members of S_1 have an incentive to merge with S_2 as long as the size of S_1, s_1, is smaller than $(n+1)/2$, regardless of the size of S_2 and of the coalitions formed by other firms on the market. □

This lemma tells us that any two associations have always incentives to merge if both are smaller than $(n+1)/2$.

Lemma 2. *Any coalition structure P such that $p > 2$ is never contractually stable whatever the decision rule for consent.*

Proof. Take any P such that $p > 2$. Then, there exists at least two coalitions $S_1 \in P$ and $S_2 \in P$ such that $s_1 < (n+1)/2$ and $s_2 < (n+1)/2$. >From Lemma 1 we have that all members of S_1 and S_2 have incentives to merge; and this merger does not request the consent of any other player than those involved with the merger. □

Lemma 3. *The symmetric coalition structure P such that $p = 2$ is never contractually stable whatever the decision rule for consent.*

Proof. Take the symmetric coalition structure $P = \{S_1, S_2\}$ where $s_1 = s_2 = n/2$. Thus, $s_1 = s_2 < (n+1)/2$, and from Lemma 1, we have that all members of S_1 and S_2 have incentives to merge; and this merger does not request the consent of any other player than those involved with the merger. □

3.1 Simple majority decision rule

Lemma 4. *Any asymmetric coalition structure P such that $p = 2$ is never contractually stable under the simple majority decision rule.*

Proof. Take any asymmetric coalition structure containing two coalitions, $P = \{S, N \setminus S\}$, where $n - 1 \geq s > n/2$. For all $i \in S$, we have

$$\begin{aligned} V_i(\{S, N \setminus S\}) &= \alpha - \lambda + \mu \left[(n+1)s - (s)^2 - (n-s)^2\right] \\ &= \alpha - \lambda + \mu \left[(3n+1)s - 2(s)^2 - n^2\right]. \end{aligned}$$

For all $j \in N \setminus S$, we have

$$\begin{aligned} V_j(\{S, N \setminus S\}) &= \alpha - \lambda + \mu \left[(n+1)(n-s) - (s)^2 - (n-s)^2\right] \\ &= \alpha - \lambda + \mu \left[(n-1)s - 2(s)^2 + n\right]. \end{aligned}$$

Since $s > n/2$, we have $V_i(\{S, N \setminus S\}) > V_j(\{S, N \setminus S\})$, $i \in S$ and $j \in N \setminus S$.

[**Case 1.**] Suppose that $n/2 < s \leq (2n-1)/3$.

We will show that there always exists a subcoalition T of $N \setminus S$ ($T \subset N \setminus S$) who has incentives to leave $N \setminus S$ to join the coalition S. That is, we consider the deviation from $P = \{S, N \setminus S\}$ to $P' = \{S \cup T, (N \setminus S) \setminus T\}$. We will show that all members of S and T prefer P' to P and that this deviation is not blocked by former partners of T in $N \setminus S$. For all $i \in S \cup T$, we have

$$\begin{aligned} &V_i(\{S \cup T, (N \setminus S) \setminus T\}) \\ &= \alpha - \lambda + \mu \left[(n+1)s + (n+1)t - (s+t)^2 - (n-s-t)^2\right] \\ &= \alpha - \lambda + \mu \left[(n+1)s - (s)^2 - (n-s)^2 + t\,(3n+1-4s-2t)\right]. \end{aligned}$$

For all $j \in (N \setminus S) \setminus T$, we have

$$
\begin{aligned}
&V_j(\{S \cup T, (N \setminus S) \setminus T\}) \\
&= \alpha - \lambda + \mu \left[(n+1)(n-s) - (n+1)t - (s+t)^2 - (n-s-t)^2\right] \\
&= \alpha - \lambda + \mu \left[(n+1)(n-s) - (s)^2 - (n-s)^2 + t\,(n-1-4s-2t)\right].
\end{aligned}
$$

