

Bargaining with Revoking Costs ^{*}

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Abstract

A simple two stage bilateral bargaining game is analyzed. The players simultaneously demand shares of a unit size pie in the first stage. If the demands add up to more than one, both players, in the second stage, simultaneously choose whether to stick to their demand or accept the other's offer. While both parties sticking to their offers leads to an impasse, accepting a lower share than the original demand is costly for each party. The set of pure strategy subgame perfect equilibria of the game is characterized for continuous payoff functions strictly increasing in the pie share and continuous cost functions, strictly increasing in the amount conceded. Higher cost functions are shown to improve bargaining power. The limit equilibrium prediction of the model, as the cost functions are made arbitrarily high, selects a unique equilibrium in the Nash Demand Game.

1 Introduction

A trade union leader who announces a demand in a negotiation with the management may risk losing his job if he accepts a lower share than the demand. The President of a country may face a tougher re-election prospect if she fails to achieve her publicly announced demand in a domestic or international bargaining situation. More generally, backing down from an initial demand made in some bargaining scenarios may entail a cost. While seemingly a weakness, these costs may actually confer greater bargaining power to the party facing these costs. If this cost makes the party prefer an impasse to concession, following incompatible offers, the said party can force a concession from her opponent who does not face such costs. The cost of revoking an earlier demand therefore gives a bargainer an ability to partially commit herself to a stated demand. I study a simple model of bilateral bargaining to identify and characterize the relationship between such revoking costs and bargaining power.

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Two players bargain over a unit sized pie in a two stage game. Each player announces a demand in the first stage. If the demands add up to less than one, they split midway between their demands. Otherwise, in the second stage, each party chooses simultaneously whether to stick to their own demand or accept the offer of the other player. Both parties sticking to their incompatible offers results in an impasse. Accepting the other players offer, however, is costly, with the cost increasing in the amount by which the accepted share is lower than the demanded share.

The set of pure strategy subgame perfect equilibrium is characterized in terms of the cost functions. The highest and lowest equilibrium payoffs for a given player are shown to increase with an increase in the revoking cost functions. The set of equilibria is shown to shrink with higher cost functions. Indeed, as the cost functions are made arbitrarily high the limit of the equilibrium set is shown to make a unique equilibrium selection in the limit game which can be interpreted as the Nash demand game, Nash(1953). The model captures the insight that a bargainer wishes to make it difficult for herself to concede to a lower offer. Interestingly it shows how making a greater demand for oneself results in making concession more difficult for the other party, giving the latter higher commitment ability. The equilibria, as a result, are characterized by a tradeoff between the twin needs of higher shares and greater commitment ability.

A number of papers have formally analyzed the effect of commitment tactics on bargaining outcomes following the insights found in Schelling(1956). Crawford(1982) and Ellingsen and Miettinen(2008) analyze a two stage game like above, with the difference that each bargainer makes the initial demand while still uncertain about the revoking cost which, unlike the present model, is a constant and independent of the demanded and accepted shares. The bargainers find out their own revoking cost before taking the second stage actions. These papers focus on the role that commitment tactics play in generating delay in bargaining. In the present model, the complete information structure results in efficiency all too readily.

In terms of objective, this paper follows Muthoo(1996, 1999) and Leventoglu and Tarar(2005). In Muthoo(1996), the first paper to formally analyze the effect of revoking costs on bargaining power, following incompatible offers in the first stage, the bargaining outcome is selected by the Nash Bargaining Solution (NBS) applied to a modified utility possibility set(UPS). A given share of the pie is mapped to this UPS with players paying a cost for a share lower than their demand. The analysis is carried out for convex cost functions and concave utility functions that are strictly increasing and twice continuously differentiable and common knowledge in the first stage. Leventoglu and Tarar(2005) conduct their analysis of the game with linear cost and utility functions, by explicitly modeling the second stage as a Rubinstein bargaining game (Rubinstein(1982)). Both papers relate an increase in revoking costs to increased bargaining power, with unique equilibrium predictions. By retaining the second stage game of Crawford(1982) the present model avoids specifying particular bargaining protocols while providing a simple framework to analyze the effect of any strictly increasing and differentiable revoking cost function on bargaining power. The present

model mirrors the comparative static results of Muthoo(1996) and provides further support for the equilibrium selection argument in the Nash Demand Game. While the above two papers capture scenarios where bargainers have the ability to renegotiate endlessly after their initial demand, the present model studies the opposite benchmark, where bargainers cannot make offers beyond the initial demand they partially commit to. Besides providing a transparent and simple analysis of the tradeoff between higher demands and greater commitment, the model allows for extensions where both parties sticking to incompatible demands leads to a next round of negotiation where the players get a chance to change their demand commitments. The simple two stage framework can be used as the stage game of a repeated game, to capture scenarios like international negotiations where each party gets to change their publicly announced demands after the failure of an earlier round of negotiation, but backing down from the most recent stated demand in a given round of negotiation incurs a cost. Given the general class of cost functions which the paper studies one could model scenarios where these cost functions change over time.

