Base Ball sacrifice play strategies: towards the Nash Equilibrium based strategies

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Abstract. In this paper the sacrifice play as a base ball strategy is quantified. In addition, the Nash Equilibrium (NE) to identify base ball winning strategies twice, when the team plays on offense as well as defensively is introduced. The aim is to identify situations and conditions during the course of a game, such that the sacrifice plays apply is opportune; alongside, to apply the Nash equilibrium model for identifying strategies in order to augment the eventual success of a team in the game, as a result from these strategies application. In multiplayer games the analysis of strategies usage is of high complexity; hence it is relevant the EN automation for simulating the strategies applicability in multiplayer games.

Keywords: *Analysis of strategies, Base Ball sacrifice plays, Multiplayer games, Nash Equilibrium.*

1. INTRODUCTION

The game of base ball is considered one of the most popular team games around the world [1]; this is due to the different strategies employed by each team to win the match. The strategies are taken during the game, watching every moment of the game as well as the current status of the team. The little research that has the game of base ball, let us note that it is a new field of study, not only because it is a very popular sport, if not the form in which the strategies are determined depending on the situation of the game , decision-making in team games as such are crucial to the results obtained.

The main aim of this work is: to identify the situations and conditions during the course of a base ball game, such that for success of the game, be appropriate to apply the Nash Equilibrium model of the team strategy, when the team plays on offense an when it play on defense. We asked: How to model the Nash Equilibrium in the base ball game, particularly inside of team to the offensive and defensive. We gave a solution trough developing a base ball simulator in three main components:

1. We constructed a context-free grammar and the stack automata, which recognized the language generates by the grammar.
2. We build a generator of base ball plays randomly.
3. We introduce the Nash Equilibrium algorithm for finding the winning collective strategies in some situations of the game.

The rest of this paper is organized as follows: Section 2 reviews the relevant literature. Section 3 describes the simulator of base ball game including the finite state automaton as well as the generator of base ball plays. Section 4 describes an algorithm to find out the Nash Equilibrium. Sección 5 describes the experimental stage, whereas in Section 6 the discussions and a brief related work overview is given, closing the paper in Section 7 with conclusion.

2. ANTECEDENTS

2.1 Game Theory and strategies

Game theory is an area of applied mathematics that uses mathematical models to study interactions formalized incentive structures (called games), to carry out decision-making processes [8], [9], [10]. Initially the Theory of Games had its main applications in economics, but is now applied to a large number of areas such as information technology, politics, biology and philosophy, among others. Game theory experienced a substantial growth and was formalized for the first time from the work of John von Neumann and Oskar Morgenstern [11], before and during the Cold War, largely due to its application in military strategies. Definitions of need concepts in the present Theory of Games deployment are next introduced.

Decision Making is an applied science that has gained considerable importance and has been the basic theme of Operational Research [12]. In the last few years have incorporated Artificial Intelligence techniques in their analysis. It involves formal analysis, computer simulation of individual behavior in games, the documentation is based on verifiable data and statistics, and experimental results are documented in the same way to support the conclusions. Decision making is the process of selecting a course of action among alternatives, is the backbone of the planning [13].

A game can be defined as a course of events, which consists of a series of actions by the players. For the game to be susceptible to mathematical analysis must also take a set of rules established without ambiguity, and the outcome of the game.

Strategies are organized and weighted set of actions for advantaging on some process or projects [13],[14], [16]. For a player participating in a game its strategies are defined as the set of rules that determine their actions for all situations that arise in the game.

Strategy profile is a set of strategies for each player that fully specifies all actions in a game. A strategy profile should include only one strategy for each player.

Deviation strategy profile: a profile is fixed for each player, and it will change every strategy by setting the strategies of others. If any player is found greater benefit by diverting its strategy, the profile is set to be ruled by a profile dominated.

A profile that is dominated, in which any deviation of any player, the value of the benefit of the deviation is greater than the fixed profile.

A normal form game [18] is defined as G = ($S_1, ..., S_n, u_1, ..., u_n$), where:

- n is the number of players ($1, ..., n$).
- ($S_1, ..., S_n$) is the set of strategies for each player.
- ($u_1, ..., u_n$) are the functions of benefit (payoff) of each player.

The payoff *or benefit function* allows calculating the benefit obtained for every possible strategy profile in the game, receives as parameters a profile of strategies and returns a numerical quantity representing the players' motivation [18].

The multiplayer games are those where two or more players participate, each player has his set of strategies used during the game. Players may be individual opponents, grouped in teams or to form a single team. If there is cooperation between the players, the game is more complex.

Zero-sum describes a situation in which the gain or loss of a participant is exactly balanced by the losses or gains of the other participants [17]. In other words, we say that a game is a zero sum game if the sum of rewards is zero. In zero-sum games the players pursuing the goals are completely opposite.

Base ball game is a team game, multiplayer, where the main tool for success is finding the most appropriate strategies that lead to win the game [1]. The game of base ball is characterized by a dual game, i.e., cooperative and uncooperative. This is because team members are encouraged to act individually, but in turn must cooperate for the benefit of the team. In the thesis proposed, the case study is the game of base ball because they provide ideal situations to define different strategies that can be efficiently simulated by a computer program in order to observe, study and understand the behavior of equipment under these strategies.

2.2 Nash Equilibrium

Nash Equilibrium (EN) is a central concept in Game Theory, essential to formalize cooperation between players on a team with the goal to win the game in dispute. To win as a team, it requires the design of collective strategies as a positive combination of individual strategies. The EN allows characterizing the collective strategy such that any player, individually, is attractive to act differently from what the collective strategy directed. The EN is the foundation to formalize the coordination of the players, so that each act to enhance the benefit of the team, leaving the possibility that closed had the option to act otherwise, individually, but to go in prejudice itself.

The Nash Equilibrium is used in this work to find out strategies profiles that are solution sets of a game that involves two or more players, taking into account that the profiles should be the best answer to each player conditional strategies others [18]. In real life, in practice, it often during the development of a collective game a player is encouraged individually to defraud the other or others, even after he had promised to cooperate. This is the crux of the dilemma, but surprisingly both players would get a better result by working together.

The prisoner's dilemma is a classic illustrative example, which in its classic statement describes the situation where the police arrested two suspects without sufficient evidence to charge a crime. Following are separate visits each and are offered the same deal: If one confesses and his accomplice not, the accomplice is sentenced to ten years and the first will be released. Symmetrically, if one confesses and the accomplice remains silent, the first will receive the penalty and the accomplice who goes free. If both confess, both will be sentenced to six years. If both deny it, all you can do is locking them up for six months for a misdemeanor charge. This can be summarized as the **Table 1**.