(a) Members of coalition S will obtain a higher payoff when t firms join their coalition only if

$$t < \frac{3n+1-4s}{2} = \frac{n-s}{2} + n - \frac{3s-1}{2}.$$

Notice that $(3n+1-4s)/2 \geq 1$ if and only if $s \leq (3n-1)/4$.
(b) Members of T have incentives to join S if and only if

$$t(3n+1-4s-2t) > (n+1)(n-2s).$$

Since $s > n/2$, the right-hand side of the above expression is negative, and therefore, a sufficient condition is $3n+1-4s-2t > 0$, which is the same condition for having members in S obtaining a higher payoff when accepting t new members.
(c) Firms belonging to $(N \setminus S) \setminus T$ are worse off in P' than in P. Thus, members of T need to have a majority in $N \setminus S$ in order to be allowed to leave coalition $N \setminus S$. That is, $t > (n-s)/2$.
$>$From **(a)**, **(b)**, **(c)**, the deviation from P to P' by $S \cup T$ will not be blocked if and only if

$$\frac{n-s}{2} < t \leq \frac{n-s}{2} + n - \frac{3s-1}{2}.$$

This interval is well defined only if $s < (2n-1)/3$.[1]

[Case 2.] Suppose that $(2n+1)/3 < s$.

We will show that there always exists a subcoalition S' of S ($S' \subset S$) who has incentives to leave alone t former partners. That is, we consider the deviation from $P = \{S, N \setminus S\}$ to $P'' = \{S', T, N \setminus S\}$ where $S' = S \setminus T$. Two conditions are required so that this deviation is not blocked.
(a) Members of S who deviate need to have the majority within S; that is, $t < s/2$.

[1] For $s > (3n-1)/4$, there is no positive t that will make members of S accept the deviation from P to P'. However, when $(2n-1)/3 < s \leq (3n-1)/4$, the t members that are accepted by S are not sufficiently large to be a majority in $N \setminus S$.

(b) Members of S' have to be better off in P'' than in P. For all $i \in S \setminus T$,

$$
\begin{aligned}
&V_i(\{S', T, N \setminus S\}) \\
&= \alpha - \lambda + \mu \left[(n+1)(s-t) - (s-t)^2 - (t)^2 - (n-s)^2\right] \\
&= \alpha - \lambda + \mu \left[(n+1)s - (s)^2 - (n-s)^2 - t\,(n+1-2s+2t)\right].
\end{aligned}
$$

For all $i \in S$,

$$V_i(\{S, N \setminus S\}) = \alpha - \lambda + \mu \left[(n+1)s - (s)^2 - (n-s)^2\right].$$

Therefore, members of $S \setminus T$ are better off in P'' if and only if $s - t > (n+1)/2$. Thus, the deviation from P to P'' by $S \setminus T$ will not be blocked if and only if

$$1 \leq t < \min\{s/2, s - (n+1)/2\}.$$

Since $s - (n+1)/2 > 1$ for $n \geq 8$, there always exists a coalition size t such that the deviation is not blocked.[2] □

Lemma 5. *The grand coalition $P^* = \{N\}$ is never contractually stable under the simple majority decision rule.*

Proof. We will show that, from the grand coalition $\{N\}$, there always exists a subcoalition S of N ($S \subset N$) who has incentives to leave N. That is, we consider the deviation from $P^* = \{N\}$ to $P''' = \{S, N \setminus S\}$. We will show that (i) all members of S prefer P''' to P^*; (ii) this deviation is not blocked by former partners, that is, members of $N \setminus S$. Take S such that the size of coalition S, s, is the integer closest to $(3n+1)/4$. If two integers are equally close to $(3n+1)/4$, the coalition size can take on those two values. Two conditions are required so that this deviation is not blocked.

(a) Members of S who deviate need to have the majority within N. Since the size of S is the integer closest to $(3n+1)/4$, we have that $n/2 < s$.

[2]For $n = 3$, $\{N\}$ is the unique contractually stable association structure under the simple majority decision rule. For $4 \leq n \leq 5$, there is a unique contractually stable association structure under the simple majority decision rule: $\{S, N \setminus S\}$ with $s = n - 1$. For $n = 6$, the unique contractually stable association structures under the simple majority decision rule are $\{S, N \setminus S\}$ with $n - 2 \leq s \leq n - 1$. For $n = 7$, the unique contractually stable association structures under the simple majority decision rule are $\{S, N \setminus S\}$ with $s = n - 2$.

