Strategic Voting in a Jury Trial with Plea Bargaining

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Abstract

We study a model of the criminal court process focusing on the interaction between plea bargaining and a jury trial. A prosecutor and a defendant participate in plea bargaining while anticipating possible outcomes of the jury trial. We assume that plea bargaining produces a bias in which the jury believes the defendant is less likely to be guilty if the case goes to trial. Consequently, the bias alters the trial outcome which is assumed to follow a strategic voting model. We find that the equilibrium behavior in the court process with plea bargaining and a jury trial resembles the equilibrium behavior in the separate jury model. However, unlike in the case of jury model, the jurors may act as if they have the prosecutor's preference against convicting the innocent and acquitting the guilty.

1 Overview

1.1 Introduction

The U.S. criminal court system has numerous steps which allow attorneys and defendants to actively participate throughout the process. Although the details differ from state to state, in general the criminal court process consists of an arrest, preliminary hearings, plea negotiations, a jury trial, and a verdict. Most people think that all sentences are delivered by a jury trial, but a significant number of cases are resolved in a pre-trial stage. Plea bargaining is one such case where a defendant is allowed to plead guilty in exchange for a lenient charge. Plea bargaining is so prevalent that, among 88,094 defendants during 2006, 76,778 (or 87%) were terminated by pleading guilty or no-contest.[1] The fact that litigation ends in the plea bargain stage in

[1] Bureau of Justice Statistics in Office of Justice Programs, U.S. Department of Justice. `http://www.ojp.usdoj.gov/bjs`

the vast majority of cases causes people to believe that a trial is not important. However, this conclusion is inaccurate since the trial directly follows when the participants in the plea bargain fail to reach an agreement.

In fact, research in legal studies suggests that plea bargaining and a jury trial closely interact with each other, and this interaction plays a significant role in the entire court process. Although most cases are settled before the jury trial, participants in the plea bargain anticipate possible outcomes of the jury trial if they fail to reach an agreement. In this sense, a primary role of a jury trial may be allocating bargain power to each side of plea bargain participants rather than handling cases directly.[2] On the other hand, the jury trial hinges on the consequences of the plea bargain. The incentive to plead guilty differs if the defendant is truly guilty or innocent, so cases with innocent defendants tend to go to the trial. The jury trial incorporate this selection bias in its verdict.[3]

In this paper, we study a model of a criminal court process focusing on the interaction between plea bargaining and a jury trial. While the previous literature studies either plea bargaining assuming an exogenously given trial behavior, or a jury trial assuming an exogenous litigation process, our model allows the plea bargain and the jury trial to interact with each other in a unified model. A prosecutor and a defendant participate in plea bargaining while anticipating possible outcomes of the jury trial, and the jurors incorporate that the defendant whom they face denied the crime and pleaded not guilty. The pleading decision and the jury trial behavior resembles a signaling game. Given a plea bargain punishment, a defendant, as a sender, signals his type by pleading either guilty or not guilty. Then the jury, as a receiver, updates the belief on the sender's type and determines conviction probabilities.

Consider for example that the jurors believe a certain proportion of the defendants in the jury trial are guilty. During the trial, each juror obtains additional information on the defendant and decides whether to vote for conviction or acquittal. Intuitively, a guilty defendant will have a higher chance to be convicted than an innocent defendant. The conviction probabilities (one for guilty and the other for innocent defendants) become higher as jurors believe that more guilty defendants come to trials.

Given a plea bargain offer by the prosecutor, a defendant compares the offer and the outcome

[2]Mnookin and Kornhauser (1979) represent this observation as "Bargaining in the shadow of the law".

[3]The strategic voting model captures this as a belief on the prior probability of a guilty defendant which in turn affects the conviction probability.

of the jury trial. Basically, a defendant pleads guilty if the bargain offers less punishment than the expected punishment from the jury trial. However, since this pleading decision affects the jurors' belief on the proportion of guilty defendants, it is not clear what will be the final outcome. Our intuition is as follows. (1) If the bargain offer for guilty defendants is acceptable compared to the jury trial behavior, all guilty defendants plead guilty. Then the jurors update their belief on the proportion of the guilty defendants in the trial and lower conviction probabilities. Then the bargain offer becomes unacceptable. (2) If the bargain offer is unacceptable, the opposite story follows. As the jurors assumes that more guilty defendants come to trial, the jurors tend to increase the conviction probabilities. Then the bargain offer may become acceptable. In general, the consequences of the punishment from pleading guilty and of the expected punishment from the trial would become equivalent for 'guilty' defendants. Since innocent defendants have less chance to be convicted in the trial, they will not plead guilty. Therefore, the ex-ante punishment levels for guilty and innocent defendants are the same as the conviction probabilities in the jury trial.

The prosecutor's objective is to deliver punishment to guilty defendants while minimizing mistakes of punishing innocent defendants. To achieve the objective, the prosecutor controls the level of the plea bargain offer. Observations in the previous paragraph imply that the prosecutor may want to manipulate the jury trial behavior so that it renders the expected levels of punishment ideal. We later show that such manipulation leads each juror to vote as if she has the prosecutor's preference against convicting the innocent and acquitting the guilty. However, such manipulation is possible only if the prosecutor cares more than jurors about the mistakenly delivered punishment to innocent defendants.

Our study generalizes the strategic voting model beyond the jury trial to the criminal court process. In strategic voting literature, it is a convention to assume that litigation is exogenously given. However, defendants and prosecutors actively participate in pre-trial stages, so implications of the strategic voting model may not be directly applicable to the entire court process. By attaching a model on plea bargaining to the strategic voting model, we show that the model can be nicely extended to cover the complete court process.

As an example, we compare two voting paradigms, the unanimity and non-unanimous rules. Feddersen and Pesendorfer (1998) compare these paradigms in a jury trial context and conclude that the unanimity rule is inferior. The probabilities of convicting the innocent and acquitting the guilty do not vanish as the number of jurors get large, whereas these probabilities vanish to zero under any non-unanimous rule. We show that this conclusion is preserved under plea bargaining.

This paper also sheds light on an economic justification of plea bargain, which is *not* motivated by saving trial costs.[4] We assume that a trial is free. Not only are explicit costs such as time and efforts excluded, but all players are also assumed to be risk neutral; they are unafraid of uncertain jury trial outcomes. Plea bargaining allows the court to screen out some guilty defendants before going to a jury trial. The accused know whether they are guilty, and plea bargaining serves as a self-selection mechanism. By doing so, it may contribute to the accuracy of the jury trial which the entire court performance hinges on.

1.2 Related Literature

Our paper shares motivations in several other papers exploring strategic behavior in a criminal court process. Related literature can be divided into those studying jury trials and those studying plea bargaining.

Jury Trial A formal study on collective decision making under uncertainty is motivated by the Condercet jury theorem (Condorcet (1785)). Suppose there are two possible true states. Condorcet models a situation in which a group of people, each of whom is partially informed about the true state, makes a decision by voting for one alternative. Although the members have a common interest to choose the true state, imperfect private information generate conflicts of interest at the time of voting. The theorem says that the group can efficiently aggregate private information and achieve a better decision with simple majority rule than if each member acts alone. An implicit assumption of the model is that each juror's voting behavior is exogenously given; each juror votes following her private information.

However, several recent researches illustrate that such action is not consistent with Bayesian Nash equilibrium behavior. (Austen-Smith and Banks (1996); Feddersen and Pesendorfer (1996)) Basic intuition departs from the fact that a vote affects the group decision only when the juror is pivotal. A strategic juror incorporate this in her voting decision and decides which to vote under the condition in which she is pivotal. Suppose for example that the voting rule is the unanimity. Even when private information is more likely from a certain alternative, her pivotal state convinces her to follow other jurors against her private information. This strategic vot-

[4] This paper is not about the legal justice of plea bargaining.

ing behavior is also evidenced by experimental studies. (Guarnaschelli, McKelvey, and Palfrey (2000); Goeree and Yariv (2010))

Motivated by the strategic voting hypothesis, Feddersen and Pesendorfer (1998) apply the model to a criminal court trial. One of the main results is the inferiority of the unanimity rule. As the number of jurors gets large, the probabilities of convicting the innocent and acquitting the guilty do not vanish to zero under the unanimity, whereas those probabilities converge to zero under any non-unanimous rule. While our paper revisits this comparison with an extension of plea bargaining and find that the inferiority persists, Coughlan (2000) illustrates somewhat opposite results under extension to mistrial or limited communication among the jurors. Coughlan points out that a disagreement under unanimity does not automatically yield acquittal, but rather mistrial; if a mistrial always results in a new trial, the probability of trial error is minimized under the unanimity rule. Meanwhile, suppose jurors have a chance to reveal private information before the final decision. If there exists a non-unanimous rule where sincere revelation and sincere voting are an equilibrium behavior, then it is also an equilibrium behavior under the unanimity rule. Since all voting mechanisms yield the same probabilities of trial errors when jurors reveal information and vote sincerely, the unanimity rule is not uniquely inferior.

There are several related papers under the context of jury deliberation. Austen-Smith and Feddersen (2003) and Gerardi and Yariv (2007) interpret jury deliberation as a bayesian communication game. In addition to finding that the inferiority of the unanimity persists, these studies show that non-unanimous voting rules with jury deliberation generate the same set of equilibrium outcomes. Thus an exact voting rule is not crucial in the final decision. An experimental study on jury deliberation by Goeree and Yariv (2010) finds that deliberation significantly diminishes the differences in the outcomes from various voting mechanisms. They also provide additional evidence of the inferiority of the unanimity rule with the lack of jury deliberation.

Plea Bargaining Most of the literature on plea bargain approaches the process via a 'bargaining' model. A jury trial costs time and effort. If the participants in a plea bargain do not want to bear additional risks, uncertainty in the trial outcome is an additional cost. Given such costs, participants in the plea bargain obtain mutual surplus by reaching an agreement, and this surplus division is a bargaining problem. A typical literature has a framework in which there are informational asymmetries and a variety of sequences by which bargaining offers are made by one or both parties (for a brief literature review on this topic, see Cooter and Rubinfeld

(1989)).