Table 1 Performance of prisoners

<table>
<tr><td></td><td colspan="3">Prisoner # 2</td></tr>
<tr><td rowspan="3">Prisoner #1</td><td></td><td>Silence</td><td>Confess</td></tr>
<tr><td>Silence</td><td>6 month both</td><td>the prisoner # 2 is released, the prisoner # 1 received 10 year</td></tr>
<tr><td>Confess</td><td>the prisoner # 1 is released, the prisoner # 2 received 10 year</td><td>6 years both</td></tr>
</table>

For the prisoner's dilemma, and following the definition, we find the Nash Equilibrium. This should list all the profiles of possible strategies and see if the profile drawn up a strategy for a player, the other strategies maximize the other player payments. **Table 2** shows the pay off matrix for players.

Table 2 payoff prisoner's dilemma

	Prisoner # 2		
Prisoner #1		Silence	Confess
	Silence	2, 2	0,3
	Confess	3, 0	1 ,1

The Prisoner's Dilemma presents four profiles as solutions of Nash Equilibrium of the game: (*silence, silence*), (*silence, confess*), (*confess, silence*) and (*confess, confess*). We begin by analyzing the profile ((*silent, silent*) and assume that is a Nash Equilibrium. If the prisoner # 1 provides that the prisoner # 2 will play silence. Did the prisoner # 1 should continue thinking of going silent? The answer is no. Because set the strategy silence for prisoner # 2, the prisoner # 1 will prefer to deviate from the strategy outlined for him in the profile proposed as a solution to the strategy since confess gets a higher payment u_1 (*confess, silence*) = 3> 2 = u_1 (*silence, silence*). This argument also applies to the prisoner # 2 (by symmetry of the game), concluding that the profile (*silence, silence*) is not a Nash equilibrium because any prisoner, may shift their strategy and get the most benefit. Suppose that is proposed as a Nash equilibrium solution profile (*confess, silence*). In this case, if the prisoner # 2 knew that the prisoner #1 was going to play confess, he would play the strategy should thus confess it maximizes its usefulness in this particular case u_2 (*confess, confess*) = 1> 0 = u_2 (*confess, silence*). Therefore, the profile (*confess, silence*) is not a Nash equilibrium. The case (*silence, confess*) is similar to the previous position of exchanging prisoners. Finally, (*confess, confess*). This is a Nash equilibrium profile because prisoners have no incentive to unilaterally deviate from a strategy proposed. If any of the prisoners decide to continue silence as strategy alone would lose value in relation to the profile (*confess, confess*), since u_1 (*silence, confess*) = 0 <1 = u_1 (*confess, confess*) and u_2 (*confess, silence*) = 0 <1 = u_2 (*confess, confess*). In **Table 3** shows the prefer deviations of the players.

Table 3 Deviations of Prisoner's Dilemma

		Prisoner # 2	
		Silence	Confess
Prisoner #1	Silence	2, 2	0,3
	Confess	3, 0	1 ,1

It can be seen with the above analysis shows that the profile (*confess, confess*) is a Nash equilibrium profile because they set the profile, no prisoner has an incentive to deviate from its strategy.

3. THE BASE BALL FORMAL MODELLING

The game of base ball is a bat-and-ball sport played between two teams of 9 players each. It is considered a strategic game hence decision making is a main element to find out the set of strategies to win the game [1], **Error! Reference source not found.**.

3.1 Context-free grammar

A thorough analysis conducts to identify the basic plays that are performed in base ball, see **Table 4** for each play name and abbreviation used in this paper. These plays are ordered and weighted based on their frequency of occurrence, i.e., based on the frequency at which these plays occur in real life. In **Fig. 1** the ordered set of plays based on their occurrence is shown:

$$s^i \geq b^i \geq f^i \geq co^i \geq o^i \geq p^i \geq ce \geq hi^i \geq a1^i \geq a2^i \geq d^i \geq dp^i \geq a3^i \geq a4^i \geq ca^i \geq r^i \geq$$
$$fs^i \geq h^i \geq tb^i \geq bp^i \geq bg^i \geq w^i \geq tp^i \geq t^i \geq bo^i$$

Fig. 1 Played ordered

Once obtained all the basic plays, we proceeded to develop a context-free grammar that generates the formal language to describe the base ball game; the language is recognized by the corresponding deterministic stack automaton, hence the game is algorithmic modeled by this finite state machine. The context-free grammar contains the terminal and non-terminal elements, as well as the rules that describe the base ball simple and complex plays being modeled by combining the terminals and non-terminals elements –in turn, that the plays are correct compound is verified:

- V is the alphabet
- $\sum$ (the set of terminals) is a subset of V
- R (the set of rules) is a finite set of $(V - \sum) \times V^*$
- B (is the initial symbol) is a element of $V - \sum$
- The members of V - ∑ are called non-terminals.

Table 4 shows the set of terminal elements, which represent simple plays and the complex plays in the base ball game.

Table 4 ∑ = Terminals symbols

Simples plays
b^i: ball
bo^i: *bolk*
bg^i: base hit
bp^i: base on balls
d^i: doublet
f^i: foul
dp^i: double *play*
fs^i: sacrifice fly
co^i: contact of ball
h^i: homerun
hi^i: *hit*
r^i: stolen base
s^i: strike
t^i: triple
tb^i: bunt
tp^i: triple play
w^i: wild pitch
played dependent on others
$a1^i$: movement to base 1
$a2^i$: movement to base 2
$a3^i$: movement to base 3
$a4^i$: movement to home
ce: change of equipment
o^i : out
p^i: punched

Table 5 shows the complete set of non-terminals that represent different types of words or clauses in sentences. **Table 6** shows some grammar rules.

Table 5 (V - ∑) = Non-terminals symbols

A: ball	Action is performed by contact of
B:	Bat
B3:	Bat with three *outs*
M:	Movement
MH:	Movement of *home run*
MR:	Movement by stolen base
MG: balls	Movement by base hit o base on
MD	Movement by doublet
MT	Movement by triplet
R:	Steal
T:	Transition