Interestingly, the rationale for extreme divisions being ruled out in this model and the equilibrium strategies are similar in spirit to findings in Kambe(1999) and Abreu and Gul(2000), where by making a lower demand a player can force her opponent to make the initial mass acceptance in the second stage war of attrition. The commitment possibilities in the latter two papers, however, are generated by the presence of behavioral types.

The rest of the paper is as follows. Section 2 presents the formal model. Section 3 analyzes the special case of the model where the payoff and cost functions are linear. The intuition behind the equilibrium strategies in the general model can be found here. Further, it is easier to foresee the comparative statics and limit arguments for the general model, by analyzing the linear specification. Section 4 characterizes the equilibrium set for the general model. Section 5 deals with comparative statics and the limit predictions of the model as the cost functions are made arbitrarily high. Section 6 concludes.

2 The Bargaining Game

Two players, 1 and 2, play a two stage game. In the first stage, player i chooses a level of demand $z_i \in [0, 1)$. If $z_1 + z_2 \leq 1$ the game ends with the division $(z_1 + \frac{1-z_1-z_2}{2}, z_2 + \frac{1-z_1-z_2}{2})$. The corresponding payoffs are $\pi_1(z_1 + \frac{1-z_1-z_2}{2}), \pi_2(z_2 + \frac{1-z_1-z_2}{2})$, where π_i is the payoff function for player i . If $z_1 + z_2 > 1$ then the following second stage simultaneous move game is played.

	<i>Accept</i>	<i>Stick</i>
<i>Accept</i>	$\pi_1(x_1) - c_1(z_1 - x_1), \pi_2(x_2) - c_2(z_2 - x_2)$	$\pi_1(1 - z_2) - c_1(d), \pi_2(z_2)$
<i>Stick</i>	$\pi_1(z_1), \pi_2(1 - z_1) - c_2(d)$	$\pi_1(0), \pi_2(0)$

where $x_i = z_i + \frac{1-z_i-z_j}{2}$ and $d = z_1 + z_2 - 1$.

The interpretation of this game is as follows. If and when the two players make incompatible demands ($z_1 + z_2 > 1$), player i must choose whether to stick to her own demand or accept j 's offer, which must be less. However, there is a cost attached to accepting a division of the pie which is less than the share demanded in the first stage. This can happen if either player i *Accepts* while j *Sticks* or if both players choose *Accept*. This feature is captured by the cost function c_i for player i . So if player i had initially demanded z_i which was incompatible with player j 's demand, z_j , then accepting j 's offer in the second stage while j sticks to his offer would give player i a payoff of $\pi_i(1 - z_j) - c_i(z_i - (1 - z_j))$. If both players choose to *Accept* in the second stage following incompatible offers (z_1, z_2) , then player i gets a compromise share $x_i = z_i + \frac{1 - z_i - z_j}{2}$ with a payoff of $\pi_i(x_i)$ and also pays the cost for accepting a lower share, $c_i(z_i - x_i)$. Note that since the second stage game is played only if $z_1 + z_2 > 1$, it must be true that $x_i < z_i$. Finally if both players decide to stick to their incompatible demands they get their disagreement payoff, $(\pi_1(0), \pi_2(0))$. The following assumptions are met by the payoff and cost functions in the rest of the note.

A1. For $i \in \{1, 2\}$, π_i is a strictly increasing and continuously differentiable function. Further $\pi_i(0) = 0$ and $\pi_i(1) = 1$.

A2. For $i \in \{1, 2\}$, c_i is a strictly increasing and continuously differentiable function with $c_i(0) = 0$.

This completes the description of the two stage bargaining game.