It is undeniable that plea bargaining originated as a way of avoiding jury trial costs at the beginning. However, its welfare effects on other than trial costs have received less attention. Grossman and Katz (1983) show that the plea bargain serves as an insurance and screening device. In the former role, it protects the innocent and society against cases where a trial process produces incorrect findings and delivers severe punishment. Although innocent defendants may be led by punishment falsely to plead guilty, the punishment will be lenient in this case. In the latter role, plea bargain sorts the guilty and innocent as a self-selection mechanism. Since the mechanism ensures that violators of the law are indeed punished, it may contribute to the accuracy of the legal system.

Priest and Klein (1984) study litigation rather than plea bargaining, but it is one of the closest studies to our paper. Priest and Klein model a litigation process that clarifies the relationship between the set of disputes settled and the set litigated. An important assumption is that the potential litigants produce rational estimates of the likely decision by possibly biasing the belief of the jury. The paper shows the disputes selected for litigation are determined endogenously, and they may differ from a representative sample of the set of all disputes. The motivation coincides with of our paper in the sense that jury trial models disregarding endogenous settlement may give inaccurate implications. While Priest and Klein informally model how the biased jury belief affects the jury decision, we construct a jury decision process through exploiting the strategic voting model.

2 The Model

A criminal court process begins with a prosecutor indicting a suspect. We assume that the defendant is either guilty (G) or innocent (I), which occur with equal probabilities. [5]

1. Plea Bargaining:

   The prosecutor suggests a take-it-or-leave-it plea bargain offer with $\theta \in [0, 1]$ proportion of the original charge. The defendant can plead either *guilty* or *not guilty*. If the defendant pleads guilty, the case terminates and the punishment θ is delivered. Otherwise, the plea bargain is withdrawn, and the case goes to a jury trial. A plea bargain gives the defendant an opportunity to avoid the judgment of conviction on the original charge.

[5] We refer prosecutors and defendants male, and jurors female.

2. A Jury Trial:

   Our jury model is based on a strategic voting hypothesis in Feddersen and Pesendorfer (1998). A jury consists of $n(n > 1)$ jurors and a voting rule $\hat{k}(0 < \hat{k} \leq n)$. During the trial, each juror interprets testimony by witnesses. We follow much of the strategic voting literature and represent this interpreting that each juror receives a private signal g or i which is positively correlated with the true states as given by

$$P[g|G] = P[i|I] = p, \quad P[i|G] = P[g|I] = 1 - p \tag{1}$$

   where $p \in (.5, 1)$; a juror receives a correct signal with probability p and the incorrect signal with probability $1 - p$.

   The jury reaches a decision by casting votes simultaneously. Each juror can vote for either conviction or acquittal. If the number of conviction votes is larger than the voting rule $\hat{k}$, the defendant is convicted (C). Otherwise, the defendant is acquitted (A). The punishment accompanied by C and A are normalized by 1 and 0 respectively. Consequently, the punishment by pleading guilty becomes θ.

Our model assumes that all players behave rationally where each acts to maximize an appropriately defined utility function. The defendant's utility changes negatively by the amount of punishment; -1 if he is convicted, 0 if he is acquitted, and $-\theta$ if he pleads guilty. He is assumed to be risk neutral; if he perceive that he will be convicted with probability s, then the ex ante utility of going to trial is $s \cdot 1 + (1 - s) \cdot 0$. The defendant wants to minimize the punishment and thus maximize his expected utility.

All jurors have an identical preferences. We normalize the utility functions so that correct judicial decisions incur no utility gain or loss: $u[C|G] = u[A|I] = 0$. Given this normalization, convicting innocent or acquitting guilty defendants incur utility losses, $u[C|I] = -q$ and $u[A|G] = -(1 - q)$, respectively where $q \in [.5, 1)$.[6][7]

Finally, we assume that the prosecutor has a preference defined on $[0, 1] \times \{\text{G, I}\}$. Much like the jurors', when the punishment $h \in [0, 1]$ is delivered to a defendant, the prosecutor's utility

[6] Suppose a juror believes that the defendant is guilty with probability $\tilde{q}$. The expected utility of a guilty verdict $(-q(1 - \tilde{q}))$ is greater than or equal to the expected utility of an innocent verdict $(-(1 - q)\tilde{q})$ if and only if $\tilde{q} \geq q$. Therefore when jurors vote for conviction, they use q as the threshold level of belief that the defendant is guilty. In this respect, Feddersen and Pesendorfer (1998) term q "the threshold level of reasonable doubt."

[7] $q < 0.5$ requires additional technical conditions, but the analysis is qualitatively intact.

is given by

$$v[h|G] = -(1-q')(1-h) \quad , \quad v[h|I] = -q'\ h$$

where $q' \in [0, 1]$.

The prosecutor is assumed to act in a state's or a social planner's interest. Critics may argue that we should alternatively consider a self-interested prosecutor who would maximize for example the total sum of delivering punishment or the average conviction probability in trial. However in actual situations, because a mistakenly managed case may becomes public later, such case will affect a prosecutor's future career. A self-interested prosecutor will be concerned with false prosecutions. We represent this concern with flawed cases with a parameterized weight, q'.[8]

Figure 1 summarizes the timing of the model. (1) A prosecutor offers θ as a lenient sentence in a plea bargain. (2) The defendant pleads either guilty or not guilty. (3) If the defendant pleads guilty, a judge respects the bargain and pronounces sentence θ, and the case terminates. If the defendant pleads not guilty, the case goes to a jury trial. (4) The jury determines whether to convict or acquit. Blue and solid lines in Figure 1 captures how actions at early stages affect actions at later stages; red and dashed lines represent how anticipated outcomes of later stages affect actions at early stages, which we study in the following sections.

3 Jury Trial

Let π denote the updated prior probability that a defendant is guilty conditioned that the case comes to a trial. We assume that a jury trial has less chance to meet a guilty defendant than an innocent defendant ($\pi \leq .5$). This assumption is not lose generality. First, it is natural that each juror is more likely to vote for a conviction when she receives a guilty signal g, rather than i. (We formally show this soon.) Since guilty defendants are more likely send signal g, guilty defendants have more chance to be convicted. As defendants anticipate such jury trial outcomes, guilty defendants tend to plead guilty and are less likely to litigate compared to innocent defendants.

A map $\sigma_j : \{g, i\} \to [0, 1]$ represents a strategy of juror j. The juror votes for conviction with

[8] A government cannot perfectly observe prosecutor's effort to avoid false prosecutions, and this yields a principle-agent problem. Although in general even well-designed incentives cannot lead the prosecutor's actions and the government's interests to perfectly coincide, we do not consider the agent problem in this paper.

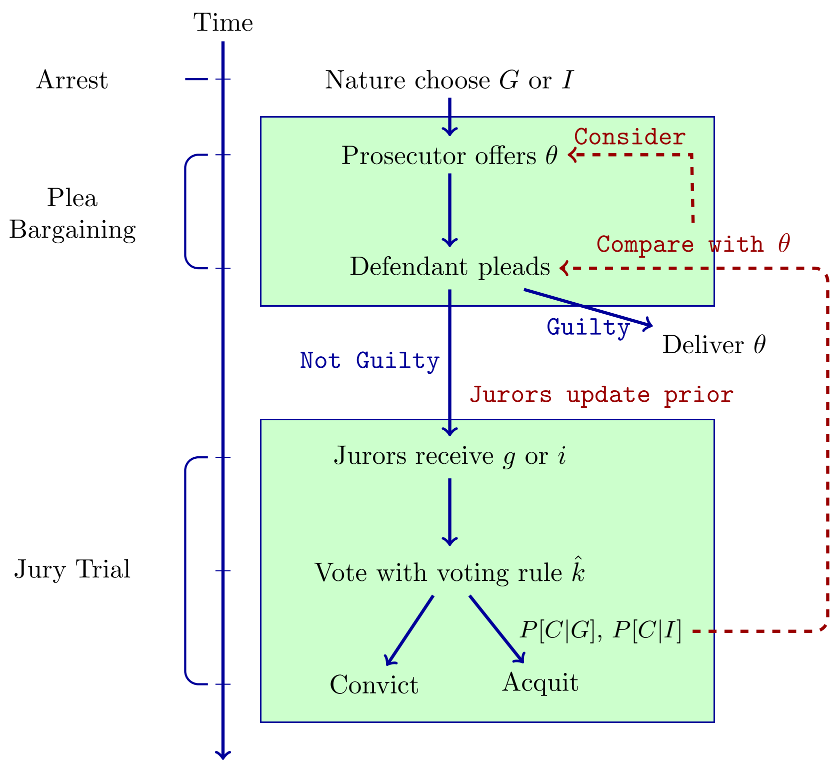


Figure 1: A Criminal Court Process.

probability $\sigma_j(g)$ when she receives a signal g; whereas, she votes for conviction with probability $\sigma_j(i)$ if the signal is i. In this paper, we consider *symmetric equilibria* in which all jurors adopt the same strategy. We denote a symmetric strategy profile as $[\sigma(g), \sigma(i)]$ without specifying a particular juror. Since the jury trial is modeled as a symmetric game, there exists at least one symmetric Nash equilibrium. We then find a symmetric equilibrium which gives all jurors the highest expected coordinated payoff. Since all jurors have the same preference for judicial decisions, especially convicting innocent and acquitting guilty people, this is a natural way of refining equilibria. We call this refined equilibrium an *Efficient symmetric Nash equilibrium*, or more succinctly an *Efficient equilibrium*.

A single juror affects the verdict only when she is in the pivotal position. [9] Assuming that the juror acts rationally, she takes into account that not only the private signal (g or i), but also the additional information from the event that she is pivotal (piv) as the evidence of guilty. The juror also knows that some defendants plead guilty, so the guilty to innocent ratio of defendants in jury trial is $\frac{\pi}{1-\pi}$. If this evidence of guilty is clear enough to exceed the reasonable doubt $(\frac{q}{1-q})$, then the juror votes for conviction. Formally the voting criteria are

$$\frac{Pr[\ piv\ |G]}{Pr[\ piv\ |I]}\ \frac{p}{1-p}\ \frac{\pi}{1-\pi} \quad \text{vs} \quad \frac{q}{1-q} \quad \text{if the signal is } g, \tag{2}$$

and

$$\frac{Pr[\ piv\ |G]}{Pr[\ piv\ |I]}\ \frac{1-p}{p}\ \frac{1-\pi}{\pi} \quad \text{vs} \quad \frac{q}{1-q} \quad \text{if the signal is } i. \tag{3}$$

The left hand side (LHS) is the likelihood ratio of guilty to innocent, given that a juror is pivotal, multiplied by the likelihood ratio of private information (g or i), times the ratio of updated prior probabilities; and the right hand side (RHS) is the ratio of reasonable doubt. If the LHS is larger than the RHS in equation (2), a juror with a private signal g has an incentive to vote for conviction; similarly, if the LHS is larger than the RHS in equation (3), a juror with a private signal i has an incentive to vote for conviction.