Table 6 Some grammar rules

B -> b^i B	Hitting can lead ball, and hit back
B -> bp^i MG B	Hitting generate base on balls, making movement and hitting return (subject to four balls before)
B -> s^i B	Hitting can generate a strike and hit back
B -> p^i B	Hitting can lead punch and hit back (subject to three strike)
B -> p^i B3	Hitting can lead punch and hit back with three out (subject three strikes and two outs before)
B -> f^i B	Hitting can generate a foul, hitting back
B -> tp^i o^k o^j o^i B3	Hitting can generate a triple play, change team
B -> co^i A	Hitting can generate contact action
A -> hi^i M B	Action can generate a hit, moving and re - bat
A-> o^i B	Action can generate one out, hitting back
A-> o^i B3	Action can generate an out, change of equipment (subject, if there are two outs before)
B -> h^i MH B	Hitting a home run can generate movement and hit back
B -> tb^i M B	Hitting can generate a bunt, moving and return to bat
Stolen base	
B -> R	Hitting can generate a stolen base (if case)
R -> r^j MR o^j B	Stealing can generate r, moving out and back to bat
B3-> ce B	Hit three out, is change of equipment
Movement of stolen	
MR -> $a2^j$ |$a3^j$|$a4^j$	
Movement home run	
MH -> $a3^j$ $a1^i$ $a4^j$ $a2^i$ a3i $a4^i$	Movement home run with man in 2 base
MH -> $a4^k$ $a3^j$ $a1^i$ $a4^j$ $a2^i$ $a3^i$ $a4^i$	Movement home run with man in 3 and 2 basis
MH -> $a4^j$ $a1^i$ $a2^i$ a3i $a4^i$	Movement home run with man in 3 base
MH -> $a4^l$ $a3^k$ $a2^j$ $a1^i$ $a4^k$ $a3^j$ $a2^i$ $a4^j$ $a3^i$ $a4^i$	Movement home run with men in 3, 2 and 1 base
Movement at base	
M -> $a4^k$ $a3^j$ $a1^i$	Movement with men on 3 y 2 base
M -> $a4^i$ $a1^i$	Movement with men on 3 base
M -> $a4^l$ $a3^k$ $a2^j$ $a1^i$	Movement with men on 3, 2 and 1 base

Strike or ball movement	
MG -> $a1^{i}$	Movement without man on base
MG -> $a2^{j}$ $a1^{i}$	Movement with man on 1 base
Double movement	
MD -> $a1^{i}$ $a2^{i}$	Movement without man on base
MD -> $a2^{j}$ $a1^{i}$ $a3^{j}$ $a2^{i}$ a,	Movement with man on 1 base
MD -> $a3^{k}$ $a2^{j}$ $a1^{i}$ $a4^{k}$ $a3^{j}$ $a2^{i}$	Movement with men on 2 and 1 base
Movement triplet	
MT -> $a4^{j}$ $a1^{i}$ $a2^{i}$ a3i	Movement with man on 3 base
MT -> $a4^{l}$ $a3^{k}$ $a2^{j}$ $a1^{i}$ $a4^{k}$ $a3^{j}$ $a2^{i}$ $a4^{j}$ $a3^{i}$	Movement with men on 3, 2 and 1 base

$$i \neq j, i \neq k, i \neq l, j \neq k, j \neq l, k \neq l$$

3.2 The finite state automaton

The stack automaton for base ball is modeled based on the structure of the base ball field; this mean the groundwork 1er, 2da, 3^{era}, home and a special base are the states of the automata; the transitions between states are given by the plays the participant players can perform. In **Fig. 2** shows the automaton for the game of base ball.

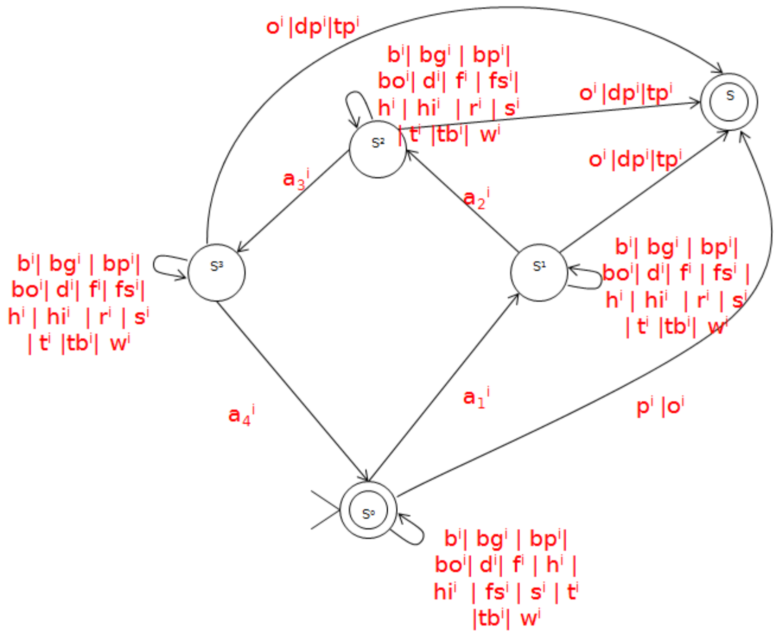

Fig. 2 Automaton of base ball

The stack automaton is ($\sum$, *S'*, Γ, s_0, δ , *H*) *consists* of:

- ∑ is the input alphabet (terminal elements),
- *S’* is the set of states {s, s_0, s_1, s_2 and s_3},
- Γ = {F’,B’,O’,ST} is the alphabet of stack symbols,
- δ = S’ x ∑ → S’ is the transitions function,
- s_0 is the initial state and
- $H = \{s_0, s\}$ is the set of halt states.

The automata analyses the strings describing the sequence of plays as well as the player who is performing them. The finite state automaton needs the stacks symbols for respective scoring the number of, strikes *ST*, fouls *F*, balls *B*, outs *O*, and the players on the bases *A1*, *A2* and *A3*. In **Table 7** shows the transitions between states, we must stress that one should use the stack to make some movement to the states, stacking and de-stacking symbols corresponding to the stack.

Table 7 Transition table

(s_0,f,nil) : (s_0,F’)	(s_2,f,nil) : (s_2,F’)
(s_0,s,nil) : (s_0,ST’)	(s_2,s,nil) : (s_2,ST’)
(s_0,b,nil) : (s_0,B’)	(s_2,b,nil) : (s_2,B’)
(s_0,bp,nil) : (s_0,nil)	(s_2,bp,nil) : (s_2,nil)
(s_0,bg,nil) : (s_0,nil)	(s_2,bg,nil) : (s_2,nil)
(s_0,bo,nil) : (s_0,nil)	(s_2,bo,nil) : (s_2,nil)
(s_0,d,nil) : (s_0,nil)	(s_2,d,nil) : (s_2,nil)
(s_0,hi,nil) : (s_0,nil)	(s_2,hi,nil) : (s_2,nil)
(s_0,h,nil) : (s_0,nil)	(s_2,h,nil) : (s_2,nil)
(s_0,fs,nil) : (s_0,nil)	(s_2,fs,nil) : (s_2,nil)
(s_0,t,nil) : (s_0,nil)	(s_2,t,nil) : (s_2,nil)
(s_0,tb,nil) : (s_0,nil)	(s_2,tb,nil) : (s_2,nil)
(s_0,w,nil) : (s_0,nil)	(s_2,w,nil) : (s_2,nil)
(s_0,a1,{F ST B}) : (s_1,A1)	(s_2,r,nil) : (s_2,nil)
(s_0,p,{F ST B}) : (s,O’)	(s_2,a3,{F ST B A2}) : (s_3,A3)
(s_0,o,{F ST B}) : (s,O’)	(s_2,o,{F ST B}) : (s,O’)
	(s_2,dp,{F ST B}) : (s,O’)
(s_1,f,nil) : (s_1,F’)	(s_2,tp,{F ST B}) : (s,O’)
(s_1,s,nil) : (s_1,ST’)	
(s_1,b,nil) : (s_1,B’)	(s_3,f,nil) : (s_3,F’)
(s_1,bp,nil) : (s_1,nil)	(s_3,s,nil) : (s_3,ST’)
(s_1,bg,nil) : (s_1,nil)	(s_3,b,nil) : (s_3,B’)
(s_1,bo,nil) : (s_1,nil)	(s_3,bp,nil) : (s_3,nil)
(s_1,d,nil) : (s_1,nil)	(s_3,bg,nil) : (s_3,nil)
(s_1,hi,nil) : (s_1,nil)	(s_3,bo,nil) : (s_3,nil)
(s_1,h,nil) : (s_1,nil)	(s_3,d,nil) : (s_3,nil)
(s_1,fs,nil) : (s_1,nil)	(s_3,hi,nil) : (s_3,nil)
(s_1,t,nil) : (s_1,nil)	(s_3,h,nil) : (s_3,nil)
(s_1,tb,nil) : (s^1,nil)	(s_3,fs,nil) : (s_3,nil)
(s_1,w,nil) : (s^1,nil)	(s_3,t,nil) : (s_3,nil)