(b) Members of S have to be better off in P''' than in P^*. For all $i \in S$,

$$V_i(\{N\}) = \alpha - \lambda + \mu\left[(n+1)(n) - (n)^2\right].$$

For all $i \in S$,

$$V_i(\{S, N \setminus S\}) = \alpha - \lambda + \mu\left[(n+1)s - (s)^2 - (n-s)^2\right].$$

Since the size of S is the integer closest to $(3n+1)/4$ and $n \geq 4$, we have that $(n+1)s - (s)^2 - (n-s)^2 > (n+1)(n) - (n)^2$. □

Proposition 1. *There is no contractually stable coalition structure under the simple majority rule.*

Since a coalitional structure that is Δ-stable under Hart and Kurz (1983) notion of stability is contractually stable under the simple majority decision rule, we have that there is no Δ-stable coalition structure.

Corollary 1. *There is no Δ-stable coalition structure.*

3.2 Unanimity decision rule

Lemma 6. *Take any coalition structure $P \neq P^*$. Then, any split of any coalition belonging to P is blocked under the unanimity decision rule.*

Proof. Take any coalition structure $P \neq P^*$. The deviation from P to $P' = P \setminus \{S\} \cup \{S_1, S_2\}$ with $S_1 \cup S_2 = S$ will be blocked because (i) at least one of the new coalitions S_1 and S_2 will have a size strictly smaller than $(n+1)/2$ and so its members will be worse off than in P, and (ii) unanimity of members of S is required. □

Lemma 7. *Take any asymmetric coalition structure P with $p = 2$. Then, any deviation from P to P^* will be blocked under the unanimity decision rule.*

Proof. For all $i \in N$, we have

$$V_i(\{N\}) = \alpha - \lambda + \mu\left[(n+1)(n) - (n)^2\right].$$

Take any asymmetric coalition structure $P = \{S, N \setminus S\}$. Without loss of generality, let $s \geq (n+1)/2$. For all $i \in S$, we have

$$V_i(\{S, N \setminus S\}) = \alpha - \lambda + \mu\left[(n+1)s - (s)^2 - (n-s)^2\right].$$

For all $i \in N \setminus S$, we have

$$V_i(\{S, N \setminus S\}) = \alpha - \lambda + \mu\left[(n+1)(n-s) - (s)^2 - (n-s)^2\right].$$

Comparing those expressions and given that $s \geq (n+1)/2$, members of S will block the deviation from $P = \{S, N \setminus S\}$ to $P^* = \{N\}$ because they are not better off in P^*. □

Lemma 8. *Take any asymmetric coalition structure* $P = \{S, N \setminus S\}$ *with* $(n+1)/2 \leq s \leq n-1$. *Then, any deviation from* $P = \{S, N \setminus S\}$ *to* $P' = \{S \cup T, (N \setminus S) \setminus T\}$ *with* $T \subset N \setminus S$ *will be blocked under the unanimity decision rule.*

Proof. Take any asymmetric coalition structure $P = \{S, N \setminus S\}$ with $(n+1)/2 \leq s \leq n-1$ and consider the deviation from P to $P' = \{S \cup T, (N \setminus S) \setminus T\}$ with $T \subset N \setminus S$. For all $i \in N \setminus S$, we have

$$V_i(\{S, N \setminus S\}) = \alpha - \lambda + \mu\left[(n+1)(n-s) - (s)^2 - (n-s)^2\right].$$

For all $i \in (N \setminus S) \setminus T$, we have

$$\begin{aligned} &V_i(\{S \cup T, (N \setminus S) \setminus T\}) \\ &= \alpha - \lambda + \mu\left[(n+1)(n-s-t) - (s+t)^2 - (n-s-t)^2\right] \\ &= \alpha - \lambda + \mu\left[(n+1)(n-s) - (s)^2 - (n-s)^2 + t(n-1-4s-2t)\right]. \end{aligned}$$

Thus, members of $(N \setminus S) \setminus T$ will block the deviation from $P = \{S, N \setminus S\}$ to $P' = \{S \cup T, (N \setminus S) \setminus T\}$ if and only if $(n-1-4s)/2 < t$. This condition is always satisfied since $(n+1)/2 \leq s$. □