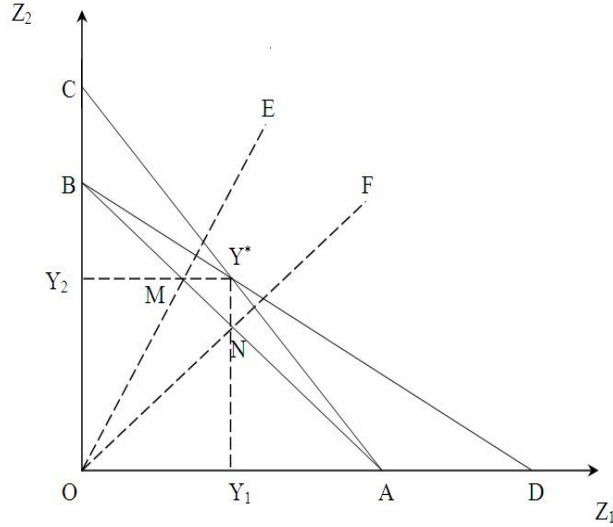


Figure 1: The Linear Model

3 The Linear Model

In this section the payoff and cost functions are assumed to be linear. In particular, $\pi_i(x) = x$ and $c_i(d) = k_i d$ where $k_i > 0$. The second stage game, therefore, is as follows

	<i>Accept</i>	<i>Stick</i>
<i>Accept</i>	$x_1 - k_1(z_1 - x_1), x_2 - k_2(z_2 - x_2)$	$1 - z_2 - k_1(z_1 + z_2 - 1), z_2$
<i>Stick</i>	$z_1, 1 - z_1 - k_2(z_1 + z_2 - 1)$	$0, 0$

Figure 1 illustrates the intuition behind Proposition 1. *BA* represents demand profiles that add up to one. *OE* and *EF* represent $z_2/z_1 = (1 + k_2)/k_1$ and $z_2/z_1 = k_2/(1 + k_1)$. *BD* is the graph of $1 - z_2 - k_1(z_1 + z_2 - 1) = 0$ while *CA* graphs $1 - z_1 - k_2(z_1 + z_2 - 1) = 0$. For points lying above (below) *BD* it must be that $1 - z_2 - k_1(z_1 + z_2 - 1) < (>)0$. Similarly points lying above (below) *CA* satisfy $1 - z_1 - k_2(z_1 + z_2 - 1) < (>)0$. Lemmas 1 and 2 show that the first stage demands in a pure strategy SPE must lie on *BA*. Incompatible offers (points above *BA*) can be separated into 4 regions. For points above *CY*D*, both players prefer *Stick* to *Accept*. In the *AY*D* region the unique NE in the second stage involves 1 playing *Accept* while 2 *Sticks*. Incompatible offers from the *CBY** region results in the unique NE (*Stick, Accept*) in the second stage. Finally for first stage offers in *BY*A* both (*Accept, Stick*) and (*Stick, Accept*) are Nash Equilibria of the second stage. Lemma 3 essentially shows that equilibrium demands cannot be in the *AN* region since player 2 would then have the incentive to deviate to a point in *AY*D*, forcing a concession from player 1 and getting a higher payoff. A symmetric argument rules out the *BM* region. Notice that making a high demand (greater than Y_1) gives player 2 greater commitment power. Indeed by making a demand which selects a point in *AY*D* player 2 ends up making *Stick* her dominant strategy in the second stage game, while leaving player 1 enough room to prefer conceding to an impasse. Demand profiles that are not ruled out as above, therefore, lie on *MN*. Proposition 1 also specifies subgame perfect strategies to support these demands.

Let (z_1, z_2) be the demands made in the first stage of a pure strategy subgame perfect equilibrium of the linear model.

Lemma 1. $z_1 + z_2 \not< 1$

Proof. This is immediate, since if $z_1 + z_2 < 1$, player i can deviate by demanding $1 - z_j$. Since $(1 - z_j, z_j)$ is still compatible, player i gets a payoff of $1 - z_j$ which is strictly higher than the original payoff z_i , as $z_1 + z_2 < 1$. \square

Lemma 2. $z_1 + z_2 \not> 1$

Proof. Suppose $z_1 + z_2 > 1$. Let the payoffs in the second stage game, which must now be played, be (y_1, y_2) . Due to the nature of the bargaining game the

outcome must be determined by a pure strategy Nash Equilibrium of the second stage game. Note that $\{Accept, Accept\}$ could never be a Nash Equilibrium of the second stage game.

Suppose the Nash Equilibrium in the second stage game for this SPE involves the strategies $\{Stick, Accept\}$. Then $y_1 = z_1$ and $y_2 = 1 - z_1 - k_2(z_1 + z_2 - 1)$. Consider what happens if player 2 deviates to making the compatible demand $\tilde{z}_2 = 1 - z_1$, in the first stage. The payoffs from this deviation are $(z_1, 1 - z_1)$. Given that $1 - z_1 > y_2$, this is a profitable deviation. So if $z_1 + z_2 > 1$ and (z_1, z_2) are demands made in a subgame perfect equilibrium, the second stage Nash Equilibrium cannot involve $\{Stick, Accept\}$. A symmetric argument rules out $\{Accept, Stick\}$. If the second stage Nash Equilibrium is $\{Stick, Stick\}$ then $y_1 = y_2 = 0$. Player i could then profitably deviate by demanding $\tilde{z}_i = \epsilon$ where $0 < \epsilon = 1 - z_j$, thereby making a compatible offer and receiving a payoff of ϵ . So irrespective of the pure strategy Nash Equilibrium in the second stage game, there is always a profitable deviation for some player if $z_1 + z_2 > 1$. \square

Lemmas 1 and 2 imply that if (z_1, z_2) are demands made in a pure strategy SPE of the bargaining game, it must be that $z_1 + z_2 = 1$.