To state the pivotal probabilities precisely, let us denote r_G as the probability of voting for conviction when the defendant is guilty, and r_I for the same probability when the defendant is innocent. Since a guilty defendant and an innocent defendant send signal g with probability p

[9] Whether a juror is pivotal or not, of course, depends not only on how the other jurors vote but also on the voting rule - unanimity, simple majority, and three-fourths, etc.

and $1-p$ respectively, we obtain

$$r_G = p\sigma(g) + (1-p)\sigma(i), \quad r_I = (1-p)\sigma(g) + p\sigma(i). \tag{4}$$

When a voting rule requires $\hat{k}$ ($1 \le \hat{k} \le n$) number of conviction votes for a guilty verdict, a juror becomes pivotal when $\hat{k}-1$ other jurors vote for conviction. Assuming that $0 < r_G, r_I < 1$, the voting criterion (2) becomes

$$\left(\frac{r_G^{\hat{k}-1}(1-r_G)^{n-\hat{k}}}{r_I^{\hat{k}-1}(1-r_I)^{n-\hat{k}}}\right)\left(\frac{p}{1-p}\right) \quad \text{vs} \quad \frac{q}{1-q}\frac{1-\pi}{\pi} \text{ if the signal is } g, \tag{5}$$

and the criterion (3) becomes

$$\left(\frac{r_G^{\hat{k}-1}(1-r_G)^{n-\hat{k}}}{r_I^{\hat{k}-1}(1-r_I)^{n-\hat{k}}}\right)\left(\frac{1-p}{p}\right) \quad \text{vs} \quad \frac{q}{1-q}\frac{1-\pi}{\pi} \text{ if the signal is } i. \tag{6}$$

When $r_G = r_I = 0$ or $r_G = r_I = 1$, (5) and (6) are not defined. We treat these strategy profiles separately from the above equations.

The above equations are necessary conditions for a jury trial equilibrium, and we use the necessary conditions to characterize equilibrium strategy profiles. An additional definition simplifies the equilibrium expression. Let us define a function $\bar{\pi}$ from $\mathbb{N}$ to $[0, q]$ as

$$\bar{\pi}(l\ ; p, q) := \frac{1}{\frac{1-q}{q}\left(\frac{p}{1-p}\right)^l + 1}$$

which is strictly decreasing in l. We can rearrange the expression and obtain

$$\left(\frac{p}{1-p}\right)^{l-1}\frac{p}{1-p} = \frac{q}{1-q}\,\frac{1-\bar{\pi}(l)}{\bar{\pi}(l)} \tag{7}$$

The function $\bar{\pi}$ maps a number of guilty signals (l) to the level of updated prior probability (π) which barely gives an enough incentive for a conviction vote. That is, $\bar{\pi}(l)$ is the minimum level of prior such that l number of guilty signals lead a juror to vote for conviction.

Although equilibria have complicated expressions, the motivation behind is straightforward. Suppose an updated prior π is less than $\bar{\pi}(\hat{k})$, which means $\hat{k}$ number of guilty signals are not enough to yield a conviction vote. If a voting rule is $\hat{k}$, a strategy profile $[\sigma(g) = 1, \sigma(i) = 0]$ is not an equilibrium. Suppose a juror receives private signal g. The pivot condition gives addition information, exactly $\hat{k}-1$ other conviction votes from $\hat{k}-1$ guilty signals. Thus the juror has a conviction voting incentive at most equal to what $\hat{k}$ number of guilty signals can motivate; possibly some acquittal votes from i signal negate $\hat{k}$ number of guilty signals.

On the other hand, suppose a strategy profile $0 < \sigma(g) < 1$, and necessarily $\sigma(i) = 0$, is an equilibrium under $\pi = \bar{\pi}(l)$ for some $\bar{\pi}(l) > \bar{\pi}(\pi)$ (or $l < \hat{k}$). The necessary condition (5) holds as an equality. Combined with (7), being pivotal ($\hat{k}-1$ other conviction votes) must give the same effect as $l-1$ guilty signals. With strategy profile $[0 < \sigma(g) < 1, \sigma(i) = 0]$, $\hat{k}-1$ conviction votes are clearly from the guilty signals; however, $n-\hat{k}$ acquittal votes may come from either guilty or innocent signal. Roughly stating, suppose $\frac{n-2\hat{k}+l}{2}$ acquittal votes are from guilty signal, and $\frac{n-l}{2}$ acquittal votes are from innocent signal in expectation. Then the number of conviction votes ($\hat{k}-1$) and acquittal votes with guilty signal ($\frac{n-2\hat{k}+l}{2}$), subtracted by the number of acquittal votes with innocent signal ($\frac{n-l}{2}$), yields $l-1$ guilty signals. $\sigma(g)$ is then determined as the ratio of $\frac{\hat{k}-1}{\hat{k}-1+\frac{n-2\hat{k}+l}{2}}$.

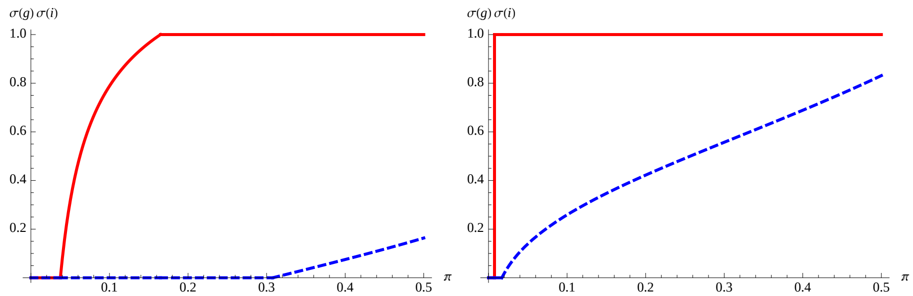
We relegate details of equilibrium computation to Appendix A, and only state equilibrium properties in Lemma 1. If the conviction probability with the signal g is strictly higher than the probability with the signal i, we say the equilibrium is *responsive*. Lemma 1 shows that a responsive equilibrium, if exists, is more efficient than a non-responsive.

Figure 2 shows an efficient equilibrium with certain pair of parameter values under the unanimity rule ($\hat{k} = n$) and a non-unanimous rule ($0 < \hat{k} < n$). Figure 3 shows the corresponding conviction probabilities. We have multiple efficient equilibria when $\pi = \bar{\pi}(n)$ under the unanimity rule. Otherwise, each π has a unique efficient equilibrium, so the conviction probabilities are single valued. Hereafter, for notational convenience, we extend inequality signs to the family of all intervals. If a real number a is larger than all b in an interval B, we denote $a > B$; if for all $a \in A$ is larger than or equal to all $b \in B$, we denote $A \geq B$.

Proposition 1 ***Jury Trial Behavior***

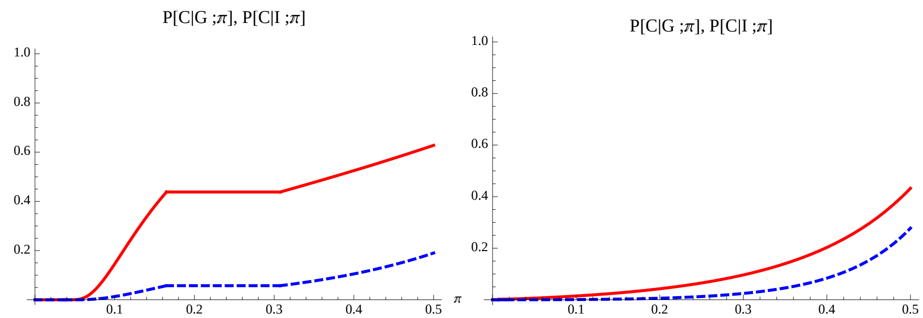
For a given π, $(\sigma(g;\pi), \sigma(i;\pi))$ denotes the set of efficient equilibrium strategy pairs. $(P[C|G,\pi], P[C|I,\pi])$ denotes the set of corresponding conviction probability pairs of a guilty and an innocent defendant, respectively.

1. *If $\pi \geq \bar{\pi}(\hat{k})$, there exists a responsive efficient equilibrium. Otherwise, the only symmetric equilibrium (thus an efficient equilibrium) is in which no juror votes for conviction.*



(a) $\hat{k} = 8$, A Non-unanimous Rule (b) $\hat{k} = 12$, The Unanimity Rule

Figure 2: Efficient symmetric equilibria of jury trial with $n = 12, p = \frac{6}{10}$ and $q = \frac{1}{2}$



(a) $\hat{k} = 8$, A Non-unanimous Rule (b) $\hat{k} = 12$, The Unanimity Rule

Figure 3: Conviction probabilities with $n = 12, p = \frac{6}{10}$ and $q = \frac{1}{2}$

2. *$P[C|G, \pi]$ and $P[C|I, \pi]$ are nondecreasing and upper hemicontinuous correspondence in π, with non-empty convex values.*

Proof: See Appendix A. ∎

4 Plea Bargaining

Distribute let ϕ_G and $(1 - \phi_G)$ denote the probability of a guilty defendant pleads guilty and not guilty, respectively. ϕ_I and $(1 - \phi_I)$ are defined similarly for an innocent defendant. Let π be the updated prior probability of guilty defendant conditioned that a case comes to a trial. Provided that not all cases terminate in plea bargaining ($\phi_G > 0$ or $\phi_I > 0$), π is determined as

$$\pi = \frac{1 - \phi_G}{(1 - \phi_G) + (1 - \phi_I)}. \tag{8}$$

If all defendants plead guilty ($\phi_G = \phi_I = 1$), the updated prior is assumed to be equal to 0.[10]

-The equilibrium presumes that a defendant know the conviction probabilities of a guilty or an innocent defendant. One can justify this assumption with defense attorneys. Although we do not have defense attorneys in the model, in real world a defendant gets advice from defense attorneys who are aware of previous judicial decisions. Indeed, it is observed that participants in plea bargaining foresee the trial, so that previous decisions in the trial significantly influence the bargaining power. (for recent studies on this topic, see Bibas (2004); Stuntz (2004)) -Note that the conviction probabilities, one for each type, in jury trial depend on the updated prior π. Conversely, conviction probabilities affect the pleading decisions, ϕ_G and ϕ_I, which in turn determine π. Therefore, given a bargain offer θ, a subgame equilibrium may be determined as a fixed point of the pair of dynamics. Moreover, plea bargaining is a signaling game. A defendant, as a sender, signals his type by pleading either guilty or not guilty. The entire jury, as a receiver, update belief of sender's type and determines conviction probabilities. The prosecutor sets an optimal bargain offer θ^*, which yields the highest equilibrium payoff.