Proposition 2. *Any asymmetric coalition structure* P *such that* $p = 2$ *is contractually stable under the unanimity decision rule.*

Proposition 3. *The grand coalition* $P^* = \{N\}$ *is always contractually stable under the unanimity decision rule.*

Proof. The grand coalition $P^* = \{N\}$ is the efficient coalition structure: $nV_i(\{N\}) > \sum_{j=1}^{m} s_j V_i(P)$ for any $P = \{S_1, S_2, ..., S_m\}$ such that $P \neq \{N\}$. Under the unanimity decision rule, any deviation from P^* to any P requires the approval of all members of N. Therefore, any deviation from P^* to any P will be blocked by at least one member of N who will be worse off in P than in P^*. □

4 Concluding remarks

Open membership	$\{n\}$
Game Γ	No SNE. All P are NE.
Game Δ	No SNE. All P are NE.
Sequential game	$\{S^*, N \setminus S^*\}$ where $s^* \simeq (3n+1)/4$

We compare now the outcomes obtained under the notion of contractual stability with those obtained under a sequential game of coalition formation with fixed payoff division proposed by Bloch (1996). A fixed protocol is assumed and the sequential game proceeds as follows. Player 1 proposes the formation of a coalition S_1 to which he belongs. Each prospective player answers the proposal in the order fixed by the protocol. If one prospective player rejects the proposal, then he makes a counter-proposal to which he belongs. If all prospective players accept, then the coalition S_1 is formed. All players in S_1 withdraw from the game, and the game proceeds among the players belonging to $N \setminus S_1$. This sequential game has an infinite horizon, but the players do not discount the future. The players who do not reach an agreement in finite time receive a payoff of zero. Contrary to the largest consistent set, this sequential game relies on the commitment assumption. Once some players have agreed to form a coalition they are committed to remain in that coalition.

Consider the following finite procedure to form coalitions. First, player 1 starts the game and chooses an integer s_1 in the interval $[1, n]$. Second, player $s_1 + 1$ chooses an integer s_2 in $[1, n - s_1]$. Third, player $s_1 + s_2 + 1$ chooses an integer s_3 in $[1, n - s_1 - s_2]$. The game goes on until the sequence $(s_1, s_2, s_3, ...)$ satisfies $\sum_j s_j = n$. For symmetric valuations, if the finite procedure yields as subgame perfect equilibrium a coalition structure with the property that payoffs are decreasing in the order in which coalitions are formed, then this coalition structure is supported by the generically unique symmetric stationary perfect equilibrium (SSPE) of the sequential game (see Bloch, 1996). This result makes easy the characterization of the SSPE outcome of the association formation game. The coalition structure consisting of a dominant association grouping around three quarters of the industry forms and the remaining firms form a smaller association is the unique SSPE outcome of the sequential game. This coalition structure is not a SNE in the game Γ: firms obtain a higher payoff in the structure $P' = \{(n/2), 1, ...1\}$ and hence the coalition structure $P = \{S^*, N \setminus S^*\}$ is not immune to a coalitional deviation in the game Γ, where s^*

is the integer closest to $(3n+1)/4$.

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A The exit structure of strategic alliances

Alliances are long-term, cooperative, relational contracts among two or more firms, and are characterized by nonfinancial investments and a profit interest by all parties. Alliances agreements contain mechanisms to regulate exit. In most instances, the termination provisions allow parties to exit an alliance only after a specified period of time or for cause, which might include a material breach of the agreement, a change in control of the counterparty, or insufficient progress on the project. By effectively locking partners into the alliance relationship, these termination provisions create incentives to mitigate opportunism.

One aspect of the exit structure is the contractual board. Many alliance contracts create a contractual board – usually called the management committee. The contractual boards formed by most alliance agreements are concerned with opportunism prevention.

Contractual boards typically are assigned the task of monitoring the alliance activities and shaping ongoing developments. The contractual board is comprised of representatives of each side, usually in equal numbers, but the absolute numbers are not so important, since unanimity is the norm. Given that alliances usually do not provide for easy dissolution, the deadlocks are dealt with according to the terms of the contracts, which ordinarily include a dispute resolution mechanism. One purpose of contractual boards, therefore, is to provide an exit option. The price of exit is the cost of the deadlock procedures.