Lemma 3. *If (z_1, z_2) is the demand profile in a pure strategy SPE of the bargaining game with $z_1 + z_2 = 1$ then $\exists \epsilon > 0$ and $i \in \{1, 2\}$ such that*

$$1 - z_i - k_j \epsilon < 0 \quad (1)$$

and

$$1 - z_j - \epsilon - k_i \epsilon > 0 \quad (2)$$

Proof. Suppose not. Let $\epsilon > 0$ and let $1 - z_i - k_j \epsilon < 0$ and $1 - z_j - \epsilon - k_i \epsilon > 0$ for some $i \in \{1, 2\}$ with (z_i, z_j) being the demands made in an SPE of the bargaining game. I will show that player j has a profitable deviation. With the present demand profile, (z_i, z_j) the payoffs are also (z_i, z_j) due to compatibility. Now suppose player j deviates to making the incompatible offer $z_j + \epsilon$. Due to incompatible offers the second stage game would have to be played. If player i chooses *Accept* then player j is clearly better off choosing *Stick*. If player i chooses *Stick* then j 's payoff from choosing *Accept* is $1 - z_i - k_j \epsilon$ which is strictly less than the 0 he gets if he *Sticks*, given the assumption above. So *Stick* strictly dominates *Accept* for player j . Given that player j will choose *Stick* player i would get $1 - z_j - \epsilon - k_i \epsilon$ if she chose *Accept* which is strictly greater than the 0 she would get if she chooses *Stick*. Consequently the unique Strict Nash Equilibrium of the second stage game following the deviation would involve i playing *Accept* and j playing *Stick*, with a payoff of $z_j + \epsilon$ for player j . Hence player j has a profitable deviation. \square

Proposition 1. $\frac{k_2}{1+k_1} \leq \frac{z_2^*}{z_1^*} \leq \frac{1+k_2}{k_1}$ and $z_1^* + z_2^* = 1$ are necessary and sufficient conditions for (z_1^*, z_2^*) to be a pure strategy subgame perfect equilibrium outcome of the bargaining game with linear payoffs and costs.

Necessity

Proof. Let (z_1^*, z_2^*) be the demands made in a pure strategy SPE of the bargaining game. From lemmas 1 and 2 it must be that $z_1^* + z_2^* = 1$. If an ϵ satisfies the conditions of lemma 3 for (z_1^*, z_2^*) it must be that $\epsilon > \frac{z_1^*}{k_1}$ (from (1), setting $i = 2$) and $\epsilon < \frac{z_2^*}{1+k_2}$ (from (2), setting $i = 2$). So it must be that $\frac{z_1^*}{k_1} < \epsilon < \frac{z_2^*}{1+k_2}$. Now, given that $\frac{z_2^*}{1+k_2}$ is bounded above by 1, such an ϵ will not exist *iff*

$$\frac{z_2^*}{1+k_2} \leq \frac{z_1^*}{k_1} \quad (3)$$

A similar argument using (1) and (2) and setting $i = 1$ shows that for profitable deviations of the kind considered in Lemma 3 not to exist, it must also be true that

$$\frac{z_1^*}{1+k_1} \leq \frac{z_2^*}{k_2} \quad (4)$$

Combining (3) and (4) gives us the necessary condition for (z_1^*, z_2^*) to be the equilibrium demands; namely

$$\frac{k_2}{1+k_1} \leq \frac{z_2^*}{z_1^*} \leq \frac{1+k_2}{k_1} \quad (5)$$

□

Sufficiency

Proof. Let (z_1^*, z_2^*) satisfy (5) and $z_1^* + z_2^* = 1$. I will construct strategies that constitute an SPE of the bargaining game using these demands. In the first stage player 1 demands z_1^* while player 2 demands z_2^* . If the second stage game is reached and if player 2 demanded $z_2 > z_2^*$ in the first stage, then player 1 chooses $\{Stick\}$ while player 2 chooses $\{Accept\}$ if $1 - z_1^* - k_2(z_2 - z_2^*) > 0$ and $\{Stick\}$ otherwise. Similarly, if player 1 demanded $z_1 > z_1^*$ in the first stage, then player 2 chooses $\{Stick\}$ while player 1 chooses $\{Accept\}$ if $1 - z_2^* - k_1(z_1 - z_1^*) > 0$ and $\{Stick\}$ otherwise.

