One-to-Many Bargaining with Endogenous Protocol

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Abstract

This paper studies the bargaining between one active player and N passive players. In each period the active player can choose any passive player to bargain with; thus, the bargaining protocol is endogenously determined. The passive players are heterogeneous in terms of their bargaining power. The set of equilibrium outcomes is characterized with two different contract forms: contingent and cash-offer contracts. It is shown that various bargaining protocols may arise in equilibria sustaining different agreements. The active player can also play one passive player off against another. We further investigate the influence of contract form on the set of equilibrium outcomes and examine the properties of Markov equilibria.

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1 Introduction

This paper studies the bargaining between one active player and N passive players on how to share the added value of a joint project that requires the cooperation of all parties. Relevant real-life situations include a real estate developer buying pieces of land from multiple owners, an employer trying to reach deals with several labor unions, and a firm acquiring a series of complementary patents, etc. A common feature of these situations is that the active player has to reach agreement with all of the passive players, who do not bargain with one another.

In contrast to most of the literature, which assumes exogenously fixed bargaining protocol, i.e., the ordering of bilateral bargaining rounds (between the active player and a passive player) and the duration of each bargaining round, in our model, the bargaining protocol is endogenously determined. More specifically, in each bargaining period, the active player chooses which passive player to bargain with; if no agreement is reached in the current period, the active player can either continue to bargain with the same passive player or move on to any other player in subsequent periods.

Another new feature of this model is that the passive players are, in general, heterogeneous in terms of their bargaining power. As in Rubinstein (1982), the right to make a proposal is the source of bargaining power, and the passive players differ in terms of the probability with which they are recognized as the proposer when they bargain with the active player.

A binding contract is signed once an agreement is reached between the active player and a passive player. Following the existing literature, we consider two types of contracts: contingent contracts and cash-offer contracts. With the former, the passive player receives the agreed payment only after the active player has reached agreement with all of the passive players and the project is finally implemented; with the latter, he is paid immediately upon agreement being reached.[1]

We aim to answer the following questions. (1) What bargaining protocols can emerge endogenously in equilibria, and of these protocols, which produces the greatest payoff for

[1] Throughout the paper, the active player is referred to as "she" and a passive player as "he".

the active player? (2) Which bargaining protocol is more plausible in terms of its simplicity and/or stationarity? (3) Finally, how are these conclusions influenced by the type of contract available to the players?

With contingent contracts, the bargaining game admits a rich set of equilibria. First, the equilibrium outcomes under different fixed bargaining protocols can arise in equilibria with endogenously determined protocols. Various beliefs about future actions off the equilibrium path result in different agreements being reached on the equilibrium path. For example, one sequence of agreements may result from the belief that the active player will never switch to another passive player until an agreement is reached, and a different sequence of agreements from the belief that she will alternate among all of the remaining passive players. In each equilibrium, the belief is correct, that is, it is indeed optimal for the active player to never switch or to alternate in case of temporary disagreement.

Moreover, there exist equilibria in which the active player plays one passive player off against another, namely, skimming equilibria. As the players become extremely patient, the active player's greatest equilibrium payoff from a skimming equilibrium can be arbitrarily close to what she could obtain in a bilateral bargaining with the weakest passive player.

The main effect of using cash-offer contracts is that the payments made in earlier bargaining periods become sunk costs for the active player, and the total surplus in subsequent bargaining remains the same. It is shown here that, in general, there exists no equilibrium in which the active player alternates among passive players. This finding is rather important, as alternate protocol is often presumed in the existing literature. Furthermore, with cash-offer contracts, impasse is an equilibrium outcome when the number of passive players is large, whereas it never occurs in equilibrium with contingent contracts.

For its strategic simplicity, we further restrict our attention to the Markov equilibrium of the bargaining game. It is shown that, in our model, any Markov equilibrium must be efficient in contrast to models with fixed protocols. The set of Markov equilibria depends on which type of contract is adopted: with contingent contracts, there is a unique Markov equilibrium in mixed strategies, whereas there are multiple Markov equilibria in pure strategies with cash-offer contracts. A particularly important finding is that, with cash-offer contracts, the active player obtains her greatest equilibrium payoff by negotiating with the passive players in ascending order of their bargaining power.

A model with endogenous protocol is more realistic than those with fixed protocols because, in reality, bargaining is rarely conducted according to some pre-determined protocol. If an active player can commit to a certain bargaining protocol, she will certainly choose the one that maximizes her equilibrium payoff. Even without such a commitment, if the same active player has to bargain with different sets of passive players over time, she will have the incentive to build up a reputation for sticking to a certain protocol. Therefore, from the policy maker's viewpoint, it may be useful to impose certain regulations on bargaining protocol to protect the passive players' interests and to reduce the likelihood of inefficient bargaining behavior.

Related Literature. Early research on one-to-many bargaining often assumes a fixed bargaining protocol.[2] This paper is most closely related to Cai (2000, 2003). Cai (2000) studies the bargaining between a railroad company and N farmers with cash-offer contracts. The farmers are located on a circle with fixed ordering. Each bargaining round between the company and one farmer consists of two periods in which each party makes one offer. If no agreement is reached, the company moves on to the next farmer on the circle. It is shown that there are multiple equilibria with endogenously determined orders of reaching agreement. Inefficient delays may arise in equilibria that satisfy a weak stationarity condition. In a later study, Cai (2003) considers a similar model with contingent contracts, showing that there are multiple Markov equilibria, some of which entail inefficient delays. A crucial feature of this study is that the active player moves on to another passive player after the rejection of her own offer. If, instead, the passive player makes the final offer in each bargaining round, then the multiplicity result does not hold.[3] This observation shows how sensitive the equilibrium

[2]See, for example, Jun (1987), Horn and Wolinsky (1988), and Stole and Zwiebel (1996).

[3]More precisely, in Cai's (2003) model, if the order of making offers is reversed in each bargaining round, then there is a unique equilibrium in which the n^{th} passive player in the queue obtains a share close to $1/2^n$ as δ goes to 1. It is equivalent to the equilibrium with a sequential protocol in our model.

outcome is to the bargaining protocol.

Menezes and Pitchford (2004) address the holdout problem in a model with one buyer and multiple sellers. They demonstrate that the complementarity of the buyer's technology is a necessary condition for equilibrium holdout. Roy Chowdhury and Sengupta (2009) consider a simultaneous bargaining protocol to investigate the role of protocol transparency in overcoming the inefficiency due to holdout. With endogenously determined protocol, our model always admits efficient equilibria; thus, the holdout problem seems to be an artifact of an assumed bargaining protocol.