(s_1,r,nil) : $(s^1,A2)$	(s_3,tb,nil) : (s_3,nil)
$(s_1,a2,\{F\ ST\ B\ A1\})$: $(s_2,A2)$	(s_3,w,nil) : (s_3,nil)
	(s_3,r,nil) : (s_0,nil)
$(s_1,o,\{F\ ST\ B\})$: (s,O')	$(s_3,a4,\{F\ ST\ B\ A3\})$: (s_0,nil)
$(s_1,dp,\{F\ ST\ B\})$: (s,O')	$(s_3,o,\{F\ ST\ B\})$: (s,O)
$(s_1,tp,\{F\ ST\ B\})$: (s,O')	$(s_3,dp,\{F\ ST\ B\})$: (s,O)
	$(s_3,tp,\{F\ ST\ B\})$: (s,O)

3.3 Classics strategies

This section introduces some base ball strategies described in the literature. The base ball, theoretically, is an infinite set of calculations, probabilities and variables. Within the strategy literature is applicable to the base ball game, which can be divided into applicable to the defense or on offense.

Strategies on the offensive:

- Batting Order
- Emerging Corridors
- Different types of bunt, such as sacrifice bunts, squeeze bunt
- Stolen of base
- Fly to sacrifice
- Hit and run
- Home run
- Hit
- Running the bases
- Doublets

On the offensive the main strategy is the appointment of the batting order. Before the game each team makes a list of the 9 players where each ones has a pre-set position at bat. The most common is to make the best players first hence they will have more opportunities to hit than those who are at the bottom of the list, but with a warning: In the first two places to put people often prefer quick legs not so good hitting, trying to get them to simply get into the bases and that the best hitters (3rd and 4th) trailers to home with a home run or a good shot to give them enough time to move forward. In addition, relevant aspects to consider on the offensive strategies, when the team has one or more runners on base:

- Stolen of base to advance the runner advanced
- Connect to hit to advance the runners

If fewer than two outs, a third strategy is available:

- Play of sacrifice to advance runners, although this involves an out.

Defensive strategies

On the defensive stage the principal aim is to inhibit the offensive strategies being applied by the opposing team.

- Base on balls (intentional)
- Double play
- Pitching (try to do less releases)
- Strategic Positioning of players

Suppose we have the following situation: there are runners on second and third base, and a dangerous player is at bat, the strategies would be:

1. Give intentionally walked,
2. instead of pitching the batter

In the following case is shown when the offensive team has runners on first and third, or first, second and third with no outs. The offensive team has the potential strategies:

1. About the defensive, and if the batter makes contact with the ball, throw the ball home to put out the more advanced player, or prevent the race.
2. Attempt a double play

Actually, while the team at bat is trying to score runs, the team in the defensive is attempting to record outs. The defense must try to predict what the next opposing team play will try, and a defense play to attempt to stop it, usually by getting outs to put the offense team away.

3.4 Qualitative analysis

In [5] a qualitative analysis of baseball strategies including entries, score and number of outs are made; as a result the following alternatives of action are future:

- Situation in score

When the team is ahead on the score, it could take more risks on the bases hence it could play more aggressively, so that the difference between markers becomes greater. On the opposite, when the team is behind on the scoreboard, it should play more conservatively, trying to keep the number of outs remaining, with the aim of moving to more advanced riders if necessary. When the score is tied or a different race, you must adjust your strategy to shape the game. If this is a low scoring game, probably have to play more aggressively to drive a race or two. If the game is very aggressive, so it is recommended to play conservative.

- Innings of the match

Other relevant factor to consider is the entries, if playing aggressively or conservatively, it often depends on whether the team is in the early, middle, or late innings. In the first innings, the main objective is to get the lead playing aggressively. Not recommended wasting outs with sacrifice bunts. Middle innings often determines the character of the game. If the game is very aggressive, your strategy should reflect that. It is recommended to play conservatively. In the late innings presented two circumstances. Late in the game, the offensive strategies will result from the score. It's played conservatively if it is losing to conserve power outs, play aggressively and if you're ahead on the scoreboard. When the team is ahead on the score in the late innings is recommended to play aggressively.

- The numbers of outs

No outs, the team must play conservatively, if the number of races and entry are practical, as they have the chance to score a few runs. With one out, the team must play aggressively trying to reach at least one run. With two outs, the possibility that the team gets a few runs is reduced significantly. If so based on the entry and the score, to be aggressive and try to get a career.

In [6], based on a statistical study of the strategies of baseball, identified certain strategies more suitable to be implemented on the basis of: entries, score and number of outs, suggesting when it is appropriate apply:

- Sacrifices plays

This type of move is used when less than two outs, and a player at third base, typically are used in the last innings.

- Stolen bases

The main characteristic of this type of move is to be used when one or more players stay on base and try to advance, thus avoiding moves that put them in risk.

Strategies for the defense did not present a distinction as well as strategies on offense, but it is best to try to get the outs, so you do not have to make too many pitches, and try to counter the moves of the opposing team.

Actually, in this work the previous qualitative analysis is quantitative translated at some extent. Particular mention is the conservative – aggressive tradeoff regarding the team score versus the inning order. Sacrifice plays applicability is tightly related with this analysis as it is shown in Section 4.

3.5 The random generator of plays

The main aim for deploying a generator of plays is construct the strings that simulate the entire base ball match, where the strings must to have a valid sequential of plays, i.e., the plays have to be generated according their probability and the sequential must be coherent. A plays generator is useful, because, it generates valid base ball strings in random, fast and easy way, which are supplied them to the base ball automata.

