The Bridge Policy Problem - Abstract

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Abstract

We study variants of an optimization problem posed by Glazer & Rubinstein [1], in which a listener decides which arguments to accept, or alternatively a transit authority decides which bridges to open. We show that a maximization version of the problem essentially admits no nontrivial approximation algorithm; for a minimization version, we give a logarithmic factor approximation algorithm, and provide a matching lower bound. Moreover, we provide dynamic programming algorithms to solve the problem optimally in certain constrained settings. Finally, we study the problem modeled as a two-person simultaneous game.

1 Introduction

Glazer & Rubinstein [1] study a persuasion problem in which a speaker is trying to persuade a listener to take some piece of advice. There are m possible states of the world (each with an associated weight), and n possible arguments (or statements) the speaker may choose among. For each possible state i, the advice is either good or bad, i.e., either applicable or not. Also for each possible state i, there is a set of statements $\sigma(i)$ that the speaker can truthfully invoke in order to make its case. The posed problem is defined from the point of view of the listener, who must choose which statements to be persuaded by, should the speaker choose to make them, and which not to. The listener will be persuaded while in world i if at least one of the states among $\sigma(i)$ has been deemed persuasive. The objective is to maximize the "net flow" of good arguments, i.e., maximizing the (weighted) difference of the number of good states that lead to persuasion minus the number of bad states that do so. When solving optimally, the objective is equivalent to minimizing the total combined number of false positive errors (accepting the advice when it does not apply) and false negative errors (rejecting the advice when it does).

The authors also present an analogous problem in terms of n bridges and m people. Each person i has a weight, which can be either positive or negative, or *good* or *bad*. For each person p there is a set of bridges $\sigma(p)$ that p is capable of using, if they happen to be open. The problem is to choose a *bridge policy*—to decide which bridges to open and which to close—in order to maximize the (weighted) net flow across the river, or equivalently, to again minimize the total combined number of false positive and false negative errors.

2 Problem settings and results

Let TP, FP, TN, FN indicate the numbers of true positives, false positives, true negatives, and false negatives, respectively. Then the two objectives are maximizing $TP - FP$ and minimizing

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$FP+FN$, respectively. Glazer & Rubinstein take for granted that the problem is solvable by brute-force enumeration. We show, however, that this approach quickly becomes infeasible. Specifically, we show that solving the problem optimally is NP-hard, and we provide hardness of approximation results. Reducing from the Maximum Independent Set (MIP) problem [2], we obtain the following:

Proposition 2.1 *The net flow variant of the Bridge Policy problem is NP-hard to solve and NP-hard to approximate with factor better than* $n^{1-\epsilon}$.

NP-hardness of optimally solving the min error setting follows from the hardness of the net flow setting: maximizing $TP - FP$ is the same as minimizing $FP - TP + g = FP + FN$ (where g is the weighted number of goods). Hardness of approximation, however, does not follow. The min error setting turns out to be Set Cover-hard [2]:

Proposition 2.2 *The min error problem is NP-hard to solve and NP-hard to approximate with ratio* $(1-\epsilon)\ln n$ *for any* $\epsilon > 0$ *(unless NP has* $n^{O(\log\log n)}$*-time algorithms).*

It turns out that there is a close affinity between the Set Cover problem and the min error variant of this problem. By a nontrivial modification of Set Cover greedy algorithm, we obtain an efficient algorithm whose approximation ratio essentially matches this lower bound.

Moreover, for multiple "convex" settings we provide optimal algorithms that solve the problem in polynomial time, specifically people and bridges are associated with discrete locations (e.g., along a river), and either 1) each person's bridge set forms an interval, or 2) each bridge's person set forms an interval. Here a set of entities (either bridges or people) forms an interval if for any two items contained in the set, any third item positioned between them is also in the set. It is natural to assume that each person has a position and can reach bridges only within some neighborhood, one the one hand, or that each bridge attracts only nearby people, on the other. For both cases, we give left-to-right quadratic-time dynamic programming algorithms.

3 Simultaneous games

The above setting can be construed as a game in which first Row moves, and then Column moves:

- Row: the listener decides which arguments to credit (resp., the policymaker decides which bridges to open)
- Column: the speaker chooses an argument to make in each possible world (resp., the opponent chooses bridges at which to position each bad)

The value for Row can be defined as e.g. either the net flow or min error objectives given above; the value for Column can be defined as the fraction of world states in which Row is persuaded, or equivalently the fraction of people able to cross the bridge. In ongoing research, we are studying equilibria for the setting in which Row and Column move simultaneously.

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References

[1] K. Glazer and A. Rubinstein. A study in the pragmatics of persuasion: A game theoretical approach. *Theoretical Economics*, 1:395–410, 2006.

[2] V. V. Vazirani. *Approximation Algorithms.* Springer, 2004.