There has also been a number of studies that attempt to endogenize bargaining protocol in various settings.[4] Noe and Wang (2004), for example, consider the bargaining between one buyer and two sellers with a general value structure and endogenous ordering. They investigate the strategic role played by the confidentiality of the bargaining ordering. An important insight is that, by conducting private negotiations, the buyer can create strategic uncertainty, by which she may obtain a greater equilibrium payoff than she would in public negotiations.[5]

A crucial feature of Noe and Wang's model is that the buyer bargains with each seller only once; thus, when the bargaining ordering is public or in pure strategy, there is a unique equilibrium by backward induction. In our model, the active player can bargain with a passive player repeatedly until agreement is reached, and she also has greater degrees of freedom in choosing the bargaining protocol. Thus, although we focus on public negotiations, multiple equilibria still arise. More importantly, the uncertainty-induced payoff advantage for the buyer also hinges on the assumption of one-shot bargaining. Without a deterministic

[4] Board and Zwiebel (2005) study a finite-horizon bilateral bargaining model in which the players compete for the right to make a proposal via auction. They show that the alternating-offers protocol may arise in equilibrium. Suh and Wen (2009) consider a multilateral bargaining model in which any pair of players can bargain with each other for a partial bilateral agreement. The players have to agree on who will leave the bargaining after each round; hence, the protocol is endogenously determined.

[5] This model is nicely extended by Krasteva and Yildirim (2010), who examine the joint effects of confidentiality and offer deadlines.

deadline, a seller can always choose to hold out until he is the last one to reach agreement; thus, the buyer cannot benefit from conducting private negotiations.

The rest of the paper is organized as follows. Section 2 outlines the one-to-many bargaining model with endogenous protocol. Section 3 analyzes the bargaining game with contingent contracts, and Section 4 considers cash-offer contracts. Section 5 discusses the implications of the results for the pattern of unionization in Horn and Wolinsky's (1988) framework. Section 6 contains further discussion and concludes the paper.

2 A Bargaining Model with Endogenous Protocol

There are $N+1$ players: active player A and N passive players indexed by $i \in \{1, 2, ..., N\}$. Player A has a project with a commonly known surplus normalized to one. To undertake this project, she needs the cooperation of all of the passive players. Hence, player A has to bargain with each passive player over the payment to be made in exchange for his cooperation.

The bargaining takes place over time divided into periods of equal length. In each period $t \in \{0, 1, 2, ...\}$, player A first chooses with whom to bargain. Then, either player A or the chosen player i makes an offer, and the other party responds with acceptance or rejection. The offer is simply the size of the payment that player i shall receive. If the offer is accepted, the two parties sign a binding contract, and player A moves on to bargain with the other passive players; if it is rejected, bargaining proceeds in the next period, and player A again chooses with whom to bargain. After player A has reached agreement with all of the passive players, the project is implemented immediately and the surplus is realized.

In each bargaining period, the proposer is randomly selected. More precisely, the probability that player i is recognized as the proposer is $p_i \in (0, 1)$, and the probability that player A is recognized is $1 - p_i$.[6] As the allocation of the right to make a proposal determines the relative bargaining power in a noncooperative bargaining framework, recognition probability

[6] We ignore the uninteresting cases with $p_i = 0$ or 1 for some i. More specifically, if $p_i = 0$, then player i has no influence at all on the bargaining outcome; if $p_i = 1$, then player A weakly prefers impasse to any other bargaining outcome.

p_i is the measure of the relative bargaining power of player i with respect to player A.

A bargaining outcome is denoted by $(s_i, t_i)_{i=1}^N$, where $s_i \in [0,1]$ is the agreed payment to player i, and t_i is the period in which this agreement is reached. Let $T = \max_i \{t_i\}$ be the date on which the final agreement is reached. As it takes at least N periods to reach agreement with all of the passive players, the bargaining outcome is inefficient if and only if $T > N - 1$. All players discount future payoffs with a common discount factor $\delta \in (0,1)$.

We consider two different types of contracts: a contingent contract, by which a passive player receives his payment only after agreement has been reached with all players and the project is finally implemented, and a cash-offer contract, by which a passive player receives his payment right away. With contingent contracts, player i's payoff from outcome $(s_i, t_i)_{i=1}^N$ is $\delta^T s_i$, and player A's is $\delta^T \left(1 - \sum_{i=1}^N s_i\right)$. If there is an impasse, i.e., $T = \infty$, then the project is not implemented and everyone gets a payoff of zero. With cash-offer contracts, player i receives his payment s_i in period t_i, and thus his payoff is $\delta^{t_i} s_i$ and player A's is $\left(\delta^T - \sum_{i=1}^N \delta^{t_i} s_i\right)$. In this case, if the bargaining encounters an impasse before any agreement has been reached, then everyone gets a payoff of zero; if some agreements have been reached prior to an impasse, then player A's payoff is negative, as the agreed payments have been made and thus become sunk costs. Clearly, this would not happen in any equilibrium.

The bargaining game described above is a well-defined extensive form game with perfect information and nature's move. Histories and strategies can be defined as usual. We adopt the subgame perfect equilibrium (henceforth equilibrium) as the solution concept. After the first $N - 1$ agreements have been reached, the bargaining between player A and the last passive player becomes a simple variant of the Rubinstein game. It is well known that this game has a unique equilibrium with immediate agreement, as stated in the following Lemma.

Lemma 1 *If player A has reached agreement with every passive player $j \neq i$ on s_j, then in the continuation game between player A and player i, there is a unique equilibrium: (i) with contingent contracts, player A (player i resp.) offers $s_i = \delta p_i \left(1 - \sum_{j \neq i} s_j\right)$ ($\hat{s}_i = [1 - \delta(1 - p_i)] \left(1 - \sum_{j \neq i} s_j\right)$ resp.), and the offer is accepted; (ii) with cash-offer contracts, player A (player i resp.) offers $s_i = \delta p_i$ ($\hat{s}_i = 1 - \delta(1 - p_i)$ resp.), and the offer is accepted.*

Note that with contingent contracts, the previously agreed payments are contingent upon the final agreement. Hence, in the final round, on the bargaining table is the total surplus less the sum of the agreed payments. With cash-offer contracts, however, payments are made immediately upon agreement and thus become "sunk" from the perspective of the remaining passive players. Hence, the total surplus in the final bargaining round remains one.

3 Contingent Contracts

In this section, we characterize the equilibria of the bargaining game with contingent contracts. An immediate observation is that impasse is never an equilibrium outcome.

Lemma 2 *With contingent contracts, impasse is not an equilibrium outcome.*

Proof. Denote by R_k the remaining surplus after the first $k \in \{0, 1, ..., N-1\}$ agreements in any equilibrium. By definition, $R_0 = 1$. If $R_{N-1} > 0$, then, by Lemma 1, player A and the last passive player reach an immediate agreement, by which both receive positive payoffs. Next, using backward induction and the assumption of $0 < p_i < 1$, it is easy to establish that if $R_k > 0$, then $0 < R_{k+1} < R_k$ for any $k \in \{0, 1, ..., N-2\}$. The key step is to note that a passive player will never reject an offer greater than δR_k and the active player will never reject a demand less than $(1-\delta) R_k$. Hence, with contingent contracts, impasse is never an equilibrium outcome. ■

In the following, it is first shown that various bargaining protocols may arise in equilibria sustaining different sets of agreements.[7] Then, we provide a general formulation of bargaining protocol. Finally, we examine the properties of Markov equilibria. For expositional ease, the rest of this section focuses on the case with two passive players. All of the results can easily be extended to the case with $N \geq 3$ passive players.

[7] The equilibrium characterization focuses on efficient equilibria. After the multiplicity of efficient equilibrium outcomes is established, it is straightforward to construct equilibria with an inefficient delay.