The generator generates random plays and verifies that:

- Be valid plays in base ball
- Be produced on base of their frequency of occurrence, and
- Each play is assigned its probability of occurrence.

Numbers are generated randomly, and each number is associated with a play of base ball. In **Fig. 3** shows the scheme of generation of base ball plays. The numbers that were generated are bounded to the number of plays, i.e. only generates 0 to *m*, where *m* is the number of simple plays, see **Table 4** as the dependent plays are formed through of simple plays.

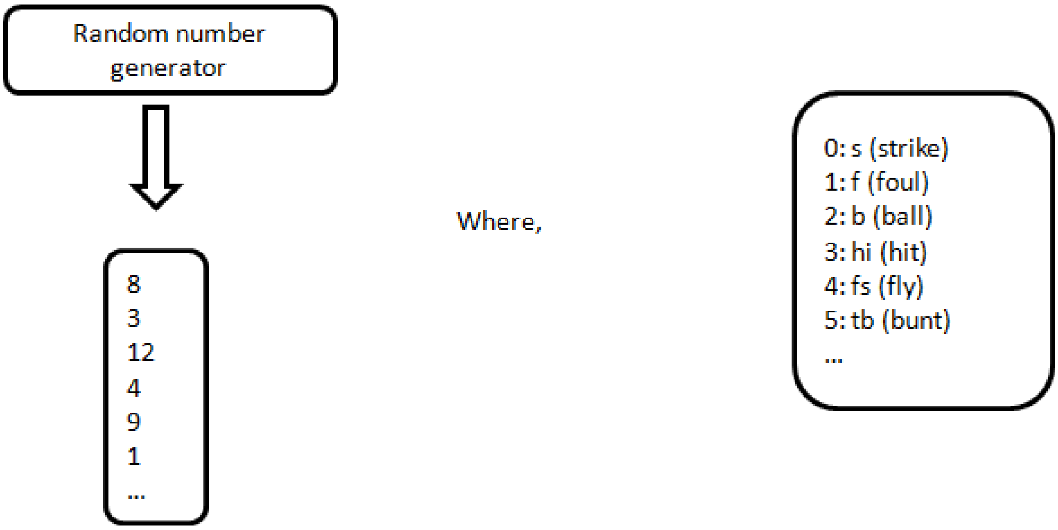

Fig. 3 Scheme of generation of base ball games

Each play is applied to flip probability function which returns only zero or one, with a probability *p* given. If $p = 0.5$ is likewise return a true (1) or false (0), which feeds on the generation of Gaussian random numbers with zero mean and standard deviation sigma. The probabilistic function receives as parameters the probability of the play and since this probability is decided whether the play is performed. It is noteworthy that the plays are not equally likely. In **Fig. 4** shows the scheme of probabilistic function.

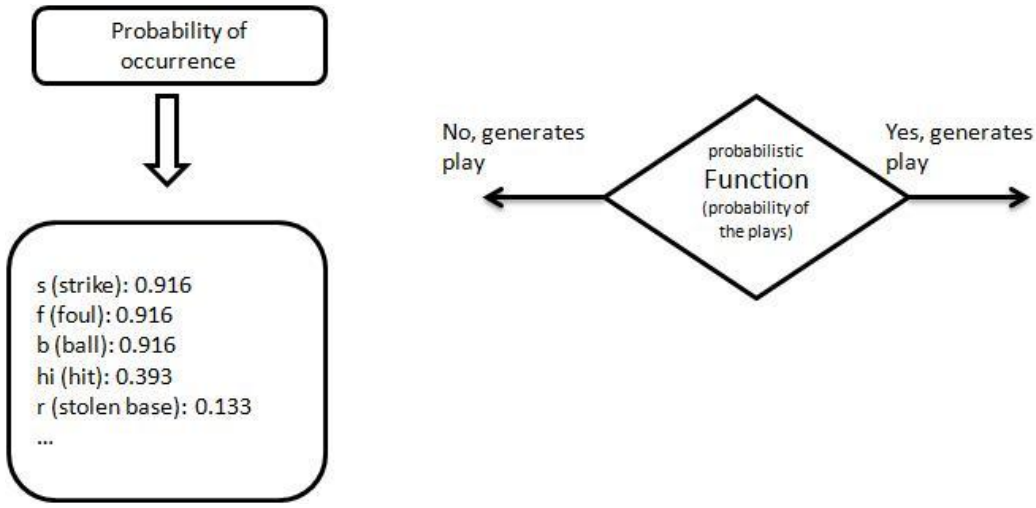

Fig. 4 Scheme of the probabilistic function

The generator of plays has a module for generating and validating these chains, i.e., after going through the process of generating play through random numbers and probability function should be to create the chain with the play that was generated. The way to do this is as follows: in the right end of a string, empty (ε) at the start, are concatenated has made plays, each new play is concatenated with an indication of the player who does it. There are plays that are dependent on others, which may be generated if and only if there is a sequence of previous plays. In **Fig. 5** shows the creation of chains of base ball, through the plays that can be done. In **Table 8** shows the algorithm for creating chain to simulate the entire base ball match.

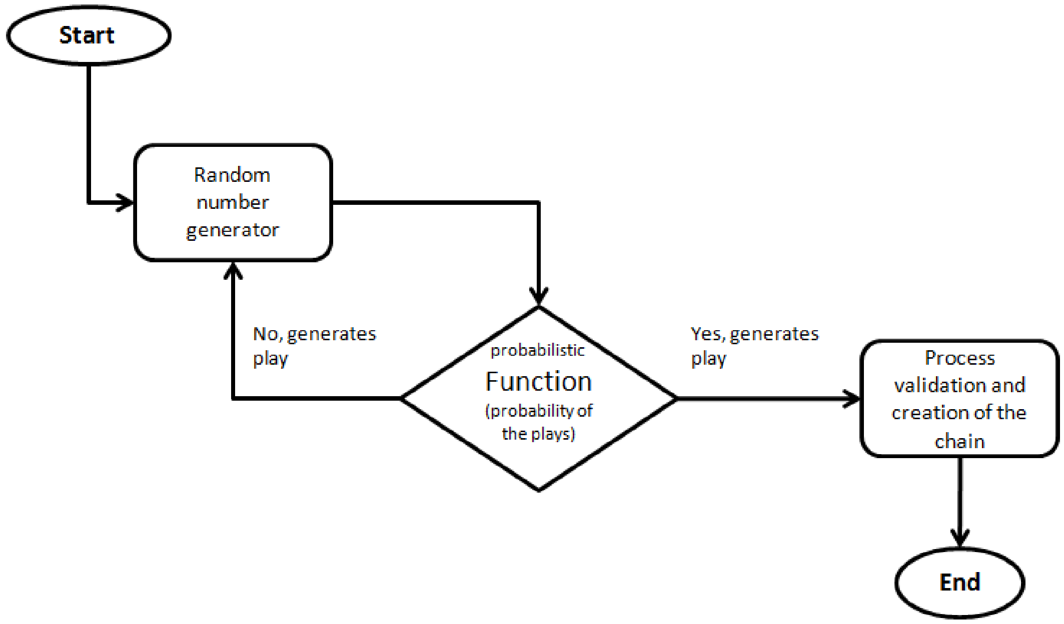

Fig. 5 General scheme of the generation and construction of chains

Table 8 Algorithm for plays generation

<u>Algorithm of generator plays</u>:

Step 1: Numbers are generated randomly in the range {0,…, *m*}, where *m* is the number of simple moves in base ball, to each number is associated a play.