The jury trial delivers punishment equal to either 0 (acquittal) or 1 (conviction), whereas a plea bargain can deliver any punishment, $h \in [0, 1]$. Delivering punishment to a guilty defendant and dismissing punishment with an innocent defendant give zero utility; dismissing punishment with a guilty defendant changes utility by $-(1 - q')$, and delivering punishment to an innocent defendant changes utility by $-q'$. Although the prosecutor's utility has a similar format to the jurors', we do not interpret q' as a level of reasonable doubt; we treat it as relative weights on incorrect decisions.

Suppose a prosecutor offers a defendant an opportunity to plead guilty with charge $\theta \in [0, 1]$. Given conviction probabilities (P_G, P_I), the defendant compares θ with either P_G or P_I, and decides whether to plead guilty or to go to trail. If θ is larger than P_G, no guilty defendant has an incentive to plead guilty ($\phi_G = 0$); similarly, if θ is larger than P_I, no innocent defendant pleads guilty. The updated prior probability, $\pi = \frac{1-\phi_G}{(1-\phi_G)+(1-\phi_I)}$, reflects pleading decisions.[11] Conversely, jurors incorporate the updated prior in their voting behavior, thus conviction probabilities, $(P_G, P_I) \in (P[C|G, \pi], P[C|I, \pi])$ are changed by π, which become

[10] Although we can derive this assumption by applying an equilibrium refinement, D1 (Cho and Kreps, 1987), the assumption appeals naturally in the litigation context without such a technical approach. We will state later more precisely about the refinement in Footnote 12.

[11] Note that we also assumed $\pi = 0$ in case of $\phi_G = \phi_I = 1$.

basis of pleading decision.

Given θ, the subgame equilibrium is determined as a fixed point in this pair of two dynamics. The prosecutor wants to set θ which yields the highest subgame equilibrium payoff. We summarize the prosecutor's problem as an optimization under constraints.

$$\max_{\theta\in[0,1]} -\frac{1}{2}q'\Big(\phi_I\theta + (1-\phi_I)P_I\Big) - \frac{1}{2}(1-q')\Big(\phi_G(1-\theta) + (1-\phi_G)(1-P_G)\Big) \tag{9}$$

$$\text{such that} \quad \begin{array}{ll} (a.1) & \phi_G \in \arg\min_{\phi'\in[0,1]} \phi'\theta + (1-\phi')P_G \\ (a.2) & \phi_I \in \arg\min_{\phi'\in[0,1]} \phi'\theta + (1-\phi')P_I \\ (b) & (P_G, P_I) \in (P[C|G,\pi], P[C|I,\pi]) \\ (c) & \pi = \begin{cases} 0 & \text{if } \phi_G = \phi_I = 1 \\ \frac{1-\phi_G}{(1-\phi_G)+(1-\phi_I)} & \text{otherwise.} \end{cases} \end{array}$$

The first term in the object function is the utility changed by mistakenly delivered punishment to the innocent with plea bargaining and the jury trial, respectively. The second term is the utility changed by mistakenly *undelivered* punishment to the guilty. Condition $(a.1)$ and $(a.2)$ represent pleading decisions by a guilty and an innocent defendant, respectively. Condition (b) requires the pair of conviction probabilities to be consistent with equilibrium outcome under π, and (c) captures how the prior is updated. The prosecutor wants to maximize the equilibrium payoff with adjusting θ.

Lemma 2 allows us to simplify the above problem. Given a θ, suppose we have $\theta < P[C|G,\pi]$ in an equilibrium. It is necessary that a guilty dependent must plead guilty, and the jury faces no guilty defendant: $\pi = 0$. However, $P[C|G;\pi = 0] = 0$ implies that $\theta < P[C|G,\pi]$ must not be true. Whereas if $\theta > P[C|G,\pi]$, no defendant pleads guilty, and the updated prior must be equal to the initial prior ($\pi = .5$). Therefore, $\theta \in P[C|G;\pi]$ is a necessary condition when the equilibrium π is in $[0, .5)$. The following lemma formally states this observation which we will use to simplify the prosecutor's problem.

Lemma 2 *Suppose the prosecutor offers θ for pleading guilty. Given that the jury choose an efficient equilibrium, one of the followings and only one must be true.*[12]

[12] In this paper, we assume that the updated prior probability π is equal to 0 when all defendants plead guilty. Indeed, Perfect Baysian Equilibrium allows any level of $\pi \in [0,1]$ as the belief off-the equilibrium path $\phi_G = \phi_I = 1$. The condition D1 in Cho and Kreps (1987) can refine the equilibrium to have $\pi = 0$. However, since the assumption is quite natural and refinement is not a major concern of this paper, we may not go over the D1 condition in detail.

- $\theta > P_G$ *and* $\phi_G = \phi_I = 0$ *and* $\pi = .5$.
- $\theta = P_G$ *with* $P_G \in P[C|G, \pi]$ *with* (ϕ_G, ϕ_I) *obtaining such* π.

Proof: Clearly, if $\theta > P_G$, and necessarily $\theta > P_I$, then no defendant pleads guilty. $\phi_G = \phi_I = 0$ and $\pi = .5$ must be true.

Otherwise, we have $\theta \leq P_G$ for some π with $P_G \in P[C|G; \pi]$. Suppose, toward a contradiction, that $\theta < P_G$. All guilty defendants plead guilty, so that $\pi = 0$. Since $P[C|G; \pi = 0] = 0$, $\theta < P_G \in P[C|G; \pi = 0]$ must not be true. Therefore, $\theta = P_G \in P[C|G; \pi]$.

In case of $\theta = P_G$, unless $\theta = 0$, a pair of conviction probabilities $(P_G, P_I) \in (P[C|G, \pi], P[C|I, \pi])$ has $\theta = P_G > P_I$. Thus an innocent defendant pleads not guilty: $\phi_I = 1$. A guilty defendant is indifferent whether to go to trial or to plead guilty, so any $\phi_G \in [0, 1]$ is a best response. Any such ϕ_G which incurs $\pi \in [\pi_m, \pi_M]$ constitutes an equilibrium.

If $\theta = 0$, then $\theta = P_G = P_I = 0$. Any pair of (ϕ_G, ϕ_I) incuring $\pi \in [0, \bar{\pi}(\hat{k})]$ constitutes an equilibrium. ∎

Note that a single θ may bring multiple equilibria with different pairs of (ϕ_G, ϕ_I). However, the prosecutor only cares about the conviction probabilities, and the lemma 2 shows that the conviction probabilities are uniquely determined by θ.

The equilibrium payoff in the case of $\theta > P[C|G; .5]$ is the same as the payoff of $\theta = \bar{P}$ where $\bar{P}$ is the maximum value of $P[C|G; .5]$. Therefore, without loss of generality, we may assume that $\theta \in [0, \bar{P}]$ and $\theta \in P[C|G; \pi]$. Moreover, $\theta > 0$ implies $\theta = P_G > P_I$ and so $\phi_I = 0$. If $\theta = 0$, $P[C|G; \pi] = P[C|I; \pi] = 0$. Therefore, we can rewrite the first term in (9) as $-\frac{1}{2}q' P_I$ without loss of generality.

Lastly, there exist a continuous and strictly monotone increasing function ν, such that $P_I = \nu(P_G)$ if and only if (P_G, P_I) is an equilibrium conviction probabilities with some π. Equilibrium pair of conviction probabilities, (P_G, P_I), is obtained from equilibrium strategy profile $(\sigma(g), \sigma(i))$, as $P_G = \sum_{k=\hat{k}}^{n} r_G^k (1 - r_G)^{n-k}$ and $P_I = \sum_{k=\hat{k}}^{n} r_I^k (1 - r_I)^{n-k}$ where $r_G = p \cdot \sigma(g) + (1 - p)\sigma(i), r_I = (1 - p) \cdot \sigma(g) + p \cdot \sigma(i)$. By the properties of equilibrium strategy shown in the proof of Lemma 1, either $\sigma(g) = 1$ or $\sigma(i) = 0$ must be true. Given the value of $\sigma(g)$ (or $\sigma(i)$), the map from $\sigma(i)$ ($\sigma(g)$) to conviction probabilities (P_G, P_I) is continuous and strictly increasing. By partitioning the domain of equilibrium strategy profiles $[\sigma(g), \sigma(i)]$ with $[\sigma(g) = 1, \sigma(i)]$ and $[\sigma(g), \sigma(i) = 0]$, a continuous and strictly increasing function ν from P_G to

P_I is constructed as a composite function of the inverse function from P_G to $\sigma(g)$ (or $\sigma(i)$) and the function from $\sigma(g)$ ($\sigma(i)$) to P_I.

Replacing $\phi_I = 0, \theta = P_G$ and $P_I = \nu(P_G)$ and , we simplifies the prosecutor's problem (9) as

$$\max_{\theta \in [0,\bar{P}]} -\frac{1}{2}q'\nu(\theta) - \frac{1}{2}(1-q')(1-\theta)$$

A closed form solution of this optimization is not our concern. Rather, we focus on the motivations behind the equilibrium.