3.1 Equilibria with Different Underlying Protocols

Different equilibrium agreements can be sustained by different conjectured underlying bargaining protocols. In equilibrium, the conjecture is correct; in other words, it is optimal for the active player to stick to that protocol. Two common protocols used in one-to-many bargaining are sequential and alternate protocols. With the former, the active player bargains with the passive players one after another according to a pre-specified order, and she never moves on to another passive player before reaching agreement with the current one. With the latter, in contrast, the active player alternates among the passive players, that is, if no agreement is reached in the current period, she will bargain with another passive player in the next period. In our model, both protocols may emerge in equilibrium.

We first describe an equilibrium, referred to as *equilibrium with a sequential protocol*, in which the active player will never switch bargaining opponent until the first agreement has been reached. More precisely, on the equilibrium path, player A randomly chooses one of the passive players to begin with, and they reach agreement immediately. Before the first agreement has been reached, any passive player chosen to bargain in period t believes that he will bargain with player A again in period $t+1$ if no agreement is reached in the current period.

In bargaining for the first agreement, player i always asks for $1-\delta(1-p_i)$ and accepts any offer that is no less than δp_i, and player A always offers δp_i and accepts any demand no greater than $1-\delta(1-p_i)$. After the first agreement is reached, player A and the remaining passive player immediately reach agreement as specified in Lemma 1. Thus, there are two equilibria with sequential protocol, depending on who reaches the first agreement. Whereas both passive players would prefer to be the first to bargain, player A's expected payoff is $\delta(1-p_1)(1-p_2)$ in both equilibria. Hence, she is indifferent about which passive player she bargains with for the first agreement, and she also has no incentive to switch to another player during the bargaining process, which justifies the aforementioned passive player's belief. It also implies that any ordering of the passive players can be sustained in an equilibrium with a sequential protocol. Hence, we have the following proposition.

Proposition 1 *With contingent contracts, any ordering of the passive players can be sustained in an equilibrium with a sequential protocol.*

The following corollary shows that, of all possible equilibria, the active player obtains the smallest expected payoff from that with a sequential protocol. By the same token, the passive player who reaches the first agreement with the active player obtains the greatest expected payoff in this equilibrium.

Corollary 1 *With contingent contracts, player A's expected equilibrium payoff is at least* $\delta(1-p_1)(1-p_2)$*, and player* i*'s is at most* δp_i*.*

Proof. With contingent contracts, an agreed conditional payment can be equivalently viewed as a share of the realized surplus when the project is implemented. Lemma 1 shows that if player A reaches the first agreement with player i on a share of s_i^*, then her expected share in the continuation game is $(1-p_j)(1-s_i^*)$. If player i's expected share in any equilibrium is no greater than p_i, i.e., $Es_i^* \leq p_i$, then player A's expected share in any equilibrium must be no less than $(1-p_1)(1-p_2)$, i.e., $Es_A^* \geq (1-p_1)(1-p_2)$.

Denote as M_i the supremum of player i's expected equilibrium share, and denote as R_m the infimum of the expected remaining share after the first agreement; that is, $Es_i^* \leq M_i$ and $Es_A^* \geq (1-p_j) R_m$. The following inequalities are self-explanatory:

$$\begin{aligned} R_m &\geq (1-p_i)(1-\delta M_i) + p_i \delta R_m \quad \text{and} \\ M_i &\leq (1-p_i)\delta M_i + p_i(1-\delta R_m), \end{aligned}$$

by which we obtain $M_i \leq p_i$. Thus, $Es_A^* \geq (1-p_1)(1-p_2)$. It follows that player A's expected equilibrium payoff is at least $\delta(1-p_1)(1-p_2)$ and player i's is at most δp_i. ■

If there are N passive players with identical bargaining power $p_i = 1/2$, as is often assumed in the literature, then the active player's expected share in an equilibrium with a sequential protocol is $1/2^N$, which is less than that of all but one of the passive players. This equilibrium outcome is not *renegotiation-proof* because player A, after reaching the final agreement, has the incentive to renegotiate the first one.[8]

Next, we describe a renegotiation-proof equilibrium, referred to as *equilibrium with an alternate protocol*, in which the active player alternates in the choice of whom to bargain with. In other words, if player A bargains with player i in period t, then she will switch to player $j \neq i$ in period $t+1$ regardless of whether agreement has been reached. In bargaining for the first agreement, player i always asks for $\hat{s}_i$ and accepts any offer that is no less than $\widetilde{s}_i$, and player A always offers $\widetilde{s}_i$ and accepts any demand no greater than $\hat{s}_i$. In the equilibrium, the following indifference conditions hold:

$$\begin{aligned} \widetilde{s}_i &= \delta p_i \left[(1-p_j)(1-\widetilde{s}_j) + p_j (1-\hat{s}_j) \right] \quad \text{and} \\ (1-p_j)\,\hat{s}_i &= \delta (1-p_i) \left[(1-p_j)(1-\widetilde{s}_j) + p_j (1-\hat{s}_j) \right], \end{aligned}$$

by which we obtain

$$\widetilde{s}_i = \frac{\delta p_i (1-p_j)}{1-\delta p_1 p_2} \qquad \text{and} \qquad \hat{s}_i = \frac{(1-\delta) + \delta p_i (1-p_j)}{1-\delta p_1 p_2}.$$

After the first agreement has been reached, player A and the remaining passive player immediately reach agreement, as specified in Lemma 1. Being the first one to reach agreement, player i's expected share is

$$E\left(s_i^{ij}\right) = \frac{p_i (1-\delta p_j)}{1-\delta p_1 p_2},$$

where the superscript ij refers to the order of reaching agreement. If player i is the second one to reach agreement, then his expected share is

$$E\left(s_i^{ji}\right) = \frac{p_i (1-p_j)}{1-\delta p_1 p_2}.$$

Again, each passive player would prefer to be the first one to reach agreement. However, as δ tends to 1, the difference between $E\left(s_i^{ij}\right)$ and $E\left(s_i^{ji}\right)$ vanishes. It is also easy to check that player A's expected share is

$$E(s_A) = \frac{(1-p_1)(1-p_2)}{1-\delta p_1 p_2}.$$

[8] Stole and Zwiebel (1996) explicitly incorporate renegotiation-proofness into their bargaining model and obtain a unique noncooperative equilibrium outcome that is equivalent to the Shaley value of the corresponding cooperative game.

Thus, player A is indifferent about her choice of bargaining opponent in period 0, moreover, were there a disagreement, she would have no incentive to deviate from the alternate protocol.

Proposition 2 *With contingent contracts, any ordering of the passive players can be sustained in an equilibrium with an alternate protocol.*

Conceivably, there exist equilibria with general alternate protocols, by which the active player bargains with player i for T_i periods before switching to player j. The equilibrium agreements can be derived in a similar way. For any finite T_1 and T_2, it is easy to see that the equilibrium outcome converges to the same limit as the equilibrium with the basic alternate protocol (i.e., $T_1 = T_2 = 1$).