To see why these strategies constitute an SPE of the bargaining game, note first that neither player has any incentive to demand a lesser amount. Now consider player i 's incentives to deviate by demanding $z_i > z_i^*$. If in the second stage player i is required by the strategies to play $\{Accept\}$ then it must be that $1 - z_j^* - k_i(z_i - z_i^*) > 0$. Given that player j 's strategy requires j to $\{Stick\}$, i would do strictly worse by deviating to $\{Stick\}$. Further, given that i chooses $\{Accept\}$ player j can do no better than play $\{Stick\}$ as is required by his strategies. In other words the off equilibrium strategies induce a strict Nash Equilibrium of the second stage game when i demands $z_i > z_i^*$ and $1 - z_j^* - k_i(z_i - z_i^*) > 0$. So by deviating to z_i , i gets a payoff of $1 - z_j^* - k_i(z_i - z_i^*)$ which is strictly less than the payoff of $1 - z_j^*$ she was guaranteed under the original strategies. Now, if the deviation z_i is such that $1 - z_j^* - k_i(z_i - z_i^*) < 0$ the strategies require i to $\{Stick\}$ which is indeed her dominant strategy in this case. By the fact that

(z_1^*, z_2^*) satisfies (5) it must be the case that $\nexists \epsilon > 0$ such that $1 - z_j^* - k_i \epsilon < 0$ and $1 - z_i^* - \epsilon - k_j \epsilon > 0$. However the deviation z_i is such that setting $\epsilon = z_i - z_i^*$ we get $1 - z_j^* - k_i \epsilon < 0$. So (5) implies that $1 - z_i^* - \epsilon - k_j \epsilon \leq 0$. Substituting for ϵ we get $1 - z_i - k_j(z_j^* - (1 - z_i)) \leq 0$. The left hand term in this inequality is the payoff j gets from choosing *Accept* while choosing *Stick* gives him 0. Therefore, j 's optimal action continues to be $\{Stick\}$ as suggested by the strategies. The pure Nash Equilibrium in the second stage after such deviations, thus, involve a payoff of $(0, 0)$, which makes i strictly worse off. As a result i has no incentive to deviate from the specified strategies. Hence, the strategies specified above constitute an SPE of the bargaining game. \square

4 The General Model

In this section the only assumptions imposed on the payoff and cost functions are **A1** and **A2**.

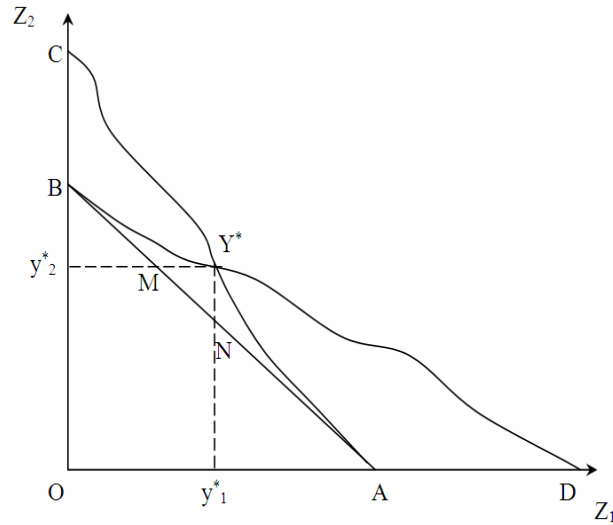


Figure 2: The General Model

Figure 2 captures the workings of Proposition 2. Note that the coordinates of a given point in the figure correspond to the shares of the pie demanded by each party. BA is the same as Fig. 1. BD is the graph for $\pi_1(1 - z_2) - c_1(z_1 + z_2 - 1) = 0$, while CA graphs $\pi_2(1 - z_1) - c_2(z_1 + z_2 - 1) = 0$. The intuition for why first stage demands must lie on MN is exactly the same as in the linear case, as can be seen by comparing this with Fig. 1. The only substantial addition for the general model is to show that the curves, BD and CA , which are generated by the particular payoff and cost functions, have a unique intersection point. This is indeed true given **A1** and **A2** and is established by Lemmas 4 and

5. Proposition 2, also provides subgame perfect strategies that can support demand profiles which lie on MN .

The following lemmas help pin down the exact characterization of the equilibrium demands.