In many partnerships, the parties agree that the partnership will endure for a particular term or specified undertaking. Of course, even in such arrangements, the partners are allowed to exit, but if a partner leaves the partnership under circumstances not sanctioned by the partnership agreement, the departing partner may be subject to damages for breach of contract. The result is a form of lock in that attempts to discourage opportunistic exit.

Alliances partners are not subject to default rules. The termination structure of alliances is entirely contractual, and as we would expect, alliance partners often strive to obtain the benefits of lock in without constructing a suicide pact. [While it is not uncommon for an alliance agreement to be terminable at the will of the larger party, that right typically requires substantial forewarning. For instance, the Exclusive License and Collaboration Agreement between MedImmune Inc. and Critical Therapeutics Inc. (July 30, 2003) provides that MedImmune has the right to terminate on six months notice.] Most alliances have termination provisions that are tied to the completion of a specified undertaking. Prior to that event, the partners may exit only for cause, a term that typically includes breach of the alliance agreement and may include other events.

If those were the only termination provisions, the exit structure of alliances would look very much like partnerships for term. Another method of exit that is commonly employed in alliances, however, is exit via deadlock. This strategy is implemented through the contractual board. Two functions for such boards: (1) improved information flow and (2) improved coordination on strategic-level decisions by forcing consensus.

Alliance partners create a contractual board and define its authority, specify its regular meetings, and provide decision making and dispute resolution rules.

One of the key advantages of alliances over partnerships or corporations is that the partners have a built-in check against opportunism. An alliance partner who feels put upon has the power to force deadlock, which triggers a substantial process that is capable of addressing the opportunistic behavior or providing a means of exit.

One very interesting feature of alliances is the contractual board. The contractual board is an important vehicle for discovering and disseminating information about the activities of the alliance. Moreover, the contractual board also play a somewhat surprising role in the exit structure of many alliances. They serve as a means of exit without breach.

B Side payments

Definition 3. An association structure P is contractually stable under side payments and the unanimity decision rule if for any $S \subseteq N$, P' obtainable from P via S such that $V_i(P') \geq V_i(P)$ for all $i \in S$ and $V_j(P') > V_j(P)$ for some $j \in S$, we have that

$$\sum_{\substack{l \in S(k) \\ \text{with } S(k) \in P}} V_l(P') \leq \sum_{\substack{l \in S(k) \\ \text{with } S(k) \in P}} V_l(P) \qquad \text{for all } k \in S.$$

Proposition 4. *Allowing for side payments among partners, the contractually stable association structures under the unanimity decision rule are*

- *any asymmetric association structure $P = \{S, N \setminus S\}$ with $(2n-1)/3 < s \leq n-1$.*
- *the grand association structure $P^* = \{N\}$.*

Proof. First, we show that the t members of T can compensate the $n-s-t$ members of $(N \setminus S) \setminus T$ when they deviate jointly with members of S from $P = \{S, N \setminus S\}$ to $P' = \{S \cup T, (N \setminus S) \setminus T\}$. Indeed, we have

$$\sum_{\substack{l \in N \setminus S \\ \text{with } N \setminus S \in P}} V_l(P') > \sum_{\substack{l \in N \setminus S \\ \text{with } N \setminus S \in P}} V_l(P)$$

where

$$\begin{aligned} \sum_{l \in N \setminus S, N \setminus S \in P} V_l(P') &= (n-s)(\alpha - \lambda) + \mu t \left[(n+1)(s+t) - (s+t)^2 - (n-s-t)^2\right] \\ &\quad + \mu(n-s-t)\left[(n+1)(n-s-t) - (s+t)^2 - (n-s-t)^2\right] \end{aligned}$$

and

$$\sum_{l \in N \setminus S, N \setminus S \in P} V_l(P) = (n-s)(\alpha - \lambda) + \mu(n-s)\left[(n+1)(n-s) - (s)^2 - (n-s)^2\right].$$