Clearly, the active player obtains a greater payoff in an equilibrium with an alternate protocol than in one with a sequential protocol.[9] Could the active player's equilibrium payoff be even greater? Cai (2003) specifies a Markov equilibrium in which player A's payoff goes to $3/8$ as δ goes to 1. Recall that in his model, the switch to a different passive player is made after the rejection of player A's offer. Hence, it may be more costly for a passive player to reject an offer than for player A to do so. More precisely, player A's rejection causes one period of delay, whereas a passive player's rejection causes three periods of delay if no agreement will be reached in the next bargaining round, which translates into a relative bargaining power ratio of $1:3$. Therefore, as δ tends to 1, one passive player's equilibrium share can be forced down to $1/4$, whereas player A and the other passive player split the remainder of the pie equally. With an endogenous bargaining protocol, the active player can further exploit this "skimming" effect.

We now describe a *skimming equilibrium.* Player A bargains with player 1 in period 0. She will switch to player 2 if her offer is rejected and will switch back after $T \geq 1$ periods of bargaining with player 2, during which no agreement will be reached.[10] Taking this as the

[9] If $p_1 = p_2 = 1/2$, then the equilibrium with an alternate protocol induces an equal split among all three players as δ tends to 1.

[10] More specifically, during the T periods of bargaining with player 2, each party makes non-serious offers so that no agreement can be reached.

underlying protocol, player A and player 1 will reach immediate agreement on either

$$\widetilde{s}_1 = \frac{1 - (1 - p_1)\,\delta^{T+1}}{p_1 + (1 - p_1)\sum_{t=0}^{T} \delta^t}$$

if player 1 proposes, or

$$\hat{s}_1 = \frac{\delta^{T+1} p_1}{p_1 + (1 - p_1)\sum_{t=0}^{T} \delta^t}$$

if player A proposes.

The T periods of fruitless bargaining with player 2 serve as punishment for player 1. If player A deviates during this punishment by switching back early, then the continuation equilibrium is the one with a sequential protocol in which player 1's expected share is p_1. Player 2 has the same interest as player A in carrying out the punishment. Thus, the length of punishment, T, is chosen such that

$$\delta^{T+1}\left[(1 - p_1)(1 - \hat{s}_1) + p_1 (1 - \widetilde{s}_1)\right] \geq \delta (1 - p_1),$$

where the left-hand side is the total payoff that player A and player 2 receive from carrying out the punishment, and the right-hand side is their total payoff from deviating from it. The condition can be simplified as

$$\delta^T \geq (1 - p_1) + p_1 \left(\sum_{t=0}^{T} \delta^t\right)^{-1}.$$

Observe that, for any $T \geq 1$, the foregoing condition is satisfied when δ is sufficiently close to 1, which leads to the following proposition.

Proposition 3 *With contingent contracts, for any integer $T \geq 1$, there is a skimming equilibrium, as described above, when δ is sufficiently close to* 1.

As δ tends to 1, the maximal T in a skimming equilibrium tends to infinity; thus, player 1's minimal expected share tends to zero and player A's maximal expected share tends to $1 - p_2$. Similarly, player A can play player 1 off against player 2, and her maximal expected share tends to $1 - p_1$ in the limit. Hence, when the players are extremely patient, the active player's greatest equilibrium payoff is arbitrarily close to what she could obtain from a bilateral bargaining with the weakest passive player. At the same time, a passive player's smallest equilibrium payoff tends to 0.

Corollary 2 *With contingent contracts, as δ tends to 1, the upper bound of the active player's expected equilibrium payoff tends to $1 - \min\{p_1, p_2\}$, and the lower bound of each passive player's expected equilibrium payoff tends to 0.*

3.2 A General Formulation

Although we have established the upper and lower bounds of each player's equilibrium payoff, our characterization of the equilibrium bargaining protocols is far from exhaustive. We now provide a general formulation of bargaining protocol and show that there is a vast multiplicity of equilibrium bargaining protocols.

Each session of consecutive bargaining periods with a specific passive player is now referred to as a *bargaining round*. Denote an underlying bargaining protocol as a sequence of natural numbers $\{K_s\}_{s=1}^{\infty}$, where K_s is the number of periods that the s^{th} bargaining round lasts before any agreement is reached.[11] When s is odd (even resp.), bargaining takes place between player A and player 1 (player 2 resp.). If an agreement is reached during any round of the bargaining, the active player immediately switches to another passive player to bargain over the remaining surplus.

Note that $K_1 = 0$ if player A chooses player 2 in period 0. Meanwhile, if $K_1 = 0$, then it must be that $K_2 > 0$. To accommodate the sequential protocol, we allow $K_s = \infty$. For example, $K_1 = \infty$ (or $K_1 = 0$ and $K_2 = \infty$) if player A adopts the sequential protocol starting from player 1 (or player 2). The basic alternate protocol corresponds to $K_s = 1$ for any s.

This formulation specifies only the active player's reduced strategies. For example, if $K_s = \infty$, then it does not specify $K_{s'}$ for any $s' > s$. It suffers no loss of generality though, once we assume that following the active player's deviation from the equilibrium protocol, the continuation equilibrium will be one with a sequential protocol. As the active player obtains the smallest payoff in equilibria with a sequential protocol, this constitutes the most severe

[11] If no agreement has been reached after K_s periods, bargaining enters the $(s+1)^{th}$ round, which lasts for at most K_{s+1} periods.

equilibrium punishment, which sustains the largest possible set of equilibrium protocols.

Finally, it is important to note that this formulation does not include the protocol adopted in a skimming equilibrium, in which the decision about whether to switch to another passive player depends on who has rejected the offer. A bargaining protocol in the form of $\{K_s\}_{s=1}^{\infty}$ is considered to be history-independent.

Proposition 4 *With contingent contracts, any bargaining protocol in the form of* $\{K_s\}_{s=1}^{\infty}$ *can be sustained in an equilibrium, and there is a unique sequence of equilibrium agreements associated with this protocol.*

Proof. As explained above, it is easy to sustain an arbitrary bargaining protocol in an equilibrium as any deviation from the protocol will be punished by switching to an equilibrium with a sequential protocol, in which the active player receives her smallest payoff. See the Appendix for the uniquely determined equilibrium outcome associated with an arbitrary protocol. ■

A closely related question is what would be the equilibrium outcome in a model with a fixed protocol in the form of $\{K_s\}_{s=1}^{\infty}$. For regular protocols, such as sequential and alternate protocols, it is easy to derive the equilibrium outcome. Doing so becomes more complicated when the bargaining protocol is irregular.[12] For example, with $\{K_s = s\}_{s=1}^{\infty}$, the number of periods in each bargaining round increases uniformly, and it is not immediately clear what the equilibrium agreements would be. The main difficulty is that we cannot establish a closed loop of indifference conditions as with the sequential or alternate protocol. Although our aim here is to characterize the set of self-enforcing bargaining protocols, our result also provides an answer to this question. A rather surprising finding is that any irregular bargaining protocol is associated with a unique equilibrium outcome that converges to the same limit as the equilibrium with the basic alternate protocol.

[12] See the Appendix for the categorization of regular and irregular bargaining protocols.

3.3 Markov Equilibria

When a bargaining game has multiple subgame perfect equilibria, stationary or Markov equilibrium is often proposed as a plausible refinement on the grounds that strategic simplicity is a desirable feature of equilibria.[13] In the one-to-many bargaining setting, the number of remaining passive players changes during the bargaining process, and thus Markov equilibrium is the proper solution concept.