Lemma 4. *There exists a unique $\bar{y}_j \in (0, 1)$ such that $\pi_i(1 - y_j) = c_i(y_j)$.*

Proof. Let $g_i(y_j) = \pi_i(1 - y_j) - c_i(y_j)$. Note that $g_i(0) = 1$ and $g_i(1) = -c_i(1) < 0$. Further, g_i is a strictly decreasing and continuous function. Consequently by the intermediate value theorem there exists \bar{y}_j such that $g_i(\bar{y}_j) = 0$. Further, given that g_i is strictly decreasing, $\bar{y}_j \in (0, 1)$. \square

Lemma 5. *There exists a unique (y_1, y_2) with $y_i \in (0, 1)$ that solves,*

$$\pi_1(1 - y_2) = c_1(y_1 + y_2 - 1) \quad (6)$$

and

$$\pi_2(1 - y_1) = c_2(y_1 + y_2 - 1) \quad (7)$$

Proof. Define the function $\hat{y}_1(y_2) = c_1^{-1}(\pi_1(1 - y_2)) + 1 - y_2$ for all $y_2 \in \{0, 1\}$ such that $\exists d > 0$ with $c_1(d) = \pi_1(1 - y_2)$.

Note that $\hat{y}_1(1) = 0$. By lemma 4 there exists $\bar{y}_2 \in (0, 1)$ such that $\hat{y}_1(\bar{y}_2) = 1$. Further, given **A1** and **A2**, \hat{y}_1 is a well defined, continuously differentiable and strictly decreasing function on $[\bar{y}_2, 1]$ with

$$\frac{\partial \hat{y}_1}{\partial y_2} = \frac{-\pi_1'(1 - y_2)}{c_1'(c_1^{-1}(\pi_1(1 - y_2)))} - 1 < -1 \quad (8)$$

Similarly define the function $\hat{y}_2(y_1) = c_2^{-1}(\pi_2(1 - y_1)) + 1 - y_1$ for all $y_1 \in [0, 1]$ such that $\exists d > 0$ with $c_2(d) = \pi_2(1 - y_1)$. By the same arguments as before, \hat{y}_2 is a continuously differentiable strictly decreasing function on the corresponding $[\bar{y}_1, 1]$ with

$$\frac{\partial \hat{y}_2}{\partial y_1} = \frac{-\pi_2'(1 - y_1)}{c_2'(c_2^{-1}(\pi_2(1 - y_1)))} - 1 < -1 \quad (9)$$

Let $\tilde{y}_2 : [0, 1] \rightarrow \Re$ be defined by $\tilde{y}_2(y_1) = \hat{y}_1^{-1}(y_1)$.

Note that $\tilde{y}_2(0) = 1$ while $\tilde{y}_2(1) = \bar{y}_2$. Also \tilde{y}_2 is a continuous and strictly decreasing function with

$$-1 < \frac{\partial \tilde{y}_2}{\partial y_1} = \frac{1}{\frac{-\pi_1'(1 - y_2)}{c_1'(c_1^{-1}(\pi_1(1 - y_2)))} - 1} < 0 \quad (10)$$

Therefore $\tilde{y}_2(\bar{y}_1) < 1$, since $\bar{y}_1 \in (0, 1)$.

Consequently $(\hat{y}_2 - \tilde{y}_2)(\bar{y}_1) = 1 - \tilde{y}_2(\bar{y}_1) > 0$.

Also, $(\hat{y}_2 - \tilde{y}_2)(1) = 0 - \bar{y}_2 < 0$.

Finally, the function $(\hat{y}_2 - \tilde{y}_2)$ is a strictly decreasing function of y_1 on $[\bar{y}_1, 1]$ as can be seen by subtracting the fraction in (10) from that in (9), the former

being strictly greater than -1 , the latter strictly less than -1 and both being negative.

Therefore by the intermediate value theorem and the fact that $(\hat{y}_2 - \tilde{y}_2)$ is a strictly decreasing function of y_1 on $[\bar{y}_1, 1]$, there exists a unique $y_1^* \in (\bar{y}_1, 1)$ such that $(\hat{y}_2 - \tilde{y}_2)(y_1^*) = 0$. Let $y_2^* = \hat{y}_2(y_1^*)$. $y_2^* \in (0, 1)$ since $y_1^* \in (\bar{y}_1, 1)$. Further, $y_2^* = \tilde{y}_2(y_1^*) \Rightarrow y_1^* = \hat{y}_1(y_2^*)$. Therefore, (y_1^*, y_2^*) solves (6) and (7) and does so uniquely amongst any (y_1, y_2) with $y_1 \in [\bar{y}_1, 1]$. The proof concludes by showing that (7) cannot hold for any $y_1 < \bar{y}_1$.