Let us revisit the criteria of a juror's conviction vote.

$$\frac{Pr[\ piv\ |G]}{Pr[\ piv\ |I]}\frac{p}{1-p} \quad \text{vs} \quad \frac{q}{1-q}\frac{1-\pi}{\pi} \quad \text{with signal } g$$

$$\frac{Pr[\ piv\ |G]}{Pr[\ piv\ |I]}\frac{1-p}{p} \quad \text{vs} \quad \frac{q}{1-q}\frac{1-\pi}{\pi} \quad \text{with signal } i$$

When they face either an updated prior π and the ratio of reasonable doubt $\frac{q}{1-q}$, the conviction voting probabilities are same as when an updated prior .5 and the ratio of reasonable doubt equal to $\frac{q}{1-q}\frac{1-\pi}{\pi}$. That means we can interpret the prosecutor's strategy as manipulating jurors' reasonable doubts whereas the updated prior is fixed to .5. The prosecutor may adjust θ, thereby obtaining π such that the distorted reasonable doubt yields an ideal trial equilibrium ideal.

Since the prosecutor is not allowed to force a juror to take a certain strategy, he can at best lead to one of the efficient equilibria. Intuitively it is best to manipulate the jurors' reasonable doubts to be the same as prosecutor's . However, the prosecutor's manipulation is available only in one direction; he can only induce $\pi \leq .5$ and increase the RHS: reasonable doubt. In the case where jurors care more about punishing innocent ($q > q'$), the prosecutor has no choice but to set $\pi = .5$. The following proposition summarizes this intuition. We relegate the proof of proposition 3 to the Appendix B

Proposition 3 *1. If $q > q'$ the prosecutor offers $\theta = \bar{P}$. All defendants plead not guilty ($\pi = .5$), and the equilibrium of the jury trial is equivalent to the equilibrium of the traditional jury model.*

2. If $q \leq q'$, the prosecutor offers θ^* such that the equilibrium behavior of the jury is the same *as the equilibrium behavior of the traditional jury model; however, unlike in the traditional jury model, the jurors behave as if they have the prosecutor's preference, q'.*

Proof: See Appendix B ∎

The Unanimity vs. Non-unanimous Rules. We can directly apply proposition 3 to the previous voting literature, and examine if the results are robust as we extend the model to include plea bargaining. Feddersen and Pesendorfer (1998) compares two voting paradigms: the unanimity and non-unanimous rules, solely based on the jury trial model. Their main conclusion is that the unanimity rule is inferior to any non-unanimous rule in the sense that, as the number of jurors gets large, the probability of convicting innocent or acquitting guilty does not converge to 0; whereas both probabilities converge to 0 under any non-unanimous rule. Their model assumes that the prior π is fixed, so that the jury faces the same proportion of guilty defendants regardless of their performance: conviction probabilities.

Corollary 4 shows that the main result in Feddersen and Pesendorfer (1998) is robust to the plea bargaining. As the proposition 3 shows that the equilibrium behavior in the criminal court model is similar to an equilibrium in the traditional jury model, the inferiority of the unanimity rule is preserved in this extension.

Corollary 4 *Suppose a criminal court has plea bargaining and a jury trial with n number of jurors. If the jury requires n conviction votes for conviction, we call it unanimity rule. Otherwise if the jury requires $\hat{k} = \alpha n$ $(0 < \alpha < 1)$, we call it a non-unanimous rule.*

- *If a jury trial uses the unanimity rule, the expected punishment for a guilty defendant converges to $1-\left(\frac{(1-\tilde{q})(1-p)}{\tilde{q}p}\right)^{1-\frac{p}{2p-1}}$; for an innocent defendant, it converges to $\left(\frac{(1-\tilde{q})(1-p)}{\tilde{q}p}\right)^{\frac{p}{2p-1}}$ as $n \to \infty$, where $\tilde{q} = \max\{q, q'\}$.*
- *If the jury trial uses a non-unanimous rule with a fixed α, the expected punishment for a guilty defendant converges to one and the expected punishment for an innocent defendant converges to zero as $n \to \infty$.*

Proof: Note that we have a responsive equilibrium when $\pi > \bar{\pi}(\hat{k})$ ($\hat{k} = n$ for the unanimity rule). Since $\bar{\pi}(l)$ is decreasing on l, we have responsive equilibrium for all $\pi > 0$ as $n \to \infty$.

For the unanimity rule, first we can prove that for a fixed π and p, a responsive equilibrium yields the probability of conviction converges to $1-\left(\frac{(1-q)(1-p)\pi}{qp(1-\pi)}\right)^{1-\frac{p}{2p-1}}$ for a guilty defendant, and to $\left(\frac{(1-q)(1-p)\pi}{qp(1-\pi)}\right)^{\frac{p}{2p-1}}$ for an innocent defendant. The convergence results are directly from the proposition 2 and 3 in Feddersen and Pesendorfer (1998). It is easy to check that parameter values here satisfy all the conditions assumed for the propositions.

Lastly, we have either $\frac{\pi}{1-\pi} = 1$ (if $q > q'$) or $\frac{1-q}{q}\frac{\pi}{1-\pi} = \frac{1-q'}{q'}$ (if $q \leq q'$) for all jury size n. The corollary for the unanimity rule follows as we replace $\frac{1-q}{q}\frac{\pi}{1-\pi} = \frac{1-\tilde{q}}{\tilde{q}}$, where $\tilde{q} = \max\{q, q'\}$

The convergence result for non-unanimous rules is also derived with a similar fashion. ∎

5 Jury Selection: Peremptory Challenges

So far we have assumed that each juror's preference, or reasonable doubt, is given and the preference is same for all jurors. However, once we accommodate jury selection process, this assumption does not hold; jury selection itself presumes that preferences are not identical, and preferences are not exogenous. We relax these two assumptions.

Jury selection consists of two categories: *challenge for cause* and *peremptory challenges*. The prosecutor and defense attorneys (or the defendant herself in this paper) ask general and specific questions of the potential jurors. When there is an obvious reason for not rending an impartial decision in the case, the potential juror is excluded as *challenge for cause argument*. In addition, each side is allowed to challenge without specifying any reason, which is called *peremptory challenges*. At the federal level, one to three challenges per jury are allowed per jury to each side, and we study this peremptory challenge.

In order to add jury selection in the model, we first assume that reasonable doubt, q, is a random variable with a continuous distribution F on $[.5, 1]$. A simplified jury selection begins as the prosecutor and the defendant designate two thresholds $\underline{q}$ and $\bar{q}$, respectively. Each juror is sequentially drawn from the original distribution F with truncated support, $[\underline{q}, \bar{q}]$.

An equilibrium of the jury trial with jury selection yields heterogeneous jurors' preferences, unless the jury selection generates thresholds $\underline{q} = \bar{q}$ so that only a single preference is selected. Although a jury trial with heterogeneous preferences looks to be a bit complicated, it turns out that equilibrium outcome is not much different from an outcome with a homogenous preference. Given that $\tilde{F}$ is the continuous distribution of truncated jurors' preferences, there are at most two level of reasonable doubts, q_g and q_i ($q_m \leq q_g \leq q_i \leq q_M$), such that a juror with q_g or q_i may use a mixed strategy when she receives g or i signal, respectively. Jurors with other doubts use pure strategies in an equilibrium. We can reproduce this equilibrium as a mixed strategy equilibrium with a single preference. Although this single preference representation is possible only if we have a relatively small preference dispersion, jury selection allows us to assume this condition.

Lemma 5 *Suppose jurors are selected from an independent and identical preference distribution* $\tilde{F}$ *over* $[q_m, q_M]$. *If*

$$\frac{q_m}{1-q_m}\left(\frac{p}{1-p}\right)^2 > \frac{q_M}{1-q_M},$$

there exists a single representative preference level such that any equilibrium conviction probabilities with original heterogeneous preferences can be reproduces as an equilibrium of a trial with the homogeneous representative preference.

Proof: See Appendix C ∎

Lemma 5 implies that we can model a jury selection process simply as a bargaining over a single representative preference q; jury selection is then summarized as a single valued function of π and q'. For a given π, the prosecutor prefers q to be close to his own preference q'; whereas the defendant prefers the highest possible level of q, which minimizes the probability of getting conviction sentence.

In all, the court process is, (i) once the prosecutor determines a pleading guilty offer (θ), (ii) the defendant pleads either guilty or nor guilty (ϕ_G and ϕ_I), (iii) in case of pleading not guilty, the prior probability is updated (π), (iv) the jury selection obtains a representative preference $q(\pi, q')$, (v) and an equilibrium outcome, (P_G, P_I), follows.

There is some difficulties to use axiomatic bargaining model in jury selection. Bargaining models assume that there exists a reservation utility level for each side. Using the reservation utilities as a reference point, the model analyzes how participants divide cooperative surplus. However, no one can leave the bargain table in jury selection, so reservation utilities are not clearly specified. This paper leaves out an explicit model for jury selection for the future study, but requires only a weak condition, called *Pareto-undominatedness*. Pareto-undomatedness only requires that the bargaining outcome, $q(\pi, q')$, should not be Pareto dominated by another q. If the jury selection process generates Pareto-undominated representative preference, we have $q' \leq q(\pi, q') \leq q_0$, where q_0 is the lowest level of preference with which all jurors vote for acquittal.

Although, we do not model the jury selection explicitly, so do not specify an exact function form of $q(\pi, q')$, it is easy to see that the conventional assumption, $\pi = .5$ in the traditional trial model, may not hold in general.

Suppose that the jury selection generates the single representative preference, $q(\pi; q')$. Voting criterion 2 and 3 implies that a jury equilibrium with an updated prior π and a reasonable doubt $q(\pi; q')$ is equivalent to an equilibrium with an updated prior equal to .5 and the ratio of reasonable doubts equal to $\frac{q(\pi;q')}{1-q(\pi;q')}\frac{1-\pi}{\pi}$. In the proof of Proposition 3, we show that when jurors perceive the prior to be equal to .5, the prosecutor's utility is maximized if the juror's reasonable doubt is equal to q'. Moreover, the prosecutor's utility is non-increasing in q for all $q > q'$. Since Pareto-undominated jury selection gives $q' \leq q(\pi, q')$, we have $\frac{q'}{1-q'} \leq \frac{q(\pi;q')}{1-q(\pi;q')}\frac{1-\pi}{\pi}$. Therefore, π^*, which minimizes the $\frac{q(\pi;q')}{1-q(\pi;q')}\frac{1-\pi}{\pi}$, yields the highest equilibrium payoff to the prosecutor. The prosecutor achieves this equilibrium by setting,

$$\theta^* \in P[C|G, \pi^*, q(\pi^*, q')].$$

Note that higher π leads to higher q_0, the lowest level of doubt where all jurors vote for acquittal in an efficient equilibrium. Since the jury selection gives $q' \leq q(\pi; q') \leq q_0$, $q(\pi; q')$ might be increasing in π. In this respect, in general, $\pi^* = .5$ may not be necessarily a minimizer of $\frac{q(\pi;q')}{1-q(\pi;q')}\frac{1-\pi}{\pi}$.