Then,

$$\sum_{l\in N\backslash S, N\backslash S\in P} V_l(P') > \sum_{l\in N\backslash S, N\backslash S\in P} V_l(P) \Leftrightarrow t > \frac{n(s+2)-s(3+4s)}{2(s+1)}.$$

This condition is always satisfied since

$$1 > \frac{n(s+2)-s(3+4s)}{2(s+1)} \text{ for } s \geq \frac{n}{2}.$$

Second, we show that the $s-t$ members of S who deviate cannot compensate the other t members of S when they deviate from $P = \{S, N\backslash S\}$ to $P' = \{S\backslash T, T, N\backslash S\}$. Indeed, we have

$$\sum_{\substack{l\in S \\ \text{with } S\in P}} V_l(P') \leq \sum_{\substack{l\in S \\ \text{with } S\in P}} V_l(P)$$

where

$$\sum_{l\in S, S\in P} V_l(P) = s(\alpha-\lambda) + \mu s\left[(n+1)s - (s)^2 - (n-s)^2\right]$$

and

$$\begin{aligned}\sum_{l\in S, S\in P} V_l(P') &= s(\alpha-\lambda) + \mu(s-t)\left[(n+1)(s-t) - (s-t)^2 - (t)^2 - (n-s)^2\right] \\ &\quad + \mu t\left[(n+1)t - (s-t)^2 - (t)^2 - (n-s)^2\right].\end{aligned}$$

Then,

$$\sum_{\substack{l\in N\backslash S \\ \text{with} N\backslash S\in P}} V_l(P') > \sum_{\substack{l\in N\backslash S \\ \text{with } N\backslash S\in P}} V_l(P) \Leftrightarrow s\left[(n+1)t - 2st + 2t^2\right] > t(n+1)(2t-s).$$

This condition is always satisfied since $(n+1) - s > 0$. □

C Conditions on payoffs

Gains are assumed to be positive, $V_i(P) > 0$ for all $i \in N$. We consider $n > 2$. We assume symmetric or identical players and equal sharing of the coalition gains among coalition members. That is, in any coalition S_i belonging to P, $V_j(P) = V_l(P)$ for all $j, l \in S_i$, $i = 1, ..., m$. So, let $V(S_i, P)$ denote the payoff obtained by any player belonging to S_i in the coalition structure P. We focus on coalition formation games satisfying the following conditions on the per-member payoffs.

(P.1) *Positive Spillovers.* $V(S_i, P\setminus\{S_1, S_2\}\cup\{S_1\cup S_2\}) > V(S_i, P)$ for all players belonging to S_i, $S_i \neq S_1, S_2$.

Condition (P.1) restricts the analysis to games with positive spillovers, where the formation of a coalition by other players increases the payoff of a player.

(P.2) *Negative Association.* $V(S_i, P) < V(S_j, P)$ if and only if $s_i > s_j$.

Condition (P.2) imposes that, in any coalition structure, small coalitions have higher per-member payoffs than big coalitions.

(P.3) *Individual Free-Riding.* $V(\{j\}, P \setminus \{S_i\} \cup \{S_i \setminus \{j\}, \{j\}\}) > V(S_i, P)$ for all $j \in S_i$, $S_i \in P$.

Condition (P.3) is related to the existence of individual free-riding incentives. That is, if a player leaves any coalition to be alone, then he is better off.

(P.4) *Efficiency.* $\nexists\, P = \{S_1, S_2, ..., S_m\} \in \mathbb{P}$ such that $P \neq \{N\}$ and $\sum_{i=1}^{m} V(S_i, P) \cdot s_i \geq V(N) \cdot n$.

Finally, condition (P.4) assumes that the grand coalition is the only efficient coalition structure with respect to payoffs, where $V(N)$ denotes the payoff of any player belonging to the grand coalition $\{N\}$.

An economic situation satisfying these four conditions is a cartel formation game with Cournot competition. Let $p(q) = a - q$ be the inverse demand (q is the industry output). The industry consists of n identical firms. Inside each cartel, we assume equal sharing of the benefits obtained from the cartel's production. Once stable agreements on cartel formation have been reached, we observe a Cournot competition among the cartels. The payoff for each firm in each possible coalition structure is well defined. Firm i's cost function is given by $d \cdot q_i$, where q_i is firm i's output and d ($a > d$) is the common constant marginal cost. As a result, the per-member payoff in a cartel of size s is, for all firms belonging to S,

$$V(S, P) = \frac{(a-d)^2}{s \cdot (p+1)^2},$$

where p is the number of cartels within P.