A Markov strategy is a strategy that depends only on payoff-relevant variables. In the current model, these include the number of passive players who have not reached agreement and the remaining surplus. Thus, the active player's strategy is Markovian if among the same set of remaining passive players and with the same available surplus, (i) she always chooses the same one to bargain with or randomizes with the same probabilities, and (ii) in bargaining with a specific passive player, she always makes the same offer and accepts the same set of demands. A Markov equilibrium is a subgame perfect equilibrium in Markov strategies.[14] Lemma 3 establishes the efficiency of any Markov equilibrium.

Lemma 3 *A Markov equilibrium of the bargaining game with contingent contracts must be efficient.*

Proof. Lemma 2 shows that with contingent contracts, impasse is not an equilibrium outcome. If a Markov equilibrium involves an inefficient delay and the first agreement is reached in period $t > 0$, then player A can deviate by truncating her strategy from period t. Then, the same sequence of agreements will be reached without any delay, as all players adopt Markov strategies. The deviation is obviously profitable. ■

In Cai (2003), when there are two passive players, the game has three Markov equilibria for a sufficiently large discount factor, and one of them is inefficient. Lemma 3 suggests

[13] Herrero (1985) shows that in the N-player Rubinstein bargaining game, there is a unique stationary subgame perfect equilibrium. The restriction to stationary strategies (to obtain uniqueness) is sometimes considered problematic (see, for example, the discussion in Osborne and Rubinstein [1990]).

[14] The definition of Markov equilibrium does not preclude a player from deviating to non-Markovian strategies, but requires that no player can benefit from such a deviation.

that the inefficient Markov equilibrium is an artifact of the assumed bargaining protocol. More precisely, in Cai (2003), although the order of reaching agreement is endogenously determined, the bargaining order is exogenously given, and delay occurs when the two orders are inconsistent.

Lemma 4 *With contingent contracts, no Markov equilibrium exists in pure strategies.*

Proof. In a Markov equilibrium in pure strategies, player A always bargains with the same passive player, say player 1, for the first agreement. Thus, the outcome should be the same as in an equilibrium with a sequential protocol. However, if player A deviates to player 2, then the latter would expect the former to switch back to player 1 if no agreement were reached in the current period. Hence, player 2 would be willing to accept any offer greater than $\delta(1-p_1)p_2$, and player A to accept player 2's demand if it is less than $1-\delta(1-p_2)$. This makes the deviation profitable for player A. ■

Lemma 4 implies that equilibria with a sequential or alternate protocol are not Markov equilibria. Particularly, in an equilibrium with a sequential protocol, once the active player deviates and bargains with another passive player, it is crucial that the latter believes that the former will not switch again until agreement is reached, whereas a Markov equilibrium (in pure strategies) requires that after any history the active player chooses the same passive player to bargain with. The following proposition specifies the unique Markov equilibrium in mixed strategies.

Proposition 5 *With contingent contracts, there is a unique Markov equilibrium in mixed strategies in which the active player chooses each passive player with equal probability in each period before the first agreement is reached.*

Proof. See the Appendix. ■

In the unique Markov equilibrium, the active player always randomly chooses a passive player with probability 1/2 regardless of the heterogeneous bargaining power. As δ tends to 1, the Markov equilibrium outcome converges to the same limit as that of equilibria with an alternate protocol. Finally, the mixed-strategy equilibrium is robust in the following sense. If the passive players believe that the active player chooses player 1 with probability $q > 1/2$, then player 1 will ask for a greater offer and player 2 will reduce his demand, which, in turn, induces the active player to reduce q.

The fact that there is no Markov equilibrium in pure strategies raises the question of whether the Markov equilibrium notion is too restrictive in the current setting. Alternatively, we can also include the identity of the current bargaining opponent as a state variable; then, whether to switch to another opponent and which one to switch to are the decisions to make. It is easy to see that the equilibria with either sequential or alternate protocol become Markovian after the introduction of this extra state variable.

4 Cash-Offer Contracts

In many real-life situations, only a binding cash-offer contract is feasible. For example, in Coase's (1960) well-known railroad example, it is reasonable to assume that the negotiating parties are limited to cash-offer contracts. In fact, the recent literature on one-to-many bargaining has mainly focused on cash-offer contracts. This section presents the equilibrium characterization with this type of contract. The analysis is in the same vein as that in the previous section except that we defer our discussion of impasse as a possible equilibrium outcome to the end of the section.

For an arbitrary ordering of passive players, there is an equilibrium with a sequential protocol. Let player 1 be the one to reach agreement first. Player A and player 1 are effectively bargaining over a surplus of $(1 - p_2)$ because p_2 will be the expected payment to player 2 in the continuation equilibrium. Thus, in the equilibrium, player A offers $\delta p_1 (1 - p_2)$ and player 1 asks for $[1 - \delta (1 - p_1)] (1 - p_2)$. Once chosen to bargain, player 1 believes that player A will not switch to another player until an agreement has been reached. At the same time, if player A deviates by switching to player 2 before reaching the first agreement, then player 2 believes that she will switch back to player 1 in the next period.[15]

[15] This represents a subtle difference from the contingent contract case, in which whomever is chosen to

Note that although every ordering of passive players can be sustained in an equilibrium with a sequential protocol, player A obtains a greater expected payoff by bargaining with the weaker passive player first. This difference in payoffs vanishes when δ tends to 1.

Proposition 6 *Among all equilibria with a sequential protocol, player A obtains the greatest payoff when she bargains with the passive players in ascending order of bargaining power.*

Proof. Straightforward calculation. ∎

Recall that with contingent contracts, the active player obtains the smallest payoff in the equilibria with a sequential protocol. The case is different here. The active player obtains an even smaller payoff in an equilibrium with an alternate protocol when it exists, and impasse may occur when there are three or more passive players. Also, with contingent contracts, each passive player would prefer to be the first to reach agreement under a sequential protocol; here, in contrast, a passive player is better off being the last to reach agreement.

More interestingly, our next proposition shows that, with cash-offer contracts, in general there exists no equilibrium with an alternate protocol. Put alternatively, the alternate protocol is not self-enforcing in this case.

Proposition 7 *With cash-offer contracts, for δ sufficiently close to* 1*, there exists no equilibrium with an alternate protocol if either $p_1 + p_2 > 1$ or $p_1 \neq p_2$.*

Proof. See the Appendix. ∎

Being the second to reach agreement, player i receives an expected payment of p_i. Thus, in an equilibrium with an alternate protocol, if player i is chosen to bargain in period 0, then he will reject any offer less than $\delta^2 p_i$. When $p_1 + p_2 > 1$, there exists no equilibrium with an alternate protocol when δ is sufficiently close to 1 because the active player's payoff from such an equilibrium would be negative.

When $p_1 = p_2 \leq 1/2$, there is an equilibrium with an alternate protocol for any $\delta \in (0, 1)$. The active player has no preference about which passive player to bargain with first. However,

bargain believes that player A will not switch again before reaching an agreement.

this is only a non-generic case. More generally, when $p_1 \neq p_2$ and $p_1 + p_2 \leq 1$, there exists no equilibrium with an alternate protocol for any $\delta \in (0, 1)$. The reason is that the active player always finds it optimal to bargain with the stronger passive player, say player 1, first. Thus, after a disagreement with player 1, player A will not switch to player 2 in the following period. This deviation cannot be punished, as player A is already receiving her smallest payoff in the assumed equilibrium with an alternate protocol.