Let $y_1 < \bar{y}_1$. By the definition of \bar{y}_1 , it must be that $\pi_2(1 - y_1) > c_2(y_1)$.
 $\Rightarrow \pi_2(1 - y_1) > c_2(y_1 + y_2 - 1)$ for all $1 - y_1 \leq y_2 \leq 1$ as $c_2(\cdot)$ is a strictly increasing function. \square

Let (y_1^*, y_2^*) be the unique solution to (6) and (7), guaranteed by lemma 5.

Proposition 2. *Given **A1** and **A2** the demand profile in any pure strategy subgame perfect equilibrium of the bargaining game must be an element of $\{(z_1^*, z_2^*)$ s.t. $z_1^* + z_2^* = 1, z_1^* \leq y_1^*$ and $z_2^* \leq y_2^*\}$.*

Proof. The argument for $z_1^* + z_2^* = 1$ is very similar to the linear case and is therefore skipped. I will first show that (z_i, z_j) with $z_i > y_i^*$ and $z_1 + z_2 = 1$ cannot be the demand profile of a pure strategy subgame perfect equilibrium. The payoffs generated by these demands are $(\pi_i(z_i), \pi_j(z_j))$. Further, $\tilde{y}_j(z_i)$ is well defined as $z_i > y_i^* > \bar{y}_i$ and satisfies $\tilde{y}_j(z_i) > z_j$. Now, given that $z_i > y_i^*$ it must be that $(\hat{y}_j - \tilde{y}_j)(z_i) < 0$.

$\Rightarrow \hat{y}_j(z_i) < \tilde{y}_j(z_i)$.

Since $\pi_j(1 - z_i) = c_j(z_i + \hat{y}_j(z_i) - 1)$ by definition, it follows that

$$\pi_j(1 - z_i) - c_j(z_i + \tilde{y}_j(z_i) - \epsilon - 1) < 0 \quad (11)$$

for a small enough $\epsilon > 0$.

On the other hand, since $\pi_i(1 - \tilde{y}_j(z_i)) - c_j(z_i + \tilde{y}_j(z_i) - 1) = 0$ it must also be true that,

$$\pi_i(1 - (\tilde{y}_j(z_i) - \epsilon)) - c_j(z_i + \tilde{y}_j(z_i) - \epsilon - 1) > 0 \quad (12)$$

for a small enough $\epsilon > 0$.

Consider the deviation by player j involving a demand of $\tilde{y}_j(z_i) - \epsilon$ in the first stage. This leads to incompatible demands thereby leading to the second stage. Now, given (11) it is a dominant strategy for j to play {Stick}. Further (12) implies that player i would strictly prefer {Accept} to {Stick} conditional on j playing {Stick}. Consequently the unique Nash Equilibrium in the second stage would involve i accepting and j sticking to her offer. The payoff to j from this deviation is $\pi_j(\tilde{y}_j(z_i) - \epsilon)$ which is strictly greater than her original payoff. This profitable deviation rules out the possibility of the equilibrium demand profile being (z_i, z_j) with $z_i > y_i^*$ and $z_1 + z_2 = 1$.

Finally I construct a pure strategy SPE to support an element of the set $\{(z_1^*, z_2^*)$ s.t. $z_1^* + z_2^* = 1, z_1^* \leq y_1^*$ and $z_2^* \leq y_2^*\}$ as the first stage demand profile. Let $\{(z_1^*, z_2^*)$ be such an element. The strategies are as follows, Player i demands z_i^* in the first stage. If the second stage game is reached and if player

j demanded $z_j > z_j^*$ in the first stage, then i chooses {Stick}, while j chooses {Accept} if $\pi_j(1 - z_i^*) - c_j(z_i^* + z_j - 1) > 0$ and chooses {Stick} otherwise. The above strategies can be verified to be subgame perfect, using arguments similar to the linear case. This concludes the proof. \square

It can be easily verified that $(\frac{1+k_1}{1+k_1+k_2}, \frac{1+k_2}{1+k_1+k_2})$ solves (6) and (7) in the linear model of section 2. Proposition 2 then readily gives us equation (5) for this special case.