Although the primitive assumptions may not hold, the inferiority results of the unanimity in Feddersen and Pesendorfer (1998) still hold in the court with both plea bargaining and jury selection, as long as the prosecutor's bargain power in jury selection is not dominated by the defendant's.

Corollary 6 *The jury selection function $q(\pi; q')$ can be rewritten as a linear combination of $\lambda q' + (1-\lambda)q_0$ where λ may be a function of π, q', and $\hat{k} = \alpha n$. Suppose there exists $\underline{\lambda} > 0$ such that $\liminf_{n\to\infty} \lambda_n \geq \underline{\lambda}$ for all π and α. Then, with large number of jurors, the inferiority of the unanimity rule still hold in a court with both plea bargaining and jury selection.*

Proof: For a given $\alpha > 0$, q_0 converges to 1 as n increases to ∞ for every π.

$$\frac{q'}{1-q'} \leq \frac{q(\pi, q')}{1-q(\pi, q')} \leq \frac{\underline{\lambda}\, q' + (1-\underline{\lambda})q_0}{1-(\underline{\lambda}\, q' + (1-\underline{\lambda})q_0)} \leq \frac{1-\underline{\lambda}(1-q')}{1-(1-\underline{\lambda}(1-q'))}$$

for any π, α and q'.

Corollary 4 shows that the conviction probabilities converge to 1 and 0 for any π and q under non-unanimous rules; whereas both probabilities converge to some positive numbers under the unanimity rule; especially, when $q = q'$ and $q = 1 - \underline{\lambda}(1 - q')$.

The probabilities of convicting guilty $P[C|G; \pi, q]$ and innocent $P[C|I; \pi, q]$ are non-increasing correspondence of q for all given π. Therefore, the limit of the conviction probabilities with jury selection is bounded above by the limit of conviction probabilities with preference $q = q'$, and bounded below by the limit of conviction probabilities with preference $q = 1 - \underline{\lambda}(1 - q')$. For non-unanimous rules, both upper and lower bounds converge to 1 for a guilty defendant, and converge to 0 for an innocent defendant. However, for the unanimity rule, upper bounds and lower bounds have values in the open interval $(0, 1)$. The incorrect conviction and acquittal do not vanish to 0. ∎

6 Conclusion

A Proof of Lemma 1

We first find all symmetric equilibria for each updated prior π. Then we compare the equilibrium payoffs and take the most efficient equilibrium.

A.1 Find all symmetric equilibria

We can show conditions under which $[\sigma(g) = \sigma(i) = 1]$ or $[\sigma(g) = \sigma(i) = 0]$ is a symmetric equilibrium.

Clearly, $[\sigma(g) = \sigma(i) = 1]$ is a symmetric equilibrium for the voting rule $1 \leq \hat{k} < n$. Given that all other jurors vote for conviction regardless of their signals, a single juror can never be pivotal; thus, any strategy, especially vote always for conviction, is a best response. $[\sigma(g) = \sigma(i) = 1]$ is not an equilibrium when $\hat{k} = n$. Given that $n - 1$ jurors choose $\sigma(g) = \sigma(i) = 1$, a juror in the pivotal position does not receive any additional information besides her private signal from the fact that she is pivotal. Therefore, each juror only uses her private signal, but we have

$$\frac{1-p}{p} < \frac{q}{1-q} \frac{1-\pi}{\pi}.$$

A single innocent signal does not yield an enough incentive to vote for conviction against reasonable doubt and updated prior probability. Both are biased in favor of an innocent defendant: $q \geq 0.5$ and $\pi \leq .5$.

In the similar fashion, $[\sigma(g) = \sigma(i) = 0]$ is an equilibrium for $1 < \hat{k} \leq n$. When $\hat{k} = 1$, $[\sigma(g) = \sigma(i) = 0]$ is an equilibrium only if $\pi \leq \bar{\pi}(1)$. A juror does not gain any additional clue on the defendant from the event that she is pivotal. With such a low π, a single guilty signal as well as an innocent signal is not enough to convince a juror to vote for conviction.

When $0 < \sigma(g)$ and $\sigma(i) < 1$, we have $0 < r_G, r_I < 1$ and the criteria of conviction vote, (5) and (6), are well defined. For the convenience, we reproduce them here. When LHS is larger than RHS, a juror has an incentive to vote for conviction.

$$\left(\frac{r_G^{\hat{k}-1}(1-r_G)^{n-\hat{k}}}{r_I^{\hat{k}-1}(1-r_I)^{n-\hat{k}}}\right)\left(\frac{p}{1-p}\right) \quad \text{vs} \quad \frac{q}{1-q}\frac{1-\pi}{\pi} \quad \text{for signal } g$$

$$\left(\frac{r_G^{\hat{k}-1}(1-r_G)^{n-\hat{k}}}{r_I^{\hat{k}-1}(1-r_I)^{n-\hat{k}}}\right)\left(\frac{1-p}{p}\right) \quad \text{vs} \quad \frac{q}{1-q}\frac{1-\pi}{\pi} \quad \text{for signal } i$$

In the sequel, we consider each case of strategy profiles and find the range of π containing such strategy profile as an equilibrium.

- (Case 1:) $0 < \sigma(g) < 1, \sigma(i) = 0$

  A juror with signal g is indifferent between conviction and acquittal. Therefore,

  $$\left(\frac{r_G^{\hat{k}-1}(1-r_G)^{n-\hat{k}}}{r_I^{\hat{k}-1}(1-r_I)^{n-\hat{k}}}\right)\left(\frac{p}{1-p}\right) = \frac{q}{1-q}\frac{1-\pi}{\pi}$$

  Substituting in $r_G = p\sigma_g$ and $r_I = (1-p)\sigma_g$ from equation (4), we obtain

  $$\left(\frac{1-p\sigma_g}{1-(1-p)\sigma_g}\right)^{n-\hat{k}}\left(\frac{p}{1-p}\right)^{\hat{k}} = \frac{q}{1-q}\frac{1-\pi}{\pi} \tag{10}$$

  If $\hat{k} = n$ (the unanimity), the equality holds only when $\pi = \bar{\pi}(\hat{k})$. Any $\sigma_g \in (0,1)$ with $\sigma_i = 0$ is a symmetric equilibrium. When $\hat{k} < n$, $\frac{1-p\sigma_g}{1-(1-p)\sigma_g}$ is strictly decreasing in σ_g, and there is at most one σ_g satisfying the equality. It is easy to see that $\bar{\pi}(\hat{k}) \leq \pi \leq \bar{\pi}(2\hat{k}-n)$ is a necessary condition and the equilibrium strategy is $(\sigma(g), \sigma(i) = 0)$ with

$$\sigma(g) = \frac{\psi_1 - 1}{(1-p)\psi_1 - p}, \quad \text{where} \quad \begin{cases} \psi_1 = \left(\frac{1-p}{p}\right)^{\frac{\hat{k}}{n-\hat{k}}} \left(\frac{q}{1-q}\frac{1-\pi}{\pi}\right)^{\frac{1}{n-\hat{k}}} \\ \\ \bar{\pi}(\hat{k}) \leq \pi \leq \bar{\pi}(2\hat{k}-n). \end{cases}$$

- (Case 2:) $\sigma(g) = 1, \sigma(i) = 0$

  A juror with signal g has an incentive to vote for conviction; whereas a juror with signal i prefers to vote for acquittal. Substituting in $r_G = p$ and $r_I = 1-p$ to the voting criteria, we obtain

$$\left(\frac{p}{1-p}\right)^{2(\hat{k}-1)-n} \leq \frac{q}{1-q}\,\frac{1-\pi}{\pi} \leq \left(\frac{p}{1-p}\right)^{2\hat{k}-n} \tag{11}$$

  which is equivalent to $\bar{\pi}(2\hat{k}-n) \leq \pi \leq \bar{\pi}(2(\hat{k}-1)-n)$. The first inequality is for a juror with signal i; and the second inequality is for a juror with signal g. When π is between $\bar{\pi}(2\hat{k}-n)$ and $\bar{\pi}(2(\hat{k}-1)-n)$, there exists a symmetric equilibrium in which every juror follows her own signals.

- (Case 3:) $\sigma(g) = 1, 0 < \sigma(i) < 1$

  A juror with signal i is indifferent between conviction and acquittal. Substituting in $r_G = p + (1-p)\sigma_i$ and $r_I = (1-p) + p\sigma_i$, we get

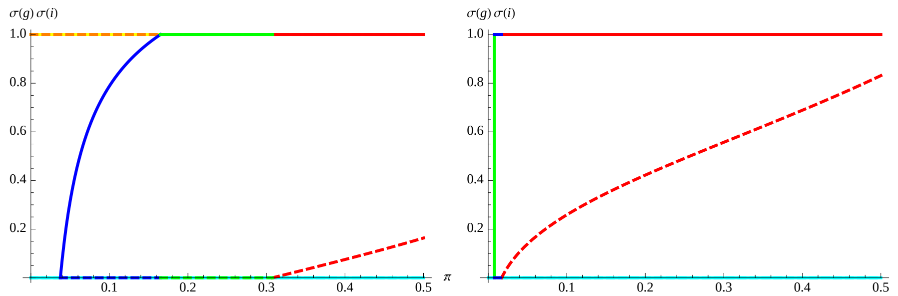
$$\left(\frac{r_G^{\hat{k}-1}(1-r_G)^{n-\hat{k}}}{r_I^{\hat{k}-1}(1-r_I)^{n-\hat{k}}}\right)\left(\frac{1-p}{p}\right) = \frac{q}{1-q}\frac{1-\pi}{\pi}$$

  which implies

$$\left(\frac{p+(1-p)\sigma_i}{(1-p)+p\sigma_i}\right)^{\hat{k}-1}\left(\frac{1-p}{p}\right)^{n-\hat{k}+1} = \frac{q}{1-q}\,\frac{1-\pi}{\pi} \tag{12}$$

  Since $\frac{p+(1-p)\sigma_i}{(1-p)+p\sigma_i}$ is strictly decreasing in σ_i, there is at most one σ_i satisfying the equality. This σ_i with $\sigma_g = 1$ forms a symmetric equilibrium. It is easy to see that such σ_i exists only if $\bar{\pi}(2(\hat{k}-1)-n) \leq \pi \leq .5$.