Finally, it is straightforward to construct skimming equilibria with cash-offer contracts. Recall that the key step is to ensure that it is optimal for the active player to carry out the punishment imposed on a passive player. With cash-offer contracts, doing so actually becomes even easier, as the lower bound of the active player's equilibrium payoff is smaller than that with contingent contracts.

Markov Equilibria. With cash-offer contracts, the equilibrium with a sequential protocol is a Markov equilibrium. However, this equilibrium differs from that with contingent contracts in terms of what happens off the equilibrium path. More specifically, if the active player deviates and bargains with another player, then this player believes that the active player will switch back in the following period, and thus no agreement can be reached and the deviation is not profitable.

Proposition 8 *With cash-offer contracts, there are multiple Markov equilibria for any* $\delta \in (0, 1)$*: (i) every equilibrium with a sequential protocol is a Markov equilibrium in pure strategies; (ii) when* $p_1 + p_2 \leq 1$*, there is also a Markov equilibrium in mixed strategies.*

Proof. Part (i) has been explained above. See the Appendix for Part (ii). ■

In the mixed-strategy Markov equilibrium, player A randomizes the choice of passive player in each period, and each player i is chosen with probability

$$q_i^* = \frac{1 - p_i}{(1 - p_1) + (1 - p_2)}.$$

As δ tends to 1, player i's expected payoff tends to p_i, and player A's expected payoff tends to $1 - p_1 - p_2$.

Note that player i's demand depends on his belief about the probability that he will be chosen to bargain in each period. If player i's perceived probability is higher than q_i^*, then his demand will be lower, and it then becomes profitable for player A to further increase the probability of choosing player i. In this sense, the mixed-strategy equilibrium is not robust, in contrast to the case with contingent contracts.

Impasse. As previously noted, with cash-offer contracts, impasse can be an equilibrium outcome. It is easy to see that impasse cannot occur when there are only two passive players.[16] Lemma 5 shows that when the two passive players are endowed with sufficiently high bargaining power, there is an equilibrium in which player A's expected payoff is 0.

Lemma 5 *In the game with two passive players and $p_1 + p_2 > 1$, when δ is sufficiently close to 1, there is an equilibrium in which player A's expect payoff is 0.*

Proof. Consider the following strategy profile. Player A randomly chooses a passive player in period 0. Before any agreement has been reached, in each period t, player A offers the chosen player i a payment of $\delta(1-p_j)$, and accepts player i's demand if it is no greater than $\delta(1-p_j)$; the chosen player i asks for $\delta(1-p_j)$, and accepts any offer that is no less than $\delta(1-p_j)$; when there is a disagreement in period t, player A switches to another passive player in period $t+1$ if and only if the disagreement is caused by her own deviation. Following this strategy profile, player A's expected payoff is 0.

It is easy to see that the chosen passive player cannot benefit from any possible deviation. If player A offers player i less than $\delta(1-p_j)$, then it is optimal for him to reject when $\delta^2 p_i > \delta(1-p_j)$. As $p_1 + p_2 > 1$, this is satisfied when δ is sufficiently close to 1. ■

When there are three passive players and $p_i + p_j > 1$ for any $i \neq j$, Lemma 5 shows that in the subgame with two remaining passive players, there is an equilibrium in which the active player's expected payoff is zero. With this as the continuation equilibrium, impasse

[16] When a passive player anticipates an impasse in an equilibrium, he is willing to accept any positive offer, and his acceptance will lead to an immediate agreement between the active player and the other passive player, which upsets the impasse equilibrium.

is an equilibrium outcome of the entire game because, if the active player agrees with any passive player on a positive payment, then her expected payoff becomes negative.

In the game with two passive players, there are multiple equilibria from which the active player obtains different payoffs. As in Cai (2000), we can construct an equilibrium for the game with three passive players, in which the active player's expected payoff is zero. This is true even when $\sum_{i=1}^{3} p_i < 1$.[17] Hence, impasse is always an equilibrium outcome when $N \geq 4$.

Proposition 9 *When δ is sufficiently close to 1, impasse is an equilibrium outcome if (i) there are at least four passive players or (ii) there are three passive players and $p_i + p_j > 1$ for any i and j.*

Proof. See the Appendix. ■

When there are many passive players, impasse becomes a possible equilibrium outcome, which gives the passive players an incentive to merge. Intuition suggests that weak negotiating parties have the incentive to merge to achieve high collective bargaining power, but here we see that negotiating parties with high bargaining power may also be willing to merge to ensure that agreement becomes possible.

5 Implications for Unionization

Horn and Wolinsky (1988) consider wage bargaining between an employer and two groups of workers. Assuming that the employer alternates in negotiating with one of the two groups, the bargaining game has a unique equilibrium. More importantly, when the two groups of workers are close substitutes, they prefer to form an encompassing union and bargain collectively; when they are strong complements, in contrast, they prefer to form two independent unions and bargain separately. In this paper, we focus on the extreme case in which the passive players are perfect complements. The existence of skimming equilibria suggests that

[17] In such an equilibrium, the passive player i reaching the first agreement may receive an expected payoff greater than p_i, which is impossible if player i is one of the two remaining passive players.

in separate bargaining, the employer may play off one group of workers against the other to reduce overall wages; thus, the two groups, even as perfect complements, still have the incentive to bargain collectively.

To be more precise, consider two unions with possibly different bargaining power, $p_1 \leq p_2$, where p_i is the relative bargaining power of union i when it bargains with the employer independently. When the two unions merge, it is unclear how the bargaining power of the encompassing union should be determined. Denote as $p = g(p_1, p_2) \in (0, 1)$ the bargaining power of the encompassing union. It is reasonable to assume that $g(p_1, p_2)$ is greater than and increasing with p_i. As a special case, let us further assume that $p = \max\{p_1, p_2\} = p_2$.

Consider the following two-stage game. In stage 1, the two unions decide whether or not to merge. A wage bargaining game with an endogenous protocol is played in the second stage.[18] If the two unions bargain separately, then the expected total wage in one skimming equilibrium approaches p_1 as δ tends to 1. If they merge and bargain collectively, then there is a unique equilibrium in which the expected total wage is p_2 for any $\delta \in (0, 1)$. Hence, if the two unions have different bargaining power, i.e., $p_1 < p_2$, then, when δ is close to 1, there is an equilibrium in which they merge in stage 1. Note that when the unions have the same bargaining power, it can never be optimal for them to merge. In other words, unions with highly asymmetric bargaining power are more likely to merge.

6 Further Discussion

This paper studies a one-to-many bargaining model in which the active player is endowed with the power to choose which passive player to bargain with in each period. Various bargaining protocols may arise as part of the active player's equilibrium strategy. In our model, the control over the protocol is highly asymmetric. Conceivably, there could be a model in which the passive players also have some control over the protocol. For example, at the beginning of each period, every passive player decides whether to make himself available for bargaining. The active player then decides which of those available to bargain with. It

[18] It is reasonable to work with contingent contracts in wage bargaining.

is not surprising that the set of equilibrium outcomes would then expand in favor of the passive players. However, from a practical point of view, we believe that it is much easier for the active player to "hide" from the passive players than the other way around.