5 Implications

5.1 Comparative Statics

As is obvious from proposition 2 the highest share for player i in equilibrium is y_i^* and the lowest, $1 - y_j^*$. To see what happens to equilibrium shares if player i 's cost function increases, consider the following setup. I fix player j 's payoff and cost functions at π_j and c_j . Player i 's payoff function is given by π_i^* , while two cost functions c_i and \hat{c}_i are considered with $c_i(d) < \hat{c}_i(d)$, for all $d > 0$. Payoff and cost functions are assumed to satisfy **A1** and **A2** respectively. Let y_i^* and $1 - y_j^*$ be the highest and lowest equilibrium payoffs for i with cost function c_i . Let the corresponding payoffs for the cost function \hat{c}_i be y_i^{**} and $1 - y_j^{**}$. Define \hat{y}_i and \bar{y}_j for the cost function c_i as in section 3. Let $\hat{\hat{y}}_i$ and $\bar{\bar{y}}_j$ be the corresponding objects for \hat{c}_i . By definition,

$$\pi_i(1 - \bar{y}_j) = c_i(\bar{y}_j) \quad (13)$$

and

$$\pi_i(1 - \bar{\bar{y}}_j) = \hat{c}_i(\bar{\bar{y}}_j) \quad (14)$$

Given **A1**, **A2** and $c_i(d) < \hat{c}_i(d)$, for all $d > 0$, it must be true that $\bar{\bar{y}}_j < \bar{y}_j$. It is also easy to verify that $\hat{\hat{y}}_i(y_j) < \hat{y}_i(y_j)$ for all $y_j \in [\bar{y}_j, 1]$. By the definition of y_j^* it must be true that $(\hat{y}_i - \tilde{y}_i)(y_j^*) = 0$. Therefore $(\hat{\hat{y}}_i - \tilde{\tilde{y}}_i)(y_j^*) < 0$. On the other hand $(\hat{\hat{y}}_i - \tilde{\tilde{y}}_i)(\bar{\bar{y}}_j) = 1 - \tilde{\tilde{y}}_i(\bar{\bar{y}}_j) > 0$. Consequently there exists $x \in (\bar{\bar{y}}_j, y_j^*)$ such that $(\hat{\hat{y}}_i - \tilde{\tilde{y}}_i)(x) = 0$. In other words $y_j^{**} = x$. Importantly, note that $y_j^{**} < y_j^*$. Further, since $y_i^{**} = \tilde{\tilde{y}}_i(y_j^{**})$ with $\tilde{\tilde{y}}_i$ being a strictly decreasing function, it is true that $y_i^{**} > y_i^*$. Therefore increasing the cost function for player i from c_i^* to c_i^{**} increases both her lowest payoff from $1 - y_j^*$ to $1 - y_j^{**}$ and her highest payoff from y_i^* to y_i^{**} . In this sense, the more costly it is to back down from the first stage demand, the greater is the player's bargaining power.

5.2 Equilibrium Selection in the Nash Demand Game

Let $c_1(d) = kc(d)$ and $c_2(d) = c(d)$, for $d \in (0, \epsilon)$ where $c(d)$ satisfies **A2** and $k > 0$ for some $\epsilon > 0$. In this case for a high enough value of $c'(0)$, the solution to (6) and (7), (y_1^*, y_2^*) will satisfy

$$\frac{\pi_1(1 - y_2^*)}{\pi_2(1 - y_1^*)} = k \quad (15)$$

Note that by choosing a high enough value of $c'(0)$, the value of $y_1^* + y_2^* - 1$ can be made lower than ϵ . At the limit as $c'(0)$ diverges to infinity, the game makes it impossible for players to back down from offers made in the first stage, thereby giving the Nash Demand Game as the limit game. Also at the limit y_1^* converges to $1 - y_2^*$. Finally, therefore, the limit of the equilibrium prediction of this model, which is the unique point (y_1^{**}, y_2^{**}) with $\frac{\pi_1(y_1^{**})}{\pi_2(y_2^{**})} = k$ and $y_1^{**} + y_2^{**} = 1$ selects a unique equilibrium in the Nash Demand Game.

6 Conclusion

The tradeoff between higher demands and higher commitment ability has been studied using a simple and transparent two stage non-cooperative model of bargaining. The ability to commit is generated by making backing down from a stated demand costly. When these costs are common knowledge and increasing in the extent of concession, higher cost functions yield greater bargaining power. The objective of this study has been to provide a simple tractable model to capture this relationship, which can then be applied to model scenarios where players have the ability to modify their commitments. While backing down in a negotiation may be costly, the breakdown of a negotiation (i.e. *(Stick, Stick)*) could lead to a new round of negotiation where the two parties get to choose new levels of demand to commit to. Given the general class of payoff and cost functions that the present analysis considers it would indeed be possible to consider the effect of changing cost structures on the bargaining outcome when parties can change their commitment after a failed round of negotiation. Making the revoking cost functions arbitrarily high makes backing down impossible at the limit, thereby resulting in the Nash Demand Game. The limit equilibrium prediction of the model is thereby shown to select a unique equilibrium in the Nash Demand Game. The equilibrium prediction is driven by the ratio of commitment costs as opposed to relative impatience.

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