  The equilibrium strategy is $(\sigma(g) = 1, \sigma(i))$ with



(a) $\hat{k} = 8$ (b) $\hat{k} = 12$

Figure 4: All symmetric equilibria of jury game with $n = 12, p = \frac{6}{10}$ and $q = \frac{6}{10}$

$$\sigma(i) = \frac{p - \psi_2(1-p)}{p\,\psi_2 - (1-p)} \quad \text{where} \quad \begin{cases} \psi_2 = \left(\frac{p}{1-p}\right)^{\frac{n-\hat{k}+1}{\hat{k}-1}} \left(\frac{q}{1-q}\frac{1-\pi}{\pi}\right)^{\frac{1}{\hat{k}-1}} \\ \\ \bar{\pi}(2(\hat{k}-1)-n) \leq \pi \leq .5 \end{cases}$$

Figure 4 summarizes all symmetric equilibria of a jury trial with $n = 12$, $p = \frac{6}{10}$, $q = \frac{6}{10}$, and $\hat{k} = 8$ and 12 when the updated prior π is in between 0 and .5. We used solid lines for $\sigma(g)$ and dashed lines for $\sigma(i)$. For each π an equilibrium strategy profile, $(\sigma(g), \sigma(i))$, has the same color.

Although these figures contain all three cases of the responsive equilibrium, we may not observe some cases when we have other pairs of parameter values. For instance, one of the threshold level, $\bar{\pi}(2(\hat{k}-1)-n)$, may not be defined or may be larger than .5. In this case, we do not have $[\sigma(g) = 1, \sigma(i) = 0]$ as an equilibrium with $\forall \pi \in [0, .5]$.

A.2 Find an efficient equilibrium

Intuitively, an equilibrium aggregating each juror's private informations must give more efficient outcome. Therefore, an equilibrium where each juror explits her private information must be payoff dominate an equilibrium in which jurors ignore signals. We confirm that the responsive equilibrium, if exists, is more efficient than non-responsive equilibrium. For a pair of equilibrium conviction probabilities, $(P_G, P_I) \in (P[C|G,\pi], P[C|I,\pi])$, expected payoff for a juror is

$$-q\,(1-\pi) \cdot P_I - (1-q)\,\pi \cdot (1-P_G)$$

The first term corresponds to the mistake of convicting innocent, and the second term corresponds to the mistake of acquitting guilty defendant.

Since $q(1-\pi)$ is larger than $(1-q)\pi$, $[\sigma(g)=\sigma(i)=0]$ gives higher payoffs than $[\sigma(g)=\sigma(i)=1]$ does. Since these two are the only symmetric equilibria for $\pi < \bar{\pi}(\hat{k})$, $[\sigma(g)=\sigma(i)=0]$ is the efficient equilibrium.

When $\pi \geq \bar{\pi}(\hat{k})$, the responsive equilibrium is more efficient than $[\sigma(g)=\sigma(i)=0]$ if and only if

$$\frac{P_G}{P_I} = \frac{\sum_{j=\hat{k}}^{n} \binom{n}{j} r_G^j (1-r_G)^{n-j}}{\sum_{j=\hat{k}}^{n} \binom{n}{j} r_I^j (1-r_I)^{n-j}} \geq \frac{q}{1-q} \frac{1-\pi}{\pi} \tag{13}$$

Note that $k' \geq k$, $r_G \geq r_I$ implies

$$\frac{r_G^{k'}(1-r_G)^{n-k'}}{r_I^{k'}(1-r_I)^{n-k'}} \geq \frac{r_G^{k}(1-r_G)^{n-k}}{r_I^{k}(1-r_I)^{n-k}} \tag{14}$$

Also, for each $x, x' > 0$ and $y, y' > 0$, if $\frac{x'}{y'} \geq \frac{x}{y}$, then $\frac{x+x'}{y+y'} \geq \frac{x}{y}$.

Therefore, (14) implies

$$\frac{\sum_{j=\hat{k}}^{n} \binom{n}{j} r_G^j (1-r_G)^{n-j}}{\sum_{j=\hat{k}}^{n} \binom{n}{j} r_I^j (1-r_I)^{n-j}} \geq \frac{r_G^{\hat{k}}(1-r_G)^{n-\hat{k}}}{r_I^{\hat{k}}(1-r_I)^{n-\hat{k}}}$$

So, to prove that inequality (13) holds, it is enough to show

$$\frac{r_G^{\hat{k}}(1-r_G)^{n-\hat{k}}}{r_I^{\hat{k}}(1-r_I)^{n-\hat{k}}} \geq \frac{q}{1-q} \frac{1-\pi}{\pi} \tag{15}$$

We proceed in each case.

- (Case 1:) $\bar{\pi}(\hat{k}) \leq \pi \leq \bar{\pi}(2\hat{k}-n)$

  By substituting in $r_G = p\sigma_g$ and $r_I = (1-p)\sigma_g$, the LHS of (15) is

$$\frac{r_G^{\hat{k}}(1-r_G)^{n-\hat{k}}}{r_I^{\hat{k}}(1-r_I)^{n-\hat{k}}} = \left(\frac{p}{1-p}\right)^{\hat{k}} \left(\frac{1-p\sigma_g}{1-(1-p)\sigma_g}\right)^{n-\hat{k}}$$

  A necessary equilibrium condition (10) implies implies that (15) holds with equality.

- (Case 2:) $\bar{\pi}(2\hat{k}-n) \leq \pi \leq \bar{\pi}(2(\hat{k}-1)-n)$

  Since $r_G = p$ and $r_I = 1-p$, the LHS of (15) is

$$\frac{r_G^{\hat{k}}(1-r_G)^{n-\hat{k}}}{r_I^{\hat{k}}(1-r_I)^{n-\hat{k}}} = \left(\frac{p}{1-p}\right)^{2\hat{k}-n}$$

From (11), equation (15) must be true.

- (Case 3:) $\bar{\pi}(2(\hat{k}-1)-n) \leq \pi \leq .5$

  Equation (12) is a necessary equilibrium condition. Noting that $\pi \leq .5$ and $p > .5$,

$$\left(\frac{p+(1-p)\sigma_i}{(1-p)+p\sigma_i}\right)^{\hat{k}-1}\left(\frac{1-p}{p}\right)^{n-\hat{k}+1} = \frac{q}{1-q}\,\frac{1-\pi}{\pi} \geq \frac{q}{1-q}.$$

  From $r_G = p + (1-p)\,\sigma_i$, $r_I = (1-p) + p\,\sigma_i$, we get

$$\frac{r_G^{\hat{k}}(1-r_G)^{n-\hat{k}}}{r_I^{\hat{k}}(1-r_I)^{n-\hat{k}}} = \left(\frac{p+(1-p)\sigma_i}{(1-p)+p\sigma_i}\right)^{\hat{k}}\left(\frac{1-p}{p}\right)^{n-\hat{k}} \geq \left(\frac{p+(1-p)\sigma_i}{(1-p)+p\sigma_i}\right)^{\hat{k}-1}\left(\frac{1-p}{p}\right)^{n-\hat{k}+1}$$

  Inequality (15) follows by the above two inequalities.

A.3 Properties of $P[C|G,\pi]$ and $P[C|I,\pi]$

When we find an efficient equilibrium, we partitioned $[0, .5]$ into (possibly less than) four closed intervals: $[0, \bar{\pi}(\hat{k})]$, $[\bar{\pi}(\hat{k}), \bar{\pi}(2\hat{k}-n)]$, $[\bar{\pi}(2\hat{k}-n), \bar{\pi}(2(\hat{k}-1)-n)]$, and $[\bar{\pi}(2(\hat{k}-1)-n), .5]$. The interval $[\bar{\pi}(\hat{k}), \bar{\pi}(2\hat{k}-n)]$ becomes a single point under the unanimity, and there exist multiple efficient equilibria. Besides that, in each interval, an efficient equilibrium strategy profile is single-valued, continuous, and non-decreasing in π.

For all non-unanimous rules, each adjacent intervals share the boundary point - $\bar{\pi}(\hat{k})$, $\bar{\pi}(2\hat{k}-n)$, and $\bar{\pi}(2(\hat{k}-1)-n)$ - so $P[C|G,\pi]$ and $P[C|I,\pi]$ are continuous and non-decreasing on $[0, .5]$. For the unanimity rule, only at $\pi = \bar{\pi}(\hat{k}=n)$, there exist multiple efficient equilibria, so $P[C|G,\pi]$ and $P[C|I,\pi]$ have multiple values. Both conviction probabilities are continuous and monotone increasing function of $\sigma(g)$ with given $\sigma(i) = 0$, and any $\sigma(g) \in [0, 1]$ constitutes an equilibrium at $\pi = \bar{\pi}(\hat{k})$. Thus, $P[C|G, \pi = \bar{\pi}(\hat{k})]$ and $P[C|I, \pi = \bar{\pi}(\hat{k})]$ are convex valued. Moreover, conviction probabilities are non-decreasing single valued function on $[0, .5]\backslash\{\pi(\hat{k})\}$, and the right limit of $\sigma(g)$ is larger than the left-limit at $\pi = \bar{\pi}(\hat{k}=n)$. Thus, the strategy profiles, thus conviction probabilities are non-decreasing in π on $[0, .5]$. Lastly, all efficient equilibria are characterized with weak inequality conditions. Therefore, an equilibrium and conviction probabilities have closed graph, thus upper hemicontinuous.

B Proof of the Proposition 3

An explicit solution formula is not our concern. Instead, we study the motivation behind this optimization.

Note that each probability is computed as

$$P_G = \sum_{k=\hat{k}}^{n} \binom{n}{k} r_G^k (1-r_G)^{n-k}$$
$$P_I = \sum_{k=\hat{k}}^{n} \binom{n}{k} r_I^k (1-r_I)^{n-k}.$$

Let θ^{info} be the conviction probability when each juror votes based only on her private signal: $\sigma(g) = 1$ and $\sigma(i) = 0$.