In our model, the contract form is assumed to be exogenously given, which seems to be a realistic description of many real-life situations. However, it would also be natural to wonder what happens if the contract form is also endogenously determined. We argue that when the players can choose between two types of contracts during the bargaining, contingent contracts are more likely to be adopted than cash-offer contracts. To see this, consider the case with two passive players. Suppose that in an equilibrium the first agreement is enforced by a cash-offer contract. It could always be replaced with a contingent contract with different terms, such that both parties will be better off if they are sufficiently patient. This is true simply because the payment in a contingent contract will not become a sunk cost, and it can force the remaining passive player to accept a smaller offer.

Finally, there are two potential modeling approaches to endogenizing bargaining protocol: one is to allow the protocol to form and evolve during the negotiation process, and the other is to let the negotiating parties choose and commit to a protocol prior to the negotiations. Although this paper adopts the former approach, it would be interesting to explore the latter approach in future research.

Appendix

Proof of Proposition 4. (General Formulation of Bargaining Protocol)

The following two cases are referred to as *regular protocols*: (1) $K_{s^*} = \infty$ for some s^*, and (2) there exist s^* and $l \geq 1$ such that $K_{s+2l} = K_s$ for any $s \geq s^*$. With a regular protocol, the equilibrium outcome can be derived by backward induction. More specifically, in Case 1, starting from period $t^* = 1 + \sum_{s=1}^{s^*-1} K_s$, the active player adopts a sequential protocol, which determines a unique continuation equilibrium outcome. Then, in each period $t < t^*$, the randomly selected proposer makes an offer such that the responder is indifferent between accepting and rejecting. By backward induction, we can uniquely determine the equilibrium outcome. In Case 2, starting from period t^*, the active player adopts a general alternate protocol, which also has a unique equilibrium outcome. Then, backward induction applies again. It is easy to see that in a general alternate protocol, as $\delta \to 1$, the equilibrium outcome converges to the same limit as the equilibrium with the basic alternate protocol.

When $\{K_s\}_{s=1}^{\infty}$ does not belong to these two cases, the main difficulty is that we cannot establish a closed loop of indifference conditions as in the equilibrium with a sequential or alternate protocol. We refer to such protocols as *irregular protocols*. Again, there are two cases to be considered.

Case 3. For any $T > 0$, there exist s^* such that $K_s > T$ for some $s > s^*$. In other words, the bargaining protocol approaches a sequential protocol as $s \to \infty$. The example with uniformly increasing length of bargaining rounds ($K_s = s$) belongs to this case.

Case 4. There exists T^* such that $K_s \leq T^*$ for any $s > 0$. Then there must exist a general alternate protocol P^* and an unbounded sequence of l_n (l_n is even) consecutive bargaining rounds (or, segment of length l_n) such that the bargaining protocol $\{K_s\}_{s=1}^{\infty}$ is consistent with P^* on each segment. In other words, the bargaining protocol approaches P^* as $s \to \infty$.

Below it is shown that, as $\delta \to 1$, the equilibrium outcome associated with an irregular protocol converges to the same limit as the equilibrium with the basic alternate protocol. Without loss of generality, assume that $K_1 > 0$.

Denote as (x_l^s, y_l^s) the equilibrium offers made in the l^{th} period of the s^{th} round of the bargaining, where $1 - x_l^s$ is the offer to the passive player of the current round and $1 - y_l^s$ is his demand. Let $B_l^s = (1 - p_i) x_l^s + p_i y_l^s$, where $i = 1$ (2 resp.) if s is odd (even resp.). In each period, the randomly selected proposer makes an offer such that the responder is indifferent between accepting and rejecting. It is easy to establish that

$$B_{K_s}^s = (1 - p_i) + \frac{\delta p_1 p_2 (1 - p_i)}{1 - p_j} B_1^{s+1}$$

and

$$B_{l-1}^s = (1 - \delta)(1 - p_i) + \delta B_l^s,$$

by which we obtain

$$B_1^s = (1 - p_i) + \frac{\delta^{K_s} p_1 p_2 (1 - p_i)}{1 - p_j} B_1^{s+1}.$$

Hence, we have

$$B_1^s = (1 - p_i) \left[1 + \sum_{n=0}^{\infty} \delta^{D_n} (p_1 p_2)^{n+1} \right],$$

where

$$D_n = \sum_{l=0}^{n} K_{s+l}.$$

As $\delta \to 1$, $B_1^s \to (1 - p_i) / (1 - p_1 p_2)$, and it follows that

$$\lim_{\delta \to 1} \left(1 - x_1^1\right) = \lim_{\delta \to 1} \left(1 - y_1^1\right) = \frac{p_1 (1 - p_2)}{1 - p_1 p_2},$$

which is the limit of player 1's expected payoff in the equilibrium with an alternate protocol.

Proof of Proposition 5. (Mixed-Strategy Markov Equilibrium: Contingent Contracts)

Formally, a mixed-strategy Markov equilibrium can be written as $\{q, (x_1, y_1), (x_2, y_2)\}$, where q is the probability that player A chooses player 1 in each period, $1 - x_i$ is the offer made to player i, and $1 - y_i$ is his demand. In the equilibrium, each player is indifferent between (1) accepting the current offer which leads to the conclusion of the bargaining in the following period, and (2) rejecting the current offer, in which case the bargaining is concluded two periods later.

Thus, the following indifference conditions hold:

$$\begin{cases} 1-x_1 = \delta q\left[(1-p_1)(1-x_1)+p_1(1-y_1)\right]+\delta(1-q)\,p_1\left[(1-p_2)\,x_2+p_2y_2\right] \\ 1-x_2 = \delta q p_2\left[(1-p_1)\,x_1+p_1y_1\right]+\delta(1-q)\left[(1-p_2)(1-x_2)+p_2(1-y_2)\right] \\ (1-p_2)\,y_1 = \delta\left[qE_{A_1}+(1-q)\,E_{A_2}\right] \\ (1-p_1)\,y_2 = \delta\left[qE_{A_1}+(1-q)\,E_{A_2}\right] \end{cases}$$

where

$$\begin{aligned} E_{A_1} &= (1-p_2)\left[(1-p_1)\,x_1+p_1y_1\right] \\ E_{A_2} &= (1-p_1)\left[(1-p_2)\,x_2+p_2y_2\right]. \end{aligned}$$

For $q \in (0,1)$, player A should be indifferent about her choice between player 1 and 2, i.e., $E_{A_1} = E_{A_2}$.

Hence, there are five unknowns and five equations. After tedious algebra, we obtain a unique solution with $q^* = 1/2$. Note that player A's randomization probability does not depend on δ and p_i. As $\delta \to 1$, player A's expected payoff goes to $\lambda(1-p_1)(1-p_2)$ and player i's goes to $\lambda p_i(1-p_j)$, where $\lambda = (1-p_1p_2)^{-1}$.