Step 2: After getting the play to be performed, using a probabilistic function to decide whether to accept the play or not, depending on the probability of occurrence of this.

Step 3: Chain is created with the play to make, including in the concatenation sequence of actions as a result of the play.

Step 4: Validation of the string as base ball play.

Step 5: If the simulation process around the base ball game is over go to Step 5, otherwise to step 1.

Step 6: End of simulation, we obtain a chain of all Party

4. TOWARDS THE NASH EQUILIBRIUM ALGORITHMIC SETTING

The Nash equilibrium is the central concept most frequently used in the analysis of sets of two or more players, to characterize the best collective strategies, such that any player, is attractive to act differently from what the collective strategy indicates. Nash equilibrium induces a stable strategic situation because of the harmful results that participants provide for any unilateral deviation. Naturally, in the evolution of such a drift, each player has to take into account the strategies of other players and in particular the activities that induce these strategies in response to each of their own actions. It must take into account, in other words, the threat embodied in the strategies of their opponents to respond optimally to them [4].

In the normal game for n player, $G=\{S_1, \ldots, S_n; u_1, \ldots, u_n\}$, the strategies $s_1^*, \ldots, s_n^*$, are an Nash Equilibrium if, for each player $\boldsymbol{i}$, s_i^* is the best answers of player i (or at least one of them) to the other strategies $\boldsymbol{n}_{-1}$, $(s_1^*, \ldots, s_{-1}^*, s_{+1}^*, \ldots, s_n^*)$.

$$u_i\,(s_1^*, \ldots, s_{i-1}^*, \boldsymbol{s_i^*}, s_{i+1}^*, \ldots, s_n^*) \geq u_i\,(s_1^*, \ldots, s_{i-1}^*, \boldsymbol{s_i}, s_{i+1}^*, \ldots, s_n^*).$$

For each possible strategy s_i, *in* S_i; that is, s_i is a solution that maximizes the payoff function

$$\underset{s_i \in S_i}{Max}\ u_i\,(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*).$$

From this definition it follows that Nash equilibrium is a profile of strategies such that no player would unilaterally deviate; hence, every player gets the most benefit with the established strategy, given the strategies of other players. Nash equilibrium consists of strategies that are optimal for each player given the strategies of other players. This does not mean that in a Nash equilibrium every player is reaching the best possible outcome, but the best result conditioned by the fact that other players play the strategies outlined for them in that profile [3].

4.1 Finite state automata

The following finite state automaton is for modeling the Nash Equilibrium; it receives strings as inputs, then processes them and determines whether these strings belong to the language the automaton recognizes. To model the strategies of the players must define the following:

- Be s_x^i any strategies of player ***i***, such that $s_x^i \in S^i$, where S^i is the set of strategies of player ***i***.

- The strategies of player ***i*** are compound by a set of actions, $s_x^i = \{a_{x1}^i, a_{x2}^i, \ldots, a_{xn}^i\}$, where $a_{xj}^i \in \sum^i$.

In **Fig. 6** shows the automata that model the set of strategies of player *i*, where:

- $\sum = \{a_{x1}, a_{x2}, \ldots, a_{xn}\}$, are the alphabet symbols
- $S' = \{s, s_0, s_1, \ldots, s_m, h_0, \ldots, h_n\}$, is the set of states
- s = is the initial state
- $\delta = S' \times \sum \rightarrow S'$, is the transition function
- $H = \{h_0, \ldots, h_n\}$, are the set of halt states.

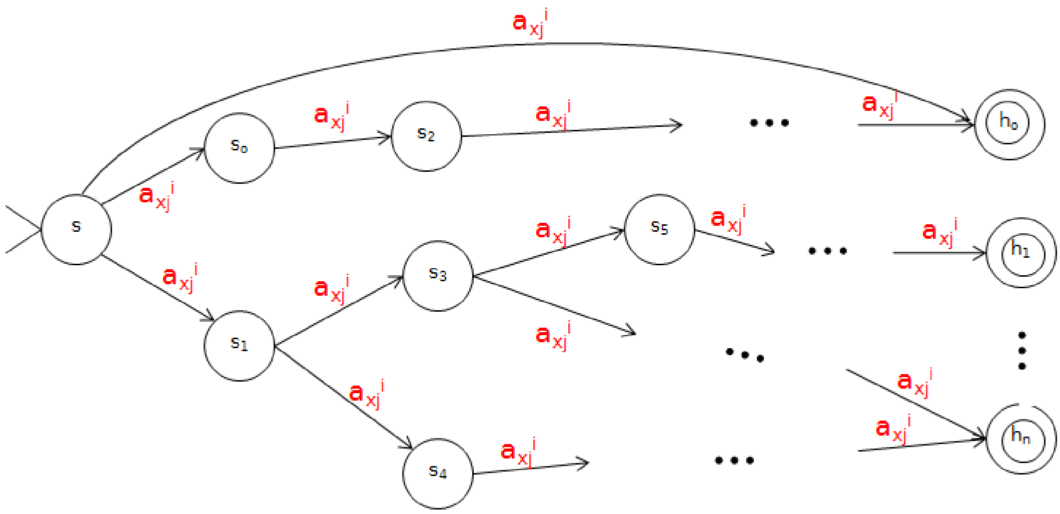

Fig. 6 Automaton for the strategies of each player *i*

For Nash Equilibrium modeling, it should be taken into account the set of strategies of all participants; thus, determine all those profiles that are part of the Nash Equilibrium. In **Fig. 7** shows automata for Nash Equilibrium, where:

- $\sum = \{a_{x1}, a_{x2}, \ldots, a_{xn}, Є, \Theta\}$, are the alphabet symbols
- $S' = \{s, s_0, s_1, \ldots, s_n, h\}$, is the set of states
- s = is the initial state
- $\delta = S' \times \sum \rightarrow S'$, is the transition function
- h is the halt state