Lemma 9. *Output cartels in a Cournot oligopoly with the inverse demand function $p(q) = a - q$ and the cost function $d(q_i) = d \cdot q_i$ satisfy (P.1)-(P.4).*

Another economic application of games with positive spillovers are economies with pure public goods. The model we study is inspired from Bloch (1997), Yi (1997) and Ray and Vohra (2001) wherein we introduce congestion. The economy consists of n agents. At cost $d_i(q_i)$, agent i can provide q_i units of the public good. Let $q = \sum_i q_i$ be the total amount of public good. The utility each agent obtains from the public good depends positively on the total amount of public good provided, but negatively on the number of coalition partners: $U_i(q) = (s)^{-\alpha} \cdot q$ for all $i \in S$, where parameter $\alpha > 0$ measures the degree of congestion. Each agent owns a technology to produce the public good, and the cost of producing the amount q_i of the public good is given by $d_i(q_i) = \frac{1}{2}(q_i)^2$. Since individual cost functions are convex and exhibit decreasing returns to scale, it is cheaper to produce an amount q of public goods using all technologies than using a single technology. In stage one the coalition formation takes place. Inside each coalition, we assume equal sharing of the production. Once a coalition structure has been formed, each coalition of agents acts noncooperatively. On the contrary, inside every coalition, agents act cooperatively and the level of public good is chosen to maximize the sum of utilities of the coalition members. That is, for any coalition structure $P = \{S_1, S_2, ..., S_m\}$, the level of public good q_{S_i} chosen by the coalition S_i solves

$$\max_{q_{S_i}} s_i \cdot \left[(s_i)^{-\alpha} \left(q_{S_i} + \sum_{j \neq i} q_{S_j} \right) - \frac{1}{2} \left(\frac{q_{S_i}}{s_i} \right)^2 \right]$$

yielding a total level of public good provision for the coalition S_i equal to $q_{S_i} = (s_i)^{2-\alpha}$, $i = 1, ..., m$. The per-member payoff in a coalition of size s_i is given by

$$V(S_i, P) = (s_i)^{-\alpha} \cdot \sum_{j=1}^{m} (s_j)^{2-\alpha} - \frac{1}{2} (s_i)^{2-2\alpha},$$

for all agents belonging to S_i, $i = 1, ..., m$.

Contrary to the cartel formation game with Cournot competition, it depends on the number of agents n and the degree of congestion α whether public goods coalitions satisfy conditions (P1)-(P4). For instance, public goods coalitions with utility function $U_i(q) = (s)^{-.15} \cdot q$ for all $i \in S$ and cost function $d_i(q_i) = \frac{1}{2}(q_i)^2$ satisfy (P.1)-(P.4) if $n \in [4, 6]$. Notice that, for $n < 4$ the condition (P.3) is violated, while for $n > 6$ it is (P.4) which is violated.

(P.5) *Inverse monotonicity.* For any given P, any $S_i, S_j \in P$ with $s_i < s_j$, and any $i \in S_i$ we have $V(S_i, P) > V(S_i, P \setminus \{S_i, S_j\} \cup \{S_j \setminus \{j\}, S_i \cup \{j\}\})$.

(N.1) *Negative Spillovers.* $V(S_i, P \setminus \{S_1, S_2\} \cup \{S_1 \cup S_2\}) < V(S_i, P)$ for all players belonging to S_i, $S_i \neq S_1, S_2$.

Condition (N.1) restricts the analysis to games with negative spillovers, where the formation of a coalition by other players decreases the payoff of a player.

(N.2) *Positive Association.* $V(S_i, P) < V(S_j, P)$ if and only if $s_i < s_j$.

Condition (N.2) imposes that, in any coalition structure, large coalitions have higher per-member payoffs than small coalitions.