Proof of Proposition 7. (Equilibrium with an Alternate Protocol: Cash-offer Contracts)

Suppose that there is an equilibrium with an alternate protocol, which can be formally written as $\{(x_1,y_1),(x_2,y_2)\}$, where $1-x_i$ is the offer to player i, and $1-y_i$ is his demand. Then, the following indifference conditions hold:

$$\begin{cases} 1-x_1 = \delta^2 p_1 \qquad \text{and} \qquad 1-x_2 = \delta^2 p_2 \\ \delta(1-p_2)-(1-y_1) = \delta\left[(1-p_2)\left(\delta(1-p_1)-\delta^2 p_2\right)+p_2\left(\delta(1-p_1)-(1-y_2)\right)\right] \\ \delta(1-p_1)-(1-y_2) = \delta\left[(1-p_1)\left(\delta(1-p_2)-\delta^2 p_1\right)+p_1\left(\delta(1-p_2)-(1-y_1)\right)\right], \end{cases}$$

from which we can solve for (x_1,y_1) and (x_2,y_2).

It is easy to check that player A's expected payoff is

$$Eu_A^{12} = \frac{\delta(1-p_1)(1-p_2) - \delta^2 p_1 p_2 (1-p_1) - \delta^3 p_1 p_2 (1-p_2)}{1-\delta^2 p_1 p_2}$$

if player 1 is the first one to reach agreement, and it is

$$Eu_A^{21} = \frac{\delta(1-p_1)(1-p_2) - \delta^2 p_1 p_2 (1-p_2) - \delta^3 p_1 p_2 (1-p_1)}{1-\delta^2 p_1 p_2}$$

if player 2 is the first one to reach agreement. Moreover,

$$\lim_{\delta \to 1} Eu_A^{12} = \lim_{\delta \to 1} Eu_A^{21} = (1 - p_1 - p_2)$$

and

$$Eu_A^{12} \geq Eu_A^{21} \text{ if and only if } p_1 \geq p_2.$$

Hence, if $p_1 + p_2 > 1$, then the equilibrium with an alternate protocol is not viable when δ is sufficiently close to 1, as player A would receive a negative payoff.

More importantly, if $p_1 \neq p_2$, then the equilibrium with an alternate protocol is not viable even when $p_1 + p_2 < 1$. This is because player A always finds it optimal to reach the first agreement with the stronger passive player, say player 1. Thus, after a disagreement with player 1, player A will not switch to player 2 in the following period. This deviation cannot be punished because player A already receives her smallest payoff from the assumed equilibrium.

Proof of Proposition 8. (Mixed-Strategy Markov Equilibrium: Cash-offer Contracts)

Formally, a mixed-strategy Markov equilibrium can be written as $\{q, (x_1, y_1), (x_2, y_2)\}$, where q is the probability that player A chooses player 1 in each period, $1 - x_i$ is the offer made to player i, and $1 - y_i$ is his demand. Similar to the case with contingent contracts, the following conditions hold:

$$
\begin{cases}
1 - x_1 = \delta q \left[(1 - p_1)(1 - x_1) + p_1 (1 - y_1) \right] + \delta^2 (1 - q) p_1 \\
1 - x_2 = \delta (1 - q) \left[(1 - p_2)(1 - x_2) + p_2 (1 - y_2) \right] + \delta^2 q p_2 \\
\delta (1 - p_2) - (1 - y_1) = \delta \left[q E_{A_1} + (1 - q) E_{A_2} \right] \\
\delta (1 - p_1) - (1 - y_2) = \delta \left[q E_{A_1} + (1 - q) E_{A_2} \right],
\end{cases}
$$

where

$$
\begin{aligned}
E_{A_1} &= (1 - p_1) \left[\delta (1 - p_2) - (1 - x_1) \right] + p_1 \left[\delta (1 - p_2) - (1 - y_1) \right] \\
E_{A_2} &= (1 - p_2) \left[\delta (1 - p_1) - (1 - x_2) \right] + p_2 \left[\delta (1 - p_1) - (1 - y_2) \right].
\end{aligned}
$$

In the mixed-strategy equilibrium, player A is indifferent about her choice between player 1 and 2, i.e., $E_{A_1} = E_{A_2}$, which can be simplified as

$$
(1 - p_1) x_1 + p_1 y_1 - \delta p_2 = (1 - p_2) x_2 + p_2 y_2 - \delta p_1.
$$

Solving for q, we obtain

$$
q^* = \frac{1 - p_1}{(1 - p_1) + (1 - p_2)}.
$$

As δ tends to 1, player i's expected payoff tends to p_i and player A's tends to $1 - p_1 - p_2$. The equilibrium is viable for any $\delta \in (0, 1)$ when $p_1 + p_2 \leq 1$.

The mixed-strategy equilibrium is not robust in the following sense. Let q be the passive players' belief about player A's strategy, and $1 - x_i(q)$ and $1 - y_i(q)$ be player i's smallest acceptable offer under this belief. If $q > q^*$ ($q < q^*$ resp.), then, given $x_i(q)$ and $y_i(q)$, player A finds it profitable to further increase (reduce resp.) q.

Proof of Proposition 9. (Impasse: Cash-offer Contracts)

It suffices to construct an equilibrium for the game with $N = 3$, in which player A's expected payoff is 0. It has been shown that, in the game with $N = 2$, there are multiple equilibria in which player A obtains different payoffs. Denote as $E_1(i, j)$ and $E_2(i, j)$ two equilibria in the subgame with passive players i and j. Denote as $U_s(i,j)$ player A's expected payoff from $E_s(i,j)$. Without loss of generality, assume that $U_1(i,j) > U_2(i,j) > 0$.

Consider the following strategy profile in the game with three passive players i, j and k:

(1) Player A randomly chooses a passive player in period 0.

(2) In period t, player A offers $\delta U_2(i,j)$ to the chosen player k and accepts player k's demand if and only if it is no greater than $\delta U_1(i,j)$.

(3) If no agreement is reached in period t, player A chooses player k again in period $t+1$.

(4) In period t, the chosen player k asks for $\delta U_1(i,j)$ and accepts any offer no less than V_k, where V_k is his continuation payoff, i.e.,

$$V_k = \sum_{l=0}^{\infty} \delta^{l+2} p_k (1-p_k)^l U_1(i,j) = \frac{\delta^2 p_k U_1(i,j)}{1-\delta(1-p_k)}.$$

(5) If player A's offer is accepted by player k in period t, then the continuation equilibrium is $E_2(i,j)$; if player k's demand is accepted, then the continuation equilibrium is $E_1(i,j)$.

With this strategy profile, the first agreement is reached when player k is recognized as a proposer for the first time, and then, $E_1(i,j)$ is played. Player A's expected payoff is 0. To verify that the strategy profile is an equilibrium, first observe that player A is indifferent about her choice of opponent to bargain for the first agreement, as her expected payoff is always 0. Second, player A cannot make an offer that is greater than $\delta U_2(i,j)$ because if such an offer is made and accepted, player A's expected payoff would be negative. Hence, it remains to show that

$$\delta U_2(i,j) < V_k = \frac{\delta^2 p_k U_1(i,j)}{1-\delta(1-p_k)}.$$

When δ is sufficiently close to 1, the foregoing condition is satisfied.

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