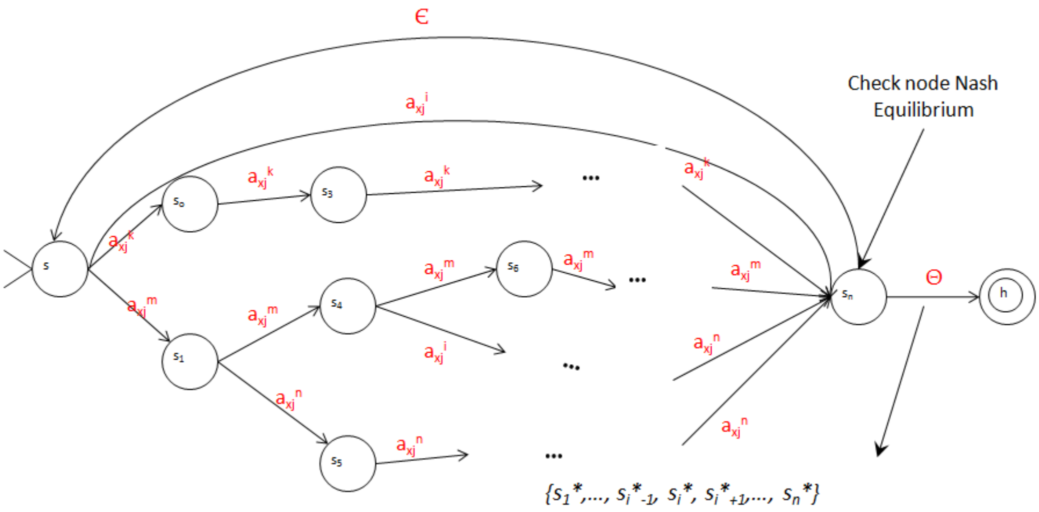

Fig. 7 Automata for Nash Equilibrium

4.2 The algorithm

For a normal form game of *n* players there are different profiles of strategies, which determine the way in which players act during the game. The profiles of strategies of a game of normal form, fulfilling the concept of Nash equilibrium are the most convenient to use; in non-cooperative games is the Nash equilibrium to strengthen cooperation among the players, in that it is those strategy profiles in which players are willing to act. Computerize the Nash equilibrium is should be determined the following:

- The number of player
- The number of strategies of players
- The number of profile of the game
- The benefit of each player for each game strategy profile

After obtaining the above base ball issues each profile need be assessed for discarding the ones whose deviations in the strategies of any player will get most benefit. In **Table 9** the algorithm to find out the Nash equilibrium strategies profiles in a game of normal form of *n* players it is shown.

Table 9 Algorithm of Nash Equilibrium for base ball strategies election

Algorithm:

Step 1: Provide the number of players, number of strategies, the number of profile, the pay off of each player for each game profile (Matrix pay off of each player)

Step 2: For each profile, the deviation are made in the strategies of each player , if any deviation is better that the strategy being analyzed, the strategy is discarded

Step 3: The profiles that have not been discarded are those that satisfy the Nash Equilibrium, are shown as the best actuation option

This algorithm obtained profiles can be used as a way for acting by the players in the game, as they are suitable profiles; the players are ready to implement these strategies because these are the best response to the strategies of other players. For detailing more precisely, how to computerize the Nash equilibrium in normal form games to *n* players. The parameters previously indicated (the number of players, the number of strategies, the number of profiles, the pay off of each player for each game profile) serve as the basis for the analysis of profiles of strategies of the game. Taking the game strategy profiles must perform all the deviations that each player could make, set a profile for testing.

A profile of strategies is a set of strategies where each player makes a formal strategy for such a profile. In a normal game for n players, there are several profiles but not everyone does the concept of Nash Equilibrium. Determining a profile of strategies for each player are made for deviations; selecting each of their strategies for each player, setting the strategies of others, and if they find that any deviations, the yield obtained is best for any player, the profile is discarded analyzed, a profile being dominated. In **Fig. 8** shows a diagram of how to make deviations for player i given a profile of strategies, it should be mentioned that deviations are due to perform for the *n* players.

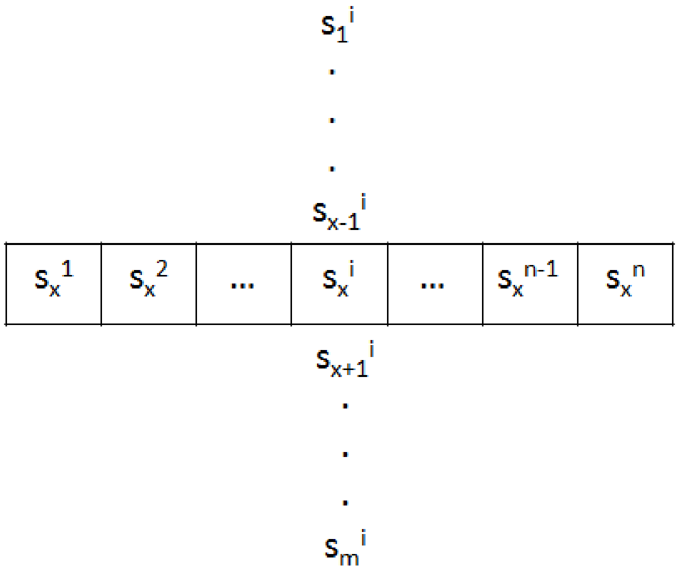

Fig. 8 Deviations in the strategies of player *i*

In a normal form game for *n* players, there may be a set of Nash equilibrium profiles, these profiles are those in which the deviations made by players, earning returns are less than or equal, i.e., in those profiles are player gets the most benefit.

4.3 Sacrifice plays

The sacrifice plays are performed in the base ball as part of a winning strategy [1], [6], whose characteristics are:

- Strategy “conservative” to gradually win.
- Strategy to increase the probability of team success.
- The place typically low-scoring players.
- Represent the team apparently lost “local minimal”, but
- It involves a “global maximum”, i.e., the team’s success at the end of the game in dispute.

To identify when the sacrifice plays are convenient to apply so to get improved results these kinds of strategies are simulated. From the empirical analysis, depending on circumstances and moments of the match, the obtained conclusions about are next given:

- The team is slightly winning
- The team is widely winning
- The team is losing with minimum margin
- The team is losing with wide margin
- Always (regardless of the score)

In regards to the moments of the match:

- In the first innings
- In the middle innings
- In the late innings

The results and observations to apply the sacrifice plays are derived from the next experimental stage, divided into sections for moments and circumstances.

5. EXPERIMENTAL STAGE

Experiments using sacrifice plays are practiced using the base ball simulator and then, the Nash Equilibrium method is applied for finding the best team strategies. The results and observations from the experimental stage by applying the sacrifice plays are divided into moments and circumstances.

Always using sacrifice plays: When the team is always used as a strategy to sacrifice plays to offensive, this led the team lost or won the game, i.e., it is difficult to identify reasons and conclusions, when all the time these moves are applied.

The team is slightly winning:

- First innings. When applied the sacrifice plays in these conditions the game, it was found that the gains were not enough, not being a significant factor in winning the game.
- Middle innings apply the sacrifice plays under these circumstances, there were no significant gains, but in general you can use to go keeping the difference on the score.

- Late Innings. In this case there was a good performance in implementing the sacrifice plays by making sure the advantage on the score, winning the match.

The team is widely winning:

- First Innings. In this case the results were not good, since the advantage gained was lost very quickly using only sacrifice plays as a strategy to start the game.
- Middle innings. In these circumstances, the sacrifice plays did not influence both within the match, although it is highly recommended when it is winning largely because the difference in score is falling.
- Late innings. In this case, we observe good behavior, using sacrifice plays to secure the advantage in the late innings.

The team is losing with a minimum margin:

- First innings. Use of sacrifice plays in these conditions yields the following:
    1. That the difference in achieving or exceeding, marker decline,
    2. On the contrary the first point, the opposing team's score increases, and consequently losing a wider margin.
- Middle innings. In this case we obtained small gain, however most can be used to go shortening marked difference in thinking in the late innings. But it is better to seek other options further moves that benefit from the team.
- Late innings. In these circumstances, there was a good performance in implementing the sacrifice plays, making often get to tie the game, or win.

The team is losing with a wide margin:

- First Inning. Under these circumstances underachieved, the profits from these games does not impact significantly on the score.
- Middle innings. In these circumstances it was observed that with these plays do not get significant gains taper differences in the scoring.

- Late innings. In this case, it was observed, the difference misbehavior is reduced but not significantly, with wide margin is lost.

In **Table 10**, shows numerical results of applying the sacrifice plays in the moments described above, with 100 runs for each case example and indentifying in gray the appropriated moment to apply them.

Table 10 Results obtained

Innings	The team is slightly winning	The team is widely winning	The team is losing with minimum margin	The team is losing with wide margin
	Win./Played	Win./Played	Win./Played	Win./Played
1-3	58/100	70/100	56/100	28/100
4-6	67/100	65/100	61/100	31/100
7-9…	86/100	89/100	81/100	38/100

Conclusions from the Table 13 by applying the sacrifices plays it is identified that the best circumstances and moments to use them are:

- When team is in the last innings and score is small; this ensures that the team what is winning, it keeps and get the victory and if the team is losing can get the best possible results, i.e., get the victory or reduce the score.

Highlights the need to find a way to determine which are most appropriate strategies the sacrifices play and other which have to be apply to the game of baseball, taking into account the number of players involved and the set of strategies that they can provide, finding collective strategies to achieve the most benefit.

5.1 Nash equilibrium and the sacrifice plays

The Nash Equilibrium indentifies the best strategies to use in some specific steps in the base ball game. The Nash Equilibrium determined using the sacrifices plays in the same factors which were found in previously experimental stage, where the sacrifices plays were used as winning strategies, and also determines other factors which it might be worth use them.

6. DISCUSSION AND RELATED WORKS

6.1 Related works using Nash Equilibrium in games

The complexity of finding Nash equilibrium in a two-player game is perhaps the outstanding open problem in algorithmic game theory [20]. In the paper of *Nash Equilibria* in random games [19] analyze the *Nash Equilibria* in two-player random games uses a simple *Las Vegas* algorithm for finding equilibrium. The algorithm is combinatorial and always finds a Nash Equilibrium; on *m x n* payoffs matrices.

A polynomial-time algorithm of Papadimitriou and Roughgarden for finding Nash equilibria in multi-player symmetric game in which each player has a small number of strategies [21]; a proof that the Lemke- Howson algorithm takes exponential time with all possible initial pivots [22].

B. von Stengel [23] did a self-contained survey of algorithms for computing Nash equilibria of two-person games. The games may be given in strategic form or extensive form. The classical Lemke-Howson algorithm finds one equilibrium of a bimatrix game, and provides an elementary proof that Nash equilibrium exists. It can be given a strong geometric intuition using graphs that show the subdivision of the players' mixed strategy sets into best-response regions. The Lemke-Howson algorithm is presented with these graphs, as well as algebraically in terms of complementary pivoting. Degenerate games require a refinement of the algorithm based on lexicographic perturbations. Commonly used definitions of degenerate games are shown as equivalent. The enumeration of all equilibria is expressed as the problem of finding matching vertices in pairs of polytopes. Algorithms for computing simply stable equilibria and perfect equilibria are explained. The computation of equilibria for extensive games is difficult for larger games since the reduced strategic form may be exponentially large compared to the game tree. If the players have perfect recall, the sequence form of the extensive game is a strategic description that is more suitable for computation. In the sequence form, pure strategies of a player are replaced by sequences of choices along a play in the game. The sequence form has the same size as the game tree, and can be used for computing equilibria with the same methods as the strategic form. The paper concludes with remarks on theoretical and practical issues of concern to these computational approaches.

6.2 Future work on simulations

- No team uses the Nash Equilibrium; when both teams play with no uses a Nash Equilibrium, there is no clearly difference between both, i.e., both teams apply strategies, but with no knowing about how and when use them.
- When the team 1 uses the Nash Equilibrium and the team 2 no. There is a clearly difference between the team who applies the Nash Equilibrium to another who no. when a team applies a Nash Equilibrium, it find the best collective strategies to use them for winning the match, and the results show a high difference.
- Both teams use the Nash Equilibrium. When both uses a Nash Equilibrium, they find the appropriated strategies to apply, the results show that both win on the same amount of match.

7. CONCLUSION

This paper examines the Nash Equilibrium applies to multiplayer games, especially to the base ball game, accounting as a particular instance the applicability of sacrifice plays. We have showed our work in different level:

1. The context-free grammar and the automaton for the base ball; the entire analysis, indentifying every things important to develop of the simulator of base ball game
2. The generator of base ball plays; the constructions and generations of strings that represents the whole match

3. The Sacrifice plays as winning strategy; Indentifying the situations for applying those plays
4. The Nash Equilibrium Algorithm; the basic idea to find the Nash Equilibrium profiles
5. The incorporation of Nash Equilibrium to the base ball simulator.

All the simulations that we obtained presents characteristics very similar to the real life, i.e., the scores are like the obtained by human, the innings did not exceed the typically numbers. The results of using sacrifice plays are well suited: the circumstances and moments for applying such plays are of opportune precision. In real base ball games the sacrifices plays are used in the same circumstances that we deduce by the simulations and as it is explained in the book Winning Strategies for Offense and Defense [5] .

Finally, the apply of Nash Equilibrium for finding the best team strategies have a positive impact in the score of the team that applied it: the team obtains significant gains compared to the team that does not apply the Nash Equilibrium.

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