When $\pi \in [\bar{\pi}(\hat{k}), \bar{\pi}(2\hat{k} - n)]$, $\sigma(i) = 0$ and only $\sigma(g)$ might varies. Then, P_G and P_I are differentiable with respect to $\sigma(g)$. (We compute either left-side or right-side derivative for the boundary points, $\{0, 1\}$.) If $\pi \in [\bar{\pi}(2(\hat{k}-1)-n), .5]$, $\sigma(g) = 1$ and only $\sigma(i)$ may change. $P[C|G]$ and $P[C|I]$ are differentiable with respect to $\sigma(i)$.

When P_G and P_I are differentiable with respect to σ, which is either $\sigma(g)$ or $\sigma(i)$, the derivative of P_G is

$$\begin{aligned}
\frac{\partial P_G}{\partial \sigma} &= \frac{\partial}{\partial \pi} \sum_{k=\hat{k}}^{n} (r_G)^k (1-r_G)^{n-k} \\
&= \sum_{k=\hat{k}}^{n-1} \left(\frac{n!}{k!(n-k)!} k r_G^{k-1} (1-r_G)^{n-k} r'_G \right. \\
&\quad \left. - \frac{n!}{k!(n-k-1)!} r_G^k (n-k)(1-r_G)^{n-k-1} r'_G \right) + n r_G^{n-1} r'_G \\
&= n r'_G \binom{n-1}{\hat{k}-1} r_G^{\hat{k}-1} (1-r_G)^{n-\hat{k}} \qquad (16)
\end{aligned}$$

With similar algebra, we get

$$\frac{\partial P_I}{\partial \sigma} = n r'_I \binom{n-1}{\hat{k}-1} r_I^{\hat{k}-1} (1-r_I)^{n-\hat{k}} \qquad (17)$$

The objective function in prosecutor's optimization problem is differentiable with respect to

$\sigma(g)$ if $\pi \in [\bar{\pi}(\hat{k}), \bar{\pi}(2\hat{k}-n)]$, and with respect to $\sigma(i)$ if $\pi \in [\bar{\pi}(2(\hat{k}-1)-n), .5]$ as

$$-q'\frac{\partial P_I}{\partial \sigma} + (1-q')\frac{\partial P_G}{\partial \sigma}$$

This derivative of the objective function is nonnegative if and only if

$$-q'\frac{\partial P_I/\partial \sigma}{\partial P_G/\partial \sigma} + (1-q') \geq 0$$

which we can rewrite using (16) and (17) as

$$\frac{r_G'}{r_I'}\left(\frac{r_G}{r_I}\right)^{\hat{k}-1}\left(\frac{1-r_G}{1-r_I}\right)^{n-\hat{k}} \geq \frac{q'}{1-q'} \tag{18}$$

We prove the Proposition 3 for each cases of θ^*. Case 1 to 4 is for the second item in the Proposition; Case 5 is for the first item.

Provided that the optimal θ^* falls into,

- (Case 1:) $0 < \theta^* < \theta^{\text{info}}$: the corresponding equilibrium yields $r_G = p\sigma(g), r_I = (1-p)\sigma(g)$. There exist a unique $\sigma(g) \in (0,1)$ such that $[\sigma(g), \sigma(i) = 0]$ obtains $\theta^* = P_G$. Since the map from θ^* to the corresponding $\sigma(g)$ is strictly increasing, first order condition implies that (18) holds with equality. That is,

  $$\left(\frac{p}{1-p}\right)^{\hat{k}}\left(\frac{1-p\sigma(g)}{1-(1-p)\sigma(g)}\right)^{n-\hat{k}} = \frac{q'}{1-q'}.$$

  Compared with jurors' voting criterion (10), we obtain

  $$\frac{q}{1-q}\frac{1-\pi}{\pi} = \frac{q'}{1-q'}$$

- (Case 2:) $\theta^{\text{info}} < \theta^* < \bar{P}$

  An equilibrium induces $r_G = p + (1-p)\sigma(i), r_I = (1-p) + p\sigma(i)$. The map from θ^* to the corresponding $\sigma(i)$ is strictly increasing. Therefore, first order condition implies (18) to hold with equality.

  $$\left(\frac{p+(1-p)\sigma_i}{(1-p)+p\sigma_i}\right)^{\hat{k}-1}\left(\frac{1-p}{p}\right)^{n-\hat{k}+1} = \frac{q'}{1-q'}.$$

  Comparing with jurors' voting criterion (12), we get

$$\frac{q}{1-q}\frac{1-\pi}{\pi} = \frac{q'}{1-q'}$$

- (Case 3:) $\theta^* = 0$

  The right-side derivative with respect to $\sigma(g)$ at $\sigma(g) = \sigma(i) = 0$ must be non-positive. Then, (18) leads to

$$\left(\frac{p}{1-p}\right)^{\hat{k}} \leq \frac{q'}{1-q'}$$

  Compared to the voting criterion (2), the juror's voting behavior is the same as when she perceives $\pi = .5$ and $q = q'$

- (Case 4:) $\theta^* = \theta^{\text{info}}$

  strategic voting is induced: $r_G = p$ and $r_I = (1-p)$. First order conditions imply that the left-side derivative is non-negative, and right-side derivative with respect to $\sigma(g)$ is non-negative at $(\sigma(g) = 1, \sigma(i) = 0)$. Then, (18) leads to

$$\left(\frac{p}{1-p}\right)^{\hat{k}}\left(\frac{1-p}{p}\right)^{n-\hat{k}} \geq \frac{q'}{1-q'}$$
$$\left(\frac{p}{1-p}\right)^{\hat{k}-1}\left(\frac{1-p}{p}\right)^{n-\hat{k}+1} \leq \frac{q'}{1-q'}$$

  which we can rewrite as

$$\left(\frac{p}{1-p}\right)^{2(\hat{k}-1)-n} \leq \frac{q'}{1-q'} \leq \left(\frac{p}{1-p}\right)^{2\hat{k}-n}$$

  Compared with (11), each juror faces exactly same voting criterion as a trial with $\pi = .5$ and $q = q'$.

- (Case 5:) $\theta^* = \bar{P}$

  The first order condition implies that the left-side derivative with respect to $\sigma(i)$ is non-negative at the equilibrium level of $\sigma(i)$ under $\theta^* = \bar{P}$. We obtain from (18)

$$\left(\frac{p+(1-p)\sigma_i}{(1-p)+p\sigma_i}\right)^{\hat{k}-1}\left(\frac{1-p}{p}\right)^{n-\hat{k}+1} \geq \frac{q'}{1-q'}$$

while the voting criterion implies

$$\left(\frac{p+(1-p)\sigma_i}{(1-p)+p\sigma_i}\right)^{\hat{k}-1}\left(\frac{1-p}{p}\right)^{n-\hat{k}+1} = \frac{q}{1-q}.$$

Therefore, $\frac{q}{1-q} \geq \frac{q'}{1-q'}$, or $q \geq q'$ is a necessary condition.

Since we have $q \leq q'$ as a necessary condition of Case 1 to 4, the prosecutor offers $\theta^* = \bar{P}$ if $q > q'$.

C Proof of Lemma 5

Without loss of generality, we assume that $q_M > q_m$.

There exists at most three intervals partitioning $[q_m, q_M]$ where each interval has the same conviction vote strategy. We denote the thresholds for the partition as q_i and q_g. An equilibrium consists of jurors with reasonable doubt in $[q_m, q_i]$ who vote only for conviction, $(q_i, q_g]$ who vote informatively, and $(q_g, q_M]$ who never vote for conviction.[13] Then we have $\tilde{F}(q_g)$ for the conviction voting probability given signal g, and $\tilde{F}(q_i)$ for the conviction voting probability given the signal i. The thresholds, if exist, are determined by

$$\left(\frac{r_G}{r_I}\right)^{\hat{k}-1}\left(\frac{1-r_G}{1-r_I}\right)^{n-\hat{k}}\frac{p}{1-p} = \frac{q_g}{1-q_g}\frac{1-\pi}{\pi} \tag{19}$$

$$\left(\frac{r_G}{r_I}\right)^{\hat{k}-1}\left(\frac{1-r_G}{1-r_I}\right)^{n-\hat{k}}\frac{1-p}{p} = \frac{q_i}{1-q_i}\frac{1-\pi}{\pi} \tag{20}$$

where $r_G = p\cdot\tilde{F}(q_g) + (1-p)\cdot\tilde{F}(q_i)$, $r_I = (1-p)\cdot\tilde{F}(q_g) + p\cdot\tilde{F}(q_i)$. When we have both thresholds, we obtain

$$\frac{q_m}{1-q_m}\left(\frac{p}{1-p}\right)^2 \leq \frac{q_i}{1-q_i}\left(\frac{p}{1-p}\right)^2 \leq \frac{q_g}{1-q_g} \leq \frac{q_M}{1-q_M}$$

Therefore, the condition

$$\frac{q_m}{1-q_m}\left(\frac{p}{1-p}\right)^2 > \frac{q_M}{1-q_M} \tag{21}$$

guarantees that there exists at most one threshold.

[13] We ignore the boundary cases which is measure zero.

If there is exactly one threshold, the $\sigma(g) = 1$ or $\sigma(i) = 0$ must be true; whereas if there exists no threshold level, either $[\sigma(g) = 1, \sigma(i) = 0]$ or $[\sigma(g) = \sigma(i) = 0]$ must be true.[14]

If (21) holds, there is a representative preference with which the efficient equilibrium outcome is the same as the equilibrium with heterogeneous preferences. For instance, in the case of one threshold, $\tilde{F}(q_i) = 0$ and $0 < \tilde{F}(q_g) < 1$, we set $q = q_g$ then $\sigma(g) = \tilde{F}(q_g)$ and $\sigma(i) = 0$ will be an efficient equilibrium with the representative preference $q = q_g$. When the original equilibrium is either strategic voting or always voting for conviction, the same equilibrium can be reproduced with any representative preference $q \in [q_m, q_M]$.

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[14] $[\sigma(g) = \sigma(i) = 1]$ is payoff dominated by $\sigma(g) = \sigma(i) = 0$

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