(N.3) *Group monotonicity for small coalitions.* For any given P, any coalitions $S_i, S_j \in P$ with $s_i > s_j$, and any subset of players $S \subseteq S_j$ we have $V(S_j, P) < V(S_i \cup S, P \setminus \{S_i, S_j\} \cup \{S_j \setminus S, S_i \cup S\})$.

(N.4) *Critical size strict monotonicity for big coalitions.* There exists a critical size $n^* \leq n$ such that for any coalition S with $s < n^*$ and for any partition P containing S we have that for any $T \in P$ with $s > t$ it holds that $V(S \cup T, P') > V(S, P)$ for $P' = P \setminus \{S, T\} \cup \{S \cup T\}$

We have symmetric players.

Lemma 10. *Any symmetric coalition structure $P \neq P^*$ is never contractually stable under the unanimity decision rule.*

Proof. By **P.4.** (efficiency) the grand coalition P^* is efficient and then, we have

$$nV(P^*) > \sum_{S \in P} sV(S, P) \text{ for all } P \in \mathbb{P},\ P \neq P^*.$$

Since P is symmetric ($P \neq P^*$), we have

$$\sum_{S \in P} sV(S, P) = nV(S, P).$$

Hence, $V(P^*) > V(S, P)$ for all $S \in P$, P symmetric, $P \neq P^*$; and no player will block the deviation from P to P^*. □

Lemma 11. *The grand coalition $P^* = \{N\}$ is contractually stable under the unanimity decision rule.*

Proof. By Lemma 10, from P^* there is no profitable deviation to any symmetric coalition structure $P \neq P^*$. Suppose we have a profitable deviation by S_1 to P which means that $V(S_1, P) > V(N)$. By **P.4.** (efficiency) we have $nV(N) > \sum_{S \in P} sV(S, P)$. Therefore, there exists $S_2 \in P$ such that $V(S_2, P) < V(N)$ and $S_2 \subset N$, so that the members of S_2 will block the deviation from P^* to P. □

Notice that if **N.2.** (positive association) is satisfied then the members of the smallest coalition of any $P \neq P^*$ will always block the deviation from P^* to P. If **P.2.** (negative association) is satisfied then the members of the biggest coalition of any $P \neq P^*$ will always block the deviation from P^* to P.

Lemma 12. *Suppose that the critical size $n^* \in [n/(k+1), n/k]$ with $k \in [0, n-1]$. Then, any coalition structure P with $p > k+1$ is never contractually stable under the unanimity decision rule.*

Proof. Suppose the contrary : there exists P with $p > k+1$ that is contractually stable under the unanimity decision rule. Since $p > k+1$ there are at least two coalitions S and T with cardinality smaller than n^* and that, by **N.4.** (critical size strict monotonicity), would like to form the coalition $S \cup T$ contradicting the first assumption. Therefore, there does not exist a coalition structure P with $p > k+1$ that is contractually stable under the unanimity decision rule. □

Notice that **N.3.** (group monotonicity for small coalitions) may enter in conflict with **N.4.** (critical size strict monotonicity). For instance, $20 = n/6 < n^* = 22 < n/5 = 24$ for $n = 120$. In $\{S_1, S_2, S_3, S_4, S_5, S_6\}$ with $s_1 = 14$, $s_2 = 15$, $s_3 = 19$, $s_4 = 20$, $s_5 = 25$, $s_6 = 26$, S_5 has incentives to merge with S_6 following **N.3.** but it contradicts **N.4.** since $25 > 22 = n^*$. If the critical size is such that coalition structures of more than two coalitions cannot be stable as in the formation of associations of firms then **N.4.** implies **N.3.**

Lemma 13. *Suppose that the critical size $n^* \in [n/(k+1), n/k]$ with $k \in [0, n-1]$. Then, a coalition structure P with $p \leq k+1$ cannot be contractually stable under the unanimity decision rule if P contains two or more coalitions with cardinality smaller than n^*.*

Proof. It is straightforward by **N.4.** (critical size strict monotonicity) that any P containing two coalitions S_1 and S_2 with cardinality smaller than n^*, $s_1 < n^*$ and

$s_2 < n^*$, is not contractually stable under the unanimity decision rule since the coalitions S_1 and S_2 have incentives to merge to form $S_1 \cup S_2$